Shape coexistence and mixing in ^{96}Zr

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I have performed a reanalysis of mixing between the first two 0^+ and 2^+ states in ^{96}Zr. My mixing amplitude for the 0^+ states is about three times a recent value, although both are small. My mixing is consistent with the $E0$ strength connecting the two 0^+ states, whereas the earlier mixing gives an $E0$ strength that is too small by an order of magnitude.

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In Sr and Zr nuclei, a rapid change in structure occurs between $N = 58$ and 60 [1,2]. Ground states of ^{98}Sr and ^{100}Zr are deformed strongly with a spherical (or near-spherical) excited 0^+ state, whereas the ground states of ^{96}Sr and ^{98}Zr are spherical with excited deformed 0^+ states. Some mixing between the two types of structures is expected and observed [1]. The trend should continue in ^{94}Sr and ^{96}Zr. In the latter, an excited deformed 0^+ state was suggested long ago [3].

For ^{98}Sr, a recent experiment [4] completed the measurement of the four $E2$ strengths connecting the first two 0^+ and 2^+ states that are necessary for a complete determination of the mixing and of the properties of the underlying basis states. A simple two-state mixing model [5] and a somewhat more sophisticated approach [4] agreed on the quantitative values of these parameters. These results also agreed with an analysis from 1980 [6] but disagreed with some conclusions in intervening years [7–9]. In this case, mixing intensities were 19.7(6)% for 0^+ and 3.84(35)% for 2^+ [4,5]. Transition matrix elements connecting the basis states were $M_g = 1.266(20)$ and $M_e = 0.292(5)e$b [5]. A second possible solution had $M_g = 1.243(20)$ and $M_e = 0.374(6)e$b with 0^+ and 2^+ mixing of 17.4% and ∼0.3%, respectively.

In ^{96}Zr, the $B(E2)$ from the first 2^+ state to the ground state (g.s.) has long been known to be 2.3(3) W.u. (Weiskopf units) [10]. The branching ratio (BR) from the second 2^+ state to the first two 0^+ states was known to be 0.280(25) [10], but the corresponding $B(E2's)$ were not known. A recent experiment [11] used inelastic electron scattering on ^{96}Zr to measure the ratio of cross sections to the first and second 2^+ states. An analysis in terms of the plane-wave Born approximation allowed a determination of the ratio of the $B(E2's)$ connecting the g.s. to these two 2^+ states and hence a value for $B(E2; 2_2^+ \rightarrow \text{g.s.})$ using the previously known value [10] for $2_1^+ \rightarrow$ g.s. This new $B(E2)$ strength together with the previously known BR [10] were used to determine $B(E2; 2_2^+ \rightarrow 0_2^+)$. The results are listed in Table I. Note that the percentage uncertainties in the last two $B(E2's)$ are larger than the uncertainty in their ratio, which comes from the BR.

Kremer *et al.* [11] performed a two-state mixing analysis of their results, in terms of spherical and deformed states. With four parameters to be determined—two mixing amplitudes and two $E2$ transition matrix elements connecting basis states—and only three known experimental quantities, the authors needed an additional condition in order to obtain a solution. They chose to assume the mixing potential matrix elements for 0^+ and 2^+ states were equal. Their results were an extremely small amount of 0^+ mixing (0.2%) and a relatively small amount of 2^+ mixing (2.5%). Their basis-state transition matrix elements were $M_{\text{sph}} = 3.54, M_{\text{def}} = 13.6$, both in $(\text{W.u.})^{1/2}$. They did not quote any uncertainties in their mixing parameters that arise from the uncertainties in the experimental $B(E2's)$. Also, something is wrong with their sign convention. For the $2_1 \rightarrow 0_1$ transition, their two contributions are destructive, but they should be constructive on quite general grounds [12]. Furthermore, their assumption of equal mixing matrix elements for the 0^+ and 2^+ states is suspect. For these reasons, I have repeated the two-state mixing analysis.

My notation for the mixing amplitudes of 0^+ and 2^+ states is the same as in Ref. [11]. Then I define $x = \beta/\alpha, y = \delta/\gamma, r = M_{\text{sph}}/M_{\text{def}}$ and deal with dimensionless ratios. I note that their value of r is 0.261, very close to the value of 0.231 that I obtained in ^{98}Sr for M_e/M_g. [Because the deformed state is the g.s. of ^{98}Sr but an excited state in ^{96}Zr, we would expect that $M_{\text{sph}}/M_{\text{def}}$ in ^{96}Zr should be similar to M_e/M_g in ^{98}Sr.] Thus, in what follows, I use $r = 0.231$. Equations to be fitted are

$$(1 + xyr)/(r + xy) = M_3/M_0 = 3.95(67) \quad \text{and}$$
$$(x - yr)/(1 + xyr) = M_2/M_3 = 0.085(7),$$

where I have taken the percentage uncertainty in the second equation from the percentage uncertainty in the BR [10].

Results with their uncertainties are listed in Table II, where they are compared with those from Ref. [11]. Note that I, too, obtain small mixing, but both of my mixing amplitudes are

TABLE I. Relevant $E2$ strengths and matrix elements in ^{96}Zr [11].

Label	Initial	Final	$B(E2)$ (W.u.)	$M(E2)$ $(\text{W.u.})^{1/2}$[a]
M_0	2_1	0_1	2.3(3)[b]	3.39(22)
M_1	2_1	0_2	Unknown	
M_2	2_2	0_1	0.26(8)[c,d]	±1.14(18)
M_3	2_2	0_2	36(11)[c,d]	13.4(21)

[a] $M^2(E2) = (2J_i + 1)B(E2; i \rightarrow f)$.
[b] Reference [10].
[c] $\text{BR}(0_2^+/0_1^+) = 0.280(25)$ [10].
[d] Reference [11].

TABLE II. Comparison of present ^{96}Zr results with those from Ref. [11].

Quantity	Ref. [11]	Present paper
$x = \beta/\alpha$	0.045	0.128(18)
$y = \delta/\gamma$	0.160	0.184(30)
$r = M_{\rm sph}/M_{\rm def}$	0.261	0.231(6)
V_0(keV)	76	199(28)
V_2(keV)	76	85(14)

larger than those of Ref. [11]. In my analysis, the $2_1 \rightarrow 0_1$ transition is constructive, unlike the situation in Ref. [11]. Finally, the mixing matrix elements V derived for 0^+ and 2^+ states are found to be quite different from each other.

It might appear that the difference in 0^+ mixing is insignificant. However, the conventional expression [7] for $E0$ strength $\rho(E0)$ connecting the two 0^+ states scales as the product $\alpha\beta$. Two values reported for $\rho^2(E0)$ in ^{96}Zr are 7.5×10^{-3} [11] and 6.9×10^{-3} [13], each with no stated uncertainty. In ^{98}Sr, $\rho^2(E0)$ is 0.053(9) [7]. Taking ratios for ^{96}Zr and ^{98}Sr, my mixing produces $\rho^2(E0) = 5.3(1.2) \times 10^{-3}$; the mixing of Ref. [11] yields 6.8×10^{-4}. Furthermore, even my mixing is less than two earlier estimates [14,15] (see Table III). Mach *et al.* [14] estimated the 0^+ mixing by considering two-proton excitations $2p_{1/2} \leftrightarrow 1g_{9/2}$. They then obtained the mixing amplitude from their ratio of β-decay ft values for the g.s. and 0^+_2 states. Considering the simplicity of their wave functions, it is remarkable that their mixing is so close to the present value. They pointed out that small mixing was consistent with a small $E0$ strength in ^{96}Zr. Hofer *et al.* [15] is a major work, containing inelastic proton and deuteron scatterings for many states of ^{96}Zr. They included both distorted-wave and coupled-channel analyses. The latter was necessary for the excited 0^+ state. They concluded that a mixing amplitude of 0.238 agreed with the coupled-channel matrix elements for the excited 0^+ state to within 15%.

TABLE III. Mixing between the first two 0^+ states in ^{96}Zr.

Source	β
Mach *et al.* [14]	0.20(5)
Hofer *et al.* [15]	0.238
Reference [11]	0.045
Present paper	0.127(18)

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