Widths of low-lying levels of ^{17}Na

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I have used a potential model and results from an earlier shell-model calculation to compute expected widths for decays of the first three states of ^{17}Na. I compare them with a recent experiment that observed a large peak near the region of the predicted $3/2^+$ and $5/2^+$ states.

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I. INTRODUCTION

With mirror symmetry, the structures of the levels of ^{17}C and ^{17}Na are the same. However, the energy spacings (and even level ordering) can be significantly different because of the so-called Thomas-Ehrman effect in which the energy of an s state is lower in the proton-excess member of a mirror pair. This effect is well known, and it is remarkably well reproduced in potential-model calculations. Various groups [1–3] have exploited this symmetry for this mirror pair by tailoring an interaction to reproduce levels of ^{17}C and then using that interaction to calculate levels of ^{17}Na. Different approaches have produced somewhat varying results that have been summarized elsewhere [4]. Here I report widths for the dominant decay branches of the first three levels of ^{17}Na.

II. CALCULATIONS AND RESULTS

A simple shell-model calculation [5] of ^{16}C in a space containing only $1d_{5/2}$ and $2s_{1/2}$ (abbreviated d and s throughout) as active nucleons produces six states—two 0^+, two 2^+, 3^+, and 4^+. These calculations reproduce fairly well the energy levels, the strengths in the reaction ^{14}C(t,p) [5,6] and the $B(E2)$ from the first 2^+ state to the ground state (g.s.) [7]. These six states then serve as cores for an $(sd)^3$ shell-model calculation of ^{17}C, the results of which have already been reported [4,8]. With mirror symmetry, these wave functions of ^{17}C are then the same as the wave functions of ^{17}Na. Relevant spectroscopic factors are listed in Table I. For the allowed decays of these ^{17}Na levels, I have computed single-particle (sp) widths in a Woods-Saxon potential well, having r_0, a, $r_{0c} = 1.26$, 0.60, 1.40 fm. The well depth was adjusted to reproduce the calculated energies of the levels, and the sp widths were computed from the phase shifts. Predicted widths for various decays then were computed from the expression $\Gamma_{\rm calc} = S\Gamma_{\rm sp}$. These results are listed in Table II.

Note that the $1/2^+$ state is predicted to be very wide—$\Gamma_{\rm calc} \sim 1.1$ MeV, and its only important decay branch is to the g.s. via $\ell = 0$. The computed width is large because the decay has $\ell = 0$ and the spectroscopic factor is large. The $3/2^+$ and $5/2^+$ states have reasonably large $S(\ell = 0)$ but for decay to the 2^+ state. Because the decay energy is lower, the computed widths are considerably smaller. Because I have no $d_{3/2}$ nucleons in my simple shell model, the $3/2^+$ state cannot decay to the g.s., but decays to the first 2^+ state are allowed for both $\ell = 0$ and 2. With the shell-model spectroscopic factors, even though S_d is about 10 times S_s, the s-wave decay dominates by a factor of about 4. Because of the absence of g.s. decays, this state is expected to be reasonably narrow—about 125 keV. Even if $d_{3/2}$ is present at some level, it should be small—probably a few percent at most—and it would not appreciably increase the expected $3/2^+$ width.

The $5/2^+$ state has a large spectroscopic factor for decay to the g.s. and hence a large width for that $\ell = 2$ decay. Decay to the first 2^+ state by $\ell = 0$ is also reasonably large, but $\ell = 2$ decay to that state is negligible. The total width of this $5/2^+$ state should be about 480 keV.

III. COMPARISON WITH EXPERIMENT

With a secondary beam of ^{17}Ne incident on a thick ^{9}Be target, Brown *et al.* [9] produced ^{17}Na and detected $3p + ^{14}$O events. They measured an excitation-energy spectrum and observed a small peak near 3 MeV and a much stronger peak near 4.85(6) MeV. An investigation of the reaction ^{15}N(t,p) [10] established that the dominant structure of ^{17}N(g.s.) was ^{15}N(g.s.) $\times\, \nu(sd)^2_0$ so that the mirror ^{17}Ne would be ^{15}O $\times\, \pi(sd)^2$. One way to think of the reaction mechanism that produces ^{17}Na from ^{17}Ne is neutron removal from the $1p$ shell followed (or preceded) by proton addition to the sd shell. The first of these is depicted in Fig. 1. The other reaction amplitude would involve proton addition to the sd shell to make the low-lying negative-parity states of ^{18}Na and then neutron removal from the $1p$ shell.

In an earlier study of neutron removal from ^{17}Ne [11], the g.s. of ^{16}Ne was the strongest state observed, with a smaller yield for the first 2^+ state, and an appreciable strength for

TABLE I. Spectroscopic factors [4] for ^{16}C $+ n = ^{17}$C.

Core	sp	$1/2^+$	$3/2^+$	$5/2^+$
0^+_1	s	0.465		
	d		∼0	0.821
0^+_2	s	0.534		
	d		∼0	0.0008
2^+_1	s		0.174	0.197
	d	0.507	1.647	0.166
2^+_2	s		0.270	0.163
	d	0.327	0.137	0.279
4^+	d		0.477	0.802
3^+	s			0.543
	d	1.167	0.371	0.0005

TABLE II. Calculated energies (MeV) and widths (keV) for $^{17}\mathrm{Na} \rightarrow {}^{16}\mathrm{Ne} + p$.

Initial	E_p[a]	Final	ℓ	E decay	S[a]	Γ_{sp}	$S\Gamma_{sp}$
$1/2^+$	3.02	0^+	0	3.02	0.465	2460	1140
$3/2^+$	3.22	0^+	2	3.22	∼0	280	
	3.22	2^+	0	1.53	0.174	570	99
	3.22	2^+	2	1.53	1.65	15	25
$5/2^+$	3.47	0^+	2	3.47	0.821	350	290
	3.47	2^+	0	1.78	0.197	960	190
	3.47	2^+	2	1.78	0.166	30	5

[a]Reference [4].

another 2^+ state at 6.18 MeV. Thus, at low excitation energy in ^{17}Na, we would expect to see states that correspond to an sd-shell proton coupled to the g.s. and/or first-excited state of ^{16}Ne. Furthermore, addition of a d proton is strongly favored kinematically over s addition. Because the $1/2^+$ g.s. of ^{17}Na is predominantly $^{16}\mathrm{Ne} \times s$, it should thus be quite weak. However, the $3/2^+$ and $5/2^+$ states should both be strong. The $3/2^+$ state is dominated by the structure $2^+ \times d$, and S is quite large 1.65. The $5/2^+$ state has a large $S = 0.82$ for g.s. $\times\, d$. A rough estimate of the relative $3/2^+$ or $5/2^+$ population to be expected in the experiment of Brown *et al.* [9] involves the product $\sigma_{-1n}S$, where the first factor is the neutron removal cross section from ^{17}Ne to the g.s. or 2^+ state of ^{16}Ne, and the second factor is the spectroscopic factor for $^{16}\mathrm{Ne} + p \rightarrow {}^{17}\mathrm{Na}$. Thus, we expect

$$\sigma(3/2^+)/\sigma(5/2^+) \sim [\sigma_{-1n}(2^+)S(2^+ \times d \rightarrow 3/2^+)]/ [\sigma_{-1n}(\mathrm{g.s.})S(\mathrm{g.s.} \times d \rightarrow 5/2^+)].$$

In the spectra in Ref. [11], the acceptance-corrected $2^+/$g.s. ratio in ^{16}Ne is 0.36(2) [12]. The ratio in another investigation [13] is about 0.31 [14]. With an S ratio of about 2, I would thus expect the ratio in Ref. [9] to be about 0.7, i.e., about 41% of the total yield to correspond to the $3/2^+$ state, and the remainder to correspond to the $5/2^+$ state. As noted above, the $1/2^+$ state should be very weak and quite broad.

To convert the ^{17}Na resonance energy [9] of $E_{3p} = 4.85(6)$ MeV to E_p, we need E_{2p} for $^{16}\mathrm{Ne} \rightarrow {}^{14}\mathrm{O} + 2p$. The latest mass evaluation [15] has $E_{2p} = 1.401(20)$ MeV, whereas Ref. [13] reports $E_{2p} = 1.466(20)$ MeV. The difference in the two is about the same as the uncertainty in the ^{17}Na peak energy. In what follows, I have used the simple average

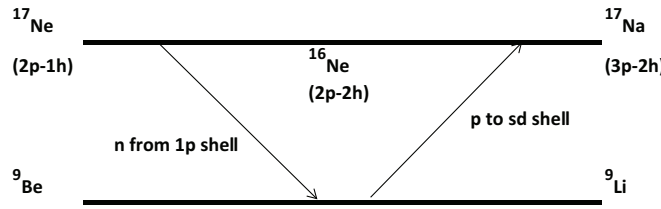

FIG. 1. Simple picture of the reaction mechanism for producing ^{17}Na from ^{17}Ne.

of the two. Thus, the peak energy is $E_p = 3.42(6)$ MeV for $^{17}\mathrm{Na} \rightarrow {}^{16}\mathrm{Ne} + p$. It is gratifying that this energy is in the range between my predictions of 3.22 and 3.47 for the $3/2^+$ and $5/2^+$ states, respectively. [Recall that the $1/2^+$ state, predicted at 3.02 MeV, is expected to be weak and quite broad.] Combining the estimated yield above and my predicted energies, the cross-section weighted peak energy would be 3.37 MeV. The agreement with the experimental peak energy is much better than could reasonably be expected. Brown *et al.* [9] reported that their peak was too broad to correspond to a single narrow state. It would be interesting to compare my predicted energies and widths with the properties of that peak.

The apparent very weak peak near 3 MeV is probably not the g.s. because the splitting from the other states is too large. Also, it seems too narrow to be the g.s. Several additional states are expected at higher excitation energies, but those resonances would be broad and overlapping. That may explain the absence of structure at higher energies in the experimental spectrum. In addition, the resolution width increases with energy, and the efficiency decreases.

IV. SUMMARY

I have used a potential model and results from an earlier shell-model calculation to compute expected widths for decays of the first three states of ^{17}Na. A recent experiment has observed a large peak near the region of the predicted $3/2^+$ and $5/2^+$ states. A simple reaction model suggests relative yields to be expected for the first three levels. Comparison of the present widths with experimental data could provide an additional test of the current model.

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