

Decay widths of bottomonium states in matter: A field theoretic model for composite hadrons

Amruta Mishra*

Department of Physics, Indian Institute of Technology, Delhi, Hauz Khas, New Delhi, 110 016, India

S. P. Misra†

Institute of Physics, Bhubaneswar, 751005, India

(Received 11 December 2016; revised manuscript received 11 April 2017; published 13 June 2017)

We compute the in-medium partial decay widths of the bottomonium states to open bottom mesons ($B\bar{B}$) using a field theoretical model for composite hadrons with quark constituents. These decay widths are calculated by using the explicit constructions for the bottomonium states and the open bottom mesons (B and \bar{B}) and the quark-antiquark pair creation term of the free Dirac Hamiltonian written in terms of the constituent quark field operators. These decay widths in the hadronic medium are calculated as arising from the mass modifications of the bottomonium states and the B and \bar{B} mesons, obtained in a chiral effective model. The decay amplitude in the present model is multiplied with a strength parameter for the light quark pair creation, which is fitted from the observed vacuum partial decay width of the bottomonium state, $\Upsilon(4S)$ to $B\bar{B}$. The effects of the isospin asymmetry, the strangeness fraction of the hadronic matter on the decay widths, arising due to the mass modifications due to these effects, have also been studied. There is observed to be appreciable effects from density, and the effects from isospin asymmetry on the partial decay widths of $\Upsilon \rightarrow B\bar{B}$ are observed to be quite pronounced at high densities. These effects should show up in the asymmetric heavy ion collisions in Compressed baryonic matter (CBM) experiments planned at the future facility at FAIR. The study of the Υ states will, however, require access to energies higher than the energy regime planned at the CBM experiment. The density effects on the decay widths of the bottomonium states should show up in the production of these states, as well as in dilepton spectra at the Super Proton Synchrotron (SPS) energies.

DOI: [10.1103/PhysRevC.95.065206](https://doi.org/10.1103/PhysRevC.95.065206)**I. INTRODUCTION**

The study of medium modifications of properties of hadrons is a topic of research which has attracted a lot of attention in recent years in strong interaction physics, in particular because of its relevance to the heavy ion collision experiments. Matter at high temperatures and/or densities is produced in ultrarelativistic heavy-ion collision experiments and the properties of hadrons in such a medium are modified, consequences of which can show up in the experimental observables of these high-energy nuclear collisions.

The open-charm (bottom) mesons, D (\bar{B}) and \bar{D} (B), are made up of a heavy-charm (bottom) quark (antiquark) and a light [up (u) or down (d)] antiquark (quark), and their mass modifications in the hadronic medium are due to their interaction with the light-quark condensate in the quantum chromodynamics (QCD) sum rule framework [1,2]. The in-medium properties of the open-charm mesons have been studied quite extensively by hadronic frameworks, e.g., the quark meson coupling (QMC) model [3–6] as well as the coupled channel approach [7–11]. Within a hadronic framework using pion exchange [12], a study of the open-charm and open-bottom mesons is observed to lead to an attractive interaction of the \bar{D} and B mesons in the nuclear matter, suggesting that these mesons can form bound states with the atomic nuclei. The \bar{D} -nucleon interactions have

recently been studied using a description of the hadrons with quark and antiquark constituents [13], where the field operators of the constituent quarks are written in terms of a constituent quark mass, which arises from dynamical chiral symmetry breaking [13–18].

In the effective hadronic model, constructed by generalizing chiral SU(3) model to the charm and bottom sectors, the mass modifications of these open-charm mesons [19–21] and the open-bottom mesons [22] arise due to their interactions with the light hadrons, namely the baryons (nucleons and hyperons) and the scalar mesons. On the other hand, the hidden charm and bottom mesons, i.e., the charmonium [21,23–25] and bottomonium states [26], have the masses modified in the hadronic medium due to the interactions with the gluon condensates in the medium. The gluon condensates of QCD is mimicked by a scalar dilaton field [21,25], within the effective hadronic model, and the medium modifications of the heavy quarkonium states, i.e., the charmonium [21] and bottomonium states [26], are studied by medium modification of the dilaton field within the model. Using a field theoretical model for composite hadrons with quark and antiquark constituents [27–29], the partial decay widths of the charmonium states to $D\bar{D}$ pair, as well as of the decay $D^* \rightarrow D\pi$, in matter have been studied [30], using the medium modifications of these hadrons using the effective hadronic model [21]. These decay widths were compared with the partial decay widths using the 3P_0 model [21,31], where a light quark-antiquark pair is assumed to be created in the 3P_0 state [32–35], and the light quark (antiquark) combines with the heavy charm antiquark (quark) of the decaying charmonium state, to produce the $D\bar{D}$ pair.

*Electronic address: amruta@physics.iitd.ac.in

†Electronic address: misrasibapasrad@gmail.com

In the present work, we study the medium modification of the decay widths of the bottomonium states to $B\bar{B}$ pair in the strange hadronic medium, due to the mass modifications of these hadrons calculated in the effective chiral model [22,26].

The outline of the paper is as follows: In Sec. II, we describe briefly the field theoretical model for the hadrons with quark constituents, which is used in the present work to calculate the partial decay widths of the bottomonium states to open-bottom mesons ($B\bar{B}$ pair). The decay widths are calculated by using explicit constructions of the bottomonium states [$\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, and $\Upsilon(4S)$] and the B and \bar{B} mesons in terms of the quark and antiquark constituents. We then calculate the matrix element of the S-matrix in the lowest order to compute the decay widths of the bottomonium states to $B\bar{B}$ (B^+B^- or $B^0\bar{B}^0$) pair. The matrix element, however, is multiplied with a parameter, which is fitted from the observed vacuum decay width of $\Upsilon(4S) \rightarrow B\bar{B}$. In the present work, the partial decay widths for the decay of the bottomonium states, $\Upsilon(NS)$, $N=1,2,3,4$, to $B\bar{B}$, are calculated using the field theoretic model for composite hadrons and their medium modifications have been studied as arising from the changes in the masses of these mesons in the hadronic medium. In Sec. III, we briefly describe the effective hadronic model, which has been used to investigate the masses of the open-bottom mesons (B and \bar{B}) and of the Υ states. The in-medium masses of the B and \bar{B} mesons in the strange hadronic medium arise due to their interactions with the baryons and the scalar mesons [22]. On the other hand, the mass modifications of the bottomonium states [26] arise due to the medium modification of the scalar dilaton field, which is incorporated in the effective hadronic framework, to simulate the scale symmetry breaking of QCD through scalar gluon condensate. In Sec. IV, we discuss the results obtained in the present investigation. Using the explicit constructions for the bottomonium states ($\Upsilon(NS)$, $N=1,2,3,4$) and using the quark pair creation term of the free Dirac Hamiltonian written in terms of the constituent quark field operators, the decay widths of the bottomonium states to $B\bar{B}$ pair are calculated within the present model. In Sec. V, we summarize the results for the medium modifications of these decay widths and discuss possible outlook.

II. THE MODEL FOR COMPOSITE HADRONS

The model used in the present work for calculating the partial decay widths of the bottomonium states to $B\bar{B}$ describes the hadrons comprising of the quark and antiquark constituents. In the present section, we shall very briefly describe the model so as to apply the same for investigating these decay widths.

The field operator for a constituent quark for a hadron at rest at time, $t=0$, is written as

$$\begin{aligned} \psi(\mathbf{x}, t=0) &= (2\pi)^{-3/2} \int [U(\mathbf{k})q_I(\mathbf{k}) \exp(i\mathbf{k} \cdot \mathbf{x}) \\ &\quad + V(\mathbf{k})\tilde{q}_I(\mathbf{k}) \exp(-i\mathbf{k} \cdot \mathbf{x})] d\mathbf{k} \\ &\equiv Q(\mathbf{x}) + \tilde{Q}(\mathbf{x}). \end{aligned} \quad (1)$$

In the above, $q_I(\mathbf{k}) = q_r(\mathbf{k})u_r$ and $\tilde{q}_I(\mathbf{k}) = \tilde{q}_s(\mathbf{k})v_s$ are the two component quark annihilation and antiquark creation

operators. The operator $q_r(\mathbf{k})$ annihilates a quark with spin r and momentum \mathbf{k} , whereas $\tilde{q}_s(\mathbf{k})$ creates an antiquark with spin s and momentum \mathbf{k} , and these operators satisfy the usual anticommutation relations,

$$\{q_r(\mathbf{k}), q_s(\mathbf{k}')^\dagger\} = \{\tilde{q}_r(\mathbf{k}), \tilde{q}_s(\mathbf{k}')^\dagger\} = \delta_{rs} \delta(\mathbf{k} - \mathbf{k}'). \quad (2)$$

In Eq. (1), $U(\mathbf{k})$ and $V(\mathbf{k})$ are given as

$$U(\mathbf{k}) = \begin{pmatrix} f(|\mathbf{k}|) \\ \boldsymbol{\sigma} \cdot \mathbf{k} g(|\mathbf{k}|) \end{pmatrix}, \quad V(\mathbf{k}) = \begin{pmatrix} \boldsymbol{\sigma} \cdot \mathbf{k} g(|\mathbf{k}|) \\ f(|\mathbf{k}|) \end{pmatrix}, \quad (3)$$

where the functions $f(|\mathbf{k}|)$ and $g(|\mathbf{k}|)$ satisfy the constraint [27],

$$f^2 + g^2 \mathbf{k}^2 = 1, \quad (4)$$

as obtained from the equal-time anticommutation relation for the four-component Dirac field operators. These functions, for the case of free Dirac field of mass M , are given as

$$f(|\mathbf{k}|) = \left(\frac{k_0 + M}{2k_0} \right)^{1/2}, \quad g(|\mathbf{k}|) = \left(\frac{1}{2k_0(k_0 + M)} \right)^{1/2}, \quad (5)$$

where $k_0 = (|\mathbf{k}|^2 + M^2)^{1/2}$. In the above, M is the constituent quark mass, which is calculated from dynamical chiral symmetry breaking and, in general, can be momentum dependent [13–17]. Using a four-point interaction for the quark operators, as in the Nambu-Jona-Lasinio model, the constituent quark mass turns out to be momentum independent [18]. Also, a recent study [13] shows the momentum dependence of $M(\mathbf{k})$ calculated within a color confining model, to be appreciable only at high momenta. In the present work of the study of decay widths of the bottomonium states to open-bottom mesons, we shall assume the constituent quark mass to be momentum independent. We shall also take the approximate forms (with a small momentum expansion) for the functions $f(|\mathbf{k}|)$ and $g(|\mathbf{k}|)$ of the field operator as given by $g(|\mathbf{k}|) = 1/[2k_0(k_0 + M)]^{1/2} \simeq 1/(2M)$ and $f(|\mathbf{k}|) = (1 - g^2 \mathbf{k}^2)^{1/2} \approx 1 - [(g^2 \mathbf{k}^2)/2]$ [30].

The field operator for the constituent quark of hadron with finite momentum is obtained by Lorentz boosting the field operator of the constituent quark of hadron at rest, which requires the time dependence of the quark field operators. As in the bag model, the time dependence is given by assuming the constituent quarks to be occupying fixed energy levels [27,28], so that for the i th quark of a hadron of mass m_H at rest, we have

$$Q_i(x) = Q_i(\mathbf{x}) \exp(-i\lambda_i m_H t), \quad (6)$$

where λ_i is the fraction of the energy (mass) of the hadron carried by the quark, with $\sum_i \lambda_i = 1$. For a hadron in motion with four-momentum p , the field operators for quark annihilation and antiquark creation, for $t=0$, are obtained by Lorentz boosting the field operator of the hadron at rest and are given as [29]

$$\begin{aligned} Q^{(p)}(\mathbf{x}, 0) &= (2\pi)^{-3/2} \int d\mathbf{k} S[L(p)] U(\mathbf{k}) Q_i(\mathbf{k} + \lambda \mathbf{p}) \\ &\quad \times \exp[i(\mathbf{k} + \lambda \mathbf{p}) \cdot \mathbf{x}] \end{aligned} \quad (7)$$

and

$$\begin{aligned} \tilde{Q}^{(p)}(\mathbf{x}, 0) &= (2\pi)^{-3/2} \int d\mathbf{k} S[L(p)] V(-\mathbf{k}) \tilde{Q}_I(-\mathbf{k} + \lambda\mathbf{p}) \\ &\times \exp[-i(-\mathbf{k} + \lambda\mathbf{p}) \cdot \mathbf{x}]. \end{aligned} \quad (8)$$

In the above, λ is the fraction of the energy of the hadron at rest, carried by the constituent quark (antiquark). In Eqs. (7) and (8), $L(p)$ is the Lorentz transformation matrix, which yields the hadron at finite four-momentum p from the hadron at rest and is given as [28]

$$L_{\mu 0} = L_{0\mu} = \frac{p^\mu}{m_H}, \quad L_{ij} = \delta_{ij} + \frac{p^i p^j}{m_H(p^0 + m_H)}, \quad (9)$$

where, $\mu = 0, 1, 2, 3$ and $i = 1, 2, 3$, and the Lorentz boosting factor $S[L(p)]$ is given as

$$S[L(p)] = \left[\frac{(p^0 + m_H)}{2m_H} \right]^{1/2} + \left[\frac{1}{2m_H(p^0 + m_H)} \right]^{1/2} \vec{\alpha} \cdot \vec{p}, \quad (10)$$

where, $\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$ are the Dirac matrices.

III. PARTIAL DECAY WIDTHS OF THE BOTTOMONIUM STATES TO $B\bar{B}$ PAIR IN THE COMPOSITE MODEL OF THE HADRONS

The partial decay widths of the bottomonium states, $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, and $\Upsilon(4S)$ to $B\bar{B}$ in the hadronic matter are studied in the present investigation. The medium modifications of these decay widths calculated in the present work arise due to the medium modifications of the decaying bottomonium state and the outgoing B and \bar{B} mesons in the hadronic medium. In vacuum, the masses of the bottomonium states, $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\Upsilon(4S)$, and the open-bottom mesons, $B(\bar{B})$, are given as 9 460.3 MeV, 10 023.26 MeV, 10 355.2 MeV, 10 579.4 MeV, and 5 279 MeV, respectively. Hence, in vacuum, the lowest Υ state, which can decay to $B\bar{B}$ is $\Upsilon(4S)$. However, the masses of the Υ states as well as B and \bar{B} mesons are modified in the hadronic medium, due to which the partial decay widths of the bottomonium states to $B\bar{B}$ pair are modified in the medium. In the hadronic matter, the modification of the B meson mass turns out to be different from the medium

modification of the \bar{B} meson mass, due to their difference in the interactions with the hadronic matter. The modifications of the masses of the open-bottom mesons arise due to the interactions with the nucleons, hyperons, as well as scalar mesons in the strange hadronic matter [22]. These in-medium masses have been calculated within an effective hadronic model, where the chiral SU(3) model has been generalized to SU(5) to derive the interactions of the B and \bar{B} mesons with the light hadrons [22]. The bottomonium states are, on the other hand, modified due to their interactions with the gluon condensates in the hadronic medium. The in-medium masses of these states [$\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, and $\Upsilon(4S)$] have been calculated within the same effective hadronic model, where the effect of scale symmetry breaking of QCD through the scalar gluon condensates are simulated by a scalar dilaton field within the hadronic model [26]. For the case of the Υ -state at rest decaying to $B(\mathbf{p})\bar{B}(-\mathbf{p})$, the magnitude of \mathbf{p} is given by

$$|\mathbf{p}| = \left(\frac{m_\Upsilon^2}{4} - \frac{m_B^2 + m_{\bar{B}}^2}{2} + \frac{(m_B^2 - m_{\bar{B}}^2)^2}{4m_\Upsilon^2} \right)^{1/2}. \quad (11)$$

In the above, the medium modifications of the masses of the bottomonium state and the B and \bar{B} mesons are considered, so as to calculate the decay width of $\Upsilon \rightarrow B\bar{B}$ in the strange hadronic medium.

The explicit construct for the state for the bottomonium state Υ with spin projection m at rest as

$$|\Upsilon_m^{Nl}(\vec{0})\rangle = \int d\mathbf{k}_1 b_1^i(\mathbf{k}_1)^\dagger a_m^{Nl}(\Upsilon, \mathbf{k}_1) \tilde{b}_I^i(-\mathbf{k}_1) |\text{vac}\rangle, \quad (12)$$

where i is the color index of the quark (antiquark) operators. In the present investigation, we shall assume the harmonic oscillator wave functions for the bottomonium states and shall study the medium modifications of the decay widths of the bottomonium states, $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, and $\Upsilon(4S)$, arising from the mass modifications of the bottomonium states as well as of the B and \bar{B} mesons.

For $\Upsilon(N S)$ ($N = 1, 2, 3, 4$) [36],

$$a_m^{NS}(\Upsilon, \mathbf{k}_1) = \sigma_m u_{NS}(\mathbf{k}_1), \quad (13)$$

where

$$u_{1S}(\mathbf{k}_1) = \frac{1}{\sqrt{6}} \left(\frac{R_{\Upsilon(1S)}^2}{\pi} \right)^{3/4} \exp \left[-\frac{1}{2} R_{\Upsilon(1S)}^2 \mathbf{k}_1^2 \right], \quad (14)$$

$$u_{2S}(\mathbf{k}_1) = \frac{1}{\sqrt{6}} \sqrt{\frac{3}{2}} \left(\frac{R_{\Upsilon(2S)}^2}{\pi} \right)^{3/4} \left(\frac{2}{3} R_{\Upsilon(2S)}^2 \mathbf{k}_1^2 - 1 \right) \exp \left[-\frac{1}{2} R_{\Upsilon(2S)}^2 \mathbf{k}_1^2 \right], \quad (15)$$

$$u_{3S}(\mathbf{k}_1) = \frac{1}{\sqrt{6}} \sqrt{\frac{15}{8}} \left(\frac{R_{\Upsilon(3S)}^2}{\pi} \right)^{3/4} \left(1 - \frac{4}{3} R_{\Upsilon(3S)}^2 \mathbf{k}_1^2 + \frac{4}{15} R_{\Upsilon(3S)}^4 \mathbf{k}_1^4 \right) \exp \left[-\frac{1}{2} R_{\Upsilon(3S)}^2 \mathbf{k}_1^2 \right], \quad (16)$$

$$u_{4S}(\mathbf{k}_1) = -\frac{1}{\sqrt{6}} \sqrt{\frac{35}{4}} \left(\frac{R_{\Upsilon(4S)}^2}{\pi} \right)^{3/4} \left(1 - 2R_{\Upsilon(4S)}^2 \mathbf{k}_1^2 + \frac{4}{5} R_{\Upsilon(4S)}^4 \mathbf{k}_1^4 - \frac{8}{105} R_{\Upsilon(4S)}^6 \mathbf{k}_1^6 \right) \exp \left[-\frac{1}{2} R_{\Upsilon(4S)}^2 \mathbf{k}_1^2 \right]. \quad (17)$$

In the above, the factor $\frac{1}{\sqrt{6}}$ refers to normalization factor arising from degeneracy factors due to color (3) and spin (2) of the quarks and antiquarks.

The B^0 and \bar{B}^0 states, with finite momenta, are explicitly given as

$$|B^0(\mathbf{p}')\rangle = \int d^3\mathbf{k} d_1^{\dagger}(\mathbf{k}_3 + \lambda_1 \mathbf{p}')^\dagger u_B(\mathbf{k}_3) \tilde{b}_1^{\dagger}(-\mathbf{k}_3 + \lambda_2 \mathbf{p}') d\mathbf{k}_3 \quad (18)$$

and

$$|\bar{B}^0(\mathbf{p})\rangle = \int d^3\mathbf{k} b_1^{\dagger}(\mathbf{k}_2 + \lambda_2 \mathbf{p})^\dagger u_B(\mathbf{k}_2) \tilde{d}_1^{\dagger}(-\mathbf{k}_2 + \lambda_1 \mathbf{p}) d\mathbf{k}_2. \quad (19)$$

In the above,

$$u_B(\mathbf{k}) = \frac{1}{\sqrt{6}} \left(\frac{R_B^2}{\pi} \right)^{3/4} \exp\left(-\frac{R_B^2 \mathbf{k}^2}{2}\right). \quad (20)$$

To calculate the partial decay widths of the decay process $\Upsilon \rightarrow B\bar{B}$, we need to know the values of λ_1 and λ_2 , the fractions of energy of the hadron carried by its constituent quark and antiquark. These are calculated by assuming that the binding energy of the hadron as shared by the quark and antiquark are *inversely* proportional to the quark and antiquark masses [28,30]. The energies of $d(\bar{d})$ and $\bar{b}(b)$ in $\bar{B}(B)$ meson are then given as [28]

$$\omega_1 = M_d + \frac{M_b}{M_b + M_d} (m_B - M_b - M_d) \quad (21)$$

and

$$\omega_2 = M_b + \frac{M_d}{M_b + M_d} (m_B - M_b - M_d), \quad (22)$$

with

$$\lambda_i = \frac{\omega_i}{m_B}. \quad (23)$$

The motivation for the assumption that the contributions from the quark (antiquark) to the binding energy of the hadron to be inversely proportional to the mass of the quark (antiquark) as in Eqs. (21) and (22) is as follows. In fact, in general,

the contributions to the binding energy of the bound state composed of particles of 1 and 2, with masses m_1 and m_2 , are assumed to be given as μ/m_i , $i = 1, 2$, multiplied by the binding energy of the bound state, where μ is the reduced mass of the system, calculated from $1/\mu = 1/m_1 + 1/m_2$. In other words, the contributions from the particles to binding energy are inversely proportional to their masses, and the total binding energy is the sum of the individual contributions, i.e., $BE = [(\mu/m_1) + (\mu/m_2)] \times BE = BE$, as it should be. The reason for making this assumption comes from the example of hydrogen atom, which is the bound state of the proton and the electron. As the mass of proton is much larger as compared to the mass of the electron, the binding energy contribution from the electron is $\frac{\mu}{m_e} \times BE \simeq BE$ of hydrogen atom, and the contribution from the proton is $\frac{\mu}{m_p} \times BE$, which is negligible as compared to the total binding energy of hydrogen atom, since $m_p \gg m_e$. With this assumption, the binding energies of the heavy-light mesons, e.g., D and \bar{D} mesons [30], as well as for B and \bar{B} mesons, mostly arise from the contribution from the light quark (antiquark).

We next evaluate the matrix element of the quark-antiquark pair creation part, $\int \mathcal{H}_{Q\bar{Q}}(\mathbf{x}, t = 0) d\mathbf{x}$, of the Dirac Hamiltonian density, between the initial and the final states for the reaction $\Upsilon \rightarrow \bar{B}^0(\mathbf{p}) + B^0(\mathbf{p}')$. The Dirac Hamiltonian density is given as

$$\mathcal{H} = \psi(x)^\dagger (-i\vec{\alpha} \cdot \vec{\nabla} + \beta m_Q) \psi(x), \quad (24)$$

where $\vec{\alpha}$ and β are the Dirac matrices, with $\vec{\alpha}$ defined following Eq. (10) and $\beta = \text{diag}(I, -I)$. In the above, $\psi(x)$ is the field operator for the constituent quark, Q with mass m_Q , which is given by Eq. (1) for $t = 0$. The relevant part of the quark pair creation term for the decay process $\Upsilon \rightarrow \bar{B}^0(\mathbf{p}) + B^0(\mathbf{p}')$ is through the $d\bar{d}$ creation. From Eqs. (7) and (8), we can write down $\mathcal{H}_{d\bar{d}}(\mathbf{x}, t = 0)$ and then integrate over \mathbf{x} to obtain the expression

$$\int \mathcal{H}_{d\bar{d}}(\mathbf{x}, t = 0) d\mathbf{x} = \int d\mathbf{k} d\mathbf{k}' d_1^{\dagger}(\mathbf{k} + \lambda_1 \mathbf{p}')^\dagger U(\mathbf{k})^\dagger S[L(p')]^\dagger \delta(-\mathbf{k}' + \lambda_1 \mathbf{p} + \mathbf{k} + \lambda_1 \mathbf{p}') \times (\boldsymbol{\alpha} \cdot (\mathbf{k} + \lambda_1 \mathbf{p}') + \beta M_d) S[L(p)] V(-\mathbf{k}') \tilde{d}_1^{\dagger}(-\mathbf{k}' + \lambda_1 \mathbf{p}), \quad (25)$$

where $S[L(p)]$ and $S[L(p')]$ in the above equation correspond to the hadrons with finite momenta, p and p' , i.e., \bar{B}^0 and B^0 mesons, and M_d is the constituent mass of the d quark. $S[L(p)]$ has already been defined in Eq. (10), and $U(\mathbf{k})$ and $V(\mathbf{k})$ are given by Eq. (3).

From Eq. (25), we can then evaluate that

$$\langle \bar{B}^0(\mathbf{p}) | \langle B^0(\mathbf{p}') | \int \mathcal{H}_{d\bar{d}}(\mathbf{x}, t = 0) d\mathbf{x} | \Upsilon(NS)_m(\vec{0}) \rangle = \delta(\mathbf{p} + \mathbf{p}') \int d\mathbf{k}_1 A_m^{\Upsilon(NS)}(\mathbf{p}, \mathbf{k}_1), \quad (26)$$

using the explicit forms of the Υ -states and \bar{B}^0 and B^0 states. We obtain the form of $A_m^{\Upsilon}(\mathbf{p}, \mathbf{k}_1)$, including summing over color,

$$A_m^{\Upsilon(NS)}(\mathbf{p}, \mathbf{k}_1) = 3u_{\bar{b}^0}(\mathbf{k}) u_{B^0}(\mathbf{k}) \cdot \text{Tr} [a_m(\Upsilon(NS), \mathbf{k}_1) U(\mathbf{k})^\dagger S(L(p'))^\dagger (\boldsymbol{\alpha} \cdot \vec{\mathbf{q}} + \beta M_d) S(L(p)) V(-\mathbf{k})], \quad (27)$$

where, $\mathbf{k} = \mathbf{k}_1 - \lambda_2 \mathbf{p}$, $\vec{\mathbf{q}} = \mathbf{k}_1 - \mathbf{p}$ and $\mathbf{p}' = -\mathbf{p}$.

We shall now simplify $A_m^{\Upsilon(NS)}(\mathbf{p}, \mathbf{k}_1)$. First, since the $B(\bar{B})$ mesons are completely nonrelativistic, we shall be assuming that $S[L(p)]$ and $S[L(p')]$ as identity. The integral in the right-hand side of Eq. (26) can be written as

$$\int d\mathbf{k}_1 A_m^{\Upsilon(NS)}(\mathbf{p}, \mathbf{k}_1) = 3 \int d\mathbf{k}_1 u_{\bar{b}^0}(\mathbf{k}) u_{B^0}(\mathbf{k}) \cdot \text{Tr} [a_m(\Upsilon(NS), \mathbf{k}_1) B(\mathbf{k}, \vec{\mathbf{q}})], \quad (28)$$

where

$$B(\mathbf{k}, \tilde{\mathbf{q}}) = \boldsymbol{\sigma} \cdot \tilde{\mathbf{q}} - (2(\mathbf{k} \cdot \tilde{\mathbf{q}})g^2 + f(\mathbf{k}))\boldsymbol{\sigma} \cdot \mathbf{k}. \quad (29)$$

We use the approximate forms of f and g at small momentum, i.e., $f \approx 1 - \frac{g^2 \mathbf{k}^2}{2}$ and $2M_d g \approx 1$, for simplifying the integral given by Eq. (28). After simplification, this integral can be written as

$$\int d\mathbf{k}_1 A_m^{\Upsilon(N S)}(\mathbf{p}, \mathbf{k}_1) = 6c_{\Upsilon(N S)} \exp[(a_{\Upsilon(N S)} b_{\Upsilon(N S)}^2 - R_B^2 \lambda_2^2) |\mathbf{p}|^2] \int d\mathbf{k}_1 T_m^{\Upsilon(N S)}(\mathbf{p}, \mathbf{k}_1), \quad (30)$$

where, $T_m^{\Upsilon(N S)}(\mathbf{p}, \mathbf{k}_1)$, for $N = 1, 2, 3, 4$, are given as

$$\begin{aligned} T_m^{\Upsilon(1 S)}(\mathbf{p}, \mathbf{k}_1) &= \frac{1}{2} \text{Tr}[\sigma_m B(\mathbf{k}, \tilde{\mathbf{q}})], \\ T_m^{\Upsilon(2 S)}(\mathbf{p}, \mathbf{k}_1) &= \frac{1}{2} \text{Tr}[\sigma_m B(\mathbf{k}, \tilde{\mathbf{q}})] \cdot \left(\frac{2}{3} R_{\Upsilon(2 S)}^2 \mathbf{k}_1^2 - 1 \right), \\ T_m^{\Upsilon(3 S)}(\mathbf{p}, \mathbf{k}_1) &= \frac{1}{2} \text{Tr}[\sigma_m B(\mathbf{k}, \tilde{\mathbf{q}})] \cdot \left(1 - \frac{4}{3} R_{\Upsilon(3 S)}^2 \mathbf{k}_1^2 + \frac{4}{15} R_{\Upsilon(3 S)}^4 \mathbf{k}_1^4 \right), \\ T_m^{\Upsilon(4 S)}(\mathbf{p}, \mathbf{k}_1) &= \frac{1}{2} \text{Tr}[\sigma_m B(\mathbf{k}, \tilde{\mathbf{q}})] \cdot \left(1 - 2 R_{\Upsilon(4 S)}^2 \mathbf{k}_1^2 + \frac{4}{5} R_{\Upsilon(4 S)}^4 \mathbf{k}_1^4 - \frac{8}{105} R_{\Upsilon(4 S)}^6 \mathbf{k}_1^6 \right). \end{aligned} \quad (31)$$

In the above, the parameters $a_{\Upsilon(N S)}$ and $b_{\Upsilon(N S)}$ are given as

$$a_{\Upsilon(N S)} = \frac{1}{2} R_{\Upsilon(N S)}^2 + R_B^2; \quad b_{\Upsilon(N S)} = R_B^2 \lambda_2 / a_{\Upsilon(N S)}, \quad (32)$$

with $R_{\Upsilon(N S)}$ as the radius of the bottomonium state, $\Upsilon(N S)$ ($N = 1, 2, 3, 4$), and

$$\begin{aligned} c_{\Upsilon(1 S)} &= \frac{1}{6\sqrt{6}} \cdot \left(\frac{R_{\Upsilon(1 S)}^2}{\pi} \right)^{3/4} \cdot \left(\frac{R_B^2}{\pi} \right)^{3/2}, & c_{\Upsilon(2 S)} &= \frac{1}{6\sqrt{6}} \sqrt{\frac{3}{2}} \left(\frac{R_{\Upsilon(2 S)}^2}{\pi} \right)^{3/4} \cdot \left(\frac{R_B^2}{\pi} \right)^{3/2}, \\ c_{\Upsilon(3 S)} &= \frac{1}{6\sqrt{6}} \sqrt{\frac{15}{8}} \left(\frac{R_{\Upsilon(3 S)}^2}{\pi} \right)^{3/4} \cdot \left(\frac{R_B^2}{\pi} \right)^{3/2}, & c_{\Upsilon(4 S)} &= \frac{1}{6\sqrt{6}} \left(\frac{\sqrt{35}}{4} \right) \left(\frac{R_{\Upsilon(4 S)}^2}{\pi} \right)^{3/4} \cdot \left(\frac{R_B^2}{\pi} \right)^{3/2}. \end{aligned} \quad (33)$$

We now change the integration variable to \mathbf{q} in Eq. (28) with the substitution $\mathbf{k}_1 = \mathbf{q} + b_{\Upsilon(N S)} \mathbf{p}$ and write

$$\int A_m^{\Upsilon(N S)}(\mathbf{p}, \mathbf{k}_1) d\mathbf{k}_1 = 6c_{\Upsilon(N S)} \exp[(a_{\Upsilon(N S)} b_{\Upsilon(N S)}^2 - \lambda_2^2 R_B^2) |\mathbf{p}|^2] \cdot \int \exp(-a_{\Upsilon(N S)} \mathbf{q}^2) T_m^{\Upsilon(N S)} d\mathbf{q}, \quad (34)$$

where $T_m^{\Upsilon(N S)}$ in the above equation is the expression $T_m^{\Upsilon(N S)}(\mathbf{p}, \mathbf{k}_1)$ given by Eq. (31), rewritten in terms of \mathbf{q} . We next proceed to simplify the above integral, by using the fact that the terms odd in \mathbf{q} in Eq. (34) will vanish. Also, using $\mathbf{q}_i \mathbf{q}_j G(|\mathbf{q}|) \equiv \frac{1}{3} \delta_{ij} \mathbf{q}^2 G(|\mathbf{q}|)$ and $\mathbf{q}_i \mathbf{q}_j \mathbf{q}_k \mathbf{q}_m G(|\mathbf{q}|) \equiv \frac{1}{15} (\delta_{ij} \delta_{km} + \delta_{ik} \delta_{jm} + \delta_{im} \delta_{jk}) \mathbf{q}^4 G(|\mathbf{q}|)$, where, $G(|\mathbf{q}|)$ is an even function of \mathbf{q} , $T_m^{\Upsilon(N S)}(\mathbf{p}, \mathbf{q})$ in the above integrand can be recast into the form

$$T_m^{\Upsilon(N S)}(\mathbf{p}, \mathbf{q}) \equiv p^m [F_0^{\Upsilon(N S)}(|\mathbf{p}|) + F_1^{\Upsilon(N S)}(|\mathbf{p}|) |\mathbf{q}|^2 + F_2^{\Upsilon(N S)}(|\mathbf{p}|) (|\mathbf{q}|^2)^2 + F_3^{\Upsilon(N S)}(|\mathbf{p}|) (|\mathbf{q}|^2)^3 + F_4^{\Upsilon(N S)}(|\mathbf{p}|) (|\mathbf{q}|^2)^4]. \quad (35)$$

The coefficients $F_i^{\Upsilon(N S)}$ ($i = 0, 1, 2, 3, 4$, and $N = 1, 2, 3, 4$) are given as

$$\begin{aligned} F_0^{\Upsilon(1 S)} &= (\lambda_2 - 1) - 2g^2 |\mathbf{p}|^2 (b_{\Upsilon(1 S)} - \lambda_2) \left(\frac{3}{4} b_{\Upsilon(1 S)}^2 - \left(1 + \frac{1}{2} \lambda_2 \right) b_{\Upsilon(1 S)} + \lambda_2 - \frac{1}{4} \lambda_2^2 \right), \\ F_1^{\Upsilon(1 S)} &= g^2 \left[-\frac{5}{2} b_{\Upsilon(1 S)} + \frac{2}{3} + \frac{11}{6} \lambda_2 \right], \quad F_2^{\Upsilon(1 S)} = 0, \quad F_3^{\Upsilon(1 S)} = 0, \quad F_4^{\Upsilon(1 S)} = 0, \\ F_0^{\Upsilon(2 S)} &= \left(\frac{2}{3} R_{\Upsilon(2 S)}^2 b_{\Upsilon(2 S)}^2 |\mathbf{p}|^2 - 1 \right) F_0^{\Upsilon(1 S)}, \quad F_1^{\Upsilon(2 S)} = \frac{2}{3} R_{\Upsilon(2 S)}^2 F_0^{\Upsilon(1 S)} + \left(\frac{2}{3} R_{\Upsilon(2 S)}^2 b_{\Upsilon(2 S)}^2 |\mathbf{p}|^2 - 1 \right) F_1^{\Upsilon(1 S)} \\ &\quad - \frac{8}{9} R_{\Upsilon(2 S)}^2 b_{\Upsilon(2 S)} g^2 |\mathbf{p}|^2 \left[\frac{9}{4} b_{\Upsilon(2 S)}^2 - b_{\Upsilon(2 S)} \left(2 + \frac{5}{2} \lambda_2 \right) + 2\lambda_2 + \frac{1}{4} \lambda_2^2 \right], \\ F_2^{\Upsilon(2 S)} &= \frac{2}{3} R_{\Upsilon(2 S)}^2 g^2 \left[-\frac{7}{2} b_{\Upsilon(2 S)} + \frac{2}{3} + \frac{11}{6} \lambda_2 \right], \quad F_3^{\Upsilon(2 S)} = 0, \quad F_4^{\Upsilon(2 S)} = 0, \\ F_0^{\Upsilon(3 S)} &= \left(-1 + \lambda_2 + \frac{g^2 |\mathbf{p}|^2}{2} (b_{\Upsilon(3 S)} - \lambda_2)^2 (3b_{\Upsilon(3 S)} + \lambda_2 - 4) \right) \left(1 - \frac{4}{3} R_{\Upsilon(3 S)}^2 b_{\Upsilon(3 S)}^2 |\mathbf{p}|^2 + \frac{4}{15} R_{\Upsilon(3 S)}^4 b_{\Upsilon(3 S)}^4 |\mathbf{p}|^4 \right), \end{aligned} \quad (37)$$

$$\begin{aligned}
 F_1^{\Upsilon(3S)} &= \frac{4}{3}R_{\Upsilon(3S)}^2(1 - \lambda_2)\left(1 - \frac{2}{3}b_{\Upsilon(3S)}^2R_{\Upsilon(3S)}^2|\mathbf{p}|^2\right) + \frac{g^2}{6}(3b_{\Upsilon(3S)} - 7\lambda_2 + 4) \\
 &\quad + \frac{g^2|\mathbf{p}|^2R_{\Upsilon(3S)}^2}{9}\left[(-3b_{\Upsilon(3S)} + 7\lambda_2 - 4)b_{\Upsilon(3S)}^2 + 4(3b_{\Upsilon(3S)} - \lambda_2 - 2)(b_{\Upsilon(3S)} - \lambda_2)(-2b_{\Upsilon(3S)} + 3\lambda_2)\right] \\
 &\quad + \frac{2g^2|\mathbf{p}|^4R_{\Upsilon(3S)}^4b_{\Upsilon(3S)}^2}{45}\left[(3b_{\Upsilon(3S)} - 7\lambda_2 + 4)b_{\Upsilon(3S)}^2 + 4(3b_{\Upsilon(3S)} - 4\lambda_2)(3b_{\Upsilon(3S)} - \lambda_2 - 2)(b_{\Upsilon(3S)} - \lambda_2)\right] \\
 F_2^{\Upsilon(3S)} &= \frac{4}{15}(\lambda_2 - 1)R_{\Upsilon(3S)}^4 - \frac{2}{9}g^2R_{\Upsilon(3S)}^2(9b_{\Upsilon(3S)} - 7\lambda_2 + 4) \\
 &\quad + \frac{g^2R_{\Upsilon(3S)}^4|\mathbf{p}|^2}{15}\left[8b_{\Upsilon(3S)}^3 - \frac{8}{3}b_{\Upsilon(3S)}(b_{\Upsilon(3S)} - \lambda_2)(3b_{\Upsilon(3S)} + \lambda_2 - 4) + 2(b_{\Upsilon(3S)} - \lambda_2)^2(3b_{\Upsilon(3S)} + \lambda_2 - 4)\right. \\
 &\quad \left.+ 14b_{\Upsilon(3S)}^2(b_{\Upsilon(3S)} - \lambda_2) - \frac{88}{15}b_{\Upsilon(3S)}^2(3b_{\Upsilon(3S)} - \lambda_2 - 2)\right], \\
 F_3^{\Upsilon(3S)} &= \frac{2g^2}{45}R_{\Upsilon(3S)}^4(15b_{\Upsilon(3S)} - 7\lambda_2 + 4), \quad F_4^{\Upsilon(3S)} = 0,
 \end{aligned} \tag{38}$$

and,

$$\begin{aligned}
 F_0^{\Upsilon(4S)} &= \frac{1}{2}(b_{\Upsilon(4S)} - 1)(b_{\Upsilon(4S)} - \lambda_2)(3b_{\Upsilon(4S)} + \lambda_2 - 4)g^2|\mathbf{p}|^2 \\
 &\quad \times \left(1 - 2R_{\Upsilon(4S)}^2b_{\Upsilon(4S)}^2|\mathbf{p}|^2 + \frac{4}{5}R_{\Upsilon(4S)}^4b_{\Upsilon(4S)}^4|\mathbf{p}|^4 - \frac{8}{105}R_{\Upsilon(4S)}^6b_{\Upsilon(4S)}^6|\mathbf{p}|^6\right), \\
 F_1^{\Upsilon(4S)} &= \frac{g^2}{6}[9(b_{\Upsilon(4S)} - 1) - 2(3b_{\Upsilon(4S)} - \lambda_2 - 2)] + \frac{g^2|\mathbf{p}|^2R_{\Upsilon(4S)}^2}{3} \\
 &\quad \times \left[(-5b_{\Upsilon(4S)} + 3)(3b_{\Upsilon(4S)} + \lambda_2 - 4)(b_{\Upsilon(4S)} - \lambda_2) - 9b_{\Upsilon(4S)}^2(b_{\Upsilon(4S)} - 1) + 2b_{\Upsilon(4S)}(3b_{\Upsilon(4S)} - \lambda_2 - 2)(3b_{\Upsilon(4S)} - 2)\right] \\
 &\quad + \frac{4g^2|\mathbf{p}|^4R_{\Upsilon(4S)}^4b_{\Upsilon(4S)}^2}{15}\left[(7b_{\Upsilon(4S)} - 5)(3b_{\Upsilon(4S)} + \lambda_2 - 4)(b_{\Upsilon(4S)} - \lambda_2)\right. \\
 &\quad \left.+ \frac{9}{2}(b_{\Upsilon(4S)} - 1)b_{\Upsilon(4S)}^2 - b_{\Upsilon(4S)}(5b_{\Upsilon(4S)} - 4)(3b_{\Upsilon(4S)} - \lambda_2 - 2)\right] \\
 &\quad - \frac{8g^2|\mathbf{p}|^6R_{\Upsilon(4S)}^6b_{\Upsilon(4S)}^4}{105}\left[\frac{1}{2}(9b_{\Upsilon(4S)} - 7)(3b_{\Upsilon(4S)} + \lambda_2 - 4)(b_{\Upsilon(4S)} - \lambda_2)\right. \\
 &\quad \left.+ \frac{3}{2}b_{\Upsilon(4S)}^2(b_{\Upsilon(4S)} - 1) - \frac{1}{3}b_{\Upsilon(4S)}(3b_{\Upsilon(4S)} - \lambda_2 - 2)(7b_{\Upsilon(4S)} - 6)\right], \\
 F_2^{\Upsilon(4S)} &= \frac{1}{3}g^2R_{\Upsilon(4S)}^2(-9b_{\Upsilon(4S)} - 2\lambda_2 + 5) + \frac{4}{5}g^2R_{\Upsilon(4S)}^4|\mathbf{p}|^2 \\
 &\quad \times \left[b_{\Upsilon(4S)}^2(7b_{\Upsilon(4S)} - 5) + \frac{1}{6}(3b_{\Upsilon(4S)} + \lambda_2 - 4)(b_{\Upsilon(4S)} - \lambda_2)(7b_{\Upsilon(4S)} - 3)\right. \\
 &\quad \left.- \frac{2}{15}b_{\Upsilon(4S)}(3b_{\Upsilon(4S)} - \lambda_2 - 2)(21b_{\Upsilon(4S)} - 10)\right] \\
 &\quad + \frac{4}{5}g^2R_{\Upsilon(4S)}^6|\mathbf{p}|^4b_{\Upsilon(4S)}^2\left[-\frac{1}{7}b_{\Upsilon(4S)}^2(9b_{\Upsilon(4S)} - 7) - \frac{4}{15}b_{\Upsilon(4S)}(b_{\Upsilon(4S)} - \lambda_2)(3b_{\Upsilon(4S)} + \lambda_2 - 4)\right. \\
 &\quad \left.- \frac{1}{3}(b_{\Upsilon(4S)} - 1)(b_{\Upsilon(4S)} - \lambda_2)(3b_{\Upsilon(4S)} + \lambda_2 - 4) + \frac{2}{105}b_{\Upsilon(4S)}(3b_{\Upsilon(4S)} - \lambda_2 - 2)(45b_{\Upsilon(4S)} - 28)\right], \\
 F_3^{\Upsilon(4S)} &= \frac{2g^2}{15}R_{\Upsilon(4S)}^4(15b_{\Upsilon(4S)} + 2\lambda_2 - 5) + \frac{4}{5}g^2R_{\Upsilon(4S)}^6|\mathbf{p}|^2
 \end{aligned}$$

$$\begin{aligned}
 & \times \left[-\frac{4}{5}b_{\Upsilon(4S)}^3 - (b_{\Upsilon(4S)} - 1)b_{\Upsilon(4S)}^2 - \frac{2}{21}b_{\Upsilon(4S)}(b_{\Upsilon(4S)} - \lambda_2)(3b_{\Upsilon(4S)} + \lambda_2 - 4) \right. \\
 & \left. - \frac{1}{21}(b_{\Upsilon(4S)} - 1)(b_{\Upsilon(4S)} - \lambda_2)(3b_{\Upsilon(4S)} + \lambda_2 - 4) + \frac{2}{105}b_{\Upsilon(4S)}(3b_{\Upsilon(4S)} - \lambda_2 - 2)(27b_{\Upsilon(4S)} - 10) \right], \\
 F_4^{\Upsilon(4S)} &= -\frac{4g^2 R_{\Upsilon(4S)}^6}{35 \times 9}(21b_{\Upsilon(4S)} + 2\lambda_2 - 5). \tag{39}
 \end{aligned}$$

On performing the integration over \mathbf{q} , Eq. (34) yields

$$\int A_m^{\Upsilon(NS)}(\mathbf{p}, \mathbf{k}_1) d\mathbf{k}_1 = A^{\Upsilon(NS)}(|\mathbf{p}|)\mathbf{p}_m, \tag{40}$$

where $A^{\Upsilon(NS)}(|\mathbf{p}|)$ is given as

$$\begin{aligned}
 A^{\Upsilon(NS)}(|\mathbf{p}|) &= 6c_{\Upsilon(NS)} \exp \left[(a_{\Upsilon(NS)} b_{\Upsilon(NS)}^2 - R_B^2 \lambda_2^2) |\mathbf{p}|^2 \right] \cdot \left(\frac{\pi}{a_{\Upsilon(NS)}} \right)^{3/2} \\
 & \times \left[F_0^{\Upsilon(NS)} + \frac{3}{2a_{\Upsilon(NS)}} \cdot F_1^{\Upsilon(NS)} + \frac{15}{4a_{\Upsilon(NS)}^2} \cdot F_2^{\Upsilon(NS)} + \frac{105}{8a_{\Upsilon(NS)}^3} \cdot F_3^{\Upsilon(NS)} + \frac{105 \times 9}{16a_{\Upsilon(NS)}^4} \cdot F_4^{\Upsilon(NS)} \right]. \tag{41}
 \end{aligned}$$

With $\langle f|S|i \rangle = \delta_4(P_f - P_i)M_{fi}$, we then have for bottomonium state, $\Upsilon(NS)$ of spin m ,

$$M_{fi} = 2\pi \cdot (-iA^{\Upsilon(NS)}(|\mathbf{p}|))\mathbf{p}_m. \tag{42}$$

In the present work, we shall be studying the in-medium decay widths of the bottomonium state, $\Upsilon(NS)$ to $B\bar{B}$ pair, arising due to the mass modifications of the bottomonium state and the B and \bar{B} states.

The expression obtained for the partial decay width of the bottomonium state decaying at rest to $B^0\bar{B}^0$ pair, after averaging over spin, is given as [30]

$$\begin{aligned}
 \Gamma[\Upsilon(NS) \rightarrow B^0\bar{B}^0] &= \gamma_{\Upsilon}^2 \frac{1}{2\pi} \int \delta(m_{\Upsilon(NS)} - p_{B^0}^0 - p_{\bar{B}^0}^0) |M_{fi}|_{av}^2 \cdot 4\pi |\mathbf{p}_{B^0}|^2 d|\mathbf{p}_{\bar{B}^0}| \\
 &= \gamma_{\Upsilon}^2 \frac{8\pi^2}{3} |\mathbf{p}|^3 \frac{p_{B^0}^0 p_{\bar{B}^0}^0}{m_{\Upsilon(NS)}} A^{\Upsilon(NS)}(|\mathbf{p}|)^2. \tag{43}
 \end{aligned}$$

In the above, $p_{B^0}^0 = (m_{B^0}^2 + |\mathbf{p}|^2)^{1/2}$, $p_{\bar{B}^0}^0 = (m_{\bar{B}^0}^2 + |\mathbf{p}|^2)^{1/2}$, and $|\mathbf{p}|$ is the magnitude of the momentum of the outgoing $B^0(\bar{B}^0)$ mesons. The decay of $\Upsilon(NS)$ to B^+B^- proceeds through a $u\bar{u}$ pair creation and the decay width Eq. (43) is modified to

$$\Gamma(\Upsilon(NS) \rightarrow B^+B^-) = \gamma_{\Upsilon}^2 \frac{8\pi^2}{3} \cdot |\mathbf{p}|^3 \frac{p_{B^+}^0 p_{B^-}^0}{m_{\Upsilon(NS)}} A^{\Upsilon(NS)}(|\mathbf{p}|)^2. \tag{44}$$

In the above, $p_{B^\pm}^0 = (m_{B^\pm}^2 + |\mathbf{p}|^2)^{1/2}$, and $|\mathbf{p}|$ is the magnitude of the momentum of the outgoing B^\pm mesons. The parameter γ_{Υ} has been introduced in the expressions for the decay widths of $\Upsilon(NS) \rightarrow B^0\bar{B}^0(B^+B^-)$, which is a measure of the production strength of the $B\bar{B}$ pair from the Υ -state through light quark antiquark pair ($d\bar{d}$ or $u\bar{u}$) creation. To study the decay width of quarkonia using a light quark pair creation model, namely, 3P_0 model, such a pair creation strength parameter, γ has been introduced in Refs. [31,35], which was fitted to the observed decay width of the meson. In the present investigation of the bottomonium decay widths, the parameter γ_{Υ} is fitted from the vacuum decay width for the channel $\Upsilon(4S) \rightarrow B\bar{B}$ [$\Upsilon(4S)$ is the lowest Υ -state which decays to $B\bar{B}$ in vacuum]. In the present work, we study the effects of the medium effects on the decay widths of the $\Upsilon \rightarrow B\bar{B}$, arising from the medium modifications of the masses of Υ as well as the B and \bar{B} mesons and explore the possibility whether the decays of the lower (excited) states of bottomonium can become kinematically possible with the medium effects.

IV. IN-MEDIUM MASSES OF OPEN-BOTTOM MESONS (B AND \bar{B}) AND BOTTOMONIUM STATES

The partial decay widths of the bottomonium states, $\Upsilon(NS)$ ($N = 1, 2, 3, 4$), to $B\bar{B}$ pair, in the hadronic medium, are calculated in the present work. The medium modifications of these decay widths arise due to the mass modifications of the decaying Υ -state and the open-bottom (B and \bar{B}) mesons. The in-medium masses of the heavy-light B and \bar{B} mesons are calculated within an effective hadronic model, which is a generalization of a chiral SU(3) model [37], based on a nonlinear realization of chiral symmetry [38–41] and broken scale invariance [42–44], to charm and bottom sectors, so as to derive the interactions of these mesons with the hadronic medium. The mass modifications of the open-bottom mesons (B and \bar{B}) arise from their interactions with the nucleons, hyperons, and the scalar mesons in the strange hadronic matter [22]. The medium modifications of the bottomonium masses, on the other hand, arise from the

medium modifications of the gluon condensate in the hadronic medium. The gluon condensate of QCD is simulated by a scale-breaking term [37,45], written in terms of a scalar dilaton field within the effective hadronic model [20,21]. Matching the trace of the energy momentum tensor in QCD to that corresponding to the scale-breaking term in the effective hadronic model gives the expression for the gluon condensate in terms of the dilaton field [20,21,25,46]. The medium modification of the gluon condensate in the hadronic matter is thus calculated from the modification of the dilaton field, using which the in-medium masses of the bottomonium states are calculated [26]. Using a QCD sum rule approach [24,47], the mass modifications of the charmonium states were calculated using the medium modifications of the gluon condensates, obtained from the medium change of the dilaton field within the effective hadronic model [25]. The leading-order perturbation calculations [48] and the framework of QCD sum rule yield the relation between medium modifications of the heavy quarkonium states (charmonium and bottomonium states) to the medium modifications of the gluon condensates [23]. The in-medium charmonium masses have been computed using the medium modifications of the gluon condensates calculated from the dilaton field in the hadronic medium, within the effective hadronic model [21]. Using the medium changes of the masses of the charmonium states, and the D and \bar{D} mesons, the partial decay widths of the charmonium states to $D\bar{D}$ have been investigated using 3P_0 model [21] as well as using the model for composite hadrons with quark and antiquark constituents [30]. In Ref. [31], using a quark-antiquark pair creation model, namely 3P_0 model, the in-medium decay widths of the charmonium states, $\psi(3686)$, $\psi(3770)$, $\chi_{c0}(3417)$, and $\chi_{c2}(3556)$ to $D\bar{D}$ pair were studied, assuming the mass drops to be the same for D as well as \bar{D} mesons in the medium and without accounting for medium modifications of the masses of the charmonium states. There were observed to be nodes for the in-medium decay widths for $\psi(3686)$, $\psi(3770)$, as well as $\chi_{c0}(3417)$, with decrease in the masses of D and \bar{D} , whereas the decay width of the charmonium state $\chi_{c2}(3556)$ showed a monotonous increase with the drop in D and \bar{D} masses. Using the mass modifications of the D and \bar{D} mesons as calculated in a chiral effective model also showed nodes for the in-medium decay widths of the charmonium states, $\psi(3686)$ and $\psi(3770)$, within the 3P_0 model, as well as in the model for composite hadrons, similar to as observed in Ref. [31]. However, the decay width of $J/\psi \rightarrow D\bar{D}$ showed a monotonous increase with the increase in density. The observed behavior of the decay widths of the charmonium states to $D\bar{D}$ in the medium is due to the competing effects from a polynomial part and an exponential part through the center of mass momentum, $|\mathbf{p}|$ of the outgoing $D(\bar{D})$ meson. Accounting for the effects of the medium modifications of the charmonium states was observed to decrease the contribution from the exponential part, as the center of mass momentum $|\mathbf{p}|$ decreases when the mass of the charmonium state drops in the medium. With the mass modifications of the charmonium states, as well as D and \bar{D} mesons, there were no nodes observed up to a density of $6\rho_0$. In the following section, using the model for composite hadrons, we shall investigate the medium effects of the bottomonium decay widths for the channel $\Upsilon \rightarrow B\bar{B}$, as arising from the

medium modifications of the masses of the B, \bar{B} mesons [22] as well as of the masses of the bottomonium states [26].

V. RESULTS AND DISCUSSIONS

In the present paper, we investigate the medium changes of the partial decay widths of the bottomonium states $[\Upsilon(N S), N = 1, 2, 3, 4]$ to $B\bar{B}$ in hadronic matter, arising due to the modifications of the masses of these hadrons calculated in an effective hadronic model. The parameters chosen for the study of these decay widths are as follows. The masses of the u and d quarks are assumed to be 330 MeV and the mass of b quark is taken to be 4 180 MeV [49] in the present work. The values of the parameters λ_1 and λ_2 are then calculated using Eqs. (21), (22), and (23) and using the mass of the $B(\bar{B})$ meson (in vacuum) as 5 279 MeV. These values turn out to be 0.1975 and 0.8025, respectively.

The parameters corresponding to the strengths of the harmonic oscillator wave functions for the $\Upsilon(N S)$ states are evaluated from the decay width $\Upsilon(N S) \rightarrow e^+e^-$, given by the formula [50,51]

$$\Gamma_{\Upsilon(N S) \rightarrow e^+e^-} = \frac{16\pi\alpha^2}{9m_{\Upsilon(N S)}^2} |\psi_{\Upsilon(N S)}(\mathbf{0})|^2, \quad (45)$$

where, $\alpha = 1/137$ is the fine structure constant, $m_{\Upsilon(N S)}$ is the mass of $\Upsilon(N S)$ in vacuum, and $\psi_{\Upsilon(N S)}(\mathbf{0})$ is the wave function of the bottomonium state, $\Upsilon(N S)$ at the origin. Using the experimental values of the leptonic decay widths of 1.34 keV, 0.612 keV, 0.443 keV, and 0.272 keV for the Υ -states, $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, and $\Upsilon(4S)$ [49], we obtain the values of the harmonic oscillation strength, $R_{\Upsilon(N S)}^{-1}$ of the bottomonium states $\Upsilon(N S)$ to be 1309.2, 915.4, 779.75 and 638.6 MeV for $N = 1, 2, 3, 4$ respectively [26]. For the study of the charmonium partial decay widths to $D\bar{D}$ pair in matter, the value of the wave function parameter, R_D of the $D(\bar{D})$ meson and the light quark pair production parameter, γ were fitted from the decay widths of $\psi'' \rightarrow D\bar{D}$ and the partial decay widths of $\psi(4040)$ to $D\bar{D}$, $D^*\bar{D}$, \bar{D}^*D , and $D^*\bar{D}^*$ [23,30]. The value of R_D^{-1} was obtained as 310 MeV [23,30]. In the present work, we assume the wave function parameter for the $B(\bar{B})$ meson, R_B to be given in terms of the parameter R_D of the $D(\bar{D})$ meson wave function, as $R_B = R_D(m_D/m_B)$, which yields the value of R_B^{-1} to be 875.6 MeV. The value of the strength of the light quark pair creation γ_Υ in the expression for the partial decay width of the Υ state to $B\bar{B}$ is fitted from the vacuum decay width of $\Upsilon(4S) \rightarrow B\bar{B}$. The experimental values of the decay widths of $\Upsilon(4S) \rightarrow B^+B^-$ and $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ as 10.516 MeV and 9.984 MeV, respectively, yield the value of γ_Υ to be 5.6.

We next calculate the decay widths of the Υ states to $B\bar{B}$ pair in matter and show the density dependence of these decay widths in Figs. 1, 2, 3, and 4, for $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$ and, $\Upsilon(4S)$, respectively. The effects of isospin asymmetry as well as strangeness of the hadronic matter on these decay widths are also illustrated in these figures. The partial decay widths for $\Upsilon(N S) \rightarrow B^0\bar{B}^0$ and $\Upsilon(N S) \rightarrow B^+B^-$ are given by Eqs. (43) and (44), where $A^{\Upsilon(N S)}(|\mathbf{p}|)$ is given by Eq. (41). The decay widths thus have an exponential as well as polynomial dependence on the center of mass

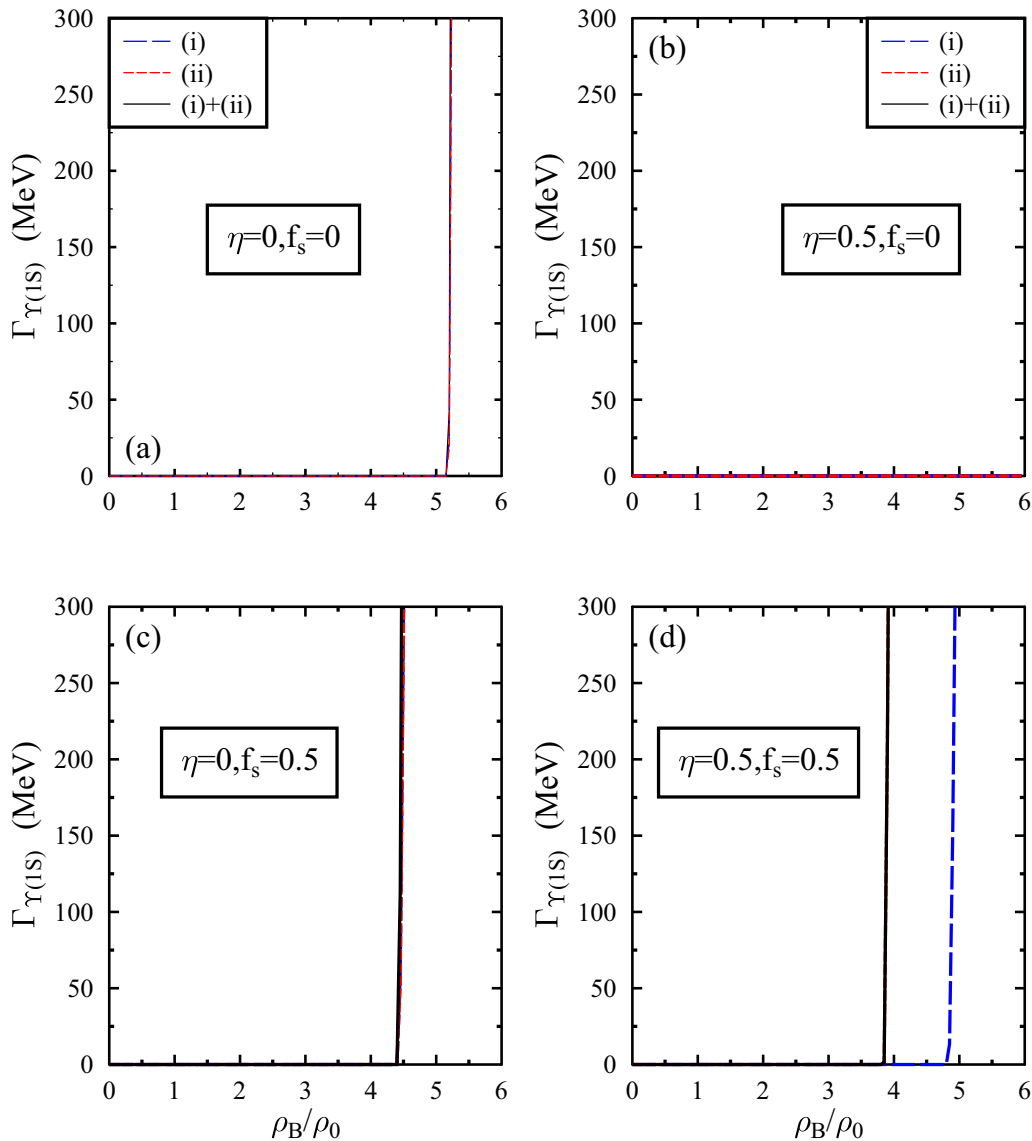


FIG. 1. The partial decay widths of $\Upsilon(1S)$ calculated using the present model for composite hadrons, to (i) $B^+ B^-$, (ii) $B^0 \bar{B}^0$, and (iii) the sum of the two channels [(i)+(ii)] in the isospin symmetric (asymmetric) nuclear matter and strange hadronic matter. In (a) and (c), the decay widths are shown for isospin symmetric ($\eta = 0$) matter, where the channels (i) and (ii) are observed to overlap due to the (almost) degenerate masses for the B mesons ($m_{B^+} \simeq m_{B^0}$), and, for the \bar{B} mesons ($m_{B^-} \simeq m_{\bar{B}^0}$) in the medium, the negligible mass difference arising due to the very small difference in their vacuum masses. The threshold density above which the decay widths become nonzero (with the center of mass momentum, $|\mathbf{p}|$ attaining a nonzero value) is observed to be higher for nuclear matter [shown in (a)] as compared to hyperonic matter [shown in (c)]. For the case of isospin asymmetric nuclear matter, the partial decay widths are seen to remain zero even up to a density of $6\rho_0$ [shown in (b)], whereas for isospin asymmetric hyperonic matter [shown in (d)], the threshold densities for the decay channels (i) and (ii) are observed to be around $4.9\rho_0$ and $3.9\rho_0$, respectively.

momentum, $|\mathbf{p}|$. To understand the observed behavior of these decay widths in matter, as obtained in the present investigation, it is useful to write these decay widths in terms of the exponential and polynomial parts, as $\Gamma(\Upsilon(NS) \rightarrow B\bar{B}) = \exp(2C_{\Upsilon(NS)}^F |\mathbf{p}|^2) \times \Gamma_{\text{polynomial}}(\Upsilon(NS) \rightarrow B\bar{B})$, where $C_{\Upsilon(NS)}^F = (a_{\Upsilon(NS)} b_{\Upsilon(NS)}^2 - R_B^2 \lambda_2^2)$ is the coefficient of $|\mathbf{p}|^2$ in the exponential part of $A^{\Upsilon(NS)}(|\mathbf{p}|)$ given by Eq. (41). In Fig. 1, the partial decay widths of $\Upsilon(1S) \rightarrow B^+ B^-$, $\Upsilon(1S) \rightarrow B^0 \bar{B}^0$, as well as the sum of these two channels are shown as functions of the baryon density in units of the nuclear matter saturation

density, ρ_0 . These are shown for the isospin symmetric as well as asymmetric nuclear and hyperonic matter cases. The decay width for $\Upsilon(1S) \rightarrow B\bar{B}$ in symmetric nuclear matter is observed to be zero up to a density of about $5.2\rho_0$, above which there is observed to be a sharp rise with density. The sharp rise of this decay width with density can be understood from the contributions of the exponential and polynomial parts as follows. With the values of $R_{\Upsilon(1S)}$ and R_B as already mentioned to be $(1309.2 \text{ MeV})^{-1}$ and $(875.6 \text{ MeV})^{-1}$, and the value of the parameter λ_2 as 0.8025, the values of $a_{\Upsilon(1S)}$ and $b_{\Upsilon(1S)}$

defined by Eq. (32) are calculated, using which the value of $C_{\Upsilon(1S)}^F$ is obtained as $-0.15 \times 10^{-6} \text{ MeV}^{-2}$. The value for the exponential part of the decay width is observed to vary very less with density (from a value of around 0.995 at a density of $5.2\rho_0$ to about 0.48 at $6\rho_0$). In the polynomial part, the contributions are from the first two terms of the expression for $A^{\Upsilon(N_S)}(|\mathbf{p}|)$ given by Eq. (41), along with the $|\mathbf{p}|$ dependent terms (modulo the exponential part) multiplying $A^{\Upsilon(N_S)}(|\mathbf{p}|)^2$ in the expressions for the decay widths of $\Upsilon(N_S) \rightarrow B^0\bar{B}^0$ and $\Upsilon(N_S) \rightarrow B^+B^-$ given by Eqs. (43) and (44), respectively. For $\Upsilon(1S) \rightarrow B\bar{B}$, it is observed that the second term within the square bracket in the expression for $A^{\Upsilon(N_S)}(|\mathbf{p}|)$ (with a value of 1.075) dominates over the first term ($|\mathbf{p}|$ dependent, which is negative and is of the order of -0.124 for a density of $6\rho_0$). Hence, the value of the sum of the two terms within the square bracket in the expression for $A^{\Upsilon(N_S)}(|\mathbf{p}|)$ stays close to unity. With increase in density, since the dependence on the exponential as well as the polynomial part of $A^{\Upsilon(N_S)}(|\mathbf{p}|)$ are very small, the dependence of the decay widths of $\Upsilon(1S) \rightarrow B\bar{B}$ is proportional to $|\mathbf{p}|^3 p_B^0 p_{\bar{B}}^0$, which is observed as a sharp rise in the decay width of $\Upsilon(1S) \rightarrow B\bar{B}$. For symmetric nuclear matter, for $\rho_B = 5.2\rho_0$, the values of the polynomial part and the exponential part are observed to be 24.3 MeV and 0.9946 (obtained from the value of $|\mathbf{p}|$ as 129 MeV), with their product (the decay width of $\Upsilon(1S) \rightarrow B^+B^-$) as 24.17 MeV. For a density of $5.24\rho_0$, the value of $|\mathbf{p}|$ as 343 MeV, gives the contributions from the polynomial and exponential parts as 456 MeV and 0.965, giving the value of the decay width of $\Upsilon(1S) \rightarrow B^+B^-$ as 440 MeV. So there is observed to be a sharp rise of the decay width with density in symmetric nuclear matter. The threshold density above which these decay widths become nonzero is observed to be smaller ($\simeq 4.4\rho_0$), with inclusion of hyperons in the medium, as seen from Fig. 1(c). This is because the masses of the $B(\bar{B})$ mesons have a larger drop in nuclear matter as compared to the mass drop in hyperonic matter. Also, the medium modifications of the open-bottom mesons dominate over the modification of the $\Upsilon(1S)$, leading to a smaller value of the center of mass momentum, $|\mathbf{p}|$ in hyperonic matter, as can be seen from the expression of $|\mathbf{p}|$ given by Eq. (11). For symmetric hyperonic matter ($f_s=0.5$), the contributions of the polynomial and the exponential parts to the decay width are observed to be 60.74 MeV and 0.99 (with $|\mathbf{p}|$ as 177 MeV) leading to the partial decay width of $\Upsilon(1S) \rightarrow B^+B^-$ as 60.13 MeV, and for a density of $4.55\rho_0$, the value is observed to increase to 630 MeV (with 661 MeV and 0.953 from the polynomial and exponential parts of the decay width). The isospin asymmetry in the hadronic matter leads to a smaller drop in the masses of the B and \bar{B} mesons, thus shifting the threshold density for the decay width to a larger value of density. For asymmetric nuclear matter, the decay width remains zero even up to a density of around $6\rho_0$, as can be seen from Fig. 1(b). With inclusion of hyperons in the medium, the threshold densities for $\Upsilon(1S) \rightarrow B^+B^-$ and $\Upsilon(1S) \rightarrow B^0\bar{B}^0$ are observed to be around $4.9\rho_0$ and $3.9\rho_0$, respectively, for $f_s=0.5$, as shown in Fig. 1(d). A similar trend is observed for the in-medium partial decay widths of the bottomonium states, $\Upsilon(2S)$ and $\Upsilon(3S)$, plotted in Figs. 2 and 3, respectively. The threshold density

above which the decay of $\Upsilon(2S)$ to $B\bar{B}$ becomes possible in the symmetric nuclear matter is around $2.6\rho_0$, which is observed to become smaller ($\simeq 2.4\rho_0$) when hyperons are included in the hadronic medium. As has been observed for the case of the partial decay width of $\Upsilon(1S) \rightarrow B\bar{B}$, for $\Upsilon(2S)$ decaying to $B\bar{B}$, there is observed to be a sharp rise in the decay width, again predominantly due to the polynomial part (proportional to $|\mathbf{p}|^3 p_B^0 p_{\bar{B}}^0$) multiplying the $A^{\Upsilon(N_S)}(|\mathbf{p}|)^2$ in the expression for the decay widths of $\Upsilon(2S) \rightarrow B\bar{B}$. This is again due to the reason that the contribution from $A^{\Upsilon(N_S)}(|\mathbf{p}|)$ from the terms in the square bracket is observed to vary from 0.83 at $2.7\rho_0$ (with a value of $|\mathbf{p}|$ to be 179 MeV) to about 0.78 at $4\rho_0$ (with a value of $|\mathbf{p}|$ to be 1121 MeV), and the exponential part is seen to vary from 0.983 at $2.7\rho_0$ to 0.515 at $4\rho_0$, due to the small value of the coefficient $C_{\Upsilon(2S)}^F$ as $-0.264 \times 10^{-6} \text{ MeV}^{-2}$. For isospin asymmetric nuclear matter (with $\eta=0.5$), the threshold densities for the decay channels $\Upsilon(2S) \rightarrow B^+B^-$ and $\Upsilon(2S) \rightarrow B^0\bar{B}^0$ are observed to be $2.6\rho_0$ and $3.5\rho_0$, respectively, as shown in Fig 2(b). These values are observed to be modified to $2.1\rho_0$ and $2.5\rho_0$ for isospin asymmetric hyperonic matter (with $\eta = 0.5$ and $f_s = 0.5$) as shown in Fig. 2(d). From Figs. 3(a) and 3(c), the threshold densities for the decay process of $\Upsilon(3S) \rightarrow B\bar{B}$ are observed to be around $1.45\rho_0$ and $1.35\rho_0$ for symmetric nuclear matter and symmetric hyperonic matter. The density dependence of the contribution from the exponential part of the decay width in symmetric nuclear matter is observed to be very small (0.966 at $1.45\rho_0$ corresponding to $|\mathbf{p}|$ as 229 MeV, to 0.79 at $1.75\rho_0$ corresponding to $|\mathbf{p}|$ to be around 652 MeV) and the value of the expression in the square bracket of $A^{\Upsilon(N_S)}(|\mathbf{p}|)$ is observed to vary from 0.21 at $1.45\rho_0$ to 0.5 at $1.75\rho_0$. Hence, the density dependence (through $|\mathbf{p}|$) of the decay widths of $\Upsilon(3S) \rightarrow B^0\bar{B}^0$ and $\Upsilon(3S) \rightarrow B^+B^-$ given by Eqs. (43) and (44), respectively, are due to the factor $|\mathbf{p}|^3 p_B p_{\bar{B}}$, multiplying the $A^{\Upsilon(N_S)}(|\mathbf{p}|)^2$ in the expression for the decay widths of $\Upsilon(3S) \rightarrow B\bar{B}$. This is the reason for the sharp rise of the $\Upsilon(3S) \rightarrow B\bar{B}$ decay width with density. With inclusion of isospin asymmetry in the medium, the densities above which the decay processes $\Upsilon(3S) \rightarrow B^+B^-$ and $\Upsilon(3S) \rightarrow B^0\bar{B}^0$ become possible, are $1.7\rho_0$ and $1.35\rho_0$ for nuclear matter, as can be seen from the Fig. 3(b), and $1.45\rho_0$ and $1.25\rho_0$ for hyperonic matter, as can be seen from Fig. 3(d).

In Fig. 4, we show the density dependence of the decay width $\Upsilon(4S) \rightarrow B\bar{B}$ for isospin asymmetric as well as isospin asymmetric nuclear and hyperonic matter. As has already been mentioned, the production strength of the light quark-antiquark pair for this decay, γ_Υ , is fitted from the experimental (vacuum) decay widths of $\Upsilon(4S) \rightarrow B^+B^-$ and $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ of 10.516 MeV and 9.984 MeV, respectively [the small difference is due to the small difference in the vacuum masses of $B^+(B^-)$ and $B^0(\bar{B}^0)$]. In isospin symmetric nuclear matter, as can be seen from Fig. 4(a), the decay width of $\Upsilon(4S) \rightarrow B^+B^-$ [which is almost identical to the decay width of $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ in the symmetric matter] is observed to increase with density reaching a value of around 70 MeV at a density of around $1.1\rho_0$. Above this value of density, there is seen to be a drop in the decay width with further increase in density, and it is observed that the decay width vanishes at a density of

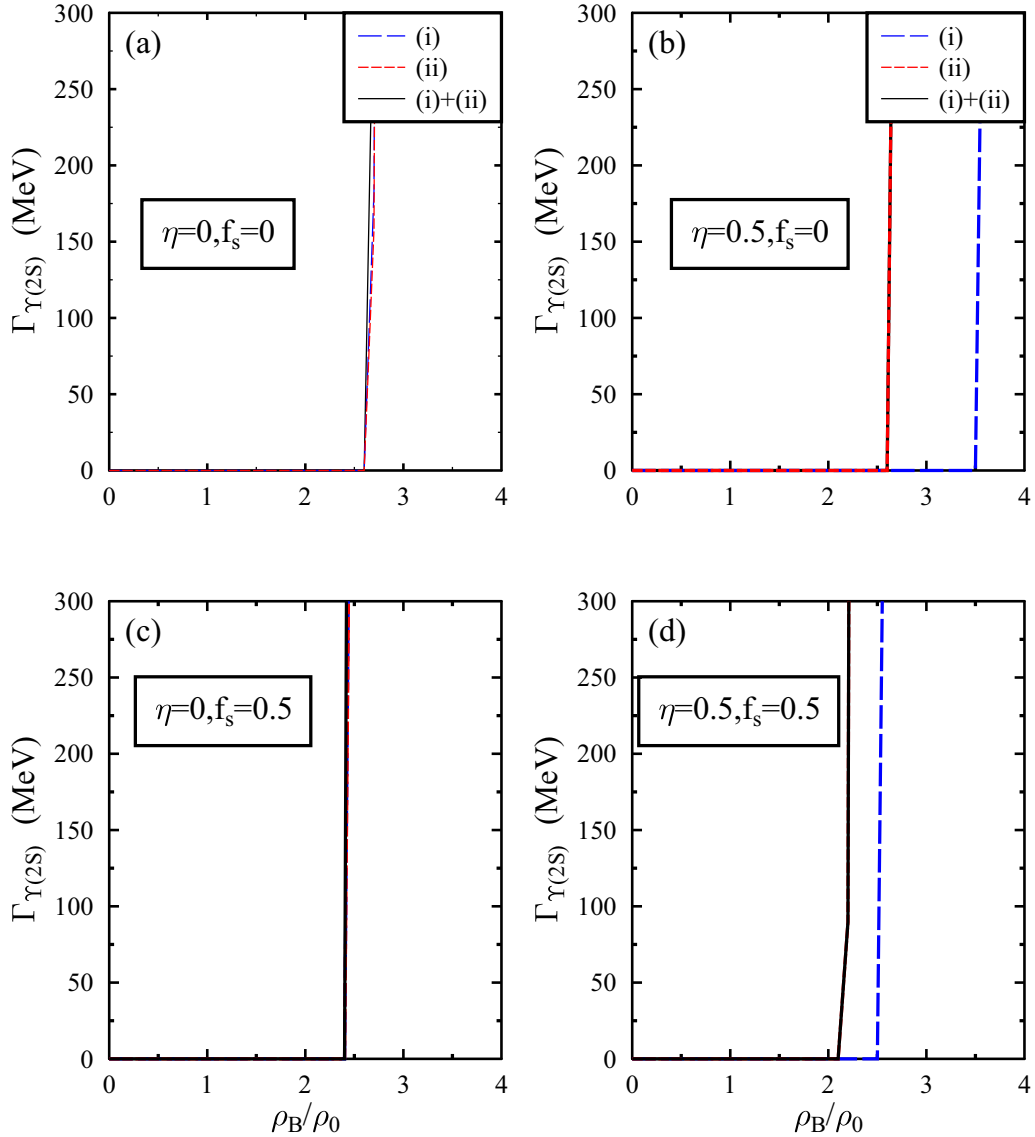


FIG. 2. The partial decay widths of $\Upsilon(2S)$ calculated using the present model for composite hadrons, to (i) B^+B^- , (ii) $B^0\bar{B}^0$, and (iii) the sum of the two channels [(i)+(ii)] in the isospin symmetric (asymmetric) nuclear matter ($f_s = 0$) and strange hadronic matter ($f_s = 0.5$). The channels (i) and (ii) overlap in the isospin symmetric ($\eta = 0$) nuclear and hyperonic matter [shown in (a) and (c)] due to the mass of the B^+ (B^-) (almost) coinciding with the mass of the B^0 (\bar{B}^0). In the symmetric (asymmetric) matter, the threshold densities, above which the decay channels for (i) and (ii) are nonzero, are observed to be higher for nuclear matter, shown in (a) and (b), as compared to hyperonic matter, shown in (c) and (d).

around $2.1\rho_0$. As the density is further increased, the decay width is also seen to increase, again reaching a maximum value of around 35 MeV at a density of around $3\rho_0$ for both the channels $\Upsilon(4S) \rightarrow B^+B^-$ and $\Upsilon(4S) \rightarrow B^0\bar{B}^0$. There is again seen to be a drop and ultimately vanishing of the decay width at a density of around $3.8\rho_0$. Above this density, there is observed to be a sharp rise in the value of the decay widths. For symmetric nuclear matter, the contribution of the exponential part of the decay width is observed to vary from 0.92 at zero density to around 0.11 at density of $4\rho_0$. The density dependence of the part of decay width corresponding to the terms in the square bracket in the expression for $A^{\Upsilon(NS)}(|\mathbf{p}|)$ given by Eq. (41), is observed to be as follows.

Its value of 0.045 at zero density, is observed to increase slowly up to a value of 0.0466 at a density of around $0.5\rho_0$, and decreases with further increase in density, becoming very small ($\sim 6.4 \times 10^{-6}$ at around $2.1\rho_0$). There is observed to be a rise in its value with density up to a density of around $3\rho_0$, followed by a drop and again reaching a value of around 6.63×10^{-6} at around $3.8\rho_0$. As the density is further increased, there is observed to be a steady increase in its value reaching a value of around 1.12 at a density of $5.9\rho_0$. The value of $|\mathbf{p}|$ is observed to increase from a value of 323 MeV at zero density to around 1630 MeV at $4\rho_0$. The density dependence of the decay width is thus due to combined effects of the contributions from the exponential term (decreasing with density), the factor

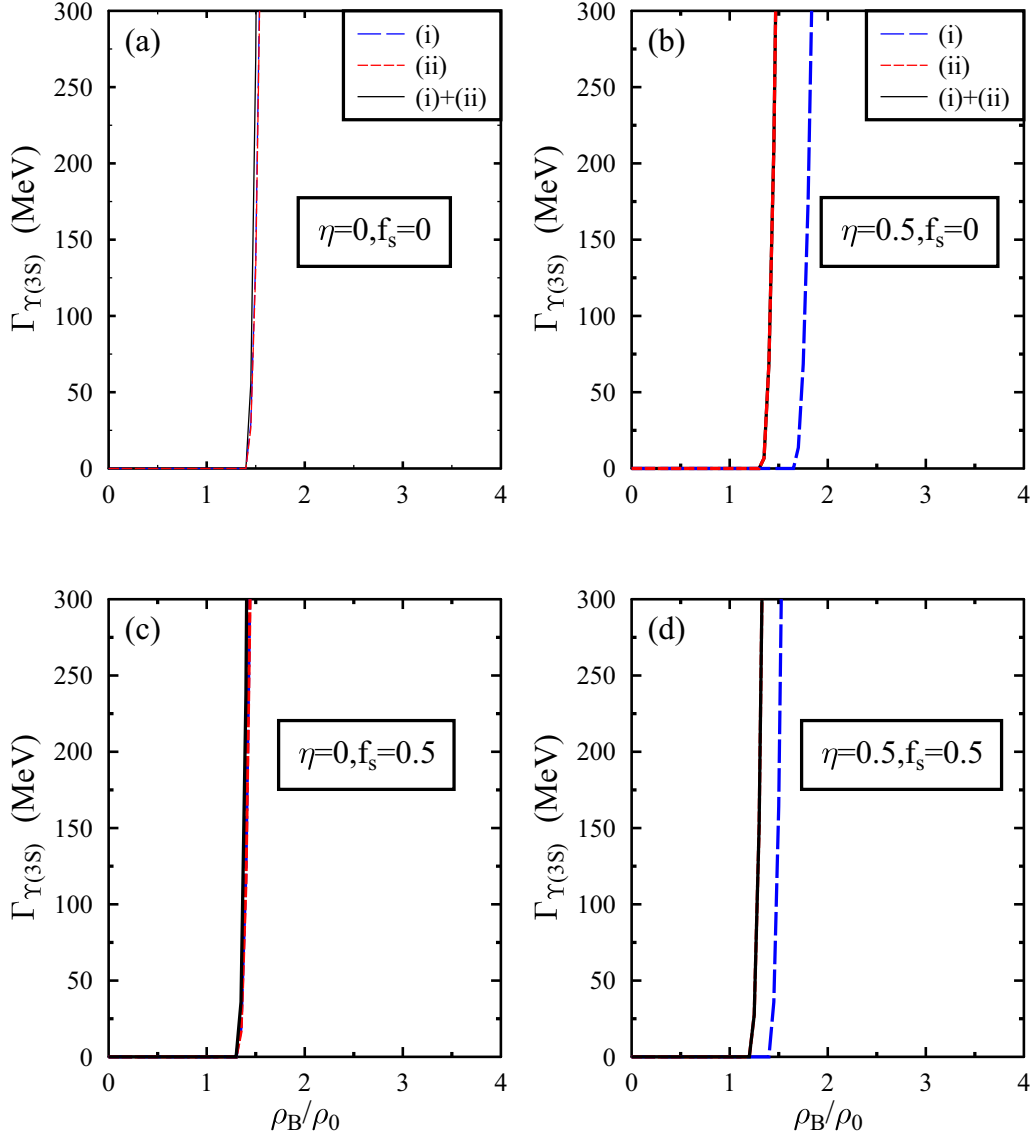


FIG. 3. The partial decay widths of $\Upsilon(3S)$ calculated using the present model for composite hadrons, to (i) B^+B^- , (ii) $B^0\bar{B}^0$, and (iii) the sum of the two channels [(i)+(ii)] in the isospin symmetric (asymmetric) nuclear matter and strange hadronic matter. The overlap of the values of the decay widths in channels (i) and (ii) in isospin symmetric matter ($\eta = 0$) in nuclear and hyperonic matter [shown in (a) and (c), respectively], are due to the mass of B^+ (B^-) being (almost) identical with the mass of B^0 (\bar{B}^0) in the medium, with the negligible difference in the mass arising due to the very small difference in the vacuum masses. In panels (b) and (d), the decay widths are plotted for the isospin asymmetric nuclear and hyperonic matter. In both the symmetric and asymmetric matter, the decay widths are observed to be nonzero above a threshold value, which is observed to be larger for the case of nuclear matter [shown in (a) and (b)] as compared to hyperonic matter [shown in (c) and (d)].

$|\mathbf{p}|^3 p_B p_{\bar{B}}$, multiplying the $A^{\Upsilon(N_S)}(|\mathbf{p}|)^2$ in the expression for the decay width of $\Upsilon(4S) \rightarrow B\bar{B}$, as well as the contribution from the terms within the square bracket in the expression of $A^{\Upsilon(N_S)}(|\mathbf{p}|)$ given by Eq. (41). The observed behavior of the decay width for the process $\Upsilon(4S) \rightarrow B\bar{B}$ in symmetric nuclear matter is thus dominantly due to the contribution of the polynomial part of the decay width, arising from the terms within the square bracket in the expression of $A^{\Upsilon(N_S)}(|\mathbf{p}|)$. The behavior of the decay width of $\Upsilon(4S) \rightarrow B\bar{B}$ with density, is observed to remain similar, with inclusion of hyperons in the medium, as shown in Fig. 4(c), but the densities at

which the decay widths reach maximum values (at densities of $0.9\rho_0$ and $2.5\rho_0$) and vanish (at densities of $1.8\rho_0$ and $3\rho_0$) are observed to be smaller than those for isospin symmetric nuclear matter. The isospin asymmetry effects on the partial decay widths of the process $\Upsilon(4S) \rightarrow B\bar{B}$ in nuclear matter and hyperonic matter are also shown in Figs. 4(b) and 4(d). In asymmetric nuclear matter, the difference in the decay widths of $\Upsilon(4S) \rightarrow B^+B^-$ and $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ are quite pronounced at high densities as can be seen in Fig. 4(b), even though the qualitative features remain the same as for symmetric nuclear matter. The decay width $\Upsilon(4S) \rightarrow B^+B^-$

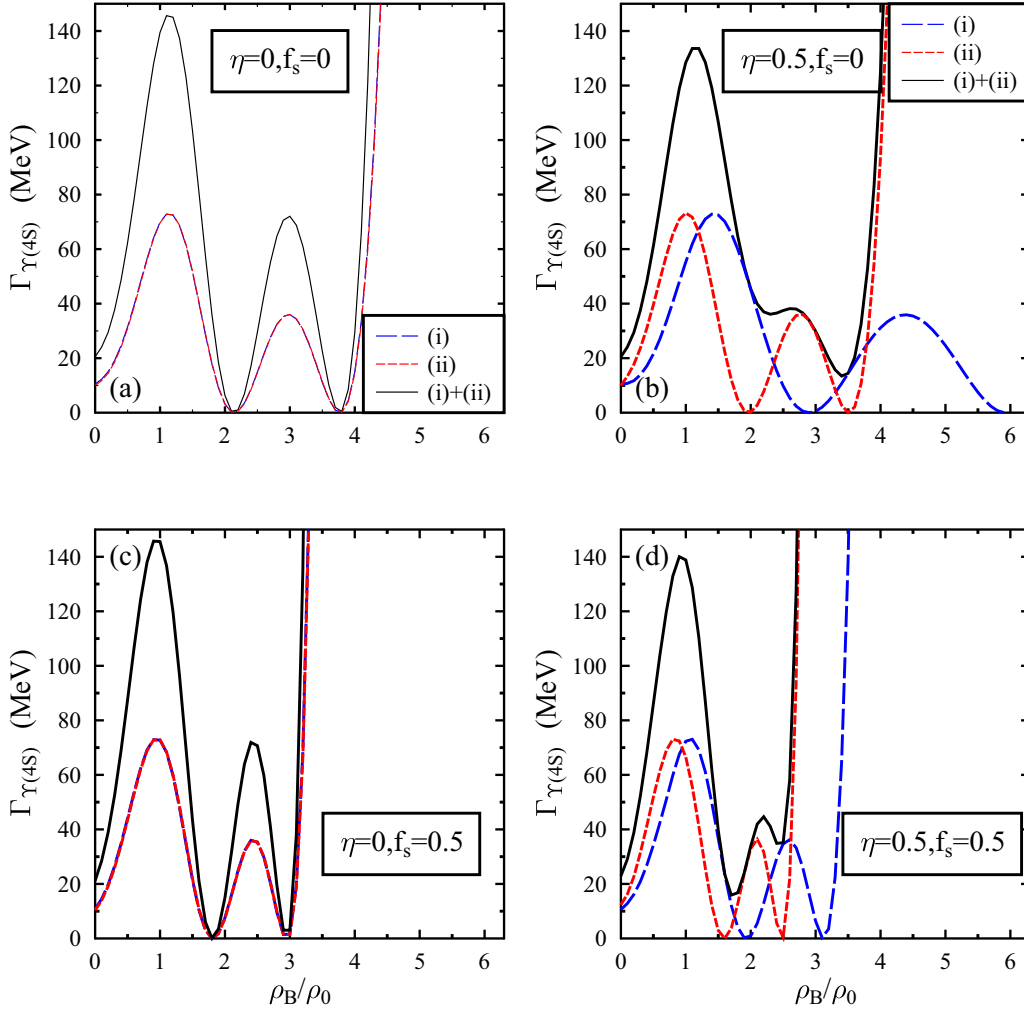


FIG. 4. The partial decay widths of $\Upsilon(4S)$ calculated using the present model for composite hadrons, to (i) B^+B^- , (ii) $B^0\bar{B}^0$, and (iii) the sum of the two channels [(i)+(ii)] in the isospin symmetric (asymmetric) nuclear matter and strange hadronic matter. The decay widths for the channels (i) and (ii) are observed to overlap for the isospin symmetric ($\eta = 0$) for nuclear matter and hyperonic matter [as shown in (a) and (c)] due to the mass of B^+ (B^-) to be close to the mass of B^0 (\bar{B}^0). There is observed to be a rise with density followed by a drop, with the decay widths reaching a value close to zero (the node), at a value of around $2.1\rho_0$ ($1.8\rho_0$) for nuclear (hyperonic) matter. With further increase in density, the value of the decay width is observed to rise, reaching a peak, followed by a drop, leading again to a value close to zero (the second node) at a density of around $3.8\rho_0$ ($3\rho_0$) for nuclear (hyperonic) matter. With further increase in density, there is observed to be a sharp rise in the decay widths. In isospin asymmetric ($\eta = 0.5$) nuclear matter ($f_s = 0$) shown in (b), the behavior of the decay widths in both the channels remain similar, but the peaks and nodes are observed to be at larger densities for channel (i) as compared to channel (ii). With the inclusion of hyperons in the isospin asymmetric medium, these peaks and nodes are observed at smaller densities [as shown in (d)] as compared to asymmetric nuclear matter [as shown in (b)].

$[\Upsilon(4S) \rightarrow B^0\bar{B}^0]$ is observed to rise with increase in density up to a density of around $1.5\rho_0$ (ρ_0) followed by a drop with further increase in density up to around $2.9\rho_0$ ($2\rho_0$) when the decay width is observed to vanish. Above the density at which there is a node, i.e., the decay width attains zero value, with further increase in density, the decay width is observed to again attain a maximum at around $4.4\rho_0$ ($2.8\rho_0$) followed by another node at density of $6\rho_0$ ($3.5\rho_0$) for the decay process $\Upsilon(4S) \rightarrow B^+B^-$ ($\Upsilon(4S) \rightarrow B^0\bar{B}^0$). The effects of isospin asymmetry are observed to be smaller for hyperonic matter [shown in Fig. 4(d)], as compared to the case of nuclear matter [shown in Fig. 4(b)], even though qualitative behavior remains the same. The values of densities at which the decay widths of

$\Upsilon(4S) \rightarrow B^+B^-$ [$\Upsilon(4S) \rightarrow B^0\bar{B}^0$] have maxima are $1.2\rho_0$ ($0.8\rho_0$) and $2.6\rho_0$ ($2.1\rho_0$) and the values where nodes are observed are $1.9\rho_0$ ($1.6\rho_0$) and $3.2\rho_0$ ($2.5\rho_0$), respectively.

As has already been mentioned the in-medium decay widths of the bottomonium states to $B\bar{B}$ have contributions of a polynomial part and an exponential part, in terms of the center of mass momentum, $|\mathbf{p}|$ defined through Eq. (11). This can be seen from the expressions of the partial decay widths of $\Upsilon(NS) \rightarrow B^0\bar{B}^0$ and $\Upsilon(NS) \rightarrow B^+B^-$ given by Eqs. (43) and (44), with $A^{\Upsilon(NS)}(|\mathbf{p}|)$ as in Eq. (41). For the decay of $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$, there is observed to be a sharp rise in the decay width with density, predominantly due to the factor $|\mathbf{p}|^3 p_B^0 p_{\bar{B}}^0$ multiplied to $A^{\Upsilon(NS)}(|\mathbf{p}|)^2$ in the

expressions of the partial decay widths of $\Upsilon(N S) \rightarrow B^0 \bar{B}^0$ and $\Upsilon(N S) \rightarrow B^+ B^-$. For the decay of $\Upsilon(N S) \rightarrow B \bar{B}$, $N = 1, 2, 3$, the effects of density arising from the exponential part as well as from the expression within the square bracket in the expression for $A^{\Upsilon(N S)}(|\mathbf{p}|)$ are observed to be very small. For the partial decay width of $\Upsilon(4 S) \rightarrow B \bar{B}$, there is seen to be an initial rise in the decay width with increase in density, followed by a decrease with further increase in the density leading to a node. As the density is further increased there is a drop again after initial increase in the decay width leading to observation of a second node. The observed behavior of the decay width of $\Upsilon(4 S) \rightarrow B \bar{B}$ is dominantly due to the behavior of the contribution of the terms within the square bracket of the expression for $A^{\Upsilon(N S)}(|\mathbf{p}|)$. In the present investigation of the in-medium decay widths of the bottomonium states, we have accounted for the medium modifications of the masses of the bottomonium states as well as the open-bottom mesons (B and \bar{B} mesons). The finding of nodes for $\Upsilon(4 S) \rightarrow B \bar{B}$ mesons should have observable consequences on the production of the hidden as well as open-bottom mesons in heavy-ion collision experiments.

VI. SUMMARY

In the present paper, we have investigated the partial decay widths of $\Upsilon \rightarrow B \bar{B}$ within a field theoretical model of composite hadrons. The medium modifications of these decay widths arise from the medium changes of the masses of the bottomonium and open-bottom mesons. The masses

of the B and \bar{B} mesons, calculated in an effective chiral model, arise due to their interactions with the baryons and the scalar mesons. On the other hand, the mass modifications of the bottomonium states in the hadronic medium are due to the medium modification of a scalar dilaton field, which is introduced in the effective hadronic model to simulate the gluon condensates of QCD. The effects of isospin asymmetry, strangeness on the bottomonium decay widths for decays $\Upsilon(N S) \rightarrow B \bar{B}$, $N = 1, 2, 3, 4$, have also been studied in the present work. The density effects are seen to be the dominant medium effects, which should have observable consequences from the dense hadronic matter created in heavy-ion collision experiments. The decay width of $\Upsilon(4 S) \rightarrow B \bar{B}$ is observed to show nodes at specific densities, similar to what was observed for the partial decay widths of the excited charmonium states to $D \bar{D}$ pair within 3P_0 model, as well as, within a field theoretical model of composite hadrons with quark and antiquark constituents. The isospin asymmetry effects are observed to be quite pronounced at high densities, leading to quite different values for the decay widths of $\Upsilon(4 S) \rightarrow B^+ B^-$ and $\Upsilon(4 S) \rightarrow B^0 \bar{B}^0$. These isospin asymmetry effects at high densities on these partial decay widths should show in the asymmetric heavy-ion collision experiments in CBM planned at FAIR. However, the study of the bottomonium states will require access to energies higher than the energy regime planned at CBM experiment. The density effects on the partial decay widths $\Upsilon \rightarrow B \bar{B}$ should show up in observables, e.g., the yield of the bottomonium states and the open-bottom mesons, as well as, in the dilepton spectra at the Super Proton Synchrotron (SPS) energies.

-
- [1] A. Hayashigaki, *Phys. Lett. B* **487**, 96 (2000).
 - [2] T. Hilger, R. Thomas, and B. Kämpfer, *Phys. Rev. C* **79**, 025202 (2009).
 - [3] P. A. M. Guichon, *Phys. Lett. B* **200**, 235 (1988); X. Jin and B. K. Jennings, *Phys. Rev. C* **55**, 1567 (1997); K. Saito and A. W. Thomas, *Phys. Lett. B* **327**, 9 (1994).
 - [4] P. K. Panda, A. Mishra, J. M. Eisenberg, and W. Greiner, *Phys. Rev. C* **56**, 3134 (1997).
 - [5] K. Tsushima, D. H. Lu, A. W. Thomas, K. Saito, and R. H. Landau, *Phys. Rev. C* **59**, 2824 (1999).
 - [6] A. Sibirtsev, K. Tsushima, and A. W. Thomas, *Eur. Phys. J. A* **6**, 351 (1999).
 - [7] L. Tolos, J. Schaffner-Bielich, and A. Mishra, *Phys. Rev. C* **70**, 025203 (2004).
 - [8] L. Tolos, J. Schaffner-Bielich, and H. Stöcker, *Phys. Lett. B* **635**, 85 (2006).
 - [9] T. Mizutani and A. Ramos, *Phys. Rev. C* **74**, 065201 (2006).
 - [10] L. Tolos, A. Ramos, and T. Mizutani, *Phys. Rev. C* **77**, 015207 (2008).
 - [11] J. Hofmann and M. F. M. Lutz, *Nucl. Phys. A* **763**, 90 (2005).
 - [12] S. Yasui and K. Sudoh, *Phys. Rev. C* **87**, 015202 (2013).
 - [13] C. E. Fontoura, G. Krein, and V. E. Vizcarra, *Phys. Rev. C* **87**, 025206 (2013).
 - [14] H. Mishra and S. P. Misra, *Phys. Rev. D* **48**, 5376 (1993).
 - [15] A. Mishra and S. P. Misra, *Z. Phys. C* **58**, 325 (1993).
 - [16] A. Mishra, H. Mishra, and S. P. Misra, *Z. Phys. C* **57**, 241 (1993).
 - [17] A. Mishra, H. Mishra, V. Sheel, S. P. Misra, and P. K. Panda, *Int. J. Mod. Phys. E* **05**, 93 (1996).
 - [18] A. Mishra and H. Mishra, *Phys. Rev. D* **69**, 014014 (2004).
 - [19] A. Mishra and A. Mazumdar, *Phys. Rev. C* **79**, 024908 (2009).
 - [20] A. Kumar and A. Mishra, *Phys. Rev. C* **81**, 065204 (2010).
 - [21] A. Kumar and A. Mishra, *Eur. Phys. J. A* **47**, 164 (2011).
 - [22] D. Pathak and A. Mishra, *Phys. Rev. C* **91**, 045206 (2015).
 - [23] S. H. Lee and C. M. Ko, *Phys. Rev. C* **67**, 038202 (2003).
 - [24] S. Kim, S. H. Lee, *Nucl. Phys. A* **679**, 517 (2001).
 - [25] A. Kumar and A. Mishra, *Phys. Rev. C* **82**, 045207 (2010).
 - [26] A. Mishra and D. Pathak, *Phys. Rev. C* **90**, 025201 (2014).
 - [27] S. P. Misra, *Phys. Rev. D* **18**, 1661 (1978).
 - [28] S. P. Misra, *Phys. Rev. D* **18**, 1673 (1978).
 - [29] S. P. Misra and L. Maharana, *Phys. Rev. D* **18**, 4103 (1978).
 - [30] A. Mishra, S. P. Misra and W. Greiner, *Int. J. Mod. Phys. E* **24**, 1550053 (2015).
 - [31] B. Friman, S. H. Lee, and T. Song, *Phys. Lett. B* **548**, 153 (2002).
 - [32] A. L. Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, *Phys. Rev. D* **8**, 2223 (1973).
 - [33] A. L. Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, *Phys. Rev. D* **9**, 1415 (1974).
 - [34] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, *Phys. Rev. D* **11**, 1272 (1975).
 - [35] T. Barnes, F. E. Close, P. R. Page, and E. S. Swanson, *Phys. Rev. D* **55**, 4157 (1997).

- [36] S. P. Misra, K. Biswal, and B. K. Parida, *Phys. Rev. D* **21**, 2029 (1980).
- [37] P. Papazoglou, D. Zschesche, S. Schramm, J. Schaffner-Bielich, H. Stöcker, and W. Greiner, *Phys. Rev. C* **59**, 411 (1999).
- [38] S. Weinberg, *Phys. Rev.* **166**, 1568 (1968).
- [39] S. Coleman, J. Wess, and B. Zumino, *Phys. Rev.* **177**, 2239 (1969).
- [40] C. G. Callan, S. Coleman, J. Wess, and B. Zumino, *Phys. Rev.* **177**, 2247 (1969).
- [41] W. A. Bardeen and B. W. Lee, *Phys. Rev.* **177**, 2389 (1969).
- [42] D. Zschesche, A. Mishra, S. Schramm, H. Stöcker, and W. Greiner, *Phys. Rev. C* **70**, 045202 (2004).
- [43] A. Mishra, K. Balazs, D. Zschesche, S. Schramm, H. Stöcker, and W. Greiner, *Phys. Rev. C* **69**, 024903 (2004).
- [44] A. Mishra, E. L. Bratkovskaya, J. Schaffner-Bielich, S. Schramm, and H. Stöcker, *Phys. Rev. C* **69**, 015202 (2004).
- [45] J. Schechter, *Phys. Rev. D* **21**, 3393 (1980).
- [46] T. D. Cohen, R. J. Furnstahl, and D. K. Griegel, *Phys. Rev. C* **45**, 1881 (1992).
- [47] F. Klingl, S. Kim, S. H. Lee, P. Morath, and W. Weise, *Phys. Rev. Lett.* **82**, 3396 (1999).
- [48] M. E. Peskin, *Nucl. Phys. B* **156**, 365 (1979).
- [49] J. Beringer *et al.* (Particle Data Group), *Phys. Rev. D* **86**, 010001 (2012).
- [50] R. V. Royen and V. F. Weiskopf, *Nuovo Cimento* **50**, 617 (1967).
- [51] S. P. Misra, S. Naik and A. R. Panda, *Pramana J. Phys.* **28**, 131 (1987).