Mass excesses of $^{21-24}$Si and ^{24}S

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I have used a recent parametrization of mirror energy differences to predict two-proton separation energies for $^{21-24}$Si and ^{24}S. Results are in good agreement with experimental values for ^{22,24}Si.

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I. INTRODUCTION

Earlier, I discovered a simple parametrization [1] of mirror energy differences (MEDs) in light nuclei. For p-shell + $2n$ nuclei, a three-parameter fit (denoted 3PF here) gave excellent agreement with experimental MEDs in terms of $P(s^2)$, the amount of s^2 occupancy. The fitting function was MED = $f[S_{2n}, P(s^2)]Z/A^{1/3}$ with $f = C + aS_{2n} - bP(s^2)$. The factor $Z/A^{1/3}$ was expressly included because any calculation of Coulomb energies will vary as Z/R. The fit produced agreement with experimental values with a root-mean-square deviation of 4 keV (better than the experimental uncertainties in some cases). That fit included all p-shell + $2n$ nuclei for which S_{2n}, S_{2p}, and $P(s^2)$ were all known—six mirror pairs in total. I then used the fit to predict (successfully, as it turned out) S_{2p} for ^{15}Ne. The sensitivities to S_{2n} and $P(s^2)$ are both easily understandable as being a consequence of the so-called Thomas-Ehrman effect in which the energy of an s state is lower in the proton-excess member of a mirror pair. This effect is extremely well reproduced by calculations in a potential model with a diffuse potential well. In the original fitting, the coefficient of the $P(s^2)$ term was $b = 0.724(6)$ MeV. In all subsequent applications, I have kept these 3PF parameters fixed. The spectroscopic factor and $P(s^2)$ are related by the equation $\Sigma S_s = 2P(s^2)$, where S_s is the spectroscopic factor for $2s_{1/2}$ neutron pickup.

In any potential-model calculation of MEDs, the result will depend on the occupancy of the s orbital because of the Thomas-Ehrman shift—hence the $P(s^2)$ dependence. And, it is well known that the Thomas-Ehrman shift depends slightly on separation energy—hence the S_{2n} term. The function I have used is thus the simplest that contains all the essential physics—the Z/R factor from Coulomb, the dependence on s occupancy, and a (slight) dependence on separation energy. With all the current research on nuclei near the proton drip line, it is important to have a simple estimate of MEDs because the usual potential-model approach is inapplicable without some detailed knowledge of the levels of the relevant core nuclei.

In the initial work, I considered only nuclei that contained no $2s_{1/2}$ nucleons in the core. Later, I investigated whether this 3PF could be extended to nuclei in which the core contained some s nucleons [2]. In $^{18-20}$Mg, the 3PF and potential-model calculations produced predictions that were virtually identical. This was extremely surprising because the potential-model calculations required energies and spectroscopic factors for several core states. For ^{19,20}Mg, the agreement with experimental MEDs was exceptional. I then used the 3PF to predict the mass excess of ^{17}Mg. Here, I use it to estimate the mass excesses of $^{21-24}$Si and ^{24}S. [It should be obvious that the assumptions of the original treatment require that the neutron-excess nucleus has even N and therefore the proton-excess mirror has even Z.]

One possibility for the fact that having s nucleons in the core does not run afoul of the Pauli principle is the following: As stated, I am considering only nuclei with even N (and hence even Z for the mirror). In the lowest state, the neutrons will tend to be coupled to 0^+, whether A is even or odd (with even N). Thus, in any term of the wave function for the last two neutrons, the s orbital is either empty or full as it can contain at most two neutrons. Thus, there is no problem with the Pauli principle. Even so, in both sets of nuclei explored so far (p-shell [1] and $N = 10$ [2] cores), the agreement was far better than I had expected.

II. CALCULATIONS AND RESULTS

I define the MED as

$$\text{MED} = S_{2n}(\text{neutron} - \text{excess nucleus}) - S_{2p}(\text{proton} - \text{excess mirror})$$

The fitting function was MED = $\{f[S_{2n}, P(s^2)]\}Z_</A^{1/3}$, where A is the mass number and $Z_<$ is the proton number of the core in the core + $2p$ nucleus. A function of the form $f = C + aS_{2n} - bP(s^2)$ was found to work extremely well [1]. The success of the 3PF when it was extended into the realm in which the core also contains sd-shell nucleons [2] has encouraged me to investigate the consequences for Si nuclei. For all the Si nuclei, $Z_<$ is 12. A recent report of the mass excess of ^{22}Si [3] has provided further motivation.

In ^{22}O, the $2s_{1/2}$ orbital is mostly empty. A shell-model calculation by Brown *et al.* [4] predicted a spectroscopic factor of $S = 0.80$ for $^{22}\text{O} \rightarrow {}^{23}\text{O}$ (g.s.) (where g.s. represents the ground state). A breakup experiment with ^{23}O reported $S(\text{g.s.}) = 0.77(10)$ [5]. A newer calculation by Brown [6] predicts $S(\text{g.s.}) = 0.7823$ and $\Sigma S(2s) = 0.8273$, implying $P(s^2) = 0.1727$ for ^{22}O. A recent experiment [3] produced a beam of ^{22}Si by fragmentation of ^{28}Si and investigated its decays. Those authors provided an estimate of the $2p$ separation energy: $S_{2p} = -108(125)$ keV. If ^{22}Si is indeed unbound to $2p$ decay, the 3PF requires $P(s^2) < 0.174$ for

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TABLE I. Present results for $2p$ separation energies of $^{21-24}$Si and ^{24}S.

Nucleus	S_{2n} (MeV) [a]	Mirror	S_{2p} (MeV)		
			Predicted	Measured [a]	ImKG [c]
^{21}N	6.75(10)	^{21}Si	−3.69(9)		−3.898(98)
^{22}O	10.66(6)	^{22}Si	−0.003(54)	−0.108(125) [b]	−0.004(57)
^{23}F	12.78(5)	^{23}Si	2.07(5)	1.79(50) from syst.	2.114(100)
^{24}Ne	14.070	^{24}Si	3.389	3.434(19)	
^{24}O	6.93(12)	^{24}S	−2.32(33)		−4.934(125)

[a]Reference [7], unless indicated otherwise.
[b]Reference [3]. The estimate from systematics is −1.2(5) MeV [7].
[c]Improved Kelson-Garvey model [10].

^{22}O—remarkably close to (but slightly larger than) the shell-model value.

Alternatively, I can use the shell-model $P(s^2)$ and the 3PF to predict the $2p$ separation energy. The result is $S_{2p}(3\text{PF}) = -3(54)$ keV where the uncertainty arises from the uncertainty in the $2n$ separation energy of ^{22}O of 10.66(6) MeV [7]. The latest mass evaluation estimates $S_{2p}(^{22}\text{Si}) = -1.2(5)$ MeV from systematics [7].

Turning now to the ^{24}Ne/^{24}Si mirror pair, I note that the $2p$ separation energy of ^{24}Si is known to be 3.434(19) MeV [7]. In a study of the ^{24}Ne(d,p) reaction (in reverse kinematics) [8], the g.s. spectroscopic factor was measured as $S = 0.80$ for $^{24}\text{Ne} \rightarrow {}^{25}\text{Ne}$ g.s. Because this value is virtually identical to the $^{22}\text{O} \rightarrow {}^{23}\text{O}$ result, I choose to use $P(s^2)$ for ^{24}Ne to be the same as for ^{22}O, viz 0.1727. The resulting 3PF prediction is then 3.389 MeV for ^{24}Si, reasonably close to the experimental value. Requiring agreement with the measured S_{2p} would have needed $P(s^2) = 0.188(6)$. It is astonishing that fit parameters obtained for nuclei from ^{12}O to ^{18}Ne [1] work so well for Mg [2] and Si [here].

I turn now to two odd-A Si nuclei—^{21}Si and ^{23}Si, neither of which has an experimental value of S_{2p}. For ^{23}Si, the estimate from systematics is 1.79(50) MeV [7]. For present purposes, because the parent nuclei have the same number of neutrons, I use the same $P(s^2)$ for these nuclei as for ^{22,24}Si, viz. 0.1727. With the experimental S_{2n} of 12.78(5) MeV for ^{23}F, the prediction is $S_{2p} = 2.07$ MeV for ^{23}Si—within the range estimated from systematics. The $2n$ separation energy of ^{21}N is 6.75(10) MeV, resulting in a prediction for ^{21}Si of $S_{2p} = -3.69$ MeV. All these results are summarized in Table I.

I have also chosen to look at ^{24}S because the spectroscopic factor for $2s_{1/2}$ pickup from its mirror $^{24}\text{O} \rightarrow {}^{23}\text{O}$ has been measured as $S(2s_{1/2}) = 1.74(19)$ [9], implying $P(s^2) = 0.87(10)$. With a separation energy of $S_{2n} = 6.93(12)$ MeV for ^{24}O, the result for ^{24}S is $S_{2p} = -2.32(33)$ MeV.

III. COMPARISON WITH OTHER PREDICTIONS

Recent predictions from an improved Kelson-Garvey (ImKG) model [10] are listed in the last column of the table. For $^{21-23}$Si, the close similarity of the two sets of predictions is amazing because the two approaches are quite different. The present 3PF analysis uses only $P(s^2)$ and S_{2n} for the mirror nucleus, whereas the ImKG model involves masses of several nearby nuclei. I see no reason why the two approaches should give similar answers. For ^{24}S, the two predictions are significantly different, as was also the case previously for ^{15}Ne. In this latter case, the 3PF prediction [1] was $S_{2p} = -2.64(24)$ MeV, and ImKG predicted −3.532(23) MeV. A recent experiment [11] reported $S_{2p} = -2.522(66)$ MeV. For both ^{24}O/^{24}S and ^{15}B/^{15}Ne, $P(s^2)$ is large. Because my 3PF depends strongly on the $2s_{1/2}$ occupancy, and ImKG depends on it only indirectly, it is clear that the two will not agree for both small and large occupancies. It appears that ImKG does not take sufficient account of the Thomas-Ehrman effect, and thus the two predictions will be expected to disagree when $P(s^2)$ is large.

A calculation for several $Z = 8$ and 20 proton-rich nuclei by Holt *et al.* [12] used two- and three-nucleon forces. For ^{22}Si, they predicted $S_{2p} = -1.63$ MeV with a calculation in the sd space and $S_{2p} = -0.12$ MeV in an expanded $sd\,f_{7/2}p_{3/2}$ space.

IV. SUMMARY

To summarize, I have used a recent parametrization to predict two-proton separation energies for $^{21-24}$Si and ^{24}S. Predictions agree well with experimental results for ^{22,24}Si. Perhaps the present predictions will spur investigations into the other nuclei.

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