

Width of $N\Delta$ and $\Delta\Delta$ states

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It is seen by a coupled-channel NN scattering calculation that in the intermediate two-baryon $N\Delta$ or $\Delta\Delta$ system the width of the state is greatly diminished due to the relative kinetic energy of the two baryons, since the internal energy of the particles, available for pionic decay, is smaller. Specifically, a state-dependent effect arises from the centrifugal barrier in $L \neq 0$ $N\Delta$ or $\Delta\Delta$ systems. The double Δ width can become even smaller than the free width of a single Δ . This has some bearing to the interpretation of the $d'(2380)$ resonance recently discovered at COSY.

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I. INTRODUCTION

Recently a clear and prominent resonance structure was observed at the WASA@COSY detector of Forschungszentrum Jülich in double pionic fusion $pn \rightarrow d\pi^0\pi^0$ [1] and later in isospin associated $pn \rightarrow d\pi^+\pi^-$ but not in the isovector channel $pp \rightarrow d\pi^+\pi^0$ [2]. Its mass is reported as 2380 MeV, somewhat below two $\Delta(1232)$ masses, and its width as 70 MeV, and in the particle zoo it has been nominated as $d'(2380)$. The structure is also seen in nonfusion reactions with isotopically freer four-body final states $NN\pi\pi$ [3,4]. Thus, along with spin-polarized measurements [5,6], the internal quantum numbers $I(J^P) = 0(3^+)$ have been also fixed.

The interpretation of this resonance has been suggested as a genuine dibaryon both without [7,8] and with explicit quark level calculations [9–11]. Considering that the resonance, whatever it is, decays mainly through $\Delta\Delta$, it is understandable that the latter calculations indicate a dominance of $\Delta\Delta$ in the state wave function (about 2/3) and the rest perhaps of a more exotic six-quark structure. The quota of the six-quark contents would decrease the width of the resonance below two times the free Δ width suggested in Refs. [7,8]. In contrast, a dynamic three-body calculation [12,13] can reproduce both the mass and width without extra explicit quark contents beyond conventional hadrons, nucleons, Δ 's, and pions. These calculations, however, contained a somewhat fictitious stable Δ' to simulate the effect of Δ in πN interaction, which might raise questions about the small width of the ensuing resonance.

It is the aim of this paper to study in a simple phenomenological way the effect of the relative kinetic energy between the two baryons to see how or if it decreases the effective decay width of the $N\Delta$ and $\Delta\Delta$ two-baryon systems. The isobar intermediate components are generated to the NN states by the coupled channels method; general reviews on this method can be found, e.g., in Refs. [14,15]. The Fourier transforms of these wave functions, first obtained from coupled Schrödinger equations in the coordinate representation, will then be used to give the momentum (and kinetic energy) distributions.

Obviously this kinetic portion is not available for the (internal) pionic decay of the Δ 's. Because, the wave function

is necessarily also spatially constrained (must die asymptotically) the kinetic energy is not arbitrary and its average is finite. This kinematic suppression of the width was taken into account long ago in calculations for $pp \rightarrow d\pi^+$ [16], but the width results were never explicitly published. Further, also a strong sensitivity can be expected on the relative orbital angular momentum of the baryons, which must give rise to quantized energy levels in closed channels. This is due to the centrifugal potential barrier—another reflection of the nonzero kinetic energy. Actually a rotational spectrum $\sim 40 L_{N\Delta}(L_{N\Delta} + 1)$ MeV was seen on top of the Δ and nucleon mass difference in a coupled channels $NN - N\Delta$ scattering calculation [17], in good agreement with the isospin one “dibaryon” masses given in Ref. [18]. This would correspond to a centrifugal barrier height for baryons approximately at one femtometer distance from each other, roughly the distance at which the $N\Delta$ wave function maximizes.

It is trivially clear that in bound $N\Delta$ or $\Delta\Delta$ states the binding energy decreases the widths due to the smaller decay phase space. It should be stressed that the above attenuation mechanism, due to the intermediate state momenta and kinetic energy, is different from this binding energy suppression and has largely been omitted. Obviously the extra narrowing of these states is highly relevant in the context of possible interpretation of the $d'(2380)$ as a $\Delta\Delta$ state. This difference is also manifest in the discussions of the present work concerning the resonant coupling in NN scattering with arbitrary external energies. In this case, in contrast to quasibound state calculations, the nonpionic two-nucleon decay is actually elastic scattering. Therefore, only pionic inelasticity involving the width is treated explicitly.

First, the system with a single Δ is treated in Sec. II to introduce the basic ideas and kinematics before proceeding to $\Delta\Delta$ of particular interest in the context of the $d'(2380)$ resonance in Sec. III.

II. $N\Delta$ STATES

In many works (e.g., on pion production reactions such as $p + p \rightarrow d + \pi^+$) the effect of the Δ is taken into account by simply including the $\Delta - N$ mass difference and width in second-order perturbation calculations into the energy denominator as $E - \Delta M + i\Gamma/2$. As a trivial consequence,

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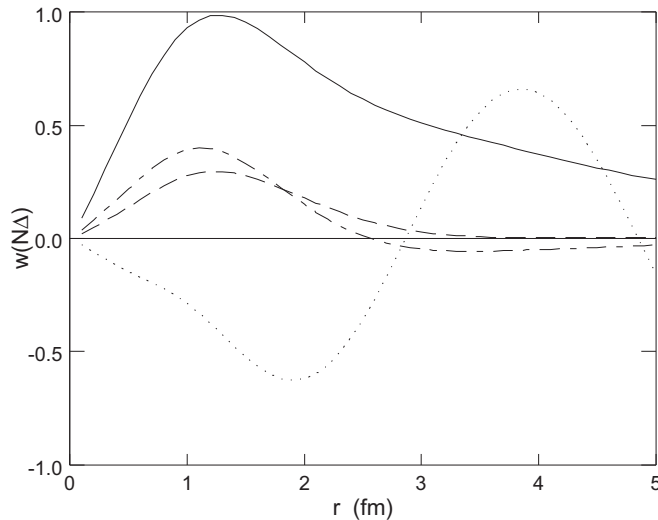


FIG. 1. The ${}^5S_2(N\Delta)$ wave function at energies 400, 578, and 765 MeV without the width (dashed, solid, and dotted curves, respectively). The dash-dotted curve has the width included at 578 MeV—the real part of the $N\Delta$ wave function shown. In this case, the normalization is associated with the NN wave function asymptotic form $u_{NN}(r) \sim \sin(kr - \pi + \delta_2)$ and the wave function is dimensionless.

in gross features this gives the energy dependence of the total cross section right, which in this example around the resonant peak is dominated by a single partial wave chain ${}^1D_2(NN) \rightarrow {}^5S_2(N\Delta) \rightarrow d\pi_{p\text{-wave}}^+$, (with also a significant contribution from ${}^3F_3(NN) \rightarrow {}^5P_3(N\Delta) \rightarrow d\pi_{d\text{-wave}}^+$, affecting importantly in differential and spin observables [19]). In the momentum (or energy) representation, this prescription is obvious and simple. However, the changes suggested in the introduction to the $N\Delta$ kinematics are not necessarily accounted for. Further, in different partial waves, the centrifugal barrier affects the magnitude of the Δ contribution and can displace the peaking, so that differential observables displaying interferences do not come out right [20].

As the present calculations are performed in the configuration space, it is also illustrative to see explicitly how the peaking itself arises in this representation with wave functions obtained from the appropriate coupled $NN - N\Delta$ Schrödinger equation [19]. For the NN angular momentum state $(LS)J$, the $N\Delta$ wave functions are then generated from the asymptotically free radial NN wave function with momentum k and normalization $u_{NN}(r) \sim kr j_L(kr + \delta_L)$. Figure 1 shows now the most important component ${}^5S_2(N\Delta)$ of the initial baryonic wave function in the pion production reaction $p + p \rightarrow d + \pi^+$. Below the nominal $N\Delta$ mass, this channel is obviously closed and exponentially decreasing as a function of the distance (dashed curve for $E_{\text{lab}} = 400$ MeV). At the $N\Delta$ threshold, lacking either positive or negative kinetic energy, the wave function becomes essentially a straight line outside the potential range $r \geq 2.5$ fm (solid curve at 578 MeV). Depending on the details of the energy and the interaction this could, in principle, be a horizontal constant,

maximizing any overlap transition integrals (in the case of this reaction with the long-ranged deuteron and relatively low energy pion). It may be noted that already at 600 MeV this line crosses the r axis at 4.6 fm, introducing the first oscillation at distances small enough to cause significant overlap reduction. At still higher energies, oscillations attain shorter wave lengths and begin to cancel the transition matrix integral (dotted curve at 765 MeV). As a consequence there is a strong peaking of the production cross section at the $N\Delta$ threshold far higher than the data [21].

However, once the Δ width is included in the equation of motion as a constant negative imaginary potential (as presented in the following discussion), the channel becomes again asymptotically closed. As can be seen (dash-dotted curve at 578 MeV), the wave function becomes strongly moderated at short distances and the oscillating wave will be exponentially attenuated at large distances with a consequent suppression of the transition at and above the $N\Delta$ threshold. So the natural inputs for the configuration space equation of motion, the Schrödinger equation, lead to similar resonance like behavior as can be obtained by explicitly forcing it by hand in the momentum and energy representation (see, e.g., Refs. [22,23]). With the closure of the channels also, similar quantization phenomena appear as for bound states but, however, smeared with the uncertainties associated with the width. As stated previously, the centrifugal barriers (or the Coulomb force if necessary) can be included with important effects on the differential observables with interfering amplitudes (for $p + p \rightarrow d + \pi^+$, see, e.g., Refs. [16,19] vs Refs. [24–26]).

The width is normally taken to be the free width of the Δ associated with the available center-of-mass NN energy. However, as will be seen, also in this dynamic input quantity the effect of the relative $N\Delta$ kinetic energy is significant and dependent on the angular momentum. This should be subtracted from the internal energy available for the Δ decay, so that effectively the two-baryon width becomes smaller than the free width (which would correspond to zero relative energy of the baryons).

Further, in these reactions in different partial waves also the $N\Delta$ centrifugal barrier directly diminishes the wave functions. Although this suppression is particularly sensitive to $L_{N\Delta}$, even the orbital angular momentum of the initial nucleons may favor transitions into $N\Delta$ in some sense. Namely, within the interaction range a reduction of the centrifugal barrier can compensate the $N\Delta$ mass difference in the excitation if $L_{N\Delta} < L_{NN}$, as seen in Ref. [17] as an explanation for $T = 1$ enhancements ($T = 0$ dibaryons). From the above considerations, it is clear that just a single number cannot account for the effective two-baryon pole position in different partial waves.

Some earlier works [27,28] attempt to take into account the $N\Delta$ recoil by plausible approximations. Mostly these end up with some scaling down of the Δ width but depending directly on the external NN energy. Although also the angular momentum part of the kinetic energy may be heuristically included, the applicable $N\Delta$ wave functions are not explicitly involved. In the most ambitious internally consistent dynamic approach, Arenhövel [29] calculates the self-energy

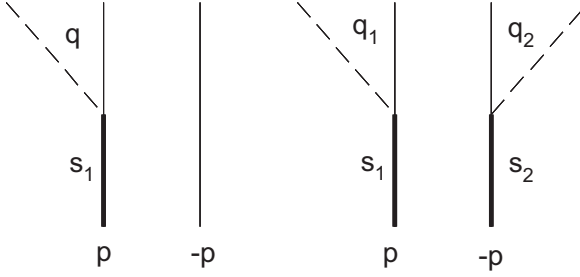


FIG. 2. Basic kinematics of the $N\Delta$ (left) and $\Delta\Delta$ (right) decays. The thick lines present the Δ 's. The relative momenta $q_{(i)}$ between the pion and the nucleon are associated with the Δ internal energy (invariant mass) by $s_i = \sqrt{q_{(i)}^2 + \mu^2} + \sqrt{q_{(i)}^2 + M^2}$. Further dependencies on the relative baryon momentum p are given in the text.

contribution to the $N\Delta$ binding energy from both one pion exchange and pion emission and absorption on the same nucleon. In addition to the width (involving an integration over momenta), due to the possibility of on-shell pions, the ensuing potential is complex. Combined with ρ exchange, he finds a ($T = 2, J^P = 2^+$) bound state with a very strongly suppressed width. In this case, the suppression is largely due to the effect of the Pauli principle on the final nucleons [29] (treated in the impulse approximation as noninteracting), but the basic result as an integral over $N\Delta$ momentum is, in principle, similar to the simpler treatment of the present work.

The aim of the present paper is less ambitious with the emphasis on the suppression of only the known and observable Δ width without paying much attention to the $N\Delta$ interaction, other than the centrifugal potential. In particular, I treat NN scattering (rather than possible bound state decay as in Ref. [29]) through the $N\Delta$ and $\Delta\Delta$ threshold energy regions. With the coupled channels, the treatment is exact at the two-body level (although the impulse approximation in Ref. [29] is certainly satisfactory with the quantum numbers relevant to the predicted bound state).

Reference [16] on the reaction $pp \rightarrow d\pi^+$ considered among other things the above discussed kinematic effects explicitly by calculating the width into the three-body final state of Fig. 2 as an average over kinematically allowed momenta

$$\Gamma_3 = \frac{2}{\pi} \frac{\int_0^{p_{\max}} |\Psi_{N\Delta}(p)|^2 \Gamma(q) p^2 dp}{\int_0^\infty |\Psi_{N\Delta}(r)|^2 r^2 dr}. \quad (1)$$

Here $\Psi_{N\Delta}(p)$ is the Fourier transform of the appropriate partial wave component of the $N\Delta$ wave function and $\Gamma(q)$ is the free $\Delta \rightarrow N\pi$ width with q as the relative $N\pi$ momentum. The maximum relative $N\Delta$ momentum which still allows the pionic decay is obtained by

$$\begin{aligned} p_{\max}^2 &= \frac{\lambda(s, (M + \mu)^2, M^2)}{4s} \\ &= \frac{[s - (M + \mu)^2 - M^2]^2 - 4M^2(M + \mu)^2}{4s} \end{aligned} \quad (2)$$

from the nucleon and pion masses M and μ , respectively, and the total c.m.s. energy \sqrt{s} . The triangle function λ is introduced in its various forms, e.g., in Ref. [30]. The physically allowed pion momentum is then constrained by the relative baryon momentum through the internal energy of the Δ

$$s_1 = (\sqrt{s} - \sqrt{M^2 + p^2})^2 - p^2 \quad (3)$$

to smaller values

$$q^2 = \frac{(s_1 - M^2 - \mu^2)^2 - 4\mu^2 M^2}{4s_1}. \quad (4)$$

Starting with a reasonable guess for the width(s), the system is solved iteratively until stable value(s) have been obtained.

Besides Γ_3 Ref. [16] and later work with πd final states also included the explicit contribution from this cross section so that the equality $\Gamma_3 + (\sigma_{d\pi}/\sigma_{\text{tot}})\Gamma_{\text{tot}} = \Gamma_{\text{tot}}$ was self-consistently satisfied, when Γ_{tot} was used in the coupled-channels calculation, giving σ_{tot} as the total inelasticity and the consequent baryon wave functions to calculate the $NN \rightarrow d\pi$ amplitudes. Here the latter term is assumed to be the two-body ($d\pi$) contribution Γ_2 to the total width. Although the present work is not aimed at pion production *per se*, this prescription is nevertheless mainly used in this section. The effect of Γ_2 is negligible for NN partial waves other than 1D_2 and 3F_3 , where it can contribute about 10–20%. It may be noted that, of course, this increases the width somewhat and thus acts against the suppression effect claimed here.

Finally, as the free Δ width input I use a fit to data [31]

$$\Gamma(q) = \frac{142 (0.81 q/\mu)^3}{1 + (0.81 q/\mu)^2} \text{ MeV} \quad (5)$$

with the characteristic p -wave resonance behavior and a soft form factor.

In addition to the limiting constraints on allowed momenta in Eq. (1), a decisive input necessary is the wave function of the $N\Delta$ intermediate state, assuming it to originate from, e.g., NN scattering. In this case, perturbation theory with Δ 's is problematic, since there are no unperturbed $N\Delta$ wave functions at hand to start with. However, the more exact coupled-channels approach offers probably the best candidates for such wave functions, and this method is used here. The coupled system of Schrödinger equations is solved for each incident nucleon state with the phenomenological Reid potential [32] as the starting point. The old age of the interaction does not matter much, since once the coupling to the excited intermediate $N\Delta$ state is invoked, additional strong attraction is gained, which must, anyway, be counteracted to avoid double counting. This is performed by changing the diagonal NN part so that the total interaction reproduces the phase shifts [33] reasonably well below the resonance (or $N\Delta$ threshold). Reference [34] presents such a change to the most important and sensitive NN states 1D_2 and 3P_1 (in the original Reid potential) to be used in this section. An extension of the potential to the necessary higher partial waves 3F_3 and 3D_3 is provided by

Day in Ref. [35]:

$$V(^3F_3) = 10.463[(1 + 2/x + 2/x^2)e^{-x}/x - (8/x + 2/x^2)e^{-4x}/x] - 729.25e^{-4x}/x + 219.8e^{-6x}/x, \quad (6)$$

$$V_C(^3D_3 - ^3G_3) = -10.463e^{-x}/x - 103.4e^{-2x}/x - 419.6e^{-4x}/x + 9924.3e^{-6}/x, \quad (7)$$

$$V_T(^3D_3 - ^3G_3) = -10.463[(1 + 3/x + 3/x^2)e^{-x}/x - (12/x + 3/x^2)e^{-4x}/x] + 351.77e^{-4x}/x - 1673.5e^{-6x}/x, \quad (8)$$

$$V_{LS}(^3D_3 - ^3G_3) = 650e^{-4x}/x - 5506e^{-6x}/x, \quad (9)$$

with $x = 0.7r$ (r in femtometers). With the $N\Delta$ coupling, as stated above, these need an additional central repulsion (fitted below the resonance at 300 and 450 MeV to the energy dependent pp and np phases of Ref. [33], including the most important $N\Delta$ or $\Delta\Delta$ component):

$$\begin{aligned} V_C(\text{coupled}) &= V_C(^3F_3) + 2700e^{-5x}/x, \\ V_C(\text{coupled}) &= V_C(^3D_3 - ^3G_3) + 2800e^{-7x}/x. \end{aligned} \quad (10)$$

The $NN \rightarrow N\Delta$ transition potential described in Ref. [16] has been fine-tuned to give the height of the $pp \rightarrow d\pi^+$ peak at the right place ≈ 580 MeV and may be trusted here, too. This peak is possibly the most sensitive probe of the transition potential. The potential involves pion and ρ -meson exchanges. The latter may be described by contact terms in recent effective field theories, but the main thing in this context is to have a transition potential which agrees with data. The role of the width, in turn, is to act as a constant imaginary ‘‘potential’’ in the $N\Delta$ channels of the coupled Schrödinger equations and produce inelasticity. It goes without saying that unitarity is not prevailed as in general not with optical potentials. At the two-baryon level, this is probably closest one can get to reality in the case of pion production. It is useful to note that besides introducing inelasticity the inclusion of the width also acts as effective repulsion; for moderate inelasticities, more imaginary interaction means less attraction.

Figure 3 shows the effective widths of the $N\Delta$ states as functions of the total c.m.s. energy, i.e., the ‘‘dibaryon’’ mass for some representative configurations. For the NN initial states 1D_2 and 3F_3 , the criterium for their choice is mainly the importance: The orbital angular momentum decrease in transitions to $^5S_2(N\Delta)$ and $^5P_3(N\Delta)$ favors the formation of $N\Delta$ (solid and dash-dotted curves, respectively). A secondary criterium was to keep the transition potential radially the same by limiting the discussion to the spin changing tensor part and thus minimizing inessential diversions (the spin-spin part is excluded in these states). It can be seen that the $N\Delta$ angular momentum tends to decrease the width as anticipated earlier. Comparison of the $^5S_2(N\Delta)$ (the highest, thin solid curve) and the $^5D_2(N\Delta)$ (the lowest, short dashes) is a striking example. It may be noted that they both originate from the same initial state $^1D_2(NN)$. The more moderate but clear effect of the initial NN angular momentum can be seen between the $^5P_3(N\Delta)$ and $^5P_1(N\Delta)$ from the $^3F_3(NN)$ and $^3P_1(NN)$ initial states (the dash-dotted and dashed curves, respectively). The straddling of the 5P_1 [the dashed curve arising from $^3P_1(NN)$] and $^5D_4(NN)$ [the dotted line from $^1G_4(NN)$] is purely accidental and due to the fact that in the latter transition the $N\Delta$ orbital angular momentum decreases from the NN , whereas in the former it does not. Therefore, the $^1G_4(NN)$ is favored as

another $T = 1$ dibaryon [17,18]. For further comparisons, also the width of the 5D_2 -wave $N\Delta$ is shown by short dashes. First, due to its large centrifugal barrier, this is much smaller than its S -wave sibling. For the reasons already discussed, its width is also smaller than that of $^5D_4(N\Delta)$, because the orbital angular momentum does not decrease in this transition. It may be still worth stating that in $^1S_0(NN) \rightarrow ^5D_0(N\Delta)$ the width is numerically negligible, because the angular momentum actually increases in the transition. In the neighborhood of the $N\Delta$ wave function maximum, the centrifugal barrier is ≈ 200 MeV, close to the $\Delta - N$ mass difference, i.e., the $N\Delta$ threshold itself.

The thick line presents the free Δ width (5) for static baryons without any centrifugal barriers. Clearly the kinematics of the intermediate baryons have a strong effect at the nominal mass 2.17 GeV of the $N\Delta$ system and above even for the S -wave $N\Delta$ (thin solid line). Far above this threshold, one might ask about the validity of the fit (5) to the width, but the softness of the form factor should rather underestimate the free width than overestimate it.

Sometimes it may be easier or also physically more meaningful and beneficial to have the pion momentum (relative to the recoil nucleon) as the primary variable. In this case, the momentum p is obtained from

$$p^2 = \frac{[s - s_1(q) - M^2]^2 - 4M^2s_1(q)}{4s} \quad (11)$$

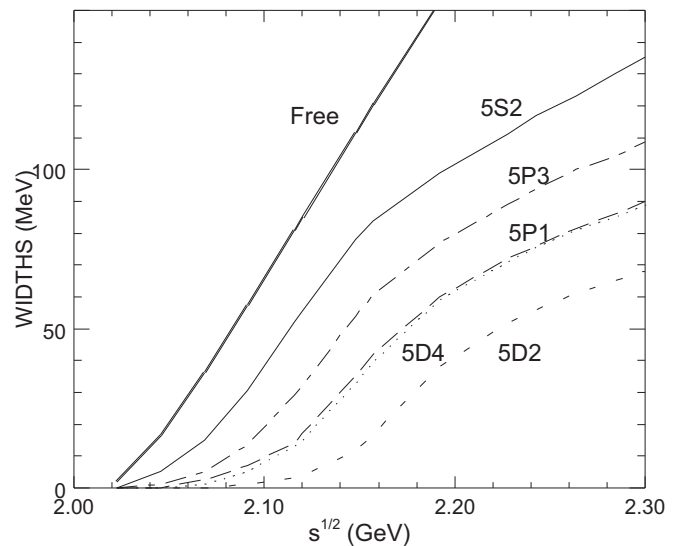


FIG. 3. The widths of a representative selection of $N\Delta$ states in NN scattering: Curves as described in the text. The free width is the thick line above the others.

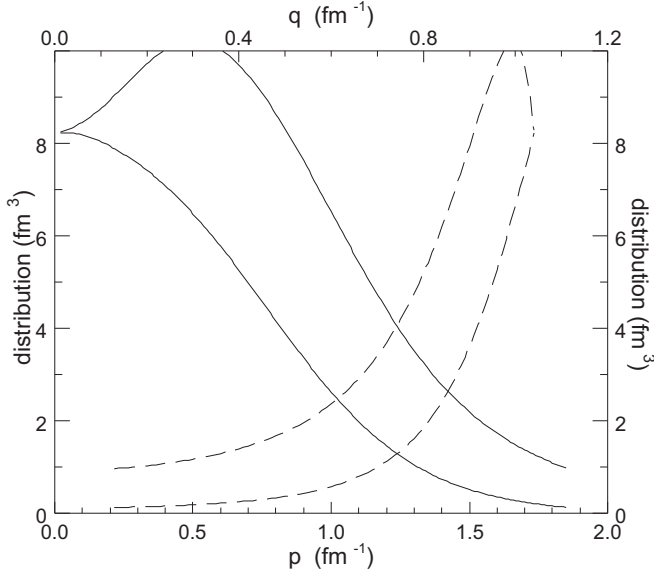


FIG. 4. Probability distributions of momenta at $E_{\text{lab}} = 578$ MeV as described in the text; the solid curves as functions of the $N\Delta$ momentum p (left y axis) and the dashed ones of the pion-nucleon momentum q (right y axis). The upper curves are for all main $N\Delta$ contributions added together and the lower ones for the dominant ${}^5S_2(N\Delta)$ alone ($p_{\text{max}} = 1.87 \text{ fm}^{-1}$ and $q_{\text{max}} = 1.04 \text{ fm}^{-1}$).

and q_{max} [with $s_{1\text{max}} = (\sqrt{s} - M)^2$] from

$$q_{\text{max}}^2 = \frac{(s - 2M\sqrt{s} - \mu^2)^2 - 4\mu^2 M^2}{4(\sqrt{s} - M)^2}. \quad (12)$$

In either presentation, the probability distribution (without the volume element $\propto p^2$) is given by the absolute square(s) of the relevant amplitude Fourier component(s) $|\Psi_{N\Delta}(p)|^2$ or $|\Psi_{N\Delta}(p(q))|^2$ shown in Fig. 4 for $E_{\text{lab}} = 578$ MeV right at the top of the $pp \rightarrow d\pi^+$ cross section [36]. The partial wave contributions are weighted by the corresponding statistical factors $(2J + 1)$. The solid curves present the dependence on p (lower abscissa and left ordinate), whereas the dashed ones are for q (upper abscissa, right ordinate). Of these curves, the lower ones include only the ${}^5S_2(N\Delta)$ component coupled to ${}^1D_2(NN)$, dominant in $pp \rightarrow d\pi^+$, whereas the upper ones have all significant smaller components up to the 3H_5 partial wave. It can be seen that the S -wave $N\Delta$ is peaked at small values of p , whereas higher angular momentum components approach zero there, but are appreciable at higher momenta, where the kinetic energy of the baryons would be large. Of course, the q dependence is opposite to p . Although the present calculation is not directly aimed at pion production observables, by, e.g., neglecting the direct NN contribution, it is conceivable that these contributions could be seen in pion production into three particles.

For a further study of the resonance-like effects of the $N\Delta$ components, Fig. 5 presents the Argand diagrams $2t = i[1 - \exp(2i\delta)]$ between E_{lab} 300 and 1000 MeV for the 1D_2 and 3F_3 partial waves, the most prominent $T = 1$ dibaryons, for which the most important $N\Delta$ configurations were quoted above and in Fig. 3. Except for the lowest and highest energies, the mesh is not evenly spaced but rather follows some experimental

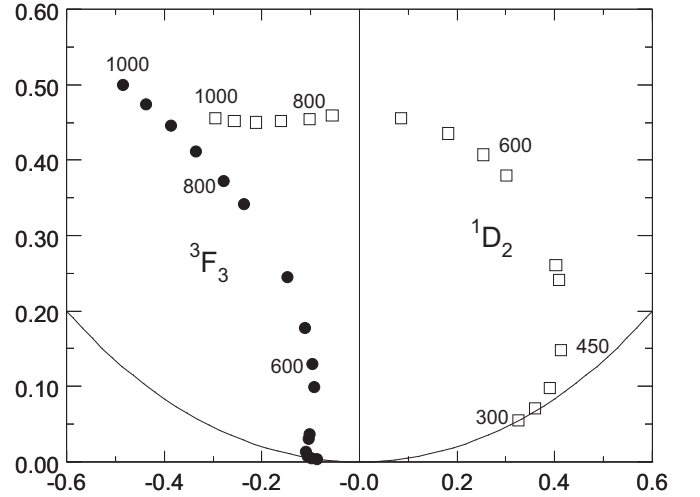


FIG. 5. Argand diagrams of two NN partial waves. Squares 1D_2 and full circles 3F_3 states. Also the unitarity circle is shown.

energies of SIN, LAMPE, and TRIUMF. It can be seen that neither is a full resonance with the phase passing $\pi/2$ nor do they go around the center of the unitarity circle.

So far also the cross section of $NN \rightarrow d\pi$ has been accounted for in the widths. To show the effect of a single $N\Delta$ channel more clearly, I constrain in the discussion to $NN({}^3F_3) \rightarrow N\Delta({}^5P_3) \rightarrow NN({}^3F_3)$ neglecting also the F -wave $N\Delta$'s. This results in about 5% decrease in the width from that shown in Fig. 3. Altogether, the neglect of the $d\pi$ and the F -wave $N\Delta$'s is a loss of less than one degree of attraction at intermediate energies of interest here. The latter neglect has practically no effect on the P -wave width.

In Fig. 6, the accumulation in the phase shift $\delta({}^3F_3)$ arising from the Reid potential (6) and the coupling to only ${}^3P_3(N\Delta)$ is presented. First, the potential itself gives a flat and relatively featureless result, which, however, agrees excellently with the analysis [33] up to the pion production threshold (dashed curve). The modification (10) is too unrealistically repulsive (dotted) but due its very short range does not change the low-energy agreement much. However, the coupling to the $N\Delta$ state returns the attraction but without the width leads to a very narrow and too high peak at ≈ 660 MeV (dash-dotted) slightly above the $N\Delta$ threshold and well in accordance with the prescription [17] quoted in the introduction. Finally, the inclusion of the width smooths the peak and gives the solid curve in good agreement with data up to 1 GeV. Actually, the deviation from the data is less than or of the same magnitude as the difference between the pp and np analyses. Also the imaginary part of the phase shift is in reasonable agreement with the data extracted from the K -matrix of Ref. [33] (triangles). It is also interesting and illuminating to note that about 100 MeV above the nominal $N\Delta$ threshold (center of mass) the coupling effect turns repulsive (the solid curve gets below the dotted) showing typical threshold cusp (or resonance) behavior. However, the smooth background potential repulsion keeps the total phase negative and thus the corresponding Argand diagram remains on the left side

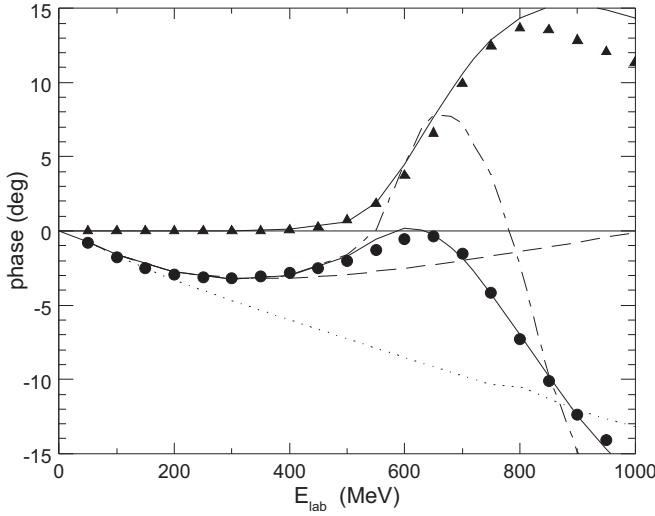


FIG. 6. The accumulation of the phase shift of the 3F_3 state. The Reid potential (6) result (dashed) and the modification (10) (dotted). Coupling to the ${}^5P_3(N\Delta)$ channel without (dash-dotted) and with the width (solid) as explained in the text. The second solid curve is the imaginary part of the phase. The data are the energy-dependent fit to pp data from Ref. [33].

of the imaginary axis in Fig. 5. In the partial wave cross section, the phase shift maximum here should then show rather as a minimum than as the “standard” maximum. Of course, this minimum in pp scattering has not much to do for the $NN \rightarrow d\pi$ reaction, where 3F_3 is the second state in importance besides 1D_2 above the threshold region, but this importance is based on the overlap of the $N\Delta$ configurations with the final $d\pi$ states—mainly ${}^5P_3(N\Delta)$ and d -wave pions.

III. $\Delta\Delta$ STATES

Conceptually, the width of a single Δ even in presence of another nucleon is quite clear. For a pair of Δ 's the situation is slightly more complex. Some works in the context of the $d'(2380)$ have considered twice the single Δ width as relevant [7,8]. However, it is difficult to see why the lifetime of two Δ 's should be only half of the lifetime of a single Δ . Rather, if one considers as the lifetime the time that is required for *both* to decay, by conditional probabilities the lifetime in this sense should be longer and the width smaller. After all, the experimental results are for the two decays with two pions.

The decay rate for particles 1 and 2 with widths Γ_1 and Γ_2 starting from time zero is $\Gamma_1 \exp(-\Gamma_1 t_1) \times \Gamma_2 \exp(-\Gamma_2 t_2)$. The total transition probability at time t is (integrating over different time orderings)

$$\begin{aligned}
 P(t) &= \Gamma_1 \Gamma_2 \left(\int_0^t e^{-\Gamma_1 t_1} dt_1 \int_0^{t_1} e^{-\Gamma_2 t_2} dt_2 \right. \\
 &\quad \left. + \int_0^t e^{-\Gamma_2 t_2} dt_2 \int_0^{t_2} e^{-\Gamma_1 t_1} dt_1 \right) \\
 &= 1 - e^{-\Gamma_1 t} - e^{-\Gamma_2 t} + e^{-(\Gamma_1 + \Gamma_2)t}. \quad (13)
 \end{aligned}$$

Thus from the time dependence of the survival probability $1 - P(t) = \exp(-\Gamma t)[\exp(-\delta t) + \exp(+\delta t) - \exp(-\Gamma t)]$ [with the notation $\Gamma = (\Gamma_1 + \Gamma_2)/2$ and $\delta = (\Gamma_1 - \Gamma_2)/2$] one may conclude that the dominant part is consistent with the decay width being the average Γ , or the single width in the case $\Gamma_1 = \Gamma_2 = \Gamma$. In view of the kinematic results of Sec. II it might be possible that even this is further decreased. However, on the other side, with the much higher energy scale of the double Δ it is also possible that the “free” width input $\Gamma(q)$ could get very large values for large momenta and the ensuing integrals would yield larger widths instead. In the absence of firm intuitive arguments an explicit estimation is required.

Now the two- Δ width is calculated as the double integral

$$\Gamma_4 = \frac{2}{\pi} \frac{\int |\Psi_{\Delta\Delta}(p)|^2 [\Gamma(q_1) + \Gamma(q_2)]/2 p^2 dp dq_1}{q_{\max} \int_0^\infty |\Psi_{\Delta\Delta}(r)|^2 r^2 dr}. \quad (14)$$

Here the maximum limit of the free variable p is obviously from the kinematics of Fig. 2 $p_{\max} = \sqrt{s/4 - (M + \mu)^2}$ and the upper limit of the pion momentum as a function of p is obtained from the maximum internal energy of particle one

$$s_{1\max} = [\sqrt{s} - \sqrt{(M + \mu)^2 + p^2}]^2 - p^2 \quad (15)$$

as

$$q_{1\max}^2 = \frac{(s_{1\max} - M^2 - \mu^2)^2 - 4\mu^2 M^2}{4s_{1\max}}. \quad (16)$$

In the pion integration, the second dependent momentum q_2 in turn is obtained from

$$q_2^2 = \frac{(s_2 - M^2 - \mu^2)^2 - 4\mu^2 M^2}{4s_2} \quad (17)$$

with $s_2 = (\sqrt{s} - \sqrt{s_1 + p^2})^2 - p^2$ and $s_1 = (M^2 + q_1^2) + (\mu^2 + q_1^2)$.

Figure 7 shows the widths for the most important ${}^7S_3(\Delta\Delta)$ state coupled to the tensor-coupled NN $I = 0$ system ${}^3S_3 - {}^3G_3$. It can be seen that at and below the two- Δ threshold the kinematic constraints with realistic wave functions cause a drastic reduction in the width. Actually at 2.38 GeV the more important 3D_3 wave would get just about 50 MeV as the width, significantly less than the reported 70 MeV. Therefore, it seems that the narrowness of the resonance cannot be used as an argument against the possibility of its being of pure $\Delta\Delta$ origin. The 3G_3 initiated state would have 13 MeV larger width, but its influence is suppressed by an order of magnitude due to the fact that to couple the S and G waves one needs to operate twice by the tensor-like transition potential. It may be possible to find dynamic origins for further inelasticity, but as in the present phenomenological calculation even the origin of the width itself is not dynamically based, such a search would be inconsistent and beyond the scope of the present work.

Finally, Fig. 8 shows the phase shifts corresponding to the initial NN partial waves 3D_3 and 3G_3 as Argand diagrams. The open boxes are the results of a calculation involving only the coupling to the S -wave $\Delta\Delta$. For the D wave, the present diagram has curvature indicative of a resonance but is significantly more open than the result from the analysis of Ref. [6] and remains mainly on the right-hand side up to

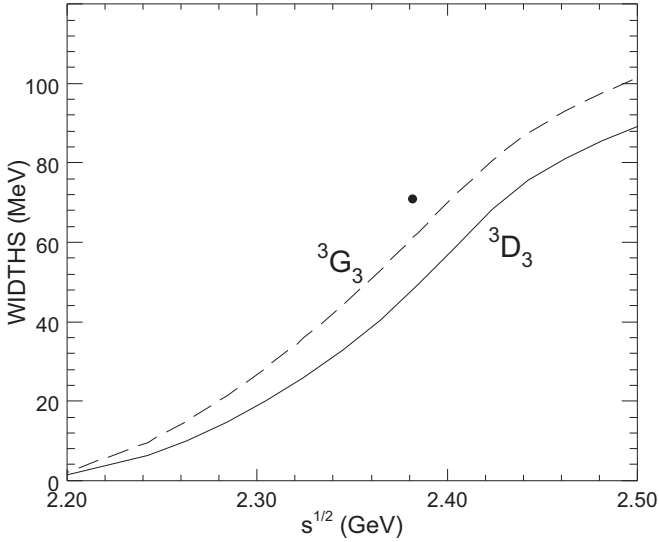


FIG. 7. The widths of the ${}^7S_3(\Delta\Delta)$ state in $I = 0$ NN scattering: The solid curve initiates from ${}^3D_3(NN)$ and the dashed one from ${}^3G_3(NN)$. The bullet shows the energy and width of the resonance reported, e.g., in Ref. [6].

the c.m. energy of ≈ 2.5 GeV. Understandably, the threshold cusp should appear rather at the double Δ mass 2.46 GeV, in agreement with the graph. (Actually the real part of the Δ pole position at 1.21 GeV, 20 MeV lower, was used in the Schrödinger equation to have the pole in the equivalent Lippmann-Schwinger equation in its place.) Like in Ref. [6], there is a small nook on the unitarity circle peaking at about 2 GeV followed by an “armpit” at 2.2 GeV. This feature appears also as more pronounced in Ref. [6]. In this calculation, the 3D_3 phase shift remains remarkably constant, varying only smoothly between 3 to 5 deg in the energy range from 150 to 1000 MeV (lab.). In Ref. [33], the phase should change sign

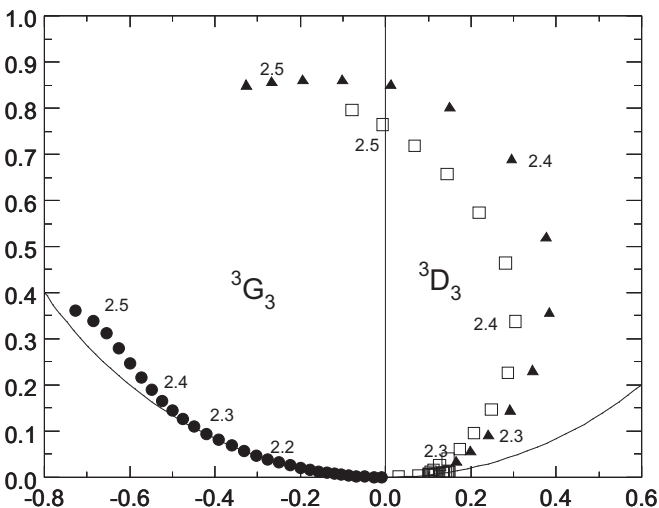


FIG. 8. Argand diagrams of two NN partial waves with a coupling to two Δ 's. Squares 3D_3 and full circles 3G_3 states. The triangles have three coupled $\Delta\Delta$ states and additional attraction in each of these channels.

above 800 MeV, but this change does not appear in the later analyses [37,38], and in agreement with the latter result the phase shift remains well as constant up to 1 GeV. The resonance and threshold regions are still fairly far above. Only well above 1 GeV in the laboratory energy an enhancement (together with inelasticity) takes place. The phase shift maximizes at about 2.43 GeV (c.m.), consistent with the doubled pole position. The G -wave result is monotonous and quite featureless and, contrary to Ref. [6], does not show any knot at 2.35 GeV. Its phase grows nearly linearly with energy and the inelasticity is small.

The weak tendency for a resonance behavior below 2.5 GeV is somewhat puzzling considering that the width input in the coupled channels is only 50 MeV in the ${}^7S_3(\Delta\Delta)$ channel (at 2.38 GeV) and restricting presently to only this single channel should rather favor a resonant behavior. As additional channels should bring more attraction, the next step might be to include also the ${}^3D_3(\Delta\Delta)$ and ${}^7D_3(\Delta\Delta)$ components (G waves should by far be negligible). A consistent calculation (adjusting also the necessary extra repulsion in the NN sector¹) gives actually slight smooth repulsion compared with the earlier one, negligible below $E_{\text{lab}} \approx 1000$ MeV and 1–2 deg in the resonance and threshold region. The overall effect is to smooth the threshold peaking, since the effective threshold of these $\Delta\Delta D$ waves is significantly higher as discussed earlier. These changes may be due to the fact that the width of the ${}^7S_3(\Delta\Delta)$ state increases by about 10 MeV in this calculation. In practice, the inclusion of the higher lying states does not change the position of the phase maximum at 2.43 GeV appreciably.

One obvious and interesting possibility is an attractive $\Delta\Delta$ interaction, which might bring the effective threshold down to the $d'(2380)$ region. For this possibility, a strong artificial test potential of about four pion strengths (in the S -wave NN potentials) is added in the $\Delta\Delta$ channels. The effect is a faster and higher rise of the phase and a subsequent faster fall after the phase maximum and a change of the sign already at 2.44 GeV. Also the position of the phase shift maximum is lowered close to 2.41 GeV. This result is shown in Fig. 8 by triangles. Also the inelasticity is increased by this attraction though the widths themselves are not changed appreciably by this addition.

Adding such an extremely strong attraction is rather a drastic act and one should question how such attraction could arise. One might speculate about a crossed two-pion exchange (with the Δ 's transforming to nucleons and back) being attractive in high pion momentum parts. Each $N\Delta\pi$ vertex has about two times the $NN\pi$ coupling strength, so the strength from the coupling coefficients alone could give a factor of 16 over the normal NN two-pion exchange (without Δ 's). However, comparisons with a real potential used here and expectations based on that are not straightforward, since unavoidably one meets on-shell pions with subsequent imaginary parts [29]. An actually dynamic calculation of the two- Δ width and an associated complex potential on the same basis would be interesting.

¹The repulsion in Eq. (10) is changed to $2700 e^{-6x}/x$, practically only a range change.

IV. CONCLUSION

The main conclusion of the present work is that the width of the $\Delta(1232)$ resonance in a two-baryon system $N\Delta$ or $\Delta\Delta$ is severely decreased due to the relative kinetic energy of the baryons and their relative angular momentum. Since the wave function is necessarily spatially confined, the expectation value of the kinetic energy is finite and out of use for (internal) decay of the particles. Further, due to this wave function confinement, the energy associated to the angular momentum barrier is quantized to roughly discrete and finite average values, also to be subtracted from the energy available to internal excitations and decays. Some obvious rules for the dependencies could be seen in Fig. 3. First, even the largest of the possible state-dependent widths are significantly smaller than the free Δ width at the energy in question (corresponding to immobile baryons). The lowest $N\Delta$ (or $\Delta\Delta$) angular momentum state has the largest width. This is associated with effective quantization of the above angular momentum energy, already phenomenologically discussed for $I = 1$ dibaryons in Ref. [17]. The higher orbital angular momentum $N\Delta$ or $\Delta\Delta$ states have increasingly smaller widths. In NN scattering with these intermediate states, also an important factor is whether the orbital angular momentum $L_{N\Delta}$ or $L_{\Delta\Delta}$ is smaller or larger than the initial L_{NN} (or equal). In the first case, the centrifugal barrier difference may partly cancel the mass barrier $M_\Delta - M_N$ or $2(M_\Delta - M_N)$, thus favoring the formation of the intermediate state, and also in this case the width is larger than in cases where the same intermediate state can be obtained from a lower L_{NN} .

As seen in Fig. 7, the effective $\Delta\Delta$ width is significantly smaller than the single Δ width at the relevant energies, at 2.38 GeV about 50 MeV, lower than the reported $d'(2380)$ width 70 MeV. This result was obtained with the most important ${}^7S_3(\Delta\Delta)$ alone. Including the D -wave $\Delta\Delta$'s increases the width to 60 MeV. Although this is just an input to an NN

scattering calculation, apparently an argument using just a comparison of the $d'(2380)$ width vs the free Δ width (not to say twice this) is not necessarily assuring for its exotic origin.

The use of the nonrelativistic Schrödinger equation might be questioned in this calculation. Relativistic kinematics has been used to get the center-of-mass NN momentum and energy to meet correctly the $N\Delta$ or $\Delta\Delta$ threshold. The subsequent nonrelativistic continuation should not, however, falsify the above rather general and obvious results, which are not sensitive to this treatment at least and in particular for the widths.

By this input alone, one cannot obtain a resonant 3D_3 structure as low as 2.38 GeV, only at the $\Delta\Delta$ threshold (the calculated phase shift maximum at 2.43 GeV using the Δ pole position as the mass). Adding arbitrarily as a test a strong attraction of pion range, it was possible to move the structure at least down to 2.41 GeV. However, the question would remain about the origin of such strong attraction, whether it could be hadronic (e.g., meson exchanges) or possibly due to coupling to a genuine six-quark configuration. In explicit discussions of the $N\Delta$ and $\Delta\Delta$ interactions, one should also include long-ranged on-shell mesons [29] leading to complex one- and two-pion potentials, which might be strong enough for (quasi)binding. Theoretically, at least the $\Delta\Delta$ threshold cusp should be there. Can one see two separate structures from different origins or have they merged together, as Bugg suggested [39,40] that resonances tend to synchronize together with thresholds?

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