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Phenomenology of muon-induced neutron yield

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The cosmogenic neutron yield Y_n characterizes the ability of matter to produce neutrons under the effect of cosmic ray muons with spectrum and average energy corresponding to an observation depth. The yield is the basic characteristic of cosmogenic neutrons. The neutron production rate and neutron flux both are derivatives of the yield. The constancy of the exponents α and β in the known dependencies of the yield on energy $Y_n \propto E_\mu^\alpha$ and the atomic weight $Y_n \propto A^\beta$ allows one to combine these dependencies in a single formula and to connect the yield with muon energy loss in matter. As a result, the phenomenological formulas for the yields of muon-induced charged pions and neutrons can be obtained. These expressions both are associated with nuclear loss of the ultrarelativistic muons, which provides the main contribution to the total neutron yield. The total yield can be described by a universal formula, which is the best fit of the experimental data.

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I. INTRODUCTION

Cosmogenic neutrons are of interest as a source of background in the underground low-background experiments. Neutrons generated in matter by cosmic-ray muons are considered cosmogenic. Neutrons generated by astrophysical, atmospheric, and solar neutrinos are also cosmogenic. The term "cosmogenic" has been associated only with neutrons from muons by virtue of their dominant role in the flux of neutrons generated at depths of up to 10 km w.e. (where "w.e." denotes water equivalent) underground by particles coming from space.

Cosmogenic neutrons (cg neutrons) can be characterized by the following values: a neutron yield Y_n [$n/\mu/(\mathrm{g/cm^2})$], a production rate $R_n(H) = I_\mu(H)Y(\overline{E}_\mu)$ ($n\mathrm{g^{-1}\,s^{-1}}$), and a neutron flux $\Phi_n(H) = R_n(H)l_n\rho = I_\mu(H)Y(\overline{E}_\mu)l_n\rho$ ($n\mathrm{cm^{-2}\,s^{-1}}$). In these expressions, $\overline{E}_\mu$ is the mean muon energy at a depth H, $I_\mu(H)$ is the muon global intensity, and $l_n\rho$ ($\mathrm{g/cm^2}$) is an attenuation length for the isotropic neutron flux. The indicated characteristics allow one to estimate the background effects caused by muon-induced neutrons in rock and setup materials using Monte Carlo simulations that take into account the configuration and dimensions of the target.

As follows from the above expressions, the main characteristic is the neutron yield Y_n. The production rate R_n and the flux Φ_n are the derivatives from the yield Y_n. The formula for the neutron yield in the material A at muon energy E_μ is

$$Y_n(A,E_\mu) = \frac{N_0}{A}\langle\sigma_{\mu A}\nu_n\rangle, \quad n/\mu/(\mathrm{g/cm^2}), \tag{1}$$

where N_0 is Avogadro's number, and $\langle\sigma_{\mu A}\nu_n\rangle$ is a mean value of the product of a μA interaction cross section and the neutron multiplicity ν_n. The multiplicity ν_n includes all the neutrons (including multiplication neutrons) that arise mainly in hadron and electromagnetic showers produced via μA interactions and developed entirely in matter. So, the product $\langle\sigma_{\mu A}\nu_n\rangle$ is a neutron production function. The cg-neutron energy spectrum that corresponds to the yield (1) is not considered here.

The yield Y_n in a line with other physical properties of matter presents an ability of matter to produce neutrons under the effect of muons. In Refs. [1,2] a universal formula (UF) was obtained for the muon-induced neutron yield: $Y_n^{\mathrm{UF}} = b^{\mathrm{UF}}E_\mu^\alpha A^\beta$, $b^{\mathrm{UF}} = 4.4\times10^{-7}\,\mathrm{cm^2/g}$, $\alpha = 0.78$, and $\beta = 0.95$. The formula is valid in the energy range from ~40 GeV up to the maximum mean muon energy underground ~400 GeV. The lower limit of the range corresponds to a depth of about 100 m w.e. This UF is the best approximation of the set of available experimental data (Table I). The UF was obtained under the assumption that the dependence of the yield on E_μ and A can be expressed as E_μ^α and A^β, where α and β are constant. The coefficient $b^{\mathrm{UF}} = 4.4\times10^{-7}\,\mathrm{cm^2/g}$ is close to the relative muon nuclear energy loss $b_n = 4.0\times10^{-7}\,\mathrm{cm^2/g}$. The UF effectiveness is shown in Fig. 1. The set of points in the coordinates $Y_n^{\mathrm{UF}} - {}^{\mathrm{ex}}Y_n$ is aligned at angle $\alpha = 45^\circ$ to the x axis (${}^{\mathrm{ex}}Y_n$ are experimental data from Table I). Obviously, if $\alpha = 45^\circ$ then $Y_n^{\mathrm{UF}} = {}^{\mathrm{ex}}Y_n$. Thus, the UF expression relates the yield to muon energy losses and nuclear properties of matter.

The yield Y_n at depths $H > 100$ m w.e. ($\overline{E}_\mu > 40$ GeV) is a sum of the components Y_n^h, Y_n^{em}, and Y_n^v. The components Y_n^h and Y_n^{em} correspond to the neutron production in hadron (h) and electromagnetic (em) showers. Components Y_n^v and Y_n^{em} present mainly photoneutrons that are produced in giant dipole resonance (GDR) by virtual photons (Y_n^v) or real photons of em showers (Y_n^{em}). The contribution of neutron production via $\mu^- A$ captures at depths greater than 100 m w.e. is negligible.

The ratio of neutron production channels has being considered by many authors [19,20]. It was shown that at energies $40 \leqslant \overline{E}_\mu \leqslant 400$ GeV the yield components for all A's are connected by the inequalities:

$$Y_n^{\mathrm{em}} \gg Y_n^v, \quad Y_n^h > Y_n^{\mathrm{em}} + Y_n^v. \tag{2}$$

II. NEUTRON PRODUCTION IN *h* SHOWERS

As follows from the inequalities (2), neutrons from h showers dominate in the total neutron yield. In the h shower, neutrons are produced mainly in deep-inelastic πA

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TABLE I. Measured neutron yield.

Experiment, Ref.	$\overline{E}_\mu$ (GeV)	H (m w.e.)	Y_{LS}	$Y_n \times 10^{-4}$ $[n/\mu/(\mathrm{g/cm^2})]$ Y_{Fe}	Y_{Cd}	Y_{Pb}
[3]	10.0 ± 6.3	20	–	0.98 ± 0.01	–	2.43 ± 0.13
[4]	10.0 ± 6.3	60	–	–	–	4.8 ± 0.6
[5]	11.0 ± 6.6	40	–	1.32 ± 0.30	–	4.03 ± 0.36
[6]	13.0 ± 7.2	20	0.20 ± 0.07	–	–	–
[7]	16.5 ± 8.1	32	0.36 ± 0.03	–	–	–
ASD, [8]	16.7 ± 8.2	25	0.47 ± 0.05	–	–	–
[5]	17.8 ± 8.4	80	–	1.69 ± 0.30	3.3 ± 0.4	5.66 ± 0.36
[4]	20 ± 9	110	–	–	–	6.8 ± 0.9
[9]	40 ± 12.6	150	–	3.31 ± 0.96	10.3 ± 4.3	11.56 ± 1.1
ASD, [8]	86 ± 18	316	1.21 ± 0.12	–	–	–
[10]	89.8 ± 2.9	610	1.19 ± 0.21	–	–	–
[11]	110 ± 21	800	–	–	–	17.5 ± 3.0
ASD, [12]	125 ± 22	570	2.04 ± 0.24	–	–	–
[13]	260 ± 8	2700	2.8 ± 0.3	–	–	–
ZEPLIN-III, [14]	260 ± 32	2850	–	–	–	58 ± 2
LSM, [15]	267^{+8}_{-11}	4850	–	–	–	27.5^{+10}_{-7}
[16]	280 ± 33	4300	–	–	–	116 ± 44
LVD, [2]	280 ± 18	3100	3.3 ± 0.5	16.4 ± 2.3	–	–
LVD, [17]	280 ± 18	3100	3.6 ± 0.3	14.3 ± 1.3	–	–
Borexino, [18]	283 ± 19	3800	3.10 ± 0.11	–	–	–
LSD, [2]	385 ± 39	5200	4.1 ± 0.6	20.3 ± 2.6	–	–

interactions of charged shower pions $\pi_s^\pm$ as well as in $\pi^- A$ captures. The h-shower structure also contains π^0 initiating the development of em subshowers. The number of neutrons in the em subshower is small compared with the hadron component of the h shower. Therefore, one can neglect neutron production in the em subshowers.

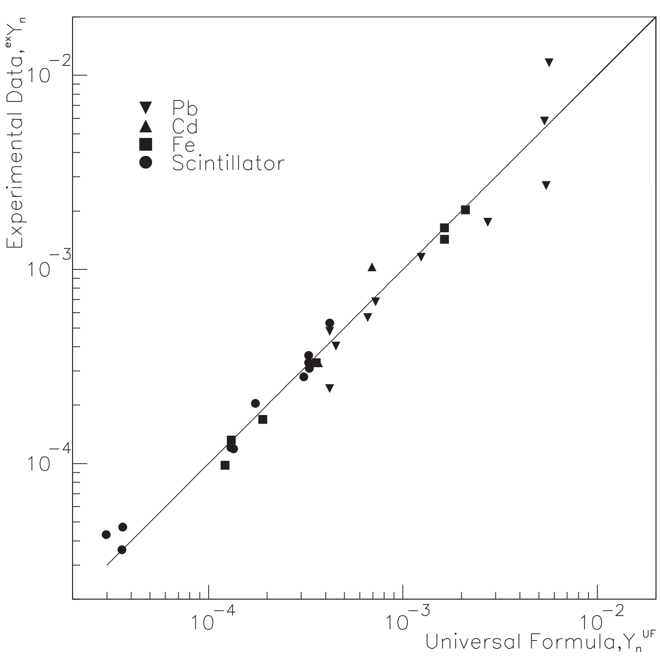

FIG. 1. Correspondence of the measured neutron yield $^{ex}Y_n$ and the values calculated using the UF.

The concept of neutron production in h showers is based on an idea of intranuclear nucleon cascade (INC). Neutrons in h showers are divided by origin into "cascade" (cas) and "evaporative" (ev) neutrons. *Cas* neutrons are produced in the fast phase of πA interaction as a result of the development of INC initiated by nucleon recoil from deep-inelastic πN collision within a nucleus. The ev neutrons appear in the subsequent πA-scattering phase. They are emitted by the excited residual nucleus $A_r^* = A - N_{cas}$; here N_{cas} is the number of cascade nucleons coming out of nucleus A, $1 \leqslant N_{cas} < A$. In the fixed-energy h shower the number $\overline{N}_{cas}$ in the πA interaction and the average number of πA interactions depend weakly on A [21,22]. The average number of ev neutrons in a πA collision, $\overline{n}_{ev}$, depends on a set of residual nuclei A_r, which is characterized by an average value of $\overline{A}_r$. Thus, the number of neutrons in the h shower $\nu_\pi^\pm n_n = \nu_\pi^\pm(\overline{n}_{cas} + a\overline{n}_{cas} + \overline{n}_{ev})$ is related to the nucleus A; besides, the average number of cas neutrons in the πA collision, $\overline{n}_{cas}$, is associated with the mother nucleus A but the value of $\overline{n}_{ev}$ is associated with the nucleus $\overline{A}_r$.

Multiplicity $\nu_\pi^\pm$ is the average number of $\pi^\pm A$ interactions in a shower, which is equal to the number of charged pions in the shower and weakly depends on A. The addend $a\overline{n}_{cas}$ ($a \ll 1$) takes into account multiplication of cascade neutrons in their nA collisions. For any A the value of $\overline{n}_{ev}$ is approximately 2 times the value of $(1 + a)\overline{n}_{cas}$.

III. THE NEUTRON YIELD OF CHARGED PIONS AND NEUTRONS IN h SHOWERS

To describe the experimental data and to present the results of calculations of the yield Y_n, the following power-law dependencies are used:

$$Y_n = c_A E_\mu^\alpha \text{ (for fixed } A), \tag{3}$$

$$Y_n = c_E A^\beta \text{ (for fixed } E_\mu), \tag{4}$$

where α and β are constant. The values of coefficients c_A and c_E and exponents α and β are defined based on the best agreement of the results of measurements or calculations with dependencies [Eqs. (3) and (4)]. They are adjustable parameters and have no physical meaning. Simple dependencies [Eqs. (3) and (4)] at correct values c_A, c_E, α, and β reflect well the tendency of the neutron yield change in a relatively small range of the mean muon energy underground from 40 to 400 GeV. Due to the constancy of exponents α and β and the independence of E_μ and A from each other, the Y_n expression can be factorized:

$$Y_n = c E_\mu^\alpha A^\beta. \tag{5}$$

In this case $c_a = cA^\beta$ and $c_E = cE_\mu^\alpha$, where c is constant.

One can also use the power-law dependencies [Eqs. (3)–(5)] for the yield component Y_n^h. Such a possibility is based on broad experimental and theoretical material obtained in the early studies of multiple processes in hadron-nucleus collisions [22–25].

According to Eq. (1), the yield Y_n^h is given by

$$Y_n^h = \frac{N_0}{A}\langle \sigma_{\mu A}^h \nu_n^h \rangle, \tag{6}$$

where $\sigma_{\mu A}^h$ is the cross section of h-shower generation and ν_n^h is the neutron multiplicity in a h shower. The cross section can be written as

$$\sigma_{\mu A}^h = \sigma_{\mu N} A^\rho, \tag{7}$$

where $\sigma_{\mu N}$ is a cross section of a deep-inelastic μN interaction and ρ is the degree of nucleon shadowing in a nucleus for virtual photons.

In the E_μ range of 10–10^4 GeV the cross section $\sigma_{\mu N}$ is constant: $\sigma_{\mu N} = 1.1 \times 10^{-28}$ cm^2. In accordance with the experimental data deep-inelastic photonuclear interaction of cosmic muons is characterized by a value of $\rho = 0.96$ [26,27]. One can transform expression (6), using formula (7) and setting $\rho = 1$:

$$\begin{aligned} Y_n^h(E_\mu, A) &= \frac{N_0}{A}\langle \sigma_{\mu N} A^\rho \nu_n^h \rangle \\ &\approx N_0 \sigma_{\mu N} \langle \nu_n^h \rangle = N_{\mu N} \langle \nu_n^h \rangle \text{ (cm}^2/\text{g)}. \end{aligned} \tag{8}$$

The number of μN interactions $N_{\mu N}$ does not depend on E_μ and, practically, on A. Hence, the dependence of the yield Y_n^h on E_μ and A is contained in the $\langle \nu_n^h \rangle$ value. As follows from the experiments [21,22,28], in deep-inelastic collisions of a particle with a nucleus the neutron number n_n weakly correlates with the particle energy and mostly depends on A. Therefore, the multiplicity $\langle \nu_n^h \rangle$ is defined by a multiplicity of pions, $\nu_\pi^\pm$, and the neutron number n_n:

$$\langle \nu_n^h(E_\mu, A) \rangle = \langle \nu_\pi^\pm(E_\mu, A) \rangle n_n(A). \tag{9}$$

The $\langle \nu_\pi^\pm \rangle$ value determines the yield $Y_\pi^\pm$ of charged pions in the h shower:

$$Y_\pi^\pm = \frac{N_0}{A}\langle \sigma_{\mu N} A^\rho \nu_\pi^\pm \rangle \approx N_{\mu N} \nu_\pi^\pm \text{ (cm}^2/\text{g)}. \tag{10}$$

The dependence of the multiplicity $\nu_\pi^\pm$ on E_μ and A can be factorized:

$$\nu_\pi^\pm(E_\mu, A) = c_\pi E_\mu^{\alpha_\pi} A^{\beta_\pi}. \tag{11}$$

Substituting Eq. (11) in Eq. (10), one finds an expression for the pion yield:

$$Y_\pi^\pm(E_\mu, A) = N_{\mu N} c_\pi E_\mu^{\alpha_\pi} A^{\beta_\pi}. \tag{12}$$

Taking into account Eqs. (8), (9), and (12), one finds

$$Y_n^h = Y_\pi^\pm(E_\mu, A) n_n(A). \tag{13}$$

IV. CORRELATION BETWEEN THE YIELD $Y_\pi^\pm$ AND THE MUON NUCLEAR ENERGY LOSS: THE PHENOMENOLOGICAL EXPRESSION FOR THE YIELD $Y_\pi^\pm$

The energy to produce pions is a portion of the muon nuclear energy loss b_n, and the energy for neutron production is taken from the shower's charged pions. Hence, the yields $Y_\pi^\pm$ and Y_n^h are associated with the b_n loss:

$$b_n = \frac{N_0}{E_{\mu A}} \int_0^{E_\mu} \varepsilon_h d\sigma(E_\mu, \varepsilon_h). \tag{14}$$

Here ε_h is the energy transferred by muons to the h shower; that is, loss b_n are connected with the generation of h showers only. This is valid for ultrarelativistic muons. Passing in Eq. (14) to the mean muon energy transfer $\bar{\varepsilon}_h$ and using formula (7), at $\rho = 1$ one finds

$$b_n = N_0 \frac{\bar{\varepsilon}_h}{E_\mu} \sigma_{\mu N} = N_{\mu N} \frac{\bar{\varepsilon}_h}{E_\mu} \text{ (cm}^2/\text{g)}. \tag{15}$$

The value $b_n = 4.0 \times 10^{-7}$ cm^2/g is constant in the E_μ range from 10 to 10^4 GeV. Consequently, the ratio $\bar{\varepsilon}_h / E_\mu$ is constant too.

Expressions (15) and (10) have the same shape and dimension with the difference that the multiplicity $\langle \nu_\pi^\pm \rangle$ is the number of charged pions in the shower $\bar{\varepsilon}_h$ only. The number of charged pions is connected with E_μ by dependence $E_\mu^{\alpha_\pi}$ [25,29]. Multiplying both sides of Eq. (15) by $E_\mu^{\alpha_\pi}$, I obtain the energy of the charged component of the muon nuclear energy loss:

$$b_n E_\mu^{\alpha_\pi} = N_{\mu N}\left(\frac{\bar{\varepsilon}_h}{E_\mu} E_\mu^{\alpha_\pi}\right), \tag{16}$$

in which the value of $\frac{\bar{\varepsilon}_h}{E_\mu} E_\mu^{\alpha_\pi}$ gives the energy of the charged component of the shower $\bar{\varepsilon}_h$ contained $\langle \nu_\pi^\pm \rangle$ pions. This energy is distributed among $\pi^\pm$ pions in acts of deep-inelastic πN scattering. Neglecting $\pi^\pm$ decays in flight, it is possible to assume that the charged component of the h shower loses all its energy through ionization ($\varepsilon_\pi^{\text{ion}}$), disintegration of nuclei in πA interactions (E_{dis}), and generation of charged pion mass ($m_\pi c^2$).

The value $\varepsilon_{1\pi}^{\rm ion}$ is the pion energy loss over the mean free path λ_π for inelastic πA reactions. The length λ_π is not connected practically with the energy of a pion and weakly depends on A. The $E_{\rm dis}$ magnitude varies in a similar way [21,28]. Energy expended per pion can be expressed as the sum $\varepsilon_{1\pi}^{\rm ion} + E_{\rm dis} + m_\pi c^2 = \varepsilon_{1\pi}(A)$; then

$$\varepsilon_{1\pi}(A)\langle \nu_\pi^{\pm}(E_\mu,A)\rangle = \frac{\overline{\varepsilon}_h}{E_\mu} E_\mu^{\alpha_\pi}. \quad (17)$$

Using Eq. (11) for $\nu_\pi^{\pm}$, one can find the equality

$$c_\pi E_\mu^{\alpha_\pi} A^{\beta_\pi} = \frac{\overline{\varepsilon}_h}{E_\mu} E_\mu^{\alpha_\pi} \frac{1}{\varepsilon_{1\pi}(A)}, \quad (18)$$

whence it follows that $c_\pi = \frac{\overline{\varepsilon}_h}{E_\mu}$ and $\varepsilon_{1\pi}$ depends on A in the following way:

$$\varepsilon_{1\pi} = 1/A^{\beta_\pi}\ (\text{GeV}). \quad (19)$$

The multiplicity $\langle \nu_\pi^{\pm}\rangle$ weakly depends on A and the type of particle projectile. The dependence $\langle \nu_\pi^{\pm}(A)\rangle$ in the form A^{β_π} at $\beta_\pi = 0.14 \pm 0.03$ was obtained in the experiment described in Ref. [23] for protons with an energy of 20–27 GeV; the value $\beta_\pi = 0.13 \pm 0.02$ was defined for π^- mesons at an energy of 17 GeV in Ref. [24]. The dependence $\nu_\pi^{\pm}$ on A is caused due to pion multiplication within a nucleus that results in a decrease in the $\overline{\varepsilon}_{1\pi}^{\rm ion}$ and $\overline{E}_{\rm dis}$ values and an increase in the fraction of the shower energy going to the pion production. The role of this process increases at increasing of A, which leads to an inverse A dependence of $\varepsilon_{1\pi}$. Taking into account Eq. (19), one finds the following for $\langle \nu_\pi^{\pm}\rangle$:

$$\langle \nu_\pi^{\pm}(E_\mu,A)\rangle = \frac{\overline{\varepsilon}_h}{E_\mu} E_\mu^{\alpha_\pi} A^{\beta_\pi}. \quad (20)$$

Substituting Eq. (20) into Eq. (10) and using Eq. (15), I arrive at the expression

$$Y_\pi^{\pm}(E_\mu,A) = b_n E_\mu^{\alpha_\pi} A^{\beta_\pi}. \quad (21)$$

The value of the exponent $\alpha_\pi = 0.75$ was defined for the first time in the extensive air showers (EAS) [25] and then confirmed by calculations [29]. Assuming $\alpha_\pi = 0.75$ and $\beta_\pi = 0.13$, one finds the expression for the yield $Y_\pi^{\pm}$:

$$Y_\pi^{\pm} = b_n E_\mu^{0.75} A^{0.13}. \quad (22)$$

In Ref. [30], the yield Y_π^{+} value for the liquid scintillator (LS) was obtained using the Monte Carlo package FLUKA:

$$Y_\pi^{+} = 4.45 \times 10^{-7} E_\mu^{0.80}. \quad (23)$$

In Ref. [31] the yield Y_π^{+} for LS has been calculated analytically at the depths of 20, 100, and 500 m w.e. to which energies $\overline{E}_\mu$ of 10.3, 22.4, and 80 GeV were attributed in Ref. [30]. One can define the values of the yield Y_π^{+} in the LS ($\overline{A} = 10.3$), using different formulas at $E_\mu = 80$ GeV: $Y_\pi^{+} = 0.86 \times 10^{-5}$ [31], 1.48×10^{-5} [30], and $Y_\pi^{+} = \frac{1}{2} Y_\pi^{\pm} = 0.72 \times 10^{-5}$ (using formula (22) while assuming $Y_\pi^{+} = Y_\pi^{-}$). The scatter of the values obtained demonstrates significant uncertainties given the calculations. One can add that the α value obtained by various authors using the Monte Carlo method is within a range from 0.6 to 0.8.

V. THE PHENOMENOLOGICAL EXPRESSION FOR THE YIELD Y_n^h

The dependence of the yield Y_n^h on E_μ and A is contained in the $\langle \nu_n^h\rangle$ value, which can be factorized as

$$\langle \nu_n^h(E_\mu,A)\rangle = c_n E_\mu^{\alpha_n} A^{\beta}. \quad (24)$$

According to Eqs. (9) and (20), the multiplicity $\langle \nu_n^h\rangle$ can be represented as

$$\langle \nu_n^h(E_\mu,A)\rangle = \langle \nu_\pi^{\pm}\rangle n_n(A) = \frac{\overline{\varepsilon}_h}{E_\mu} E_\mu^{\alpha_\pi} A^{\beta_\pi} n_n(A). \quad (25)$$

The right-hand sides of Eqs. (24) and (25) are equal to each other: $c_n E_\mu^{\alpha_n} A^{\beta} = \frac{\overline{\varepsilon}_h}{E_\mu} E_\mu^{\alpha_\pi} A^{\beta_\pi} n_n(A)$. Because the n_n value is not dependent on energy E_μ, then $\alpha_n = \alpha_\pi$ and $c_n A^{\beta-\beta_\pi} = \frac{\overline{\varepsilon}_h}{E_\mu} n_n(A)$. Hence it follows that $c_n = \frac{\overline{\varepsilon}_h}{E_\mu}$ and $n_n(A) = A^{\beta-\beta_\pi}$. Denoting $\beta - \beta_\pi = \beta_n$, substituting $n(A) = A^{\beta_n}$ in the expression for ν_n^h, and taking into account Eqs. (13) and (21), I obtain $Y_n^h(E_\mu,A) = b_n E_\mu^{\alpha_\pi} A^{\beta_\pi} A^{\beta_n}$.

Experimental data and calculations within the INC model [22] are in good agreement with the exponent $\beta_n = 0.74 \pm 0.10$. Using the value of $\beta_\pi = 0.13$ and taking into account the uncertainties of the definition of the β values, one can assume $\beta_\pi + \beta_n \approx 0.90$. In such a case I get the following expression:

$$Y_n^h(E_\mu,A) = b_n E_\mu^{0.75} A^{0.90}. \quad (26)$$

VI. THE PHENOMENOLOGICAL EXPRESSION FOR THE YIELD $Y_n^{\rm em}$

Muons initiate an em shower via δ electrons, radiative γ quanta (r), or e^+e^- pairs (p). The em shower produces a low neutron amount, but due to a high-generation cross section the em showers provide contributions to the cg-neutron yield comparable with those from h showers. Any em shower consists of electrons e^+,e^- and shower γ quanta (photons). Amounts of both showers' charged particles $N_{\rm sh}^e$ and photons $N_{\rm sh}^\gamma$ are proportional to the shower energy $E_{\rm em}$. The number of photons with an energy above 10 MeV is 2–3 times the number $N_{\rm sh}^e$. At high energies $E_{\rm em}$, hadron h subshowers appear in the em-shower structure, which are produced via photoproduction. The probability of this process is low due to the steep shower photon spectrum $P(\varepsilon_\gamma) \propto 1/\varepsilon_\gamma^2$. Contribution of h subshowers to the value of the yield $Y_n^{\rm em}$ are not considered below. In contrast to the h showers, practically all the em-shower energy is spent for a medium ionization.

The dominant neutron production process in em showers is photoproduction because, first, the photoproduction cross section is $\sim 10^2$ times the cross section of the eA-electronuclear reactions and, second, $N_{\rm sh}^\gamma > N_{\rm sh}^e$. Among photoproduction processes, the largest contribution to the yield $Y_n^{\rm em}$ is introduced by GDR producing ev neutrons. The GDR region is within the range from the nucleon binding energy in the nucleus up to $\sim$20 MeV. The GDR maximum is given by the expression $E_\gamma^{\rm max} \approx 40A^{-0.2}$ MeV. The photoabsorption cross section σ_a is given by

$$\sigma_a = \int_0^{m_\pi c^2} \sigma_{\gamma A} dE_\gamma \approx 60\frac{(A-Z)Z}{A} 10^{-27}\ \mathrm{cm}^2\,\mathrm{MeV}. \quad (27)$$

Due to the large GDR width (2 to 8 MeV) and its maximum location, the photoneutron yield weakly depends on the shape of the photon spectrum $P(\varepsilon_\gamma)$ and it is determined by the number of photons: $Y_n^{\mathrm{em}} \propto N_{\mathrm{sh}}^{\gamma} \propto E_{\mathrm{em}}$. Because $N_{\mathrm{sh}}^{\gamma} \propto E_{\mathrm{em}}$, and the em-shower generation is determined by the cross section $\sigma_{\mu A}^{\mathrm{em}}$, the Y_n^{em} yield is proportional to the em-muon energy loss:

$$Y_n^{\mathrm{em}} \propto \left(\frac{dE_\mu}{dx}\right)^{\mathrm{em}} = k_\delta + [b_r(A) + b_p(A)]E_\mu. \quad (28)$$

Here k_δ, b_r, and b_p are functions weakly dependent on E_μ. The k_δ value at E_μ above 10 GeV increases insignificantly and is practically independent of A. So one can assume that $k_\delta \approx$ const. Values b_r and b_p represent the muon energy loss:

$$b_{r,p} = \left(\frac{1}{E_\mu}\frac{dE_\mu}{dx}\right)_{r,p} = \frac{N_0}{E_\mu A}\int_0^{E_\mu} \varepsilon_{r,p}\sigma_{r,p}(E_\mu,\varepsilon)d\varepsilon, \quad (29)$$

where $\varepsilon_{r,p}$ is the γ-quantum or pair energy and $\sigma_{r,p}$ is the cross section of the respective process.

The losses b_r and b_p within the range from E_μ 40 to 400 GeV are practically independent of the energy E_μ, in this case $(\frac{dE_\mu}{dx})_{r,p} \propto E_\mu$. These losses are connected with the matter properties by the following dependence:

$$b_{r,p}(A) \propto Z^2/A \approx A^{0.94}/4 \propto A^{0.94} \approx A^{1.0}. \quad (30)$$

Having introduced into Eq. (28) the coefficient $\nu_n^{\gamma A}(A)$, which considers a neutron multiplicity at the γA absorption, and also E_μ dependence, I obtain the expression

$$Y_n^{\mathrm{em}} = c^{\mathrm{em}}\nu_n^{\gamma A}\big[k_\delta + b_r(A)E_\mu^{1.0} + b_p(A)E_\mu^{1.0}\big], \quad (31)$$

where c^{em} is a portion of em loss for producing neutrons, which is the same for all em processes.

The neutron production function was approximated in the GDR region by the expression $\sigma_a\nu_n^{\gamma A} = 5.2\times10^{-4}A^{1.8}$ MeV b [32]. Comparing this formula with Eq. (27) and assuming $(A-Z)Z/A \approx A^{1.0}/4 \propto A^{1.0}$, one can obtain the dependence $\nu_n^{\gamma A} = c_{\gamma A}A^{0.8}$, which characterizes a photoneutron multiplicity in em showers at any energy E_{em}.

One can transform Eq. (31), in accordance with Eq. (30) assuming $b_r(A) \approx a_r A^{1.0}$ and $b_p(A) \approx a_p A^{1.0}$ (the values a_r and a_p are constants) and using the expression $\nu_n^{\gamma A} = c_{\gamma A}A^{0.8}$:

$$\begin{aligned} Y_n^{\mathrm{em}}(E_\mu,A) = {} & c^{\mathrm{em}}c_{\gamma A}k_\delta A^{0.8} + c^{\mathrm{em}}c_{\gamma A}a_r A^{1.8}E_\mu^{1.0} \\ & + c^{\mathrm{em}}c_{\gamma A}a_p A^{1.8}E_\mu^{1.0}. \end{aligned} \quad (32)$$

Joining the constants in Eq. (32) in the c_δ, c_r, and c_p coefficients, one can find the dependence of the Y_n^{em} yield on E_μ and A:

$$Y_n^{\mathrm{em}}(E_\mu,A) = c_\delta A^{0.8} + c_r A^{1.8}E_\mu^{1.0} + c_p A^{1.8}E_\mu^{1.0}. \quad (33)$$

In this expression representing the neutron yield for em processes only, one can include the Y_n^{v} term relating to the nuclear muon loss and corresponding to neutron production by virtual photons. Despite the more rigid spectrum $\propto 1/E_\gamma^{v}$ in contrast to the spectrum of real photons in the em showers, virtual photons produce the overwhelming majority of neutrons also via GDR due to its large width. As a result, the expression for Y_n^{v} takes a form similar to the expression for the neutron yield in δ showers: $Y_n^{v} = c_v A^{0.8}$. Including this formula in Eq. (33) one finds the neutron yield in all the processes except for h showers:

$$Y_n^{\mathrm{ph}} = (c_\delta + c_v)A^{0.8} + (c_r + c_p)A^{1.8}E_\mu^{1.0}. \quad (34)$$

Members of this expression represent the neutrons produced via the nuclear photoeffect. These neutrons originate from primary nuclei A of matter in contrast to the h showers, where evaporative neutrons are emitted by remnants of the nuclei A_r. Starting from an energy of $E_\mu \sim 100$ GeV, the second term dominates in the yield (34), so the Y_n^{ph} yield can be represented in a form similar to the expression Y_n^{h} (26): $Y_n^{\mathrm{ph}} = cE_\mu^{\alpha}A^{\beta_{\mathrm{ph}}}$. Here exponents α and β_{ph} are slightly less than 1.0 and 1.8, respectively. Thus, the total neutron yield is the sum of the components Y_n^{h} and Y_n^{ph}:

$$Y_n \approx Y_n^{h} + Y_n^{\mathrm{ph}} = b_n E_\mu^{0.75}A^{0.90} + cE_\mu^{\alpha}A^{\beta_{\mathrm{ph}}}. \quad (35)$$

Substituting Y_n^{UF} for Y_n in Eq. (35), I get

$$b_n^{\mathrm{UF}}E_\mu^{0.78}A^{0.95} = b_n E_\mu^{0.75}A^{0.90} + cE_\mu^{\alpha}A^{\beta_{\mathrm{ph}}}. \quad (36)$$

Using the expressions Y_n^{h} and Y_n^{UF}, one can define the portion of the hadron component in the total yield Y_n as follows: $K(E_\mu,A) = Y_n^{h}/Y_n^{\mathrm{UF}} = 0.91(E_\mu)^{-0.03}A^{-0.05}$. For example, at $E_\mu = 280$ GeV the K values are enclosed within 0.68 and 0.59 for the numbers A from 12 to 207.

The UF parameters were fitted to the experimental data. This procedure takes into account the contribution of the Y_n^{em} component into the total cg-neutron yield as well as the impact of the real muon spectrum on the real Y_n value. The $\sigma_{\mu A}\nu_n$ function in Eq. (1) is not only a summary for the μA interactions but also is integrated over the muon spectrum at a depth of observation. Due to the cg-neutron yield energy dependence E_μ^{α} and a quasiflat muon spectrum deep underground, $\frac{dN_\mu}{dE} \sim \frac{1}{[E_0(H)+E_\mu]^\gamma}$, the use of monoenergetic muons with energy $\overline{E}_\mu$ in calculations results in the Y_n yield value being overestimated by 12% for $\overline{E}_\mu \sim 100$ GeV and by 5% for $\overline{E}_\mu \sim 300$ GeV if $\alpha = 0.75$ [33,34]. Nevertheless, the measured yield Y_n is attributed to the energy $\overline{E}_\mu$ because the $\overline{E}_\mu$ value is a natural physical parameter characterizing the muon flux and muon interactions underground.

It can be noted that in the high-energy h shower a large number of neutrons is produced. This is a rare event leading to significant fluctuations in the value of Y_n obtained during a finite-time measurement. Thus, the yield calculated by the UF is an asymptotic value for the yield magnitudes that are obtained in measurements.

VII. CONCLUSION

Empirical expressions $c_A E_\mu^{\alpha}$ and $c_E A^{\beta}$ are the simplest representations of the cg-neutron yield dependence on E_μ and A. Obtained by fitting to the experimental or calculated data, they reflect trends in the values $Y_n(E_\mu)$ and $Y_n(A)$ without discovering their correlation with physical processes of the neutron production by muons. The universal formula $Y_n^{\rm UF} = b_n^{\rm UF} E_\mu^\alpha A^\beta$ is also empirical due to the method of its derivation. But the UF uncovers the meaning of the coefficients c_A and c_E and points out that the neutron yield is connected with muon energy loss. The UF kernel is the phenomenological Y_n^h expression that is obtained within the framework of the concept of deep-inelastic muon scattering and πA interaction. This approach allows one to associate the yield Y_n^h with the muon nuclear energy loss and with the characteristics of neutron production in the hadron showers and to explain the origin of the exponent values α and β in the Y_n^h and $Y_n^{\rm UF}$ expressions.

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