Description of proton radioactivity using the Coulomb and proximity potential model for deformed nuclei

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(Received 30 July 2017; revised manuscript received 9 September 2017; published 25 September 2017)

Half-life predictions have been performed for the proton emitters with $Z > 50$ in the ground state and isomeric state using the Coulomb and proximity potential model for deformed nuclei (CPPMDN). The agreement of the calculated values with the experimental data made it possible to predict some proton emissions that are not verified experimentally yet. For a comparison, the calculations also are performed using other theoretical models, such as the Gamow-like model of Zdeb *et al.* [Eur. Phys. J. A **52**, 323 (2016)], the semiempirical relation of Hatsukawa *et al.* [Phys. Rev. C **42**, 674 (1990)], and the CPPM of Santhosh *et al.* [Pramana **58**, 611 (2002)]. The Geiger-Nuttall law, originally observed for α decay, studied for proton radioactivity is found to work well provided it is plotted for the isotopes of a given proton emitter nuclide with the same ℓ value. The universal curve is found to be valid for proton radioactivity also as we obtained a single straight line for all proton emissions irrespective of the parents. Through the analysis of the experimentally measured half-lives of 44 proton emitters, the study revealed that the present systematic study lends support to a unified description for studying α decay, cluster radioactivity, and proton radioactivity.

DOI: 10.1103/PhysRevC.96.034619

I. INTRODUCTION

One of the major goals that have been achieved in the case of nuclei far from stability is the observation of many new radioactive decay modes which can be used as an effective tool to extract information on nuclear structure and the internuclear potential. Since the proton binding of an element is steadily decreasing with the neutron number, one can predict the existence of a proton drip line beyond which the nuclei become unstable against proton emission [1]. Thus proton radioactivity becomes energetically possible in close analogy to α decay where the proton tunnels through the potential barrier due to the combined Coulomb and centrifugal potentials. Compared to α decay, protons experience a relatively low Coulomb barrier and a high centrifugal barrier due to their lower charge and mass [2]. The associated lifetimes are sufficiently long enough to obtain information on nuclear properties very far from the valley of β stability, and this new decay mode determines the borderline of nuclear stability for neutron-deficient nuclei [3].

The experimental confirmation of proton radioactivity was first given by Jackson *et al.* [4] in 1970 by detecting the emission of a proton from the isomeric state of ^{53}Co to the ground state of ^{52}Fe [4]. However, the second example of proton radioactivity was discovered many years after the first discovery. That is, in 1981, at the velocity filter SHIP at GSI, the proton emission from the ground state of ^{151}Lu was discovered [5]. Soon afterwards, ^{147}Tm using the isotope separator at GSI [6] and ^{109}I and ^{113}Cs using a catcher foil technique at Munich [3] were discovered. By 1984, Hofmann *et al.* [7] reported the evidence for two additional proton transitions, the isomeric decay in ^{147}Tm and the decay of ^{150}Lu. In 1994, Page *et al.* [8] discovered the proton emitter ^{112}Cs, and in subsequent years Davids *et al.* [9] reported the first proton emitter above Pb and discovered proton radioactivity from highly deformed ^{141}Ho and ^{131}Eu [10]. In addition to these decays, with the development of advanced experimental facilities and radioactive beams, a number of proton emissions from the ground state or low isomeric states have recently been discovered in the mass regions $A = 110$–180 [2,11–19], and more proton decay events will be discovered in the future.

The nuclear structure information including the shell structure and the coupling between bound and unbound nuclear states that can be obtained from the proton decay studies by the measurement of proton energy, half-life, and branching ratio underlines the importance of proton decay studies. Several theoretical approaches have been put forward to obtain the half-lives of spherical and deformed proton emitters [20–27]. In the earliest attempts, the Coulomb potential plus the Woods-Saxon potential have been used predominantly for proton decay studies [28,29]. Additionally, appropriate descriptions of proton radioactivity were provided by Zhao *et al.* [30], Yao *et al.* [31], and Ferreira *et al.* [32] within the scheme of covariant density functional theory. The microscopic calculations [33–41] of the half-lives of proton radioactivity and the phenomenological models, such as the unified fission model [42], the Gamow-like model [43], and the generalized liquid drop model [44] also are considered as standard tools to describe proton radioactivity. Calculations using all the above approaches provide excellent estimates for the lifetimes of proton radioactivity.

The main intention of this paper is to carry out systematic calculations on the proton decay half-lives of proton emitters in the ground state and isomeric state using both experimental and calculated Q values by taking into account of the centrifugal as well as the overlapping effects. The Coulomb and proximity potential model for deformed nuclei (CPPMDN) [45] and its spherical version, the Coulomb and proximity potential model (CPPM) [46], which have been applied successfully for many

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years to study α decay and cluster radioactivity [47–50] now is applied to proton radioactivity for the first time. It is expected that new regions of proton-unstable nuclei close to or beyond the proton drip line will be explored in the near future and with this we have predicted the half-lives of certain proton emitters which, so far, have not been detected experimentally.

The paper is organized in the following manner. In Sec. II the theoretical framework of the paper is presented. In Sec. III the results and discussion are given, and the main conclusions of the paper are provided in Sec. IV.

II. THE MODEL

A. The CPPMDN

In the CPPMDN, the potential-energy barrier is taken as the sum of the deformed Coulomb potential, the deformed two-term proximity potential, and the centrifugal potential for the touching configuration and for the separated fragments. For the prescission (overlap) region, a simple power-law interpolation as performed by Shi and Swiatecki [51] is used.

The interacting potential for two spherical nuclei exhibiting proton decay is given by

$$V = \frac{Z_1 Z_2 e^2}{r} + V_P(z) + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2} \quad \text{for } z > 0. \tag{1}$$

Here Z_1 and Z_2 are the atomic numbers of the daughter nucleus and the emitted proton, "z" is the distance between the near surfaces of the fragments, "r" is the distance between fragment centers and is given as $r = z + C_1 + C_2$, where C_1 and C_2 are the Süsmann central radii of the daughter nucleus and the emitted proton. The term ℓ represents the angular momentum, μ represents the reduced mass, and V_P is the proximity potential. The proximity potential V_P is given by Blocki and Swiatecki [52] as

$$V_P(z) = 4\pi\gamma b\left[\frac{C_1 C_2}{(C_1 + C_2)}\right]\Phi\left(\frac{z}{b}\right), \tag{2}$$

with the nuclear surface tension coefficient,

$$\gamma = 0.9517[1 - 1.7826(N - Z)^2/A^2]\ \text{MeV/fm}^2, \tag{3}$$

where N, Z, and A represent neutron, proton, and mass number, respectively, of the parent,Φ represents the universal proximity potential [53] given as

$$\Phi(\varepsilon) = -4.41 e^{-\varepsilon/0.7176} \quad \text{for } \varepsilon > 1.9475, \tag{4}$$

$$\Phi(\varepsilon) = -1.7817 + 0.9270\varepsilon + 0.0169\varepsilon^2 - 0.05148\varepsilon^3 \quad \text{for } 0 \leqslant \varepsilon \leqslant 1.9475, \tag{5}$$

with $\varepsilon = z/b$ where the width (diffuseness) of the nuclear surface $b \approx 1$ fm and Süsmann central radii C_i of fragments are related to sharp radii R_i as

$$C_i = R_i - \left(\frac{b^2}{R_i}\right). \tag{6}$$

For R_i we use a semiempirical formula in terms of mass number A_i as [53]

$$R_i = 1.28 A_i^{1/3} - 0.76 + 0.8 A_i^{-1/3}. \tag{7}$$

The potential for the internal part (overlap region) of the barrier is given as

$$V = a_0 (L - L_0)^n \quad \text{for } z < 0. \tag{8}$$

Here $L = z + 2C_1 + 2C_2$ and $L_0 = 2C$, where C_1, C_2, and C are the Süsmann central radii of the daughter nucleus, the emitted proton, and the parent nucleus, respectively. The constants a_0 and n are determined by the smooth matching of the two potentials at the touching point.

Using the one-dimensional WKB approximation, the barrier penetrability P is given as

$$P = \exp\left\{-\frac{2}{\hbar}\int_a^b \sqrt{2\mu(V - Q)}\,dz\right\}. \tag{9}$$

Here the mass parameter is replaced by $\mu = m A_1 A_2 / A$, where "m" is the nucleon mass and A_1, A_2 are the mass numbers of the daughter nucleus and the emitted proton, respectively. The turning points "a" and "b" are determined from the equation $V(a) = V(b) = Q$. The above integral can be evaluated numerically or analytically, and the half-life is given by

$$T_{1/2} = \left(\frac{\ln 2}{\lambda}\right) = \left(\frac{\ln 2}{\nu P}\right), \tag{10}$$

where λ is the decay constant and ν represents the number of assaults on the barrier per second.

The Coulomb interaction between the two deformed and oriented nuclei taken from Ref. [54] with higher multipole deformations included [55,56] is given as

$$V_C(r,\theta) = \frac{Z_1 Z_2 e^2}{r} + 3 Z_1 Z_2 e^2 \sum_{\lambda, i=1,2} \frac{1}{2\lambda + 1} \frac{R_{0i}^{\lambda}}{r^{\lambda+1}} Y_{\lambda}^{(0)}(\alpha_i) \times \left[\beta_{\lambda i} + \frac{4}{7}\beta_{\lambda i}^2 Y_{\lambda}^{(0)}(\alpha_i)\delta_{\lambda,2}\right], \tag{11}$$

with

$$R_i(\alpha_i) = R_{0i}\left[1 + \sum_{\lambda} \beta_{\lambda i} Y_{\lambda}^{(0)}(\alpha_i)\right], \tag{12}$$

where $R_{0i} = 1.28 A_i^{1/3} - 0.76 + 0.8 A_i^{-1/3}$. Here α_i is the angle between the radius vector and the symmetry axis of the i^{th} nuclei (see Fig. 1 of Ref. [55]), and it is to be noted that the quadrupole interaction term proportional to $\beta_{21}\beta_{22}$ is neglected because of its short-range character.

The two-term proximity potential for the interaction between a deformed and a spherical nucleus is given by Baltz and Bayman [57] as

$$V_{P2}(R,\theta) = 2\pi\left[\frac{R_1(\alpha) R_C}{R_1(\alpha) + R_C + S}\right]^{1/2}\left[\frac{R_2(\alpha) R_C}{R_2(\alpha) + R_C + S}\right]^{1/2} \times \left(\left[\varepsilon_0(S) + \frac{R_1(\alpha) + R_C}{2R_1(\alpha) R_C}\varepsilon_1(S)\right] \times \left[\varepsilon_0(S) + \frac{R_2(\alpha) + R_C}{2R_2(\alpha) R_C}\varepsilon_1(S)\right]\right)^{1/2}, \tag{13}$$

where θ is the angle between the symmetry axis of the deformed nuclei and the line joining the centers of the two interacting nuclei and α corresponds to the angle between the radius vector and the symmetry axis of the nuclei (see Fig. 5 of Ref. [57]). $R_1(\alpha)$ and $R_2(\alpha)$ are the principal radii of the curvature of the daughter nuclei, R_C is the radius of the spherical cluster, S is the distance between the surfaces along the straight line connecting the fragments, and $\varepsilon_0(S)$ and $\varepsilon_1(S)$ are the one-dimensional slab-on-slab functions.

B. Gamow-like model

In analogy to α decay and cluster radioactivity, proton emission also is considered as a quantum-mechanical tunneling process in which the emitted particle tunnels through a one-dimensional potential barrier [43].

The penetration probability calculated using the WKB approximation is as follows:

$$P = \exp\left\{-\frac{2}{\hbar}\int_{R_{\rm in}}^{R_{\rm out}}\sqrt{2\mu[V(r)-E_P]dr}\right\}, \tag{14}$$

where $\mu = mA_1A_2/A$, m is the nucleon mass, A_1,A_2 are the mass numbers of the daughter nucleus and the emitted proton, respectively, and E_P is the kinetic energy. $R_{\rm in}$ is the classical inner turning point, equal to the radius of the spherical square well in which the proton is trapped before emission,

$$R_{\rm in} = r_0\big(A_1^{1/3} + A_2^{1/3}\big), \tag{15}$$

and $R_{\rm out}$ is the exit point from the potential barrier which is determined by the condition $V(R_{\rm out}) = E_P$.

The proton-nucleus potential $V(r)$ is taken as the sum of the Coulomb potential and the centrifugal potential for the touching configuration and for separated fragments. The proton emission half-life is given by

$$T_{1/2} = \frac{\ln 2}{\nu P}, \tag{16}$$

where ν is the frequency of assaults of the proton against the potential energy barrier. In a first approximation it is given by the harmonic oscillator frequency present in the Nilsson potential [58],

$$h\nu = \frac{41}{A^{1/3}}\mathrm{MeV}. \tag{17}$$

C. The semiempirical relation

Hatsukawa *et al.* [59] derived a semiempirical relation for the prediction of α half-lives by considering it as a fission-like process. We have extended the relation of Hastukawa *et al.* [59] to calculate the half-lives of proton radioactivity.

The WKB approximation of barrier penetrability P leads to

$$P = \exp\left\{-\frac{2}{\hbar}\int_{R_0}^{R_2}\sqrt{2\mu[V(r)-Q]}dr\right\}, \tag{18}$$

where $\mu = mA_1A_2/A$, m is the nucleon mass, and A,A_1,A_2 are the mass numbers of the parent nucleus, the daughter nucleus, and the emitted proton, respectively. Q is the measured decay energy of the proton. Here, the potential is constructed by taking both the external part and the internal part of the barrier.

The half-life can be calculated using the equation given as

$$\log_{10}T = 0.27464Z_1Z_2\left\{\left[\frac{A_1A_2}{AQ}\right]^{1/2}[\arccos\sqrt{X} - \sqrt{X(1-X)}]\right\} - 20.446, \tag{19}$$

where $X = R_1/R_2 = r_0(A_1^{1/3} + A_2^{1/3})\frac{Q}{Z_1Z_2e^2}$ and r_0 is taken to be 1.2249 fm. R_1 represents the touching configuration, and R_2 is the outer turning point.

III. RESULTS AND DISCUSSION

By utilizing the CPPMDN, we have studied proton radioactivity from proton emitters in the ground state as well as in the isomeric state by analyzing 44 experimentally detected proton emitters. In the CPPMDN, the external interaction potential is constructed by taking the deformed Coulomb potential, the deformed two-term proximity potential, and the centrifugal potential. Here, in the calculation of the half-lives of the proton emitters the overlapping effects also are included for which the simple power-law interpolation is used. The proton emission is energetically possible only when the Q value of the reaction is positive and is given by

$$Q = \Delta M - (\Delta M_1 + \Delta M_2) + k\big(Z^{\varepsilon} - Z_1^{\varepsilon}\big), \tag{20}$$

where ΔM, ΔM_1, and ΔM_2 are the mass excesses of the parent nuclei, daughter nuclei, and the emitted proton, respectively. The term $k(Z^{\varepsilon} - Z_1^{\varepsilon})$ represents the screening effect of the atomic electrons [60] with $k = 8.7$ eV, $\varepsilon = 2.517$ for $Z \geqslant 60$, and $k = 13.6$ eV, $\varepsilon = 2.408$ for $Z < 60$ [61]. The Q values are calculated using the mass excess values taken from the recent mass table of Audi *et al.* [62]. The angular momentum transfer associated with the transitions is obtained from the spin-parity selection rule given as

$$|J_i - J_f| \leqslant j \leqslant |J_i + J_f|, \quad \frac{\pi_i}{\pi_f} = (-1)^{\ell}, \tag{21}$$

where J_i, J_f, π_i, and π_j are the spin and parity of the parent and daughter nuclei, respectively, and ℓ and j are the angular momentum and spin of the outgoing proton.

The half-lives of 29 proton emitters in the ground state and 15 proton emitters in the isomeric state are evaluated using our formalism CPPMDN in which the effect of quadrapole deformation (β_2) of the parent and daughter nuclei are included. The deformation values are taken from the mass table of Möller *et al.* [63]. Since the Coulomb potential is relatively low and the centrifugal potential is relatively high compared to the α decay, the proton decay half-lives are sensitive to the angular momentum ℓ associated with the transitions. In the present paper the experimental ℓ values are taken from Blank and Borge [19]. For a comparison, the half-lives also are evaluated using the CPPM, the Gamow-like model of Zdeb *et al.* (denoted as Gamow) [43] and using the semiempirical relation of Hatsukawa *et al.* (denoted as Hatsukawa) [59].

In Tables I and II, we have listed the proton radioactivity from experimentally detected proton emitters with $Z = 53$–83

TABLE I. A comparison of calculated half-lives of the proton emitters in the ground state with different theoretical models and experimental data. The experimental Q values and half-lives are taken from Ref. [19] except where noted.

Parent nuclei	ℓ value	Q value (keV)	$\log_{10} T_{1/2}$ (s)				
			CPPMDN	CPPM	Gamow	Hatsukawa	Expt. [19]
^{109}I	2	827(5)	−3.607	−3.177	−5.085	−4.515	$-4.029^{+0.0023}_{-0.0023}$
		831[a]	−3.661	−3.231	−5.137	−4.567	
^{112}Cs	2	823(7)	−2.901	−2.348	−4.273	−3.709	$-3.301^{+0.0792}_{-0.0969}$
		823[a]	−2.895	−2.343	−4.268	−3.703	
^{113}Cs	2	976(3)	−5.096	−4.513	−6.349	−5.785	$-4.777^{+0.0178}_{-0.0186}$
		985[a]	−5.202	−4.619	−6.451	−5.887	
^{117}La	2	814(11)	−2.290	−1.462	−3.390	−2.834	$-1.623^{+0.0350}_{-0.0381}$
		833[a]	−2.607	−1.779	−3.697	−3.140	
^{121}Pr	2	900(10)	−2.912	−2.087	−3.976	−3.425	$-2.000^{+0.2041}_{-0.1549}$
		904[a]	−2.969	−2.143	−4.031	−3.480	
^{130}Eu	2	1039(15)	−3.659	−2.659	−4.489	−3.950	$-3.046^{+0.1888}_{-0.1689}$
		1044[a]	−3.720	−2.720	−4.548	−4.010	
^{131}Eu	2	959(9)	−2.571	−1.579	−3.440	−2.903	$-1.670^{+0.0351}_{-0.0359}$
		964[a]	−2.649	−1.656	−3.515	−2.978	
^{135}Tb	3	1200(7)	−4.593	−3.550	−5.698	−5.166	$-3.027^{+0.1307}_{-0.1158}$
		1194[a]	−4.532	−3.489	−5.640	−5.108	
^{140}Ho	3	1106(10)	−2.807	−1.869	−4.048	−3.522	$-2.222^{+0.1761}_{-0.3010}$
		1091[a]	−2.787	−1.849	−4.028	−3.503	
^{141}Ho	3	1190(8)	−3.737	−2.875	−5.013	−4.489	$-2.387^{+0.0948}_{-0.1214}$
		1195[a]	−3.792	−2.930	−5.067	−4.542	
^{144}Tm	5	1725(16)	−7.001	−6.118	−8.870	−8.348	$-5.569^{+0.2121}_{-0.1303}$
		1711[a]	−6.978	−6.026	−8.781	−8.259	
^{145}Tm	5	1753(7)	−7.001	−6.316	−9.052	−8.532	$-5.456^{+0.0384}_{-0.0421}$
		1755[a]	−7.195	−6.329	−9.064	−8.544	
^{146}Tm	5	1210(4)	−2.585	−1.749	−4.641	−4.123	$-0.930^{+0.0230}_{-0.0243}$
		916[a]	1.461	2.302	−0.708	−0.191	
^{147}Tm	5	1073(5)	0.690	−0.087	−3.023	−2.507	$0.577^{+0.1258}_{-0.1778}$
		1074[a]	0.682	−0.095	−3.031	−2.514	
^{150}Lu	5	1283(3)	−1.223	−1.971	−4.826	−4.312	$-1.194^{+0.0364}_{-0.0398}$
		1286[a]	−1.250	−1.998	−4.852	−4.338	
^{151}Lu	5	1253(3)	−0.970	−1.664	−4.522	−4.010	$-0.896^{+0.0061}_{-0.0062}$
		1256[a]	−1.005	−1.699	−4.555	−4.043	
^{155}Ta	5	1468(15)	−3.320	−3.236	−6.013	−5.505	$-2.538^{+0.1811}_{-0.2071}$
		1466[a]	−3.302	−3.218	−5.998	−5.490	
^{156}Ta	2	1030(5)	−0.917	−0.683	−1.160	−0.655	$-0.609^{+0.0804}_{-0.0921}$
		1036[a]	−0.854	−0.620	−1.221	−0.716	
^{157}Ta	0	947(7)	1.453	1.613	0.158	0.662	$-0.523^{+0.1303}_{-0.1871}$
		957[a]	1.287	1.446	−0.004	0.500	
^{159}Re	5	1816 (20) [b]	−5.654	−5.417	−8.088	−7.584	$-4.678^{+0.0003}_{-0.0004}$
		1607[a]	−4.138	−3.901	−6.627	−6.124	
^{160}Re	2	1285(5)	−2.341	−1.988	−3.734	−3.233	$-3.060^{+0.0648}_{-0.0645}$
		1287[a]	−2.339	−1.986	−3.732	−3.231	
^{161}Re	0	1214(6)	−1.947	−1.569	−2.922	−2.421	$-3.357^{+0.0020}_{-0.0020}$
		1217[a]	−1.981	−1.603	−2.955	−2.454	
^{164}Ir	5	1844(9)	−5.590	−5.173	−7.820	−7.322	$-3.947^{+0.1900}_{-0.0116}$
		1577[a]	−3.601	−3.182	−5.904	−5.406	
^{166}Ir	2	1168(7)	−0.410	0.035	−1.759	−1.264	$-0.818^{+0.1664}_{-0.2734}$
		1178[a]	−0.542	−0.097	−1.887	−1.392	
^{167}Ir	0	1096(6)	0.167	0.621	−0.799	−0.306	$-0.959^{+0.0555}_{-0.0637}$
		1087[a]	0.298	0.752	−0.673	−0.179	
^{170}Au	2	1488(12)	−3.047	−2.953	−4.637	−4.146	$-3.493^{+0.0823}_{-0.0865}$
		1489[a]	−3.054	−2.959	−4.643	−4.152	
^{171}Au	0	1464(10)	−3.362	−3.133	−4.423	−3.933	$-4.611^{+0.0762}_{-0.0588}$
		1451[a]	−3.331	−3.102	−4.393	−3.903	
^{176}Tl	0	1282(18)	−0.832	−0.594	−1.973	−1.489	$-2.284^{+0.0244}_{-0.1362}$

TABLE I. (*Continued.*)

Parent nuclei	ℓ value	Q value (keV)	$\log_{10} T_{1/2}$ (s)				
			CPPMDN	CPPM	Gamow	Hatsukawa	Expt. [19]
		1271[a]	−0.739	−0.501	−1.883	−1.399	
^{177}Tl	0	1180(20)	0.465	0.709	−0.709	−0.226	$-1.174^{+0.1910}_{-0.3489}$
		1168[a]	0.633	0.878	−0.545	−0.063	

[a] Q values are computed using the mass excess values taken from Ref. [62].
[b] Q value and half-life are taken from Ref. [17].

in the ground state and isomeric state, respectively. We have excluded the ^{105}Sb proton emitter from our tables as the reinvestigation of the direct proton decay of ^{105}Sb reported that it is no longer a proton emitter [64,65]. For the experimentally detected proton emitters ($53 \leqslant Z \leqslant 83$), the calculations are performed using both experimental (taken from Blank and Borge [19]) and calculated Q values. Table I gives the half-lives obtained using the four theoretical approaches CPPMDN, CPPM, Gamow, and Hatsukawa for proton emitters in the ground state, and Table II gives the half-lives for proton emitters in the isomeric states [superscript (m) denotes the isomeric states] that are discovered experimentally. Consider the proton emitter ^{130}Eu for which $\ell = 2$ and the experimental and theoretical Q values are 1.039 and 1.044 MeV, respectively. When the experimental Q value is adopted, the half-life obtained using the CPPMDN is 2.19×10^{-4} s, and when the calculated Q value is used the corresponding half-life is 1.91×10^{-4} s. To compare, both values are found to agree with the experimental data (8.99×10^{-4} s) reasonably well. We also have performed the half-life evaluations using both

TABLE II. The comparison of the calculated half-lives of the proton emitters in the isomeric state with different theoretical models and experimental data. The experimental Q values and half-lives are taken from Ref. [19].

Parent nuclei	ℓ value	Q value (keV)	$\log_{10} T_{1/2}$ (s)				
			CPPMDN	CPPM	Gamow	Hatsukawa	Expt. [19]
^{141m}Ho	0	1255(8)	−5.161	−4.435	−5.687	−5.162	$-2.180^{+0.0551}_{-0.0491}$
		1265[a]	−5.263	−4.538	−5.785	−5.260	
^{146m}Tm	5	1140(4)	−1.767	−0.930	−3.848	−3.330	$-0.693^{+0.0127}_{-0.0130}$
		1056[a]	−2.647	−1.811	−4.701	−4.183	
^{147m}Tm	2	1133(3)	−2.378	−1.987	−3.772	−3.255	$-3.444^{+0.0414}_{-0.0458}$
		1135[a]	−2.405	−2.014	−3.798	−3.281	
^{150m}Lu	2	1306(5)	−3.756	−3.327	−5.057	−4.544	$-4.367^{+0.0655}_{-0.0537}$
		1307[a]	−3.758	−3.329	−5.059	−4.546	
^{151m}Lu	2	1332(10)	−4.057	−3.605	−5.320	−4.808	$-4.796^{+0.0263}_{-0.0280}$
		1387[a]	−3.801	−3.348	−5.073	−4.561	
^{156m}Ta	5	1127(7)	0.740	0.425	−2.459	−1.953	$0.930^{+0.0965}_{-0.1243}$
		1317[a]	−1.48	−1.798	−4.616	−4.109	
^{159m}Re	5	1831(20)	−5.752	−5.515	−8.183	−7.679	$-4.695^{+0.0731}_{-0.0879}$
		1817[a]	−5.659	−5.422	−8.093	−7.589	
^{161m}Re	5	1338(7)	−1.942	−1.481	−4.275	−3.774	$-0.650^{+0.0563}_{-0.0647}$
		1387[a]	−2.439	−1.978	−4.756	−4.256	
^{165m}Ir	5	1733(7)	−4.877	−4.416	−7.085	−6.588	$-3.469^{+0.0813}_{-0.1001}$
		1737[a]	−4.909	−4.448	−7.116	−6.619	
^{166m}Ir	5	1340(8)	−1.472	−0.965	−3.742	−3.247	$-0.076^{+0.1249}_{-0.1761}$
		1348[a]	−1.558	−1.051	−3.825	−3.330	
^{167m}Ir	5	1261(7)	−0.644	−0.093	−2.890	−2.396	$0.875^{+0.1206}_{-0.1675}$
		1264[a]	−0.676	−0.125	−2.921	−2.427	
^{170m}Au	5	1770(6)	−3.839	−4.238	−6.882	−6.391	$-2.980^{+0.0531}_{-0.0557}$
		1753[a]	−3.711	−4.111	−6.760	−6.268	
^{171m}Au	5	1719(4)	−3.443	−3.874	−6.526	−6.036	$-2.654^{+0.0357}_{-0.0389}$
		1712[a]	−3.391	−3.822	−6.476	−5.985	
^{177m}Tl	5	1984(8)	−6.069	−5.276	−7.842	−7.357	$-3.402^{+0.0863}_{-0.0939}$
		1955[a]	−5.885	−5.091	−7.664	−7.179	
^{185m}Bi	0	1624(16)	−5.358	−3.595	−4.853	−4.377	$-4.237^{+0.0627}_{-0.0732}$
		1614[a]	−5.268	−3.507	−4.769	−4.293	

[a] Q values are computed using the mass excess values taken from Ref. [62].

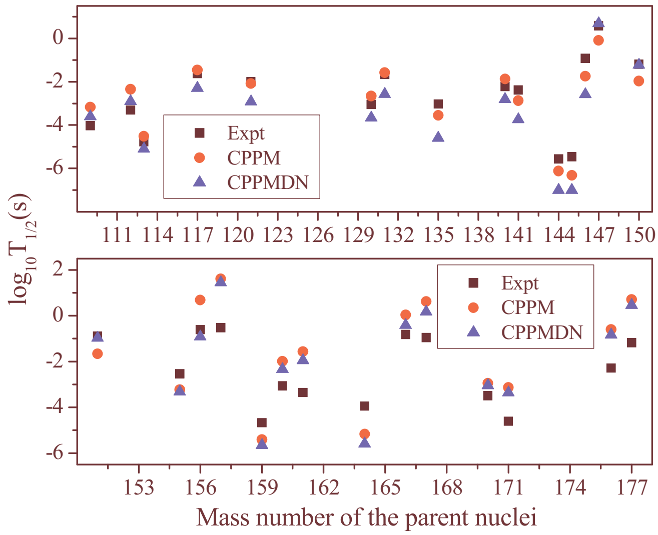

FIG. 1. The comparison of the CPPMDN and CPPM results with the experimental values for the proton emitters in the ground state.

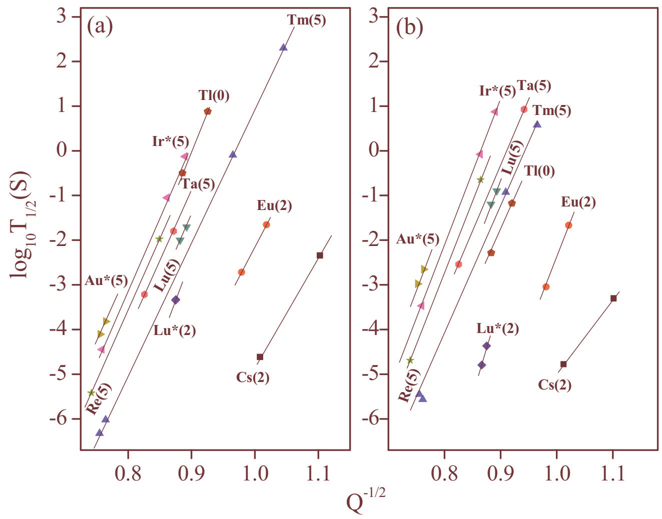

FIG. 2. G-N plots for different cases of proton radioactivity (a) using calculated half-lives and Q values and (b) using experimental half-lives and Q values. The angular momentum values ℓ associated with the transitions are given within the parentheses.

experimental and calculated Q values within the CPPM (treating both parent and daughter nuclei as spherical), Gamow, and Hatsukawa. For the last two approaches, Gamow and Hatsukawa, the deviation of half-lives from the experimental data is noted to be a little bit larger compared to the CPPMDN and CPPM. To verify this we have obtained the standard deviation of the logarithmic values of the estimated half-lives with the experimental data for the above theoretical approaches using the equation,

$$\sigma = \left\{ \frac{1}{n-1} \sum_{i=1}^{n} \left(\log_{10} T_i^{\mathrm{cal}} - \log_{10} T_i^{\mathrm{exp}} \right)^2 \right\}^{1/2}. \tag{22}$$

The standard deviation obtained is low for the CPPM and CPPMDN ($\sigma_{\mathrm{CPPM}} = 1.075$, $\sigma_{\mathrm{CPPMDN}} = 1.275$). On the other hand, for Gamow and Hatsukawa the standard deviation obtained is relatively high ($\sigma_{\mathrm{Gamow}} = 2.654$, $\sigma_{\mathrm{Hatsukawa}} = 2.265$). This high standard deviation of Gamow and Hatsukawa may be because, for the calculation of proton emission half-lives, the interaction potential (for the external part) is taken as the sum of the Coulomb and centrifugal potentials without considering the nuclear interaction potential. A significantly low standard deviation obtained for the CPPMDN and CPPM is due to the inclusion of the nuclear potential along with the Coulomb and centrifugal potentials. This implies that the nuclear part of the interaction is quite important in the study of proton radioactivity. For a better understanding of our predictions in comparison with the experimental values, a graph is plotted with the logarithmic half-lives obtained using the CPPMDN and CPPM versus the mass number of the parent nuclei and is given in Fig. 1.

The Geiger-Nuttall (G-N) law, originally observed for α decay, is studied for proton radioactivity, shown in Fig. 2. The study has shown that the G-N law is valid in the case of proton radioactivity, in analogy to α decay, provided it is plotted for the isotopes of a particular proton emitter nuclide with the same ℓ values. For example, the four isotopes of Tm, ^{144,145,146,147}Tm with the same ℓ values ($\ell = 5$) lie on the same straight line and are a perfect example of the G-N law. Even though the isotopes of a given proton emitter nuclide with the same ℓ values is limited in number, we are able to construct the trend successfully. In Fig. 2, for a comparison, we also have given the G-N plots for the experimental logarithmic half-lives versus $Q^{-1/2}$. Also, the linear behavior of the G-N plot is found to be unaltered with the inclusion of the nuclear potential, although it is meant for the pure Coulomb potential.

Since the present analysis is able to reproduce the experimental data reasonably well with a quite satisfactory standard deviation, we hope that the present approach, the CPPMDN, and the CPPM can be used as a unified model to describe α, cluster, and proton decays. This motivated us to check whether the universal curve meant for cluster decay [66] is

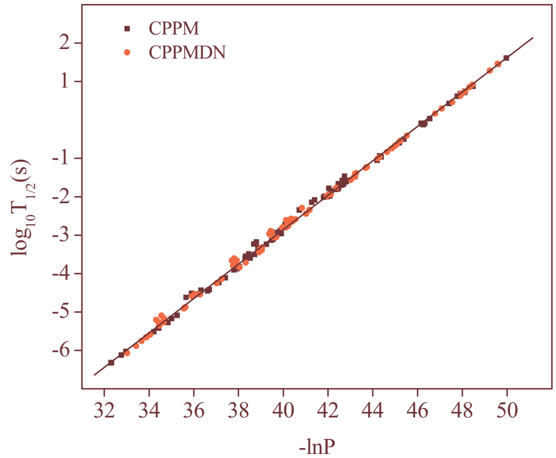

FIG. 3. A universal curve for experimental and calculated logarithmic half-lives versus a negative logarithm of penetrability for proton emissions from various parent nuclei using the CPPMDN and CPPM.

TABLE III. The prediction of the half-lives of the proton emitters that are not experimentally detected yet. The Q values are computed using the mass excess values taken from Ref. [62].

Parent nuclei	ℓ_{min}	Q value (keV)	$\log_{10} T_{1/2}$ (s)			
			CPPMDN	CPPM	Gamow	Hatsukawa
^{58}Ge	1	245	1.095	1.486	−0.480	0.195
^{89}Rh	4	518	1.436	0.572	−2.563	−1.959
^{105}Sb	2	331	9.921	10.081	7.783	8.358
^{108}I	2	610	0.582	0.983	−1.071	−0.501
^{173}Au	0	1002	2.239	2.762	1.276	1.763
^{184}Bi	1	1364	−2.506	−0.873	−2.357	−1.880
^{186}Bi	1	1124	0.662	2.240	0.663	1.137
^{159}Ta	0	385	19.619	20.009	18.166	18.667
^{168}Ir	4	507	16.132	16.644	13.797	14.288
^{169}Ir	0	636	10.042	10.555	8.888	9.378
^{179}Tl	0	764	8.165	8.522	6.904	7.384
^{180}Tl	0	392	24.203	24.319	22.428	22.906
^{193}At	0	732	11.751	11.014	9.357	9.823

valid for proton radioactivity. We have succeeded in obtaining a single straight line for all proton emissions irrespective of the parents with intercepts $= -20.799$ and slopes $= 0.449$ (shown in Fig. 3). In our previous studies [46,47] on α decay and cluster decay, we have obtained the universal curve as a single straight line for all emissions irrespective of the parents with the same slopes and intercepts. So certainly, it would be remarkable that the universal curve for cluster decay and α decay also is applicable for proton decay.

The agreement attained in reproducing the experimental data using the present analysis for proton decay and the above findings enabled us to predict the half-lives of certain proton emitters in the ground state that are not verified yet experimentally. We have predicted the half-lives of proton emitters with $Z > 50$ and a few proton emitters with $Z < 50$ using the CPPMDN, the CPPM (for the spherical case), the Gamow-like model of Zdeb *et al.* [43], and the semiempirical relation of Hatsukawa *et al.* [59]. The results are provided in Table III. The ground-state proton radioactivity has yet to be identified for elements below Sn, and our predictions on the half-lives of proton emitters ^{58}Ge and ^{89}Rh will be helpful for the detection of proton emitters with $Z < 50$ in the future. The predictions for these new proton emitters having measurable half-lives can be accessed in the future by taking advantage of the developing experimental techniques. Thus our present paper revealed that the CPPMDN and CPPM can now be used successfully as a unified model to study proton decay, α decay, and cluster decay.

IV. CONCLUSION

To summarize, the proton radioactivity from proton emitters in the ground state and isomeric state has been carried out within the CPPMDN using both experimental and calculated Q values and is found to agree with the experimental data. The decay half-lives obtained using the CPPMDN are compared with those of other theoretical approaches, the CPPM, the Gamow-like model of Zdeb *et al.* [43], and the semiempirical relation of Hatsukawa *et al.* [59]. The standard deviation obtained with respect to the experimental half-lives suggested that the present systematic study, the CPPMDN, and the CPPM can be applied successfully to study proton decay along with α and cluster decay studies. The G-N law is found to be obeyed in the case of proton emissions also but for the isotopes of a particular proton emitter nuclide with the same ℓ value. We also have found that the universal curve is valid in the case of proton decay as we obtained a single straight line with the same slopes and intercepts for all proton emitters. The good experimental correlation of the predicted half-lives encouraged us to predict the existence of new proton emitters above the proton drip line which can be detected in the future.

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