High-j neutron excitations outside ¹³⁶Xe

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The $\nu 0h_{9/2}$ and $\nu 0i_{13/2}$ strength at ¹³⁷Xe, a single neutron outside the N = 82 shell closure, has been determined using the ¹³⁶Xe(α , ³He)¹³⁷Xe reaction carried out at 100 MeV. We confirm the recent observation of the second 13/2⁺ state and reassess previous data on the 9/2⁻ states, obtaining spectroscopic factors. These new data provide additional constraints on predictions of the same single-neutron excitations at ¹³³Sn.

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I. INTRODUCTION

In this work we report on the $j^{\pi} = 9/2^{-}$ and $j^{\pi} = 13/2^{+}$ excitations outside of ¹³⁶Xe, and discuss them in context of the evolution of these single-neutron excitations across the N = 83 isotonic chain. Together with previous data [1], this provides a consistent description of these orbitals from ¹³⁷Xe (Z = 54) to ¹⁴⁵Sm (Z = 62).

The character of single-neutron excitations outside of N = 82 has been explored with nucleon transfer reactions, both in terms of the energy centroid of their strength and the fragmentation of this strength among the actual states of the nucleus.

Booth, Wilson, and Ipson [2] observed two $\ell = 6$, $13/2^+$ states populated by the (d, p) reaction on ¹³⁸Ba, ¹⁴⁰Ce, ¹⁴²Nd, and ¹⁴⁴Sm. These measurements, among others, also identified two $\ell = 5$, $9/2^-$ states outside each of the stable even N = 82isotones. Heyde *et al.* [3] and others [4–6] explored this fragmentation in terms of coupling to core vibrational states. The spectroscopic overlaps of these same high-*j* excitations were reassessed using the $(\alpha, {}^{3}\text{He})$ reaction at 51 MeV [1]. The $(\alpha, {}^{3}\text{He})$ reaction is better matched for higher angular momentum transfer and yields more reliable spectroscopic factors [7,8]. These studies of the location of the $\nu 0h_{9/2}$ and $\nu 0i_{13/2}$ centroids were prompted by new calculations exploring the evolution of single-particle excitations due to the tensor interaction [9].

Extending the systematic study of the N = 83 isotones with transfer reactions to ¹³⁷Xe has proved challenging due to xenon being a gas at room temperature. Early measurements using gas targets [10,11] did not have the necessary beam energy, nor the most suitable choice of reaction, to successfully probe the high-*j* 9/2⁻ and 13/2⁺ states. The measurement [12] of the (d, p) reaction on ¹³⁶Xe in inverse kinematics, the first of its kind, focused on the low-*j* states and again, had insufficient incident energy to probe the high-*j* states quantitatively.

The work of Allmond *et al.* [13] explored sub-barrier heavy-ion transfer reactions on both 136 Xe and 134 Te. Using

the particle- γ coincidence technique, they observed the $13/2_1^+$ state in ¹³⁷Xe for the first time. This guided the analysis of a subsequent measurement of the ¹³⁶Xe(d, p)¹³⁷Xe reaction carried out in inverse kinematics at 10 MeV/u [14]. Spectroscopic factors were determined for the two 9/2⁻ states that share the $\nu 0h_{9/2}$ strength and also the lowest $13/2^+$ state. However, the excitation energy and strength of the higher-lying $13/2^+$ state was not observed.

Most recently, Reviol *et al.* [15] observed the $13/2_2^+$ state in ¹³⁷Xe for the first time. This was done via neutron transfer induced by ⁹Be and ¹²C at energies below the Coulomb barrier using the particle- γ coincidence technique. Using these data in conjunction with shell-model calculations, they proposed a notably lower value of the $\nu 0i_{13/2}$ single-neutron energy at ¹³³Sn of 2366 keV. Contrary to previous predictions, this value lies *below* the neutron separation energy. Their estimate was largely based on the calculations reproducing the energies of the two, now observed, $13/2^+$ states in ¹³⁷Xe and other structurally relevant states in ¹³⁴Sb and ¹³⁵Sb.

A quantitative determination of the spectroscopic factors for the neutron $9/2^-$ and $13/2^+$ excitations in ¹³⁷Xe is still lacking. In the present paper, we report on a study of the ¹³⁶Xe(α ,³He)¹³⁷Xe reaction at a beam energy of 100 MeV to probe the $\ell = 5$, $9/2^-$ and $\ell = 6$, $13/2^+$ single-neutron excitations. An additional measurement of the ¹⁴⁴Sm(α , ³He)¹⁴⁵Sm reaction was included in this study to provide a consistency check between this work and that done at a lower incident α -particle energy of 51 MeV [1]. The present data allow for better predictions of the location of the $\nu 0h_{9/2}$ and $\nu 0i_{13/2}$ strength at ¹³⁵Te and ¹³³Sn, and a measure of the evolution of these single-particle excitations and the influence of the tensor interaction on the neutron single-particle states as the proton orbits are filling.

II. THE EXPERIMENT

We carried out this measurement, experiment E453, at the Research Center for Nuclear Physics at Osaka University, Japan. An α -particle beam of 100 MeV was delivered, via the so-called WS course [16,17] to the scattering chamber of

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the Grand Raiden (GR) spectrometer [18]. The dispersionmatching capabilities of that beam line were not utilized during this experiment. The outgoing ions from the reaction were momentum analyzed by the GR spectrometer and their position and identity determined with vertical drift chambers and scintillators at the focal plane. The (α , ³He) reaction was carried out on three targets. The first, a 93.8%-enriched ¹⁴⁴Sm target, prepared from its oxide, with a nominal thickness of 525 μ g/cm² mounted on a carbon backing of ~40 μ g/cm² thickness. As mentioned above, this target was used to provide a comparison with a previous measurement of the ¹⁴⁴Sm(α , ³He)¹⁴⁵Sm reaction carried out at 51 MeV [1]. The second was the ¹³⁶Xe target, enriched to greater than

99.9%, prepared in a manner identical to that described in Ref. [19], using the RCNP gas-cell target system [20]. The gas cell had PEN windows [21], a compound of only carbon, oxygen, and hydrogen isotopes. Such a choice, over other common plastics that often contain nitrogen or chlorine isotopes, was essential for this measurement in terms of leaving the measured spectra relatively free of contaminant peaks. Such windows have been used previously in charge-exchange and transfer-reactions studies on Xe isotopes [19,22]. The windows of the gas cell were 4 μ m thick. Several cells were prepared in case of failure. Typical α -particle beam currents were around 40 enA and the beam had an approximate diameter of ≤ 2 mm. Over the course of the measurement, two gas cells were used. The first cell survived approximately $9 \times 10^{15} \alpha$ particles at the nominal current, and the second received about 5×10^{15} (to the end of the experiment). The pressure and temperature of the gas volume were recorded throughout the measurement. Both remained within a few percent of their initial values, being around 16-17 kPa and 298 K, with and without beam. The target had an effective 136 Xe-gas thickness of ~850(120) $\mu g/cm^{\bar{2}}$, which was derived independently from both elastically scattered α -particle yields, described below, and from an empirical formula given in Ref. [20]. The two results were consistent at the 30% level, which in turn reflects the uncertainties on the measured absolute cross sections.

Finally, a natural carbon target, of thickness $\sim 10 \text{ mg/cm}^2$, was used at each of the angles to observe the contaminant peaks, of which, those from reactions on carbon were dominant. Residual oxygen in the carbon foils gave some indication of the location of the oxygen contaminant peaks. For the measurements on the carbon targets, the optical elements of the GR spectrometer remained the same as those for the respective Sm and Xe target.

The reaction was carried out at angles of $\theta_{lab} = 4^{\circ}$, 6° , 9° , 12° , 15° , 18° , $and 21^{\circ}$ on the Xe and C targets. Only angles of 6° and 9° were used for the ¹⁴⁴Sm target. The aperture of the GR spectrometer was 1.36 msr, subtending approximately 1.5° horizontally in the laboratory frame. Faraday cups were used to measure the beam current. For all spectrometer angles greater than $\theta_{lab} = 4^{\circ}$, a Faraday cup in the scattering chamber (SC cup) was used. At $\theta_{lab} = 4^{\circ}$ the SC cup obscured the GR spectrometer aperture. Hence at this angle, a cup placed just after the first quadrupole element of the spectrometer (Q1 cup) was used. This introduced a small systematic uncertainty which was estimated to be less than a few percent.

While absolute cross sections are not required to determine the centroids of single-particle strength, they can be of value in the analysis process by providing comparisons between different targets. Typically, absolute cross sections can be acquired by calibrating the target thickness and spectrometer aperture to well-known reaction cross sections, such as elastic scattering in the Rutherford regime. Due to the thick gas-cell target, such measurements were not possible here.

Absolute cross sections were acquired by normalizing the yield from (α, α) elastic scattering at an angle of $\theta_{lab} = 6^{\circ}$ ($\theta_{lab} = 9^{\circ}$ for the ¹⁴⁴Sm target) to cross sections from opticalmodel calculations. At such forward angles the cross section is very sensitive to the scattering angle and is estimated to be 62(12)% [33(14)% for ¹⁴⁴Sm] of the Rutherford scattering cross section, where the uncertainties take into account the variations between different parameters, which are relatively small, and uncertainties relating to those of the angle and aperture, which dominate.

The elastic scattering on ¹³⁶Xe was measured at all of the angles for which the (α , ³He) reaction was measured, with the exception of $\theta_{lab} = 4^{\circ}$. The general features of the scattering

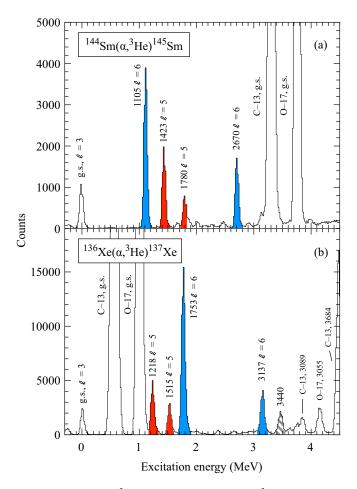


FIG. 1. Outgoing ³He spectra following the $(\alpha, {}^{3}\text{He})$ reaction on isotopes of ${}^{144}\text{Sm}$ (a) and ${}^{136}\text{Xe}$ (b) from $E_{\alpha} = 100$ MeV and $\theta_{\text{lab}} = 6^{\circ}$. The 9/2⁻ ($\ell = 5$) states are shaded red and 13/2⁺ states ($\ell = 6$) in blue. The 3440-keV state (hatched) is tentatively assigned as a third 13/2⁺ state.

data were well reproduced by the optical-model calculations, although the uncertainties in the absolute cross sections are relatively large, being around 30% for Xe and around 50% for Sm.

III. ANALYSIS AND RESULTS

The spectra from the (α ,³He) reaction on ¹⁴⁴Sm and ¹³⁶Xe are shown in Fig. 1 for a spectrometer angle $\theta_{lab} = 6^{\circ}$. The spectra are dominated by states populated via $\ell = 5$ and $\ell = 6$ transfer. Aside from these high-*j* states and the ground state, which contains more-or-less all of the $\nu 1 f_{7/2}$ strength [23], essentially no other states are populated with significant yield. The *Q*-value resolution was 65–70 keV full width at half maximum. Contaminants, resulting from reactions on carbon and oxygen, lie above the region of interest in ¹⁴⁵Sm. For Xe, the contaminant peaks fall just below the high-*j* states of interest at $\theta_{lab} = 4^{\circ}$ and 6° , and slowly move across this region as the angle becomes larger. Beyond 4° and 6° , only at 18° were all high-*j* states unobscured.

The Sm spectrum is, as expected, similar to that of Fig. 1 of Ref. [1], which shows the same reaction but carried out at a lower energy of $E_{\alpha} = 51$ MeV. As mentioned in the Introduction, this pattern is observed in other N = 83 isotones. The corresponding states are clearly seen in ¹³⁷Xe. The 3137-keV $13/2_2^+$ state, observed recently for the first time in the study of Reviol *et al.* [15], is clearly seen.

The angular distributions for the states populated in ¹³⁷Xe are shown in Fig. 2. Although the choice of reaction and incident beam energy lends itself to exquisite selectivity for populating high- ℓ states, the angular distributions are relatively featureless, particularly for any differences in the $\ell = 5$ and $\ell = 6$ shapes. As such, making robust ℓ -value assignments is

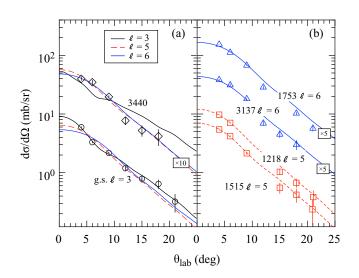


FIG. 2. Angular distributions for states populated in the 136 Xe(α , 3 He) 137 Xe reaction at 100 MeV. Panel (a) shows the ground-state $\ell = 3$ transition (circles) and the 3440-keV state with $\ell = 5$ or $\ell = 6$ transfer (×10, diamonds) and (b) the known 9/2⁻ (squares) and 13/2⁺ (×5, triangles) states. The curves are DWBA calculations fitted to the data for the respective ℓ transfers.

not possible. We rely on the previous spin and parity assignments for the 1218-keV and 1515-keV, 9/2⁻ states. Note, the latter was previously reported to lie at an excitation energy of 1590(20) keV [14]. This was observed as part of an unresolved doublet with a known $\ell = 3$ state at 1534 keV via the (d, p)reaction in inverse kinematics, where the energy resolution, statistics, and momentum matching for $\ell = 5$ were poor. Here, the state is cleanly populated and the excitation energy unambiguous. Due to the selectivity of heavy-ion transfer, the $\ell = 5, 9/2_1^-$ strength was only weakly populated in previous works [13,15], and the $9/2_2^-$ excitation was not observed. The two $13/2^+$ states, at 1753 and 3137 keV, have also been previously assigned. The experimental angular distributions for these ℓ transfers agree well with the shapes calculated with the finite-range distorted wave Born approximation (DWBA) using the code PTOLEMY [24]. The ground-state $\ell = 3$ is also well reproduced by the DWBA. While similar to the $\ell = 5$ and $\ell = 6$ shapes, the χ^2/dof of the fits for these shapes strongly favor $\ell = 3$, being 1.05, 6.4, and 11.2 for $\ell = 3$, 5, and 6, respectively.

An additional peak was observed at 3440 keV, which was populated with a cross section approximately seven times less than that of the lowest-lying $13/2^+$ state. The angular distribution for the 3440-keV state is consistent with $\ell = 5$ or $\ell = 6$ transfer, but not with $\ell = 1$ (which is poorly momentum matched in this reaction and would lead to unphysical values for the overlaps) or $\ell = 3$. Previous (d, p) studies, such as those of Ref. [14], had neither sufficient statistics nor resolution to resolve states above approximately 3 MeV in excitation energy. As we will discuss, we tentatively assign the 3440-keV state as $j^{\pi} = 13/2^+$. Two additional states with cross sections a factor of 10 times less than the dominant $\ell = 6$ state were observed above the 3440-keV state. These may also have high spin. All cross sections are included in the Supplemental Material [25].

Different parametrizations of optical-model potentials were explored for the DWBA calculations. The availability of global optical-model parametrizations for α particles in this energy regime is quite limited compared to lighter ions. We used those of Refs. [26,27] along with those derived from scattering experiments optimized for ~100 MeV energies and on nuclei close in mass to the ones studied here. The most favorable being that of Ref. [28] who derived parameters from the scattering of ~80-MeV α particles on ²⁰⁸Pb. Other works demonstrated that these same parameters reproduce angular distributions for the (α , ³He) reaction at 80 MeV [29] and 100 MeV [30] on ¹⁴⁴Sm.

For the ³He parametrizations, numerous global opticalmodel potentials have been derived from data at these masses and incident energies. We explored those from Refs. [31–33]. Overall, nine different combinations of parameters were used. For the bound-state wave function we used parametrizations derived from the Green's function Monte Carlo calculations of Brida *et al.* [34]. The target bound-state wave function was generated in the conventional way, using a Woods-Saxon potential with the depth varied to reproduce the binding energy of the transferred nucleon. The radial and diffuseness parameters of the potential were $r_0 = 1.28$ fm and a = 0.65 fm and the spin-orbit potential was taken as $V_{so} = 6$ MeV, $r_{so} =$ 1.10 fm, and $a_{so} = 0.65$ fm.

TABLE I. Excitation energies (to the nearest keV unless otherwise stated), differential cross sections (σ), and normalized spectroscopic factors (*S*) for states populated via $\ell = 5$ and $\ell = 6$ transfer in the (α , ³He) reaction on ¹³⁶Xe at $\theta_{lab} = 6^{\circ}$.

E (keV)	l	j^{π}	σ_{6°	S
1218	5	$9/2_1^-$	6.45(4)	0.51(4)
1515(5) ^a	5	$9/2^{-}_{2}$	3.67(3)	0.29(3)
1753	6	$13/2^+_1$	21.0(1)	0.78(7)
3137(1)	6	$13/2^{+}_{2}$	5.62(4)	0.22(2)
3440(15)	(6)	$(13/2_3^{+})$	2.97(3)	0.12(2)

^aThis energy of this state was previously estimated to be at 1590(20) keV in Ref. [14].

The spectroscopic factors quoted in Table I were extracted from the $\theta_{lab} = 6^{\circ}$ cross sections. The relative spectroscopic factors were derived from a common normalization N_j , such that $N_j = \sum (2j + 1)C^2S_j/(2j + 1)$ [7]. Across the nine different combinations of optical-model parametrizations, the $\theta_{lab} = 4^{\circ}$ and $\theta_{lab} = 6^{\circ}$ data, and the two different *j* values, a normalization for the ¹³⁷Xe states of 0.52(3) was found, where the quoted uncertainty is the rms spread. This spread, though small, also includes different summed strengths based on whether the previously unobserved 3440-keV state is assigned $j^{\pi} = 9/2^{-}$ or $j^{\pi} = 13/2^{+}$. The spectroscopic factors in Table I include the $13/2_3^+$ state in the normalization.

The normalized spectroscopic factors for states populated in ¹⁴⁵Sm agree well with those reported in Ref. [1]. The resulting centroids for ¹⁴⁵Sm derived from the current data are $\epsilon_{h_{9/2}} = 1531(10)$ keV and $\epsilon_{i_{13/2}} = 1589(10)$ keV (cf. Table II). These values are within 10 keV of those calculated in Ref. [1]. In the remaining discussion we adopt the spectroscopic factors, centroids, and mixing matrix elements given in Ref. [1] for ¹⁴⁵Sm as a matter of convenience, as they also include a consistent analysis of the same properties for ¹⁴³Nd, ¹⁴¹Ce, and ¹³⁹Ba.

The relative spectroscopic factors from the 136 Xe(d, p) study carried out at 10 MeV/u in inverse kinematics [14] agree within the quoted uncertainties with those of the present study. This bodes well for future (d, p) studies at comparable energies on isotopes of Te and Sn, where the α -induced reactions are likely to prove challenging.

As mentioned above, the state at 3440 keV was unexpected and its relatively large cross section and angular distribution

TABLE II. Centroids, in MeV, for the $0h_{9/2}$ and $0i_{13/2}$ excitations from 54 < Z < 62. Those for 56 < Z < 62 are from Ref. [1]. The uncertainties are less than ± 0.02 MeV unless stated otherwise.

Isotope	$\epsilon_{h_{9/2}}$	$\epsilon_{i_{13/2}}$
¹³⁷ Xe	1.327(10)	2.206(20) ^a
¹³⁹ Ba	1.407(10)	1.879(24)
¹⁴¹ Ce	1.447(10)	1.702 (52)
¹⁴³ Nd	1.493(5)	1.627(31)
¹⁴⁵ Sm	1.526(10)	1.594(29)

^aIncludes the tentative $13/2^+_3$ state.

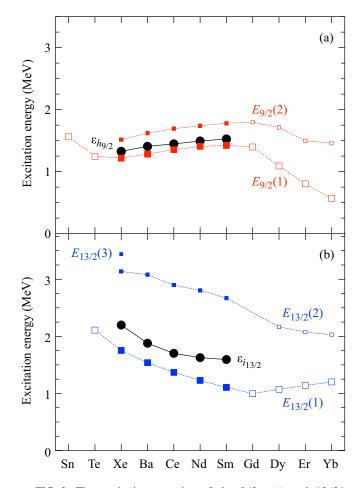


FIG. 3. The excitation energies of the $9/2^-$ (a) and $13/2^+$ (b) states at N = 83. The solid symbols are states for which spectroscopic factors from the (α ,³He) reaction are available. A third tentatively assigned $13/2^+$ state in ¹³⁷Xe is also shown. The $0h_{9/2}$ and $0i_{13/2}$ centroids, $\epsilon_{h_{9/2}}$ and $\epsilon_{i_{13/2}}$, are also shown as black dots.

suggest a high ℓ value. There is little to discriminate between $\ell = 5$ and $\ell = 6$ angular distributions at this incident beam energy (as seen in Fig. 2). The systematics of the $9/2^{-}$ and $13/2^+$ excitations are shown in Fig. 3. Were the 3440-keV state to have a $j^{\pi} = 9/2^{-}$ assignment, it would have a spectroscopic factor ~ 0.3 and the resulting centroid would lie at approximately 1885 keV, some 560 keV higher in excitation energy. This would be at odds with the systematics shown in Fig. 3. An assignment of $j^{\pi} = 13/2^+$ is perhaps most likely, with a spectroscopic factor of 0.12(2) as quoted in Table I. With the 3440 keV state included in the normalization as $j^{\pi} = 13/2^+$, and subsequent calculations of the centroid, it only shifts the energy of the $0i_{13/2}$ orbital by ~140 keV. We searched for evidence of the 3440-keV state in other works, the most promising one being that of Reviol et al. [15]. This state could possibly decay via a 1687-keV y-ray transition to the $13/2_1^+$ state or to the $13/2_2^+$ state via a 303-keV γ ray. However, the statistics in these data are limited and there appears to be no clear evidence for such decays [15,35].

The uncertainties on the absolute cross sections are of the order of 30% as discussed in Sec. II. This implies an uncertainty on the absolute spectroscopic factors of about the same order, around 30–40%, which is dominated by the uncertainties in the cross sections. However, the relative spectroscopic factors have comparably smaller uncertainties, around 5%, as seen in the rms spread of the normalizations using different optical-model parametrizations. The uncertainties shown in Table I for the cross sections are those of the statistical uncertainties and those related to the fitting procedure. Due to the high statistics of the recorded yields, these are of the order of 1% for the relative cross sections. The normalized spectroscopic factors have an uncertainty of approximately 5–10%, which reflects the variation between the different optical-model parametrizations.

IV. DISCUSSION

A. Single-particle energies and the tensor force

The $v0h_{9/2}$ and $v0i_{13/2}$ single-particle energy centroids at ¹³⁷Xe are calculated to be $\epsilon_{h_{9/2}} = 1327(10)$ keV and $\epsilon_{i_{13/2}} =$ 2206(20) keV. The latter includes the probable "third $13/2_3^+$ state" at 3440 keV. We include this in the remainder of the discussion. The uncertainties, as presented in the figures and the tables, assume this state is correctly assigned, i.e., they do not include the \sim 140 keV difference between the centroid were the assignment of the spin-parity for the 3440 keV state be in error. The centroids, $\epsilon_{h_{9/2}}$ and $\epsilon_{i_{13/2}}$, are reconstructed from the fragments as their centers of gravity, where $\epsilon_i =$ $\sum E_i S_i / \sum S_i$. They are shown in Fig. 3 along with other data. The binding energies of these centroids are shown in Fig 4. The centroids move closer together in binding energy by about 0.8 MeV going from Xe to Sm, with the binding of the $\nu 0i_{13/2}$ increasing more rapidly with increasing proton number than that of the $\nu 0h_{9/2}$ orbital.

The $\pi 0g_{7/2}$ and $\pi 1d_{5/2}$ orbitals are filling above Z = 50. The tensor interaction is attractive when the proton and neutron orbits have a different spin orientation with respect to their orbital angular momentum $(\ell + 1/2 \text{ and } \ell - 1/2)$, or vice

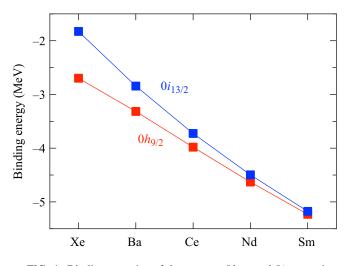


FIG. 4. Binding energies of the neutron $0h_{9/2}$ and $0i_{13/2}$ excitations, corresponding to the centroids in Fig. 3, as a function of proton number.

versa) and repulsive when the spin orientation is the same. Its magnitude increases with increasing ℓ and is greatest between orbitals with zero nodes. In the present case, the $\pi 0g_{7/2}$, $\nu 0h_{9/2}$, and $\nu 0i_{13/2}$ states are nodeless and have significant radial overlap.

The evolution of a given single-particle energy with respect to another can be expressed as $\epsilon_{j\nu} = \epsilon_{\text{initial},j\nu} + \sum V_M(j_\nu j'_\pi)N_{j\pi}$, where $\epsilon_{\text{initial},j\nu}$ is a starting value. $V_M(j_\nu j'_\pi)$ is the magnitude of the monopole interaction from the tensor force, which has the effect of decreasing the separation between the $0i_{13/2}$ and $0h_{9/2}$ orbitals by 0.18 MeV per each

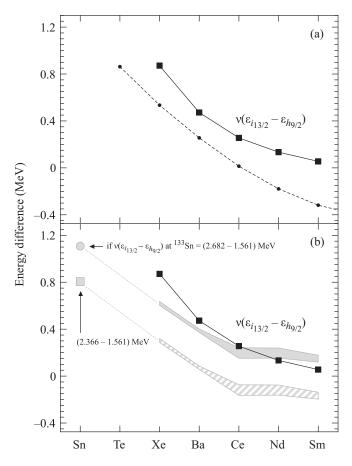


FIG. 5. (a) The differences in the centroids of the $0i_{13/2}$ and $0h_{9/2}$ single-neutron strengths (where data for Ba, Ce, Nd, and Sm are from Ref. [1]) are shown as the black squares connected by a solid line. The dashed line is the energy difference between the $13/2_1^+$ and $9/2_1^-$ states for the N = 83 isotones between Te and Sm [37]. In Panel (b) the black squares connected by the solid line are the experimental differences as in (a). The shaded and hatched areas represent calculations based on the tensor interaction [9], anchored to estimates of the $0i_{13/2}$ single-particle energy in ¹³³Sn, based on two different predictions. The shaded region is anchored to the gray circle at Z = 50, which is based on the average value of $\epsilon_{i_{13/2}}$ from Urban et al. [38] and Korgul et al. [39], 2.682 MeV. The location of the $\epsilon_{h_{9/2}}$ is taken as 1.561 MeV, the location of the observed $9/2_1^-$ state [37]. The hatched area represents a similar calculation but anchored to the gray square at Z = 50 which uses the estimate of $\epsilon_{i_{13/2}} = 2.366$ MeV of Reviol et al. [15]. The width of the shaded region reflects the uncertainties in the proton occupancies [19,36].

proton added to the $\pi 0g_{7/2}$ orbital $(N_{j\pi})$ and increasing the separation by 0.005 MeV per each proton added to the $\pi 1d_{5/2}$ orbital [1]. This effect is shown by the shaded and hatched areas in Fig. 5. The proton occupancies, $N_{j\pi}$, are taken from Wildenthal *et al.* [36] and Entwisle *et al.* [19], who studied all the stable even N = 82 isotones via proton transfer. The widths of the shaded and hatched regions reflect the uncertainties in these proton occupancies. The shaded and the hatched regions show the difference in the single-particle energies as described by the tensor interaction and known proton occupancies using two different predictions for the separation of the $0i_{13/2}$ and $0h_{9/2}$ single-particle energies at ¹³³Sn.

For both the shaded and the hatched regions in Fig. 5(b), the $0h_{9/2}$ energy in ¹³³Sn is taken to be the observed $9/2_1^{-1}$ state at 1561 keV [37]. This is likely a fair assumption as the fragmentation is expected to be minimal with only one neutron outside the doubly magic core, and with the 2^+ excitation above 4 MeV in excitation energy, nearly 3 MeV higher than in ¹³⁴Te. Most previous predictions for $\epsilon_{i_{13/2}}$ at ¹³³Sn have placed it above the neutron separation energy of $S_n = 2402(4)$ keV. The $0i_{13/2}$ energy for ¹³³Sn that results in the shaded region in Fig. 5(b) is $\epsilon_{i_{13/2}} = 2682$ keV, the average of those given by Urban *et al.* [38] and by Korgul *et al.* [39]. With these values for the single-particle energies in 133 Sn, the agreement between the calculated differences in the centroids as a function of proton number, as indicted by the shaded area, and the experimental centroids is remarkably good. Using a value of 2366 keV for the $0i_{13/2}$ energy [15] shifts the calculated differences down by \sim 370 keV, as shown by the gray hatched area. From studies of heavy-ion transfer reactions, Allmond et al. [40] observed a 2792-keV γ ray, for which it was tempting to infer this as a candidate for a $13/2^+$ state, being consistent with the result of Urban et al. within their quoted uncertainties. However, no assignment was made based on other considerations.

B. Trends in mixing matrix elements

As noted in previous works (e.g., [1–6], among others), the $0h_{9/2}$ and $0i_{13/2}$ strength is split, apparently mixing with the weak-coupling state of the $\nu 1 f_{7/2}$ coupled to the 2^+_{core} or 3^-_{core} states.

The mixing matrix elements between the single-particle and weak-coupling state can be extracted using a simple twolevel mixing model, which we describe in the Appendix. The mixing matrix elements are shown in Table III and Fig. 6 for

TABLE III. Mixing matrix elements (V) for N = 83, 54 < Z < 62, all in MeV.

		V
	$9/2^-, 2^+_{ m core} \otimes \nu f_{7/2}$	$13/2^+, 3^{\rm core} \otimes \nu f_{7/2}$
¹³⁷ Xe	0.143(5)	0.614(20) ^a
¹³⁹ Ba	0.162(6)	0.639(17)
¹⁴¹ Ce	0.151(5)	0.632(34)
¹⁴³ Nd	0.145(2)	0.686(16)
¹⁴⁵ Sm	0.162(4)	0.725(9)

^aIncludes the tentative $13/2_3^+$ state in the centroid of $\epsilon_{i_{13/2}}$.

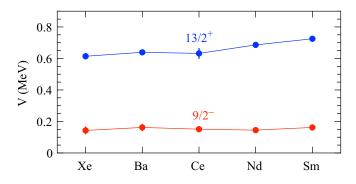


FIG. 6. The mixing matrix elements. The error bars reflect the uncertainties in the derived values.

54 < Z < 62. In this example, the $j^{\pi} = 9/2^{-}$ admixtures are assumed to be the $|0h_{9/2}\rangle$ and $|2^+, 1f_{7/2}\rangle$, and the $j^{\pi} = 13/2^+$ admixtures to be $|0i_{13/2}\rangle$ and $|3^-, 1f_{7/2}\rangle$, as in Ref. [1]. The mixing matrix elements are relatively strong, especially for the $0i_{13/2}$ state. For the $9/2^-$ state, the magnitude of the mixing matrix element is remarkably constant across these isotopes, with an average value of 0.153(9) MeV, where the rms spread across the five isotopes is given in parentheses. For the $13/2^+$ states it is a factor of ~4 larger, with an average value of 0.66(5) MeV.

Another $j^{\pi} = 13/2^+$ state at approximately the same excitation energy can also arise from the $13/2^+$ member of the weak-coupling multiplet corresponding to the core 2^+ state and $\nu 0i_{13/2}$ state, $|2^+, 0i_{13/2}\rangle$, as was discussed in other works,

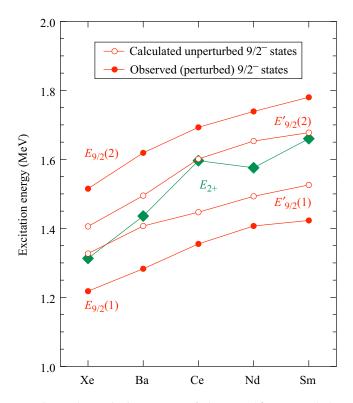


FIG. 7. The excitation energy of the core 2^+ state and the perturbed (*E*) and unperturbed (*E'*)9/2⁻ excitations; $E'_{9/2}(1) \equiv \epsilon_{h_{9/2}}$.

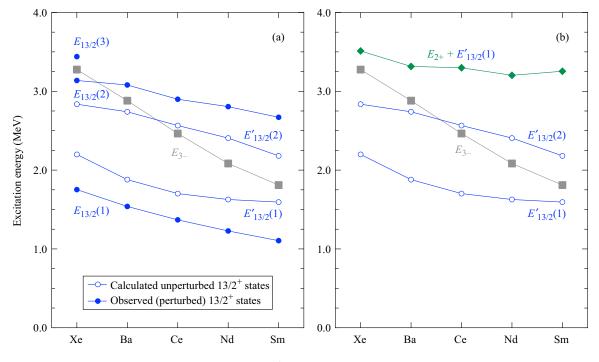


FIG. 8. (a) The measured excitation energy (*E*) of the $13/2^+$ states in N = 83 nuclei from 54 < Z < 62, the calculated unperturbed energies (*E'*), and the core (N = 82) 3⁻ energy. Panel (b) shows information on the core 2⁺ states. Note, $E'_{13/2}(1) \equiv \epsilon_{i_{13/2}}$.

e.g., Heyde *et al.* [3]. This lends further support to the assignment of the 3440-keV state as $13/2_3^+$. We note that Heyde *et al.*'s calculations of the excitation energy and amplitude of the three $13/2^+$ states in 137 Xe are remarkably close to what we observe in this measurement, although the same calculated properties for the heavier isotones are in poorer agreement with the experimental data. A third $13/2^+$ state, most likely arising from $|2^+, 0i_{13/2}\rangle$, was not seen in the heavier N = 83 isotones in the work of Ref. [1]. In that work the limit on observing such a state would be at the level of $\sim 5\%$ of the lowest-lying state. A description of the possible three $13/2^+$ states would require a more complex three-level mixing calculation.

In solving for the mixing matrix elements, one also obtains the original unperturbed values of the "single-particle state" (the "centroid," that was already discussed) and the "weakcoupling state." Figures 7 and 8(a) show these for the $9/2^$ and $13/2^+$ states, respectively.

The unperturbed $9/2^-$ energy tracks the 2^+_{core} very closely, being consistently within 0.2 MeV of it from 54 < Z < 62. The energy of the 2^+_{core} state only changes by a few tenths of an MeV across that range. The relationship between the 3^-_{core} and the unperturbed $13/2^+$ excitations is more startling. Over the same range the 3^-_{core} excitation energy changes by over an MeV. The excitation energy of the unperturbed $13/2^+_2$ state is below the 3^-_{core} by ~0.4 MeV at Xe, and at Sm it is almost 0.4 MeV above it.

Figure 8(b) also shows the 2^+_{core} in relation to the 3^-_{core} and unperturbed $13/2^+$ excitations. As Z decreases from Sm to Xe, the 3^-_{core} state rises to an energy close to the sum of the 2^+_{core} state and the $\nu 0i_{13/2}$ energies, again lending support to the notion that the 3440-keV state has $j^{\pi} = 13/2^+$.

We note that the behavior of the 3^- excitations at N = 82 appears to be unique across the chart of nuclides for semimagic nuclei. For example, at Z = 20, 50, and 82, and at N = 28 and 50, 3^- excitations are relatively constant in energy while crossing the doubly magic nuclei, unlike the 2^+ states. ¹³²Sn is

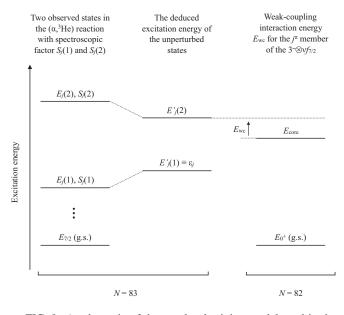


FIG. 9. A schematic of the two-level mixing model used in the analysis in Sec. IV B. This is shown for $13/2^+$ states—the same definitions apply for the $9/2^-$ states. The energies, spectroscopic factors, and weak-coupling interaction energy are defined in the accompanying text.

	$9/2^-, 2^+_{ m core} \otimes u f_{7/2}$						$13/2^+, 3^{ m core} \otimes \nu f_{7/2}$							
^{A}X	$E_{9/2}(1)$	$S_{9/2}(1)$	$E_{9/2}(2)$	$S_{9/2}(2)$	$E_{9/2}^{\prime}(1)$	$E_{9/2}^\prime(2)$	V	$E_{13/2}(1)$	$S_{13/2}(1)$	$E_{13/2}(2)$	$S_{13/2}(2)$	$E_{13/2}^{\prime}(1)$	$E_{13/2}^{\prime}(2)$	V
¹³⁶ Xe	1218	0.43	1515	0.24	1327	1406	143	1753	0.84	3137	0.15	2206	2836	614 ^a
¹³⁸ Ba	1283	0.70	1619	0.41	1407	1495	162	1539	0.60	3080	0.17	1879	2740	639
¹⁴⁰ Ce	1355	0.67	1693	0.25	1447	1601	151	1369	0.79	2899	0.22	1702	2566	632
¹⁴² Nd	1407	0.83	1739	0.29	1493	1653	145	1228	0.65	2805	0.22	1627	2406	686
¹⁴⁴ Sm	1423	0.84	1780	0.34	1526	1677	162	1105	0.66	2670	0.30	1594	2181	725

TABLE IV. Energies to the nearest keV for excitations in the N = 83 isotones between $54 \le Z \le 62$ that are relevant for the determination of the mixing matrix elements. Uncertainties are not given but can be found elsewhere in this work.

^aLike Table III, this value includes the tentative $13/2_3^+$ state in the centroid of $\epsilon_{i_{13/2}}$.

an exception, in that at N = 82 the 3⁻ excitation rises sharply from ~2.6 MeV at ¹³⁰Sn to 4.35 MeV at ¹³²Sn, as one tracks it along Z = 50. It then decreases monotonically from ¹³²Sn to ¹⁴⁶Gd, from 4.35 to 1.58 MeV, as one tracks it along N =82. Over this region, protons are filling the $\pi 0g_{7/2}$ orbital and this trend may perhaps be attributed to the increasing contribution of protons from these states being excited to the $\pi 0h_{11/2}$ orbits. The change in sign occurs approximately where these orbits are half filled. To our knowledge, no systematic shell-model studies of this excitation have been carried out over this region. Perhaps the 3⁻ excitation is not as collective as in other regions?

V. CONCLUSION

The high-j, $9/2^-$ and $13/2^+$ single-neutron excitations outside of ¹³⁶Xe have been studied via the (α , ³He) reaction at an incident energy of 100 MeV. In this work, two prominent states with $j^{\pi} = 9/2^{-}$ and two with $j^{\pi} = 13/2^{+}$ were populated. This is in line with what has been observed in the heavier N = 83 systems and can be interpreted as the result of weak coupling to core vibrational states. An additional weak fragment was observed, with a probable assignment of $j^{\pi} =$ $13/2^+$. The $\nu 0h_{9/2}$ and $\nu 0i_{13/2}$ single-particle energies were determined to lie at 1327(10) and 2206(20) KeV, respectively. They reflect a smooth continuation of the trends seen in the heavier N = 83 isotones and are well described by the action of the tensor interaction based on the assumption of a $\epsilon_{i_{13/2}} \approx 2680$ keV at ¹³³Sn. While the (d, p) reaction is not the ideal probe of such states, at 10 MeV/u it can provide valuable information as evidenced by the results obtained in a recent inverse-kinematics study, which yielded comparable information in terms of accuracy. Such experiments with radioactive ion beams of ¹³⁴Te and ¹³²Sn at energies around 10 MeV/u are likely to be possible soon.

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APPENDIX

A simple two-level mixing model was used to deduce the mixing matrix elements discussed in Sec. IV B. We describe the method for extracting these here, with notation that is consistent with the text and figures above. Figure 9 is a schematic showing the excitation energies of the perturbed (observed) states, E_j , and the unperturbed (deduced) states, E'_j , at N = 83 and the core excitation at N = 82. The relevant information for N = 83 excitation energies, perturbed and unperturbed, and spectroscopic factors are given in Table IV for $54 \le Z \le 62$.

The two wave functions for the observed states can be written as $\Psi_1 = \alpha \psi_1 + \beta \psi_2$ and $\Psi_2 = -\beta \psi_1 + \alpha \psi_2$, where Ψ are the actual wave functions and ψ are the unperturbed wave functions. The amplitudes relate to the spectroscopic factors, where $S_1 = \alpha^2$ and $S_2 = \beta^2$. These are normalized such that $\alpha^2 + \beta^2 = 1$.

Using the notation of Fig. 9, it follows that $E'_{j}(1)$, the unperturbed energy of the single-particle state, is given by

$$E'_{i}(1) = S_{i}(1)E_{i}(1) + S_{i}(2)E_{i}(2),$$
(A1)

and that of the weak-coupling state by

$$E'_{j}(2) = S_{j}(2)E_{j}(1) + S_{j}(1)E_{j}(2),$$
(A2)

where E_j are the observed energies and S_j are the normalized spectroscopic factors. The mixing matrix elements, V, are then calculated by

$$V = \frac{1}{2}\sqrt{\Delta E_j^2 - \Delta E_j'^2},\tag{A3}$$

where $\Delta E^2 = E_j(2) - E_j(1)$ and $\Delta E'^2 = E'_j(2) - E'_j(1)$. The factor of $\frac{1}{2}$ is a matter of convention, sometimes it is expressed without this factor. E_{wc} in Fig. 9 is the weak-coupling interaction energy for the member of the multiplet that has the angular momentum j.

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