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Multiplicity derivative: A new signature of a first-order phase transition in intermediate-energy heavy-ion collisions

S. Mallik,[1,2] G. Chaudhuri,[1,2] P. Das,[1,2] and S. Das Gupta[3]
[1] *Physics Group, Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700064, India*
[2] *Homi Bhabha National Institute, Training School Complex, Anushakti Nagar, Mumbai 400085, India*
[3] *Physics Department, McGill University, Montréal, Canada H3A 2T8*

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Measurement of M, the total multiplicity, for central collision between comparable mass heavy ions can provide a signature for first-order phase transition. The derivative of M with respect to E^*/A, where E^* is the excitation energy in the center of mass and A is the total mass of the dissociating system, is expected to go through maximum as a function of E^*. Theoretical modeling shows that this is the energy where the specific heat C_v maximizes, which typically happens at the first-order phase transition. The measurement of total M is probably feasible in more than one laboratory.

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Introduction. In this Rapid Communication, we suggest experiments which can provide evidence (or absence of evidence) for first-order phase transition in intermediate-energy heavy-ion collisions. Phase transitions occur in large systems and signatures of phase transition can be masked by finite sizes. In nuclear physics, the Coulomb interaction prevents formation of very large systems in the laboratory. In addition to limiting the size of nuclei, Coulomb effects further corrupt signatures of phase transition. If finite size and Coulomb effects totally mask the signature of phase transition, then no definite conclusions can be reached from the data. We suggest that the situation is not that ambiguous.

In a seminal paper, Gulminelli and Chomaz pointed out that just the effect of finite size will cause bimodality to appear in the mass distribution of composites for a first-order transition [1]. In heavy-ion collisions (HIC), many composites are produced. Let us denote by $P_m(k)$ the probability that in the mass distribution, the composite with mass k appears as the maximum mass. One can plot $P_m(k)$ as a function of k. In the case of first-order phase transition, finite size produces two maxima at two different values of k. The energy at which the two maxima achieve the same value defines the energy of the bimodal point. In thermodynamic models, instead of energy, the primary variable is the temperature. We could talk about the temperature of the bimodal point. The range of energy or temperature where two maxima are seen can be called the bimodal region. It is a small region. Bimodality has appeared in many calculations. It was shown to appear in the Boltzmann-Uehling-Uhlenbeck (BUU) transport model of central collisions between two equal ions with Coulomb forces switched off [2]. It appeared in quantum molecular dynamics calculation [3]. It is seen in the canonical thermodynamic model [4–6].

The question we ask is as follows: If the corruptive effects of Coulomb interaction is so strong that bimodality is destroyed, is there any other observable that points to vestiges of a first-order phase transition? Our answer is yes. We use the canonical thermodynamic model (CTM) [7] to establish our claim. But we need first to turn to bimodality in CTM without and with Coulomb interaction.

Two different microcanonical versions employing similar physics as CTM are the statistical multifragmentation model (SMM) by the Copenhagen group [8] and the microcanonical metropolis Monte Carlo (MMMC) by the Berlin group [9]. All these models [7–9] were very successfully used to fit many data in HIC. Results from CTM and SMM have been found to be very close [10]. Here we use CTM. We will skip all calculational details of CTM as they can be found in many places. Composites carry charge and the long-range Coulomb interactions between composites are included in Wigner-Seitz approximation [8] in SMM and are also adopted in CTM. We will use results from a previous calculation and, in particular, Fig. 1 of Ref. [5]. The example studied dissociation of a system with N and Z equal to 75. The Coulomb effects were studied by varying the strength of Coulomb interaction using a multiplicative factor x_c. $x_c = 0$ means no Coulomb interaction, and $x_c = 1$ means the actual strength of Coulomb force. An intermediate value of x_c means a reduced value of Coulomb interaction. The lesson that we learn from that work is this: For $x_c = 0$, bimodality appears. In addition, the specific heat c_v hits a maximum value at the bimodal point. For small values of x_c, the bimodality region is shrinking and the maximum value of c_v is close to the bimodal temperature but not identical. Bimodality disappears before reaching $x_c = 1$, but the usual behavior of c_v reaching a maximum at phase transition temperature continues until $x_c = 1$. So if we could measure the c_v, we would see vestiges of first-order phase transition even with the usual Coulomb force. But since measuring c_v is not a practical suggestion, is there some other measurable quantity that also maximizes when c_v does? Theoretical modeling predicts that the derivative of total multiplicity with respect to temperature displays a maximum which coincides with the maximum of c_v. This is shown in the next section. Since temperature increases with increasing beam energy, the maximum can be located in experiments.

Results. In central collisions of nearly equal mass ions, one can measure with 4π detectors the total multiplicity $M = \sum M_a$. Here a denotes the mass numbers of composites. In CTM, the derivative of M with T as a function of T is seen to have a maximum. Figure 1 (left panel) shows the total multiplicity for fragmenting system having proton number $(Z) = 82$, neutron number $(N) = 126$, and its derivative dM/dT (the right panel). Results for both real nuclei and for

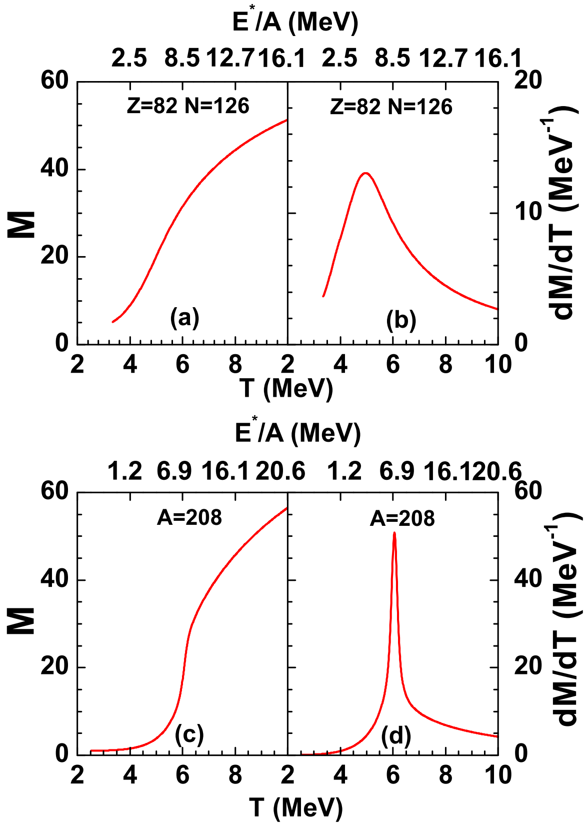

FIG. 1. Variation of multiplicity M (left panels) and dM/dT (right panels) with temperature (bottom x axes) and excitation per nucleon (top x axes) from the CTM calculation for fragmenting systems having $Z = 82$ and $N = 126$ (top panels). Bottom panels represent the same but for a hypothetical system of one kind of particle with no Coulomb interaction but the same mass number ($A = 208$). E^* is $E - E_0$, where E_0 is the ground-state energy of the dissociating system in the liquid drop model whose parameters are given in Ref. [7].

one kind of particle have been displayed in order to emphasize the effects of Coulomb interaction. The rise and the peak are much sharper in the absence of Coulomb interaction, clearly indicating the role of the long-range interaction. As the system size decreases (Fig. 2), the features become less sharp, as in $Z = 28$ and $N = 30$. The peak in dM/dT coincides with the maximum of specific heat at constant volume C_v as a function of temperature, and this is seen in Figs. 3 and 4 for $Z = 82$, $N = 126$ and $Z = 28, N = 30$ respectively. Of course, experiments do not give T directly but a plot against E^*/A will also show a nearly coincident maximum (see Figs. 1 and 2). The peak in C_v is a signature of first-order phase transition. In dM/dT, we have the peak coinciding with that of C_v and hence we are proposing it as a new method for testing the occurrence of first-order phase transition in HIC. Even where bimodality develops, it may be easier to locate the position of the maximum in the derivative of M since the bimodal region is very narrow.

It is also worth mentioning that near the maximum of dM/dT, the entropy of the dissociating system makes a higher jump than is seen far from it. This is also shown in Fig. 5. For

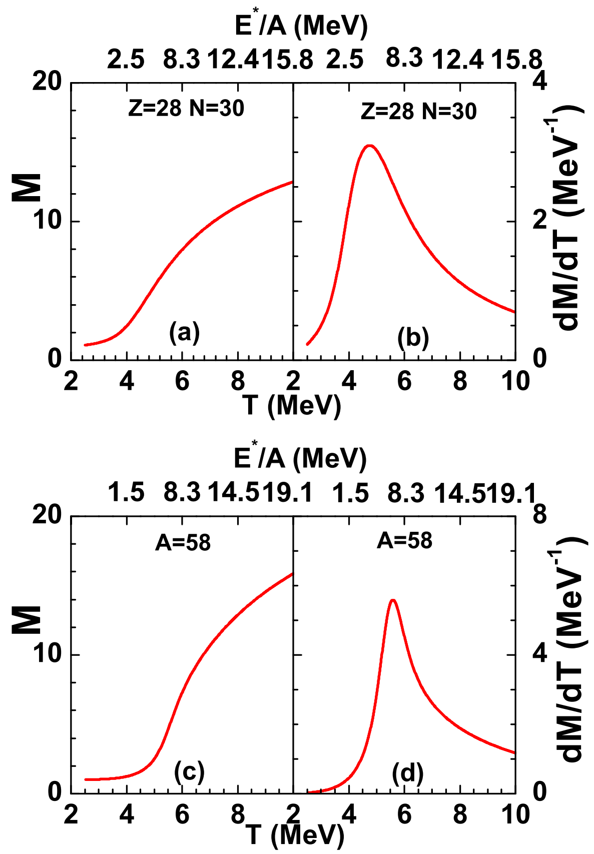

FIG. 2. Same as Fig. 1 but the fragmenting systems are $Z = 28$ and $N = 30$ (top panels) and $A = 58$ (bottom panels).

the hypothetical (one-particle) system, the increase in entropy near the maximum of dM/dT is much more pronounced (lower panel), while the Coulomb effect smears the rise in the real system (upper panel).

It is well known that composites from CTM are excited and hence will undergo sequential two-body decay [11], which will change the total multiplicity. We have examined this and found

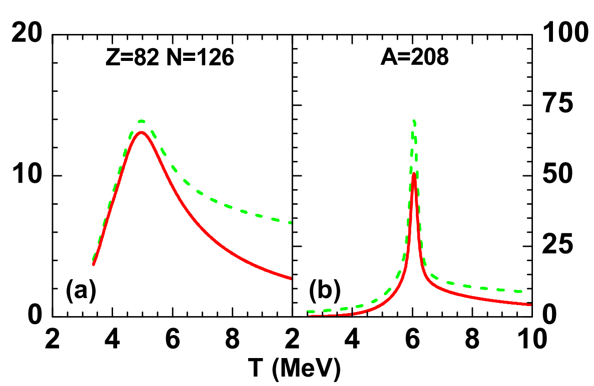

FIG. 3. Variation of dM/dT (red solid lines) and C_v (green dashed lines) with temperature from CTM for fragmenting systems having $Z = 82$ and $N = 126$ (left panel) and for hypothetical systems of one kind of particle with no Coulomb interaction of mass number $A = 208$. To draw dM/dT and C_v in the same scale, C_v is normalized by a factor of $1/50$.

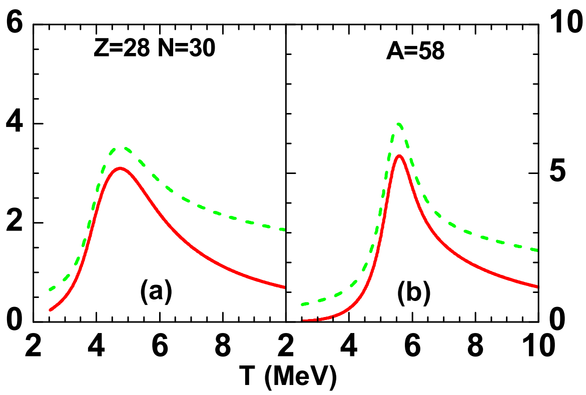

FIG. 4. Same as in Fig. 3, but the fragmenting systems are $Z = 28$ and $N = 30$ (left panel) and $A = 58$ (right panel).

that this will not alter our conclusions. In fact, sequential decay makes the peak in dM/dT sharper. This is shown in Fig. 6.

Lastly, we have examined the features of intermediate-mass fragments (composites with charge $3 \leqslant z \leqslant 20$) and it is observed that similar behavior is also displayed by M_{IMF} and its derivative, as shown in Fig. 7. It was shown earlier for an idealized system with one kind of particles that there is a dramatic increase in M_{IMF} in a short temperature interval [12]. For the sake of completeness, we have also shown here how M_{IMF} behaves with temperature with Coulomb interaction included. The peak in the derivative does not exactly coincide with that of C_v and this is expected since in M_{IMF} all the composites and nucleons are not included which are used in

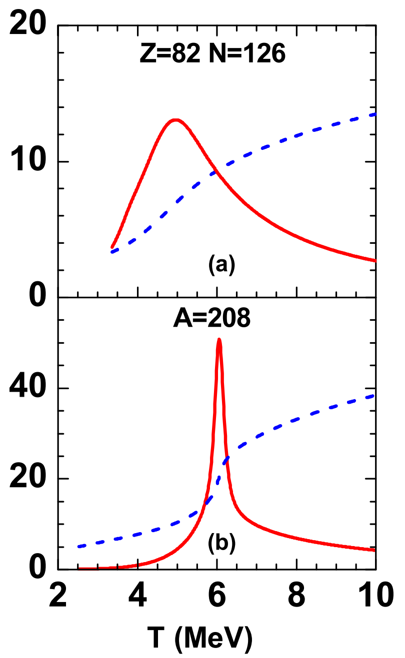

FIG. 5. Variation of entropy (blue dashed lines) and dM/dT (red solid lines) with temperature from CTM for fragmenting systems having $Z = 82$ and $N = 126$ (top panel) and for hypothetical system of one kind of particle with no Coulomb interaction of mass number $A = 208$ (bottom panel). To draw S and dM/dT in the same scale, S is normalized by a factor of $1/20$ for $Z = 82$ and $N = 126$ system and $1/50$ for hypothetical system of one kind of particle.

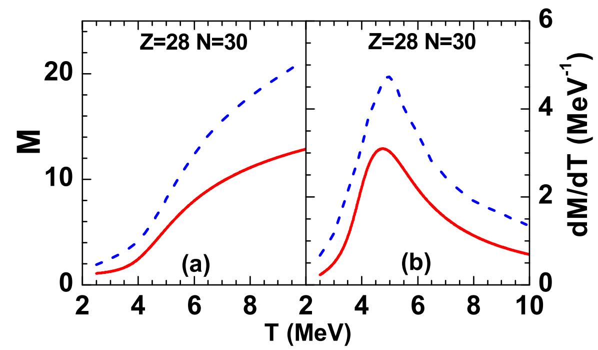

FIG. 6. Effect of secondary decay on M (left panel) and dM/dT (right panel) for fragmenting systems having $Z = 28$ and $N = 30$. Red solid lines show the results after the multifragmentation stage (calculated from CTM), whereas blue dashed lines represent the results after secondary decay of the excited fragments.

the calculation of C_v. M_{IMF} is also an important experimental observable which is measured in many situations [13–15] instead of the total multiplicity M. However, prescription of considering full M and its derivative is more precise in locating the position of the maximum of c_v, which signifies that we are at first-order phase transition.

Discussions. Establishing evidence for phase transitions in nuclear matter from data obtained from intermediate-energy heavy-ion collisions has attracted much attention in the past twenty years. Here we have used measurable dM/dE and dM_{IMF}/dE as evidence for first-order phase transition should a maximum be seen. The answer is unambiguous: It is either yes or no. Most past investigations have suffered from ambiguity. An example was trying to fit an individual M_a to $a^{-\tau} f(a^{\sigma}(T - T_c))$ [16–20]. Equally acceptable but quite approximate fits were found with very different models, so no conclusions could be made. One model that predicted first-order phase transition was the lattice gas model [21], but the property of M was not investigated. It will be interesting to pursue that.

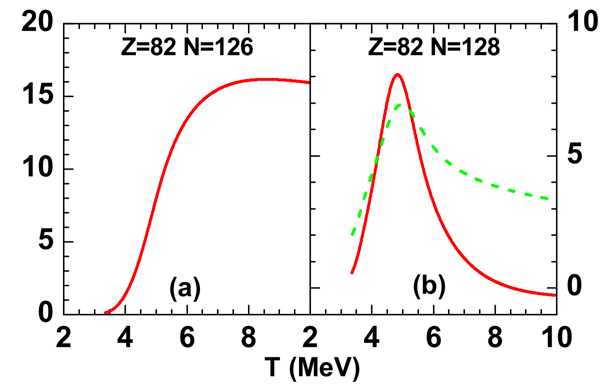

FIG. 7. Variation of intermediate-mass fragment (IMF) multiplicity M_{IMF} (left panels) and first-order derivative of IMF multiplicity dM_{IMF}/dT (right panels) with temperature from CTM calculation for fragmenting systems having $Z = 82$ and $N = 126$. Variation of C_v with temperature (T) is shown by green dashed line in right panel. To draw dM_{IMF}/dT and C_v in the same scale, C_v is normalized by a factor of $1/100$.

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