Reexamination of fission in the $A \approx 200$ mass region with excitation energy near 50 MeV

K. Mahata[1,2,*] and S. Kailas[1,3,†]
[1]*Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai 400 085, India*
[2]*Homi Bhabha National Institute, Anushakti Nagar, Mumbai 400 094, India*
[3]*Department of Atomic Energy Centre for Excellence in Basic Sciences & University of Mumbai, Mumbai 400 098, India*
(Received 26 December 2016; published 19 May 2017)

Even though the fission of nuclei in the mass region 200 with excitation energy near 50 MeV has been studied extensively, a unique description of the fission probability and prefission neutron multiplicity ($\nu_{\rm pre}$) data remains elusive. In the present work, a reexamination of the relevant data along with a new estimate of $\nu_{\rm pre}$ and fission chance distributions, obtained from the experimental fission excitation functions of neighboring Po isotopes, has been carried out. The $\nu_{\rm pre}$ from the above-mentioned method, sensitive to only the presaddle part, is significantly lower than the value obtained from neutron spectra measurements. Further, $\nu_{\rm pre}$ from the fission chance data is in good agreement with the statistical model predictions, which also accounts for the light-ion induced fission probability data up to low excitation energy (~30 MeV). From this observation, it is concluded that the presaddle dynamical effects are not significant over this excitation energy range, and the $\nu_{\rm pre}$ data determined from the neutron spectra might have a significant contribution from the near-scission emission.

DOI: 10.1103/PhysRevC.95.054616

I. INTRODUCTION

Understanding fission of a nucleus, particularly in the mass region ~ 200, continues to be challenging problem. Study of fission has relevance in studies related to superheavy elements, stellar nucleosynthesis, and nuclear energy applications. Both statistical and dynamical models are used to interpret fission observables. In both models, the potential energy surface and the nuclear level densities are the key ingredients. In general the potential energy has a macroscopic (liquid drop) part and a microscopic (shell correction) part. The nuclear level density also depends on shell correction and deformation. Even though significant progress has been made in the understanding of the fission process, there are ambiguities in choosing the parameters of the theoretical models [1–4].

Simultaneous statistical model analysis of the heavy-ion induced fission excitation function and prefission neutron multiplicity ($\nu_{\rm pre}$) data for ^{210}Po ($E^* \sim 40$–60 MeV) required lowering the fission barrier (saddle point) [2,5]. The correction due to dynamical emission with a delay of 30 zs (1 zs = 10^{-21} s) was estimated to be not so significant at these energies [2]. However, the light-ion induced fission excitation functions could not be reproduced with the same fission barrier. The statistical model calculation with the value of fission barrier [6] determined from the p and α induced fission excitation functions for ^{210}Po underpredicts the $\nu_{\rm pre}$ data extracted from the measured neutron energy spectra [5].

In a recent Letter [1], the results of calculations for fission observables using an advanced four-dimensional (4D) Langevin code [7,8] for ^{12}C + ^{194,198}Pt systems have been reported. The authors claim to have reasonably reproduced the experimental fission probability and the $\nu_{\rm pre}$ data along with other observables without incorporating a shell correction in the potential energy surface. They have also suggested that the differences between the results of the statistical model [2,5] and their results are not due to dynamical effects, and that these two sets of calculations differs in some statistical model parameters (level density parameters, particle decay width, etc.).

It seems that the experimental masses were used to calculate the excitation energy at the equilibrium deformation in Ref. [1]. So the potential energy surface at the equilibrium deformation was fixed at the experimental mass, thus having microscopic corrections. Now if one uses only the macroscopic part of the potential to describe the change in the potential as a function of deformation, it amounts to lowering the whole surface with the same microscopic corrections as that at the equilibrium deformation, rather than having no microscopic corrections (see Fig. 1). If one wants to assume that microscopic corrections are completely washed out, the potential of the equilibrium deformation should be raised from the experimental mass to the liquid drop mass. In other words, the liquid drop mass of the compound nucleus [9] or the experimental mass with back shifting [10] should be used in order to do calculations using only the macroscopic potential energy. One should also keep in mind that the ground state shell corrections (Δ_n) in this mass region are large ($\Delta_n = -10.6$ MeV for ^{210}Po), which can influence the fission as well as the particle emission widths.

As mentioned in Ref. [1], the calculations in the 1970s [10–12] could obtain consistent descriptions of experimental fission cross sections in this mass region without any shell correction at the saddle point. However, those calculations have not considered the multi-chance nature of fission. In other words, they have assumed that the fissions following neutron emissions or the prefission neutron multiplicities are negligible. The model used in Refs. [2,5] also predicted smaller $\nu_{\rm pre}$, if no shell correction at the saddle point is assumed. Hence the results in Refs. [2,5] are consistent with the results in the 1970s [10–12]. However, the experimentally measured $\nu_{\rm pre}$ data in heavy-ion induced fission were found to be

*kmahata@barc.gov.in
†Raja Ramanna Fellow

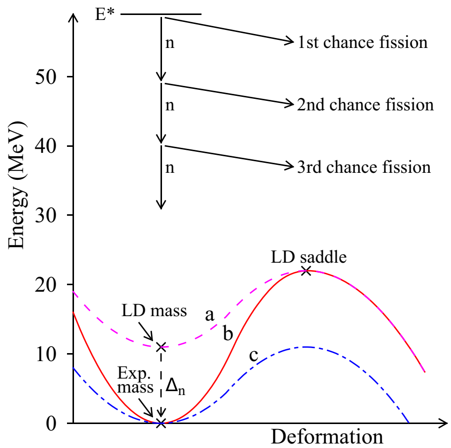

FIG. 1. A schematic representation of the potential energy as a function of deformation in mass $A \approx 200$. Different reference surfaces frequently used in the statistical model calculations are marked as "a", "b", and "c". Dominant deexcitation modes of a excited compound nucleus in this mass region are also shown by vertical and slanting arrows for neutron emission and fission, respectively.

significantly large even at an excitation energy of 50 MeV [5]. Simultaneous statistical model analysis of the heavy-ion induced fission excitation function and $\nu_{\rm pre}$ data requires lowering the fission barrier (saddle point) [2,5]. In Ref. [2], the $\nu_{\rm pre}$ data were corrected for the emission of dynamical neutrons corresponding to an assumed fission delay of 30 zs, taken from the literature [13]. It seems the saddle point has also been lowered in the advanced 4D Langevin calculation presented in Ref. [1].

In an earlier work [6], it was observed that both the heavy-ion and light-ion induced fission excitation functions for the compound nucleus ^{210}Po could be simultaneously explained. However, they failed to reproduce the experimental $\nu_{\rm pre}$ data. It was concluded that the experimental $\nu_{\rm pre}$ data may have a contribution from nonstatistical processes and that the experimental $\nu_{\rm pre}$ data should not be used in a statistical model to determine the fission barrier. The mechanism of prefission neutron emission at these lower excitation energies (~50 MeV) in this mass region should be investigated further.

II. STATISTICAL MODEL ANALYSIS

In order to highlight the sensitivity of the fission observables to the potential energy surface, we have carried out statistical model calculations with three different options (see Fig. 1): option "a" uses the liquid drop mass (M_{LD}) and liquid drop fission barrier (B_{LD}), "b" the experimental mass ($M_{\rm exp} = M_{LD} + \Delta_n$) along with a damping of the shell correction at the ground state (Δ_n) with excitation energy and shell corrected fission barrier ($B_{LD} - \Delta_n$), and "c" the experimental mass and liquid drop fission barrier.

Option "a" can be used at sufficiently high energy, when microscopic corrections are expected to wash out. This option uses the liquid drop potential energy for the equilibrium deformation as well as for the saddle deformation. Option "b" should be used for intermediate excitation energy, where the microscopic corrections are expected to be weaker than those at the ground state but not washed out completely. This option uses the shell corrected potential energy for the equilibrium deformation and the liquid drop potential energy for the saddle deformation. It is assumed that there is no shell correction at the saddle deformation and the saddle point coincides with the liquid drop saddle point. Analysis of the low energy light-ion induced fission excitation functions, which are more sensitive to the fission barrier and hence to the shell correction at the saddle point, yielded negligibly small shell correction at the saddle point [6]. The gradual washing out of the shell corrections at the equilibrium deformation is taken into account according to the Ignatyuk prescription [14] with an energy dependent damping factor (η) [6]. Option "c" is often used in the analysis of fission at intermediate and high excitation energy. As shown in Fig. 1, the saddle point in option "c" is lower by Δ_n as compared to the liquid drop saddle point. This is a special case of the more general prescription ($B_f = B_{LD} - \Delta_n + \Delta_f$) used in Refs. [2,5,6,15], where Δ_f is the shell correction (if any) at the saddle point. The option "c" can be arrived at with $\Delta_f = \Delta_n$. This option is often considered by many as a purely macroscopic calculation, which is not true.

The statistical model calculations have been carried out using a modified version of the statistical model code PACE [16]. According to the statistical model of compound nucleus decay, all possible decays are intrinsically equally likely and are governed by factors such as the relative density of levels (phase space) and transmission coefficients. In the present case, the most dominant mode of decay is sequential (3–5) neutron emission leading to ($3n$–$5n$) evaporation residue formation. As shown in Fig. 1, fission occurs from the initial compound nucleus excitation energy (first chance fission) or after emission of one or more neutrons (later chance fission). In the case of fission of nuclei with sufficient excitation energy above the fission barrier, as in the present case, quantum tunneling does not play a significant role and has not been considered. The expression used for the level density is the same as that given in Ref. [6], with entropy as given below. At the equilibrium deformation, the value of entropy, S_n, is calculated as

$$
\begin{aligned}
S_n^2 &= 4\tilde{a}_n[E_x + \Delta_n] && \text{for option "a"},\\
&= 4\tilde{a}_n[E_x + \Delta_n(1 - e^{-\eta E_x})] && \text{for option "b"},\\
&= 4\tilde{a}_n E_x && \text{for option "c"}.
\end{aligned}
$$

At the saddle point deformation, the value of entropy is calculated as

$$
\begin{aligned}
S_f^2 &= 4\tilde{a}_f[E_x + \Delta_n - B_{LD}] && \text{for option "a"},\\
&= 4\tilde{a}_f[E_x - (B_{LD} - \Delta_n)] && \text{for option "b"},\\
&= 4\tilde{a}_f[E_x - B_{LD}] && \text{for option "c"},
\end{aligned}
$$

where $E_x = E^* - \delta_p - E_{\rm rot}(J)$. Here, δ_p and $E_{\rm rot}(J)$ are the pairing energy and the rotational energy, respectively. As can be seen from the above expressions, while option "a" is an approximation of the realistic option "b" in the high energy limit, option "c" is an incorrect implementation of option "a". The asymptotic value of the level density parameter at the

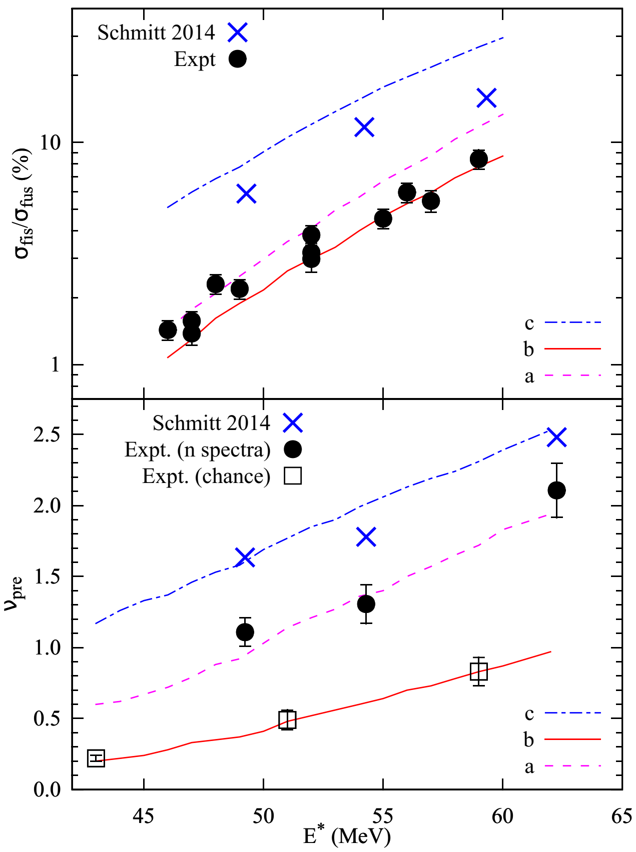

FIG. 2. Statistical model predictions for fission (top panel) and prefission neutron multiplicity (bottom panel) with options "a" (dashed line), "b" (continuous line), and "c" (dot-dashed line) are compared with the experimental data and the prediction of the advanced 4D Langevin code (Schmitt 2014 [1]) for the $^{12}C + ^{198}Pt$ system. The prefission neutron multiplicity obtained from the fission chance distribution (see text) for ^{212}Po is also shown as an open rectangle.

equilibrium deformation ($\tilde{a_n}$) is taken as $A/9$. The ratio of the asymptotic value of the level density parameter at the saddle deformation to that at the equilibrium deformation ($\tilde{a_f}/\tilde{a_n}$) is taken as 1.018, which was found to give the best fit to the data for the $\alpha + ^{206}Pb$ system [6]. Shell corrections at the ground state (Δ_n) are taken from Ref. [17]. The angular momentum dependent macroscopic (liquid drop) part of the fission barrier [$B_{LD}(J)$] and $E_{rot}(J)$ are taken from the rotating finite range model (RFRM) [18].

As can be seen from the Fig. 2, the statistical model calculation with option "a" reproduces both the experimental fission probabilities and the prefission neutron multiplicity (ν_{pre}) data [5,19,20]. Calculation with option "b" reproduces the experimental fission probabilities. However, it fails to reproduce the experimental ν_{pre} data. Calculation with option "c" overpredicts both the experimental fission probabilities and ν_{pre} data. However, they are in good agreement with the results obtained from the advanced 4D Langevin dynamical calculation [1]. The dynamical calculation of Schmitt et al. [1] also overpredicts both the experimental fission probabilities and ν_{pre} data over the entire energy range, contrary to their

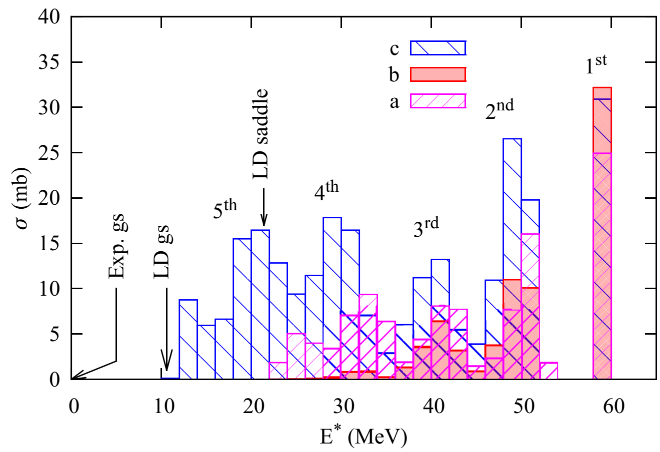

FIG. 3. Statistical model predictions of the excitation energy of the nuclei undergoing fission arising from the decay of compound nuclei with same excitation energy (59 MeV) after one or more neutron emissions for the $^{12}C + ^{198}Pt$ system with different options for the potential energy surface (see text).

claims of reasonable agreement. Good agreement between the results of the dynamical calculation [1] and the present statistical model with similar values of the fission barrier (option "c") and level density parameters indicates that the dynamical contribution in the results of the dynamical calculation [1] was not significant. It should be mentioned here that the low energy light-ion induced fission excitation functions of ^{210}Po could be reproduced by the statistical model with option "b" only [6], and the corresponding barrier height is in good agreement with that predicted by the macroscopic-microscopic finite range liquid-drop model [21].

III. MULTICHANCE FISSION AND NEUTRON MULTIPLICITY

In order to investigate further, we have studied the multichance nature of fission. Figure 3 shows the distribution of excitation energy of the nuclei undergoing fission arising from the decay of the compound nuclei with same initial excitation energy (59 MeV) after one or more neutron emissions for the $^{12}C + ^{198}Pt$ system. In the case of option "c", the saddle point is lower than the liquid drop saddle point. As a consequence, fission occurs even at energies below the liquid drop saddle point, resulting in higher ν_{pre} as compared to the other options. Like most of the models, the present statistical model calculations do not consider barrier tunneling for fission. As far as the level density at the saddle point is concerned, the options "a" and "b" are the same. However, the level density at equilibrium deformation falls off slowly with decrease in excitation energy when a realistic continuous damping of shell effect is considered (option "b") as compared to complete washing out of the shell effect (option "a"). Hence, as excitation energy decreases, neutron emission is more favored over fission in option "b" as compared to that in option "a". This reduces the higher chance fission probabilities in the case of option "b" as compared to that in the case of option "a", leading to lower ν_{pre} with option "b".

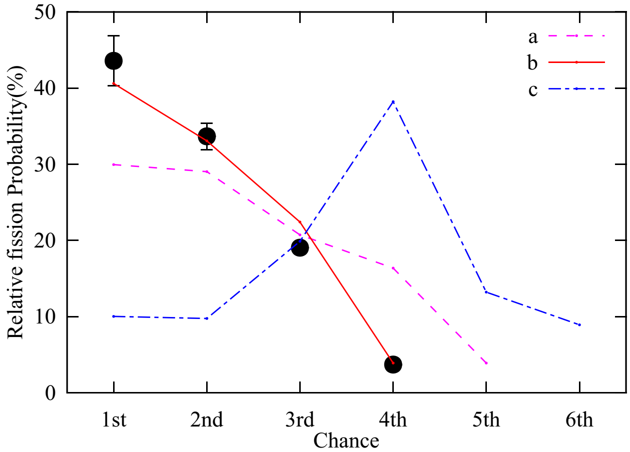

FIG. 4. Relative fission probabilities as a function of chance for ^{212}Po at $E^* = 59$ MeV, extracted from the experimental fission excitation functions of ^{3,4}He + ^{206,207,208}Pb [22], are compared with the statistical model predictions with different options for potential energy surface (see text).

We have also extracted the fission chance distribution and $\nu_{\rm pre}$ in fission of ^{212}Po at $E^* = 59$ MeV from the experimental fission excitation functions for ^{3,4}He + ^{206,207,208}Pb systems [22]. The first chance fission probability is obtained from the observed cumulative fission probabilities $P_{\rm obs}$ of neighboring isotopes: ^{212}Po at an excitation energy E_0 and ^{211}Po at $E_1 = E_0 - S_n - 2T$ as

$$P(^{212}{\rm Po}, E_0) = \frac{P_{\rm obs}(^{212}{\rm Po}, E_0) - P_{\rm obs}(^{211}{\rm Po}, E_1)}{1 - P_{\rm obs}(^{211}{\rm Po}, E_1)}, \quad (1)$$

where S_n and T are the neutron separation energy and temperature, respectively. A similar method was used earlier by Natowitz *et al.* [23]. It should be noted here that the fission probability also depends on angular momentum. Hence, the angular momentum population in ^{212}Po at E_0 should also match with that of ^{211}Po at E_1. The measured fission fragment anisotropy values for the ^{4}He + ^{206}Pb system [24] are constant in the excitation energy range 30 to 40 MeV, indicating the saturation of angular momentum population above 30 MeV. Hence, the populated angular momentum distributions in these light-ion induced reactions are expected to be same over the energy range considered here. The second chance fission probability of ^{212}Po is taken as the first chance probability of ^{211}Po at the excitation energy E_1, the third chance fission probability is taken as the first chance fission probability of ^{210}Po at the excitation energy $E_2 = E_1 - S_n - 2T$, and so on.

The extracted fission probabilities as a function of chance for ^{212}Po at an excitation energy of 59 MeV are compared with the statistical model calculations with the different options for the potential energy surface in Fig. 4. The values $\tilde{a_f}/\tilde{a}_n = 1.026$ and $B_{LD} = B_{RFRM} \times 1.10$ gives best fit to the fission excitation function of ^{212}Po with option "b", and have been used for all the options to calculate the chance distribution. The predicted distributions do not change significantly, even if the parameters are allowed to vary to fit the high energy part (> 40 MeV) of the excitation functions with the option "c". The low energy (< 40 MeV) part of the excitation functions cannot be fitted with option "c" [6]. While the calculation with option "b" reproduces the experimental chance distribution, the predicted distribution with option "c" is found to be very different.

The average number of neutron emitted before fission ($\nu_{\rm pre}$) can also be obtained from the chance distribution as

$$\nu_{\rm pre} = \frac{\sum_{i=1}^{n}(i-1) \times P_f^i}{\sum_{i=1}^{n} P_f^i}, \quad (2)$$

where P_f^i is the probability of ith chance fission. The $\nu_{\rm pre}$ value obtained from the experimental chance distribution is 0.86 ± 0.16 for ^{212}Po at an excitation energy of 59 MeV. While the result with option "b" (0.9) is in excellent agreement with the value of $\nu_{\rm pre}$ obtained from the chance distribution, predicted values with option "a" (1.35) and "c" (2.62) are found to be much larger compared to experimental value. The same observations were made at 43 and 51 MeV. For comparison, we have plotted the $\nu_{\rm pre}$ obtained from the chance distribution for ^{212}Po in Fig. 2. As can be seen, the values of $\nu_{\rm pre}$ obtained from the chance distribution for ^{212}Po are smaller than those obtained from the neutron energy spectra measurement for ^{210}Po and are in excellent agreement with the statistical model calculation with option "b" (see Fig. 2). The measured $\nu_{\rm pre}$ data were found to be similar for ^{206}Po and ^{210}Po at excitation energies around 50 MeV [5] and the statistical model also does not predict any strong isotopic dependence.

In general, the fission probability can also be influenced by the dynamics. However, for the systems and excitation energies considered here, the fission probabilities are very low and the statistical fission mean lives are very large (estimated statistical fission mean life ~ 900 zs at $E^* = 59$ MeV for ^{210}Po.). Dynamical fluctuations, much shorter in time scale, are not expected to have significant effect. The experimental fission probabilities also show a monotonic increase as a function of excitation energy, as expected from statistical competition. According to statistical theory of fission, post-saddle phenomena do not alter the fission probability. Hence, the $\nu_{\rm pre}$ extracted from the fission excitation functions can be considered as presaddle contributions. These $\nu_{\rm pre}$ values are in excellent agreement with the statistical model with option "b" (see Fig. 2), which also accounts for the available fission excitation functions in light-ion as well as heavy-ion induced reactions [6]. This indicates that the influence of dynamics in the motion from the equilibrium to the saddle deformation is not significant for the systems considered in the present study.

The $\nu_{\rm pre}$ extracted from the neutron spectra measurement [5] are found to be higher than those from chance distributions. This confirms that there are significant post-saddle contributions in the $\nu_{\rm pre}$ obtained from the neutron energy spectra measurement [5]. For the nuclei in the mass region $A \approx 200$, the saddle point is extremely deformed and close to the scission point. The saddle-to-scission motion is expected to be fast and to not contribute to $\nu_{\rm pre}$ significantly. A combined dynamical and statistical model also predicted negligibly small saddle-to-scission contribution in $\nu_{\rm pre}$ for nuclei in the mass region $A \approx 200$ with excitation energy around 50 MeV [25]. The contribution and the characteristics of scission neutrons emitted at the instant of neck rupture for actinide nuclei is being investigated [26,27]. Upper limits of 0.58 and 0.73 in $^{235}U(n_{th},f)$ and $^{252}Cf(sf)$ have been estimated, respectively [27]. Such a contribution, if present in this mass region also, would be sufficient to explain the observed difference between the ν_{pre} obtained using the two different methods.

IV. SUMMARY AND CONCLUSION

In summary, we have reexamined fission in the $A \approx 200$ mass region with excitation energy around 50 MeV. The sensitivities of fission observables to the different potential energy surfaces, frequently used in literature, have been studied. The use of experimental mass with only liquid drop deformation energy (option "c"), which might have been used in Ref. [1] and by many others, does not correspond to a pure liquid drop surface. Even though the application of a pure liquid drop surface (option "a") is justifiable at high energies, it is difficult to constrain the model parameters [2,3,6], and the conclusion drawn from such an analysis can be ambiguous. The chance distributions and the ν_{pre} values obtained from the fission excitation functions for the neighboring Po isotopes are in excellent agreement with the statistical model calculation with realistic damping of the ground state shell correction [14,28] (option "b"), which also accounts for the available fission excitation functions in both light-ion and heavy-ion induced reactions. This prescription should be employed for more accurate knowledge of fission. The ν_{pre} values obtained using the new approach are found to be smaller than those obtained from the measured neutron spectra. This confirms that there are post-saddle contributions in the ν_{pre} extracted from the neutron energy spectra. The contribution of scission neutrons, largely ignored in the study of fission of excited nuclei, should be investigated.

ACKNOWLEDGMENT

One of us (S.K.) acknowledges the support received from the Department of Atomic Energy under the Raja Ramanna fellowship.

[1] C. Schmitt, K. Mazurek, and P. Nadtochy, Phys. Lett. B **737**, 289 (2014).
[2] K. Mahata, S. Kailas, and S. S. Kapoor, Phys. Rev. C **74**, 041301(R) (2006).
[3] K. Mahata, S. Kailas, A. Shrivastava, A. Chatterjee, A. Navin, P. Singh, S. Santra, and B. Tomar, Nucl. Phys. A **720**, 209 (2003).
[4] S. E. Vigdor, H. J. Karwowski, W. W. Jacobs, S. Kailas, P. Singh, F. Foga, and P. Yip, Phys. Lett. B **90**, 384 (1980).
[5] K. S. Golda, A. Saxena, V. K. Mittal, K. Mahata, P. Sugathan, A. Jhingan, V. Singh, R. Sandal, S. Goyal, J. Gehlot, A. Dhal, B. R. Behera, R. K. Bhowmik, and S. Kailas, Nucl. Phys. A **913**, 157 (2013).
[6] K. Mahata, S. Kailas, and S. S. Kapoor, Phys. Rev. C **92**, 034602 (2015).
[7] P. N. Nadtochy, E. G. Ryabov, A. E. Gegechkori, Y. A. Anischenko, and G. D. Adeev, Phys. Rev. C **85**, 064619 (2012).
[8] P. N. Nadtochy, E. G. Ryabov, A. E. Gegechkori, Y. A. Anischenko, and G. D. Adeev, Phys. Rev. C **89**, 014616 (2014).
[9] R. Charity, J. Leigh, J. Bokhorst, A. Chatterjee, G. Foote, D. Hinde, J. Newton, S. Ogaza, and D. Ward, Nucl. Phys. A **457**, 441 (1986).
[10] L. G. Moretto, K. X. Jing, R. Gatti, G. J. Wozniak, and R. P. Schmitt, Phys. Rev. Lett. **75**, 4186 (1995).
[11] L. G. Moretto, S. G. Thompson, J. Routti, and R. C. Gatti, Phys. Lett. B **38**, 471 (1972).
[12] A. V. Ignatyuk, M. G. Itkis, V. N. Okolovich, G. N. Smirenkin, and A. S. Tishin, Sov. J. Nucl. Phys. **21**, 612 (1975) [Yad. Fiz. **21**, 1185 (1975)].
[13] D. J. Hinde, H. Ogata, M. Tanaka, T. Shimoda, N. Takahashi, A. Shinohara, S. Wakamatsu, K. Katori, and H. Okamura, Phys. Rev. C **39**, 2268 (1989).
[14] A. V. Ignatyuk, G. N. Smirenkin, and A. S. Tishin, Sov. J. Nucl. Phys. **21**, 255 (1975) [Yad. Fiz. **21**, 485 (1975)].
[15] K. Mahata, Pramana **85**, 281 (2015).
[16] A. Gavron, Phys. Rev. C **21**, 230 (1980).
[17] W. D. Myers and W. J. Swiatecki, Lawrence Berkeley Laboratory Report No. LBL-36803, 1994 (unpublished).
[18] A. J. Sierk, Phys. Rev. C **33**, 2039 (1986).
[19] A. Shrivastava, S. Kailas, A. Chatterjee, A. M. Samant, A. Navin, P. Singh, and B. S. Tomar, Phys. Rev. Lett. **82**, 699 (1999).
[20] J. van der Plicht, H. C. Britt, M. M. Fowler, Z. Fraenkel, A. Gavron, J. B. Wilhelmy, F. Plasil, T. C. Awes, and G. R. Young, Phys. Rev. C **28**, 2022 (1983).
[21] P. Möller, A. J. Sierk, T. Ichikawa, A. Iwamoto, R. Bengtsson, H. Uhrenholt, and S. Åberg, Phys. Rev. C **79**, 064304 (2009).
[22] K. Jing, Ph.D. thesis, University of California at Berkeley, 1999 (LBNL-43410).
[23] J. Natowitz, M. Gonin, M. Gui, K. Hagel, Y. Lou, D. Utley, and R. Wada, Phys. Lett. B **247**, 242 (1990).
[24] R. Chaudhry, R. Vandenbosch, and J. R. Huizenga, Phys. Rev. **126**, 220 (1962).
[25] P. Fröbrich and I. Gonthar, Phys. Rep. **292**, 131 (1998).
[26] N. Carjan and M. Rizea, Phys. Lett. B **747**, 178 (2015).
[27] R. Capote, N. Carjan, and S. Chiba, Phys. Rev. C **93**, 024609 (2016).
[28] P. C. Rout, D. R. Chakrabarty, V. M. Datar, S. Kumar, E. T. Mirgule, A. Mitra, V. Nanal, S. P. Behera, and V. Singh, Phys. Rev. Lett. **110**, 062501 (2013).