Antiproton-proton annihilation into charged light meson pairs within effective meson theory

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We revisit antiproton-proton annihilation into light mesons in the energy domain relevant to the antiproton annihilation at Darmstadt (PANDA) experiment at the GSI Facility for Antiproton and Ion Research (FAIR) [$2.25(1.5) \leqslant \sqrt{s}(p_L) \leqslant 5.47(15)$ GeV (GeV/c) where $\sqrt{s}(p_L)$ is the total energy (the beam momentum in the laboratory frame)]. An effective meson model is developed, with mesonic and baryonic degrees of freedom. Form factors are added to take into account the composite nature of the interacting hadrons. A comparison is made with the existing data for charged pion pair production and predictions for angular distributions and energy dependence in the range $3.362(5) \leqslant \sqrt{s}(p_L) \leqslant 4.559(10.1)$ GeV (GeV/c). The model is applied to $\pi^{\pm}p$ elastic scattering, using crossing symmetry, and to charged kaon pair production, on the basis of SU(3) symmetry. In all cases the results illustrate a nice agreement with the data.

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I. INTRODUCTION

Large experimental and theoretical efforts have been going on for decades to understand and classify high-energy processes driven by the strong interaction. We consider here hadronic reactions at intermediate energies, focusing on two-body final-state processes.

A beam of antiprotons is a very peculiar probe due to the fact that scattering and annihilation may occur in the same process, with definite kinematical characteristics. This process has been studied in the past, in connection with experiments at the Low Energy Antiproton Ring (LEAR) at CERN and FermiLab, where antiproton beams were available. Annihilation occurs mostly through the production of several pions, five pions being the most probable channel. This paper focuses on the annihilation reaction induced by antiprotons on a proton target, with production of two charged pions. At low energies the annihilation into light meson pairs is dominated by a few number of partial waves and the angular distribution shows a series of oscillations. Experimental data are analyzed with the use of Legendre polynomials [1]. This regime was also studied with the aim to look for resonances in the $\bar{p}p$ system.

A change of behavior appears above $\sqrt{s} = 2$ GeV, where the angular distributions become typical for peripheral processes. They are peaked at forward and backward angles, corresponding to small values of t or u, respectively (s, t, and u are the Mandelstam variables). The differential cross section $d\sigma/dt$ and the integrated cross section show a power-law behavior as a function of energy [2,3]. At even larger energies, the total cross section becomes asymptotically constant, reaching a regime where $d\sigma/dt$ is function of t only, being independent of s.

The most exhaustive data on neutral pion (and other neutral meson) production were published by the FermiLab E760 collaboration in the energy range ($2.911 \leqslant \sqrt{s} \leqslant 3.686$) GeV [4] and will be the subject of future work. Until now, no calculation has been developed to reproduce all these data. We focus here on charged pion production, which has a larger cross section. Data are scarce and do not fill with continuity a large angular or energy range [5–7]. To consolidate the model, we take into account other data sets on the crossed reaction $\pi^{\pm}p$ elastic scattering and on $\bar{p}p$ annihilation into charged kaons [6] in a comparable energy range [8,9].

According to the foreseen performances of the antiproton annihilation at Darmstadt (PANDA) experiment at the GSI Facility for Antiproton and Ion Research (FAIR) [10], a large amount of data related to light meson pairs production from $\bar{p}p$ annihilation is expected in the future. The best possible knowledge of light meson production is also requested before the experiment takes place, because pions constitute an important background for many other channels. The development of a realistic model working in the few GeV region is necessary, in particular the foreseen program on timelike form-factor measurements [11]. This program requires the detection of a lepton pair, and will benefit from a reliable estimation of the hadronic background. Pion pair production has five or six times larger cross section: pions should be effectively detected and identified [12,13]. For this aim, the model should reproduce the gross features of the data and should be expressed in a convenient form to be implemented in Monte Carlo calculations.

For the considered reaction, $\bar{p}p \to \pi^+\pi^-$, few calculations of cross section and angular distributions exist in the literature, and apply at lower energies than those of interest here. A baryon (N and Δ) t-channel exchange model has applicability below 1 GeV beam momentum [14–16]. Potential models [17] and sophisticated coupled-channel calculations

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[18] become too involved to be extended to higher energies, due, among other things, to the opening of several channels and contributions of many resonances [18–20]. Final-state interaction has been discussed in several works, such as Ref. [21]. Microscopic quark models were developed to predict relative two-body branching ratios, dynamical selection rules, or consequences of SU(2)/SU(3) symmetry (see, for example, Refs. [22,23]). Within a quark model, the authors of Ref. [24] show that relativistic effects accounting for higher partial-wave contributions should be taken into account to reproduce satisfactorily the data from Ref. [25].

The phenomenology added to take into account nonperturbative effects at low energies is, however, not unique and gave rise to different approaches (for a review, see Refs. [26,27]). The domain of applicability of the above calculations does not exceed 2 GeV.

At higher energies, a constituent interchange quark model was developed in Ref. [28]. The energy and angular dependencies can be predicted for large-angle scattering and elastic or quasi-elastic processes.

An effective Lagrangian model was recently developed at larger energies, including meson exchanges in the s channel, which qualitatively reproduces a limited set of angular distributions [29]. Parameters include cutoff and normalizations of form factors. It is assumed that Regge poles dominate the low-as well as the large-$|t|$ regions. An *ad hoc* parametrization of the nucleon Regge trajectory, that saturates at large, negative t (t is the Mandelstam variable for the transferred momentum), ensures the transition between *soft* and *hard* regimes. However, the authors warn against the application of this approach to neighboring energies, which is possibly related to the specific extrapolation of Regge trajectories in the region $t < 0$.

We develop here a model with meson and baryon exchanges in s, t, and u channels, in the energy range of the PANDA experiment at FAIR [$2.25(1.5) \leqslant \sqrt{s}(p_L) \leqslant 5.47(15)$ GeV (GeV/c)]. It is known that first-order Born diagrams give cross sections much larger than measured, because Feynman diagrams assume point-like particles. Form factors are added to take into account the composite nature of the interacting particles at vertices. Their parametrization is model dependent: parameters as coupling constants or cutoff are adjusted to reproduce the data. A "Reggeization" of the trajectories is often added to reproduce the very forward and very backward scattering angles. This class of models should be considered as a phenomenological, effective way to take into account microscopic degrees of freedom and quark exchange diagrams.

Our aim is to build a model with minimal ingredients, to calculate the basic features of charged-pion production in the energy range that will be investigated by the future experiment PANDA at FAIR. To test the adequacy of the model, we exploit also the existing $\pi^{\pm} p$ elastic-scattering data, applying crossing symmetry in order to compare the predictions based on the annihilation channel, at least in a limited kinematical range. The main requirement is that the model should be able to reproduce at the same time charged-pion production from annihilation, and $\pi^{\pm} p$ elastic scattering. It is straightforward to extend the model to charged kaon pair production, with suitable replacements of the masses of the particles. At the considered energies, one can also rely on SU(3) symmetry, which gives a prescription to connect charged kaon and pion pair production from $\bar{p}p$ annihilation. The comparison of the results with the existing angular distributions and energy dependencies shows that the model successfully reproduces the available data.

Our paper is organized as follows: In Sec. II, the formalism is derived: the definition of the variables, the kinematics, and the outline of the model. In Sec. III the relevant data are shown in comparison with the results of the model for cross sections and angular distributions. Section IV shows a straightforward application to kaon pair production. In Sec. V conclusions summarize the main findings of the paper. The relevant diagrams are calculated in the appendixes.

II. FORMALISM

A. Kinematics and cross section

We consider the annihilation reaction

$$\bar{p}(p_1) + p(p_2) \to \pi^-(k_1) + \pi^+(k_2) \tag{1}$$

in the center-of-mass system (CMS). The notation of four-momenta is shown in the parentheses. The following notations are used:

$$\begin{aligned} q_t &= (-p_1 + k_1), \quad q_t^2 = t,\\ q_u &= (-p_1 + k_2), \quad q_u^2 = u,\\ q_s &= (p_1 + p_2), \quad q_s^2 = s, \end{aligned}$$

with $s + t + u = 2M_N^2 + 2m_\pi^2$, where M_N (m_π) is the nucleon (pion) mass. The useful scalar product between four-vectors are explicitly written as

$$\begin{aligned} 2p_1k_2 &= 2k_1p_2 = M_N^2 + m_\pi^2 - u,\\ 2p_1k_1 &= 2k_2p_2 = M_N^2 + m_\pi^2 - t,\\ 2p_1p_2 &= s - 2M_N^2, \quad 2k_1k_2 = s - 2m_\pi^2,\\ p_1^2 &= p_2^2 = M_N^2 = E^2 - |\vec{p}|^2,\\ k_1^2 &= k_2^2 = m_\pi^2 = \varepsilon^2 - |\vec{k}|^2, \end{aligned} \tag{2}$$

where E (ε) is the energy of the proton (pion), and $|\vec{p}|$ and $|\vec{k}|$ are the initial and final particles momenta (moduli), respectively, in the CMS. The general expression for the differential cross section of reaction (1) is

$$\frac{d\sigma}{d\Omega} = \frac{1}{2^8\pi^2}\frac{1}{s}\frac{\beta_\pi}{\beta_p}\overline{|\mathcal{M}|^2}, \quad \frac{d\sigma}{d\cos\theta} = 2E^2\beta_p\beta_\pi\frac{d\sigma}{dt}, \tag{3}$$

where $\overline{|\mathcal{M}|^2}$ is the squared matrix element of the process averaged over the spins of the initial particles, β_p (β_π) is the velocity of the proton (pion) in the CMS. The phase volume can be transformed as $d\Omega \to 2\pi\ d\cos\theta$ due to the azimuthal symmetry of binary reactions. The total cross section is

$$\sigma = \int \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 s}\frac{|\vec{p}|}{|\vec{k}|}d\Omega. \tag{4}$$

B. Crossing symmetry

Crossing symmetry relates annihilation and scattering cross sections and states that the same amplitudes describe the

TABLE I. Correspondence between variables in the crossed scattering (s) and annihilation (a) channels.

Annihilation	Scattering
$\bar{p}(p_1)+p(p_2)\to\pi^-(k_1)+\pi^+(k_2)$	$\pi^-(-k_2)+p(p_2)\to\pi^-(k_1)+p(-p_1)$
$s_a=(p_1+p_2)^2$	$s_s=(-k_2+p_2)^2$
$t_a=(p_1-k_1)^2$	$t_s=(-k_2-k_1)^2$
$u_a=(p_1-k_2)^2$	$u_s=(p_1-k_2)^2$
$s_a=4E^2=4(M^2+\lvert\vec{p}_a\rvert^2)$	$s_s=m^2+M^2+2E_2'\epsilon_2'+2\lvert\vec{k}_s\rvert^2$
$\sigma_a=\frac{1}{4}\frac{\overline{\lvert\mathcal{M}_a\rvert^2}}{64\pi^2 s}\frac{\lvert\vec{k}_a\rvert}{\lvert\vec{p}_a\rvert}$	$\sigma_s=\frac{1}{2}\frac{\overline{\lvert\mathcal{M}_s\rvert^2}}{64\pi^2 s}\frac{\lvert\vec{k}_s\rvert}{\lvert\vec{p}_s\rvert}$

crossed processes. This means that the matrix element $\mathcal{M}(s,t)$ is the same for crossed processes at corresponding s and t values, but the variables span different regions of the kinematical space. To find this correspondence, kinematical replacements between variables should be done, as indicated in Table I. Note that the coefficients 1/2 and 1/4 in the cross-section formulas are the spin factors: $(2S_\pi+1)(2S_p+1)$ and $(2S_{\bar{p}}+1)(2S_p+1)$ for the scattering and annihilation channels, respectively, where S_π, S_p, and $S_{\bar{p}}$ are the spins of the corresponding initial particles. The incident momentum in the annihilation channel for a given invariant s is $|\vec{p}_a|=(s/4-M^2)^{1/2}$. From the equality $s_a=s_s$, the CMS momentum for $\pi^- p$ scattering, $|k_s|$, is evaluated at the same s value:

$$|\vec{k}_s|^2=\frac{1}{4s}[m^4-2m^2(M^2+s)+(M^2-s)^2], \tag{5}$$

and the amplitudes are assumed to be the same at this point. Then the cross sections for the two crossed processes are related by

$$\sigma_a=\frac{1}{2}\frac{|\vec{k}_s|^2}{|\vec{p}_a|^2}\sigma_s. \tag{6}$$

If the scattering cross section is measured at a value $s_s=s_1$ different from $s_a=s$, at small t values one should rescale the cross section by using the empirical dependence $\sigma_s\simeq \text{const.}\times s^{-2}$ [6].

C. The model, the amplitudes, and the matrix element

The formulas written above are model independent, i.e., they hold for any reaction mechanism. To calculate $\mathcal{M}$, one needs to specify a model for the reaction. In this work we consider the process (1) within the formalism of effective meson Lagrangian.

The following contributions to the cross section for the reaction (1) are calculated as illustrated in Fig. 1:

(i) baryon exchange:
- (a) t-channel nucleon (neutron) exchange, Fig. 1(a);
- (b) t-channel Δ^0 exchange, Fig. 1(b);
- (c) u-channel Δ^{++} exchange, Fig. 1(c);

(ii) s-channel ρ-meson exchange, Fig. 1(d).

The total amplitude is written as a coherent sum of all the amplitudes:

$$\mathcal{M}=\mathcal{M}_n+\mathcal{M}_{\Delta^0}+\mathcal{M}_{\Delta^{++}}+\mathcal{M}_\rho. \tag{7}$$

The dominant contribution in the forward direction is N exchange, whereas Δ^{++} mostly contributes to backward scattering. We neglect the difference of masses between the nucleons as well as between different charge states of the pion and of the Δ resonance. Scattering around $\cos\theta=0$ is sensitive to s-channel exchange of vector mesons, with the same quantum numbers as the photon. Although several resonances are present in these region, no one appears to be dominant outside its peak region. Adding resonances brings new parameters and unknown relative phases. Therefore, we limit this contribution to ρ-meson exchange. Assuming ρ is the dominant contribution, our approach is equivalent to giving more weight to the P wave. Far from their maximum, all the $L=1$ resonances give proportional contributions, and one can consider that they are effectively taken into account. The significant test of the contributing L values is contained in the experimental angular distributions. These are well reproduced by our model. The expressions for the amplitudes and their interferences are detailed in the appendix. Coupling constants are fixed from the known decays of the particles if it is possible, otherwise we use the values from effective potentials of Ref. [30]. Masses and widths are taken from Ref. [31].

The effects of strong interaction in the initial state, between proton and antiproton, coming from the exchange of vector and (pseudo) scalar mesons are essential. At larger energies they effectively lead to the Regge form of the amplitude. However, in the considered energy range, it turns out that the considered data may be reproduced only with unusual values of the Regge parameters. Thus we conclude that we are not in the Regge regime yet. However, one should realize this pre-Regge behavior taking into account some effects of intermediate nucleon (Δ resonance) off-mass-shellness.

After different attempts, we found that the best solution in this energy range, which is not asymptotic yet, is to parametrize the form factors at the $\bar{p}p$ vertexes, with a

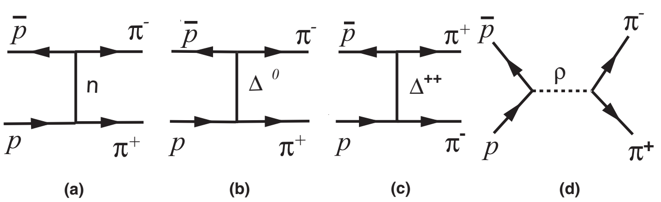

FIG. 1. Feynman diagrams for the reaction $\bar{p}+p\to\pi^-+\pi^+$ within the effective meson Lagrangian approach.

logarithmic function of x (where $x = s$ or t or u). This choice is based on a QCD derivation from Refs. [32,33] that relates the asymptotic behavior of form factors to the quark contents of the participating hadrons. It is also known that a logarithmic dependence of the $\bar{p}p$ cross section reproduces quite well the background for resonant processes [34,35]. It turns out that the function

$$F_{N,\Delta}(x) = \frac{\mathcal{N}_{N,\Delta} M_0^4}{\left[\left(x - \Lambda_{N,\Delta}^2\right) \log \frac{(x-\Lambda_{N,\Delta}^2)}{\Lambda_{\rm QCD}^2}\right]^2}, \quad x = s,t,u \tag{8}$$

acts as a very convenient form factor, where $M_0 = 3.86$ GeV is a scale parameter, which has been inserted to conserve units, $\Lambda_{\rm QCD} = 0.3$ GeV is the QCD scale parameter; $\mathcal{N}_{N,(\Delta)} = 0.361 \pm 0.006$ (0.041 ± 0.003) is a normalization constant fit on the data, $\Lambda_{N,(\Delta)} = 2.25 \pm 0.09$ (1.05 ± 0.04) is a "slope" parameter fit on the data (in GeV units).

The procedure is the following: first we apply the form factor $F_{N,\Delta}$ which depends on momentum transfer (t or u) to take into account the composite nature of the particle in the interaction point. Second, we use the factor $F_{N,\Delta}(s)$ which effectively takes into account pre-Regge regime excitations of higher resonances in the intermediate state. This leads to an effective form factor as the product $\widetilde{F}_{N,\Delta}(s,t) = F_{N,\Delta}(s)F_{N,\Delta}(t)$ [or $\widetilde{F}_{N,\Delta}(s,u) = F_{N,\Delta}(s)F_{N,\Delta}(u)$] containing the same set of parameters for the s and $t(u)$ dependencies, but different for N and Δ exchanges. The fit does not require independent parameters for s and $t(u)$ dependencies.

The ρNN vertex includes the proton structure in the vector current form with two form factors (FFs) F_1^ρ and F_2^ρ

$$\Gamma_\mu(q_s) = F_1^\rho\left(q_s^2\right)\gamma_\mu(q_s) + \frac{i}{2M_N} F_2^\rho\left(q_s^2\right)\sigma_{\mu\nu} q_s^\nu, \tag{9}$$

where $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu]$ is the antisymmetric tensor. Due to the isovector nature of the ρ, the ρNN is similar to the electromagnetic vertex γNN. However, the two form factors $F_1^\rho(q_s^2)$ and $F_2^\rho(q_s^2)$ are different from the proton electromagnetic ones. We do not attempt new parametrization but prefer to fix the form, the constants, and the parameters of $F_1^\rho(s)$ and $F_2^\rho(s)$ according to Refs. [30,36,37] as

$$F_1^\rho(s) = g_{\rho NN} \frac{\Lambda_\rho^4}{\Lambda_\rho^4 + \left(s - M_\rho^2\right)^2}, \quad F_2^\rho(s) = \kappa_\rho F_1^\rho(s), \tag{10}$$

with normalization $F_1^\rho(M_\rho^2) = g_{\rho NN}$, where the constant $g_{\rho NN}$ corresponds to the coupling of the vector meson ρ with the nucleon $[g_{\rho NN}^2/(4\pi) = 0.55]$, $\kappa_\rho = 3.7$ is the anomalous magnetic moment of the proton with respect to the coupling with ρ, and $\Lambda_\rho = 0.911$ is an empirical cutoff.

The ρ amplitude is in principle complex, since form factors in the s channel can contain some nontrivial phase. To reproduce the s dependence of the cross section, a relative phase, $\Phi = e^{i\pi\phi_\rho}$, and a normalization are added.

III. COMPARISON WITH EXISTING DATA: $\bar{p} + p \to \pi^- + \pi^+$ AND $\pi^\pm + p \to \pi^\pm + p$

The results for the the $\cos\theta$ dependence of the annihilation reaction $\bar{p} + p \to \pi^- + \pi^+$ are shown for the available data at four values of the total CMS energy (Laboratory antiproton momentum): $\sqrt{s}(p_L) = 3.362(5)$ GeV (GeV/c) [7], 3.627(6) GeV (GeV/c) [38], 3.680(6.2) GeV (GeV/c) [6], and 4.559 (10.1) GeV (GeV/c) [39]. The calculation from the present model is compared to the predictions from the constituent interchange model of Ref. [28].

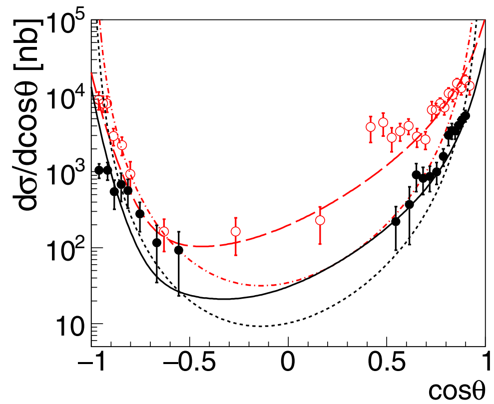

FIG. 2. Angular dependence (log scale) for the reaction $\bar{p} + p \to \pi^- + \pi^+$, for $\sqrt{s} = 3.680$ GeV from Ref. [6] (black solid circles) and for $\sqrt{s} = 3.362$ GeV from Ref. [7] (red open circles). The present calculation is shown by a long-dashed red line, and a solid black line at the corresponding energies. The calculation from the constituent interchange model of Ref. [28] is also reported as a dash-dotted red line and a dotted black line, respectively.

In the fitting procedures, only the data taken at the CERN proton synchrotron, from Refs. [6,7,39], were included, getting a χ^2/ndf = 2.16 ("ndf" is the number of degrees of freedom). The data from Ref. [38], taken at the Brookhaven National Laboratory multiparticle spectrometer (MPS), were excluded from the fit, due to several neighboring points that bias the fit. The results in this plot are therefore given with the parameters fit on the other data sets.

The data Ref. [6] (black solid circles) and from Ref. [7] (red open circles) are shown in Fig. 2, together with the results of the present calculation for the corresponding energies, solid black line and dashed red line, respectively. The calculation from the constituent interchange model of Ref. [28] is also reported as a dotted black line and a dash-dotted red line, respectively.

For the two sets of annihilation data, from Refs. [38] and [39], few points are available at forward angles. For the sake of clarity, a reduced angular region is shown in Fig. 3.

The available data from the annihilation reaction do not cover the full θ range, especially in the backward region data are scarce. Because the PANDA detector covers a 4π solid angle, we rely on the data from $\pi^\pm p$ elastic scattering with the help of crossing symmetry to fill this region. The results for elastic scattering $\pi^- + p \to \pi^- + p$ are shown in Fig. 4. The data correspond to very backward angles of the π^- emission. No other rearrangement of the parameters was done for the elastic-scattering data bringing an additional test of the reliability of the model and of the validity of crossing symmetry, at least in the kinematical region where one diagram dominates. The agreement is very good for all data sets.

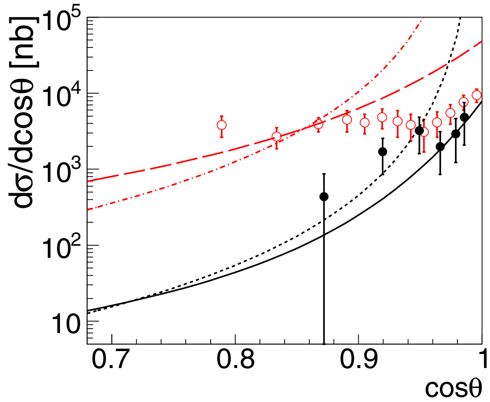

FIG. 3. Angular dependence (log scale) for the reaction $\bar{p} + p \to \pi^- + \pi^+$ for $\sqrt{s} = 3.627$ GeV [38] (red open circles), and 4.559 GeV [39] (black solid circles). The data from Ref. [38] were excluded from the fit, but they are well reproduced *a posteriori*. Lines are as in Fig. 2.

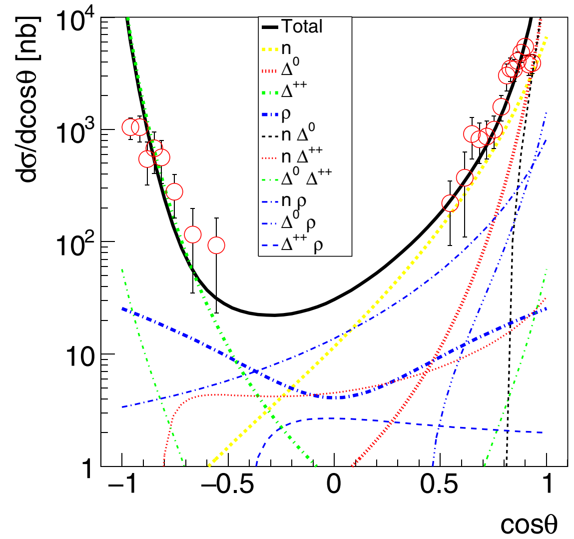

FIG. 5. Angular distribution for $\bar{p} + p \to \pi^+ + \pi^-$ for $\sqrt{s} = 3.680$ GeV. The data are from Ref. [6]. The total function (black solid line) and all components are shown on a log scale: n exchange (yellow thick short-dashed line) dominates at forward angle, followed by Δ^0 (read thick dotted line). Δ^{++} (green thick dash-dotted line) represents the largest contribution for backward angles. The ρ channel (blue thick long-dashed line) has a larger contribution for $\cos\theta \simeq 0$, relative to the dominant components. The interferences are $n\Delta^0$ (thin black short-dashed line) and $n\Delta^{++}$ (thin red dotted line), visible at forward angles, $\Delta^0\Delta^{++}$ (green thin short dash-dotted line), $n\rho$ (blue thin long-dashed dotted line), $\Delta^0\rho$ (blue thin dash-triple-dotted line), $\Delta^{++}\rho$ (blue thin long-dashed line).

The angular distribution for $\sqrt{s} = 3.680$ GeV is shown in Fig. 5. The total result (black, solid line) gives a very good description of the data from Ref. [6] (red open circles), that here can be appreciated in log scale. For this single data set $\chi^2/\text{ndf} = 1.69$.

All components and their interferences are illustrated. n exchange in t channel dominates at forward angles, followed by Δ^0 exchange. Δ^{++} represent the largest contribution for backward angles. The interferences affect the shape of the angular distribution, some of them being negative in part of the angular region. The ρs-channel exchange is small, but its contribution as well as the interferences with the other diagrams help in saturating the cross section around $\cos\theta = 0$, being relatively more important in the region $-0.5 \leqslant \cos\theta = 0.5$.

The necessary number of parameters is very limited: two for the proton and two for the Δ form factor. The phase between the s channel (ρ exchange) and the u, t channels, ϕ_ρ, after fitting, is set to unity. The relative normalization of the ρ diagram is also found consistent with unity and is set to unity.

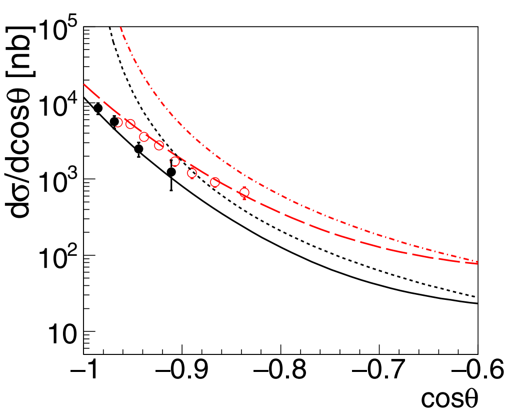

FIG. 4. Angular dependence (log scale) from the elastic reactions $\pi^- + p \to \pi^- + p$ at $\sqrt{s} = 3.463$ GeV from Ref. [8] (red open circles), and at $\sqrt{s} = 3.747$ GeV from Ref. [9] (black solid circles). Lines are as in Fig. 2.

The dominance of the nucleon and Δ^{++} exchanges in the forward and backward angles, may point out to the presence of nucleon and Δ^{++} poles in the unphysical region. At lower energies, this phenomena is similar to the peak arising from charged-pion exchange (pion pole) in the charge-exchange neutron-proton scattering (backward peak) and in the proton-antiproton to neutron-antineutron scattering (forward peak) [40]. At larger energies, the pion pole model has been also successfully applied for neutron-proton scattering in Ref. [41].

We checked that the results are quite stable against a change of the parameters in a reasonable interval. In Fig. 6 the s dependence of the cross section is shown for $\cos\theta = 0$. The long (short) dashed line corresponds to a change of $+10\%$ (-10%) of the parameters, that increases (decreases) the total function. The sensitivity of the function to these changes is s dependent, becoming negligible over 4.5 GeV. QCD gives predictions for the cross section of exclusive processes, formulated in terms of quark counting rules [2,3], that in our case is proportional to s^{-8}. Our model is reasonably consistent with these prediction in the energy range considered.

We may integrate the calculated differential cross section and give the following values $\sigma(\bar{p} + p \to \pi^+ + \pi^-) = 4.2 \pm 2.1$ mb at $\sqrt{s} = 3.362$ GeV, $\sigma = 1.4 \pm 0.8$ mb at $\sqrt{s} = 3.680$ GeV, and $\sigma = 1.0 \pm 0.5$ mb at $\sqrt{s} = 4.559$ GeV, where,

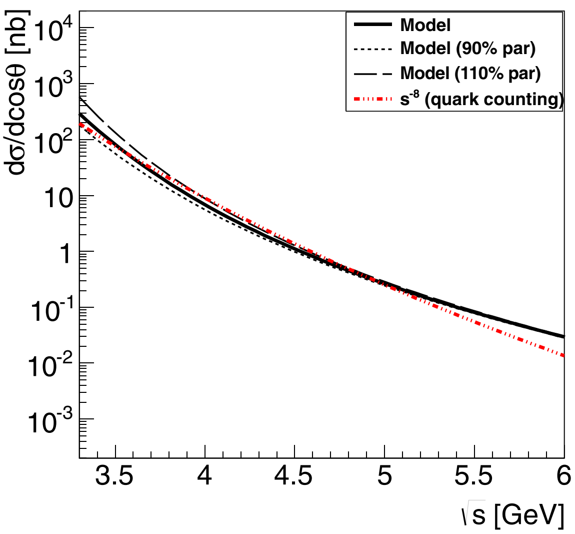

FIG. 6. s dependence for $\bar{p}+p\to\pi^++\pi^-$ for $\theta=90^\circ$. The total function (black solid line), two functions with parameters changed by $+10\%$ (black long-dashed line) and -10% (short-dashed line) from the fitted values are shown on a log scale, compared with quark-counting-rule results from Refs. [2,3] (red triple dot-dashed line).

for safety, we overestimate the error propagation on the fit parameters by 50%.

IV. EXTENSION TO STRANGE PARTICLES: $\bar{p}+p\to K^-+K^+$

A similar model, based on corresponding diagrams can be built for charged kaon pair production, changing the mass of the produced and exchanged particle, and replacing the

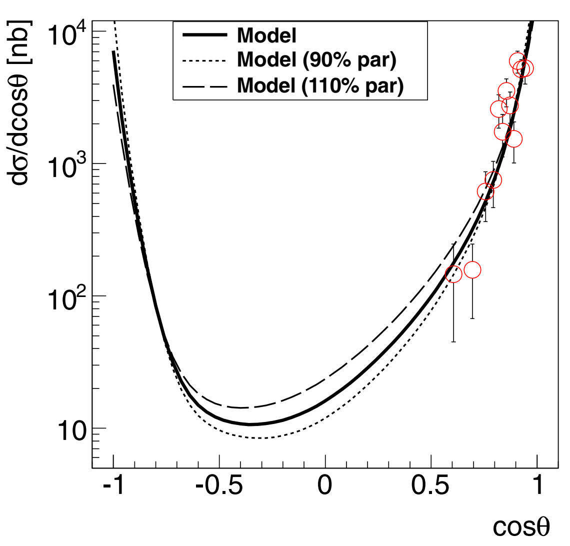

FIG. 7. Angular distribution for $\bar{p}+p\to K^++K^-$ for $\sqrt{s}=3.680$ GeV. The total function (black solid line) is shown on a log scale. The dashed (dotted) line is the result of the model corresponding to values of the parameters 10% lower (higher). The data are from Ref. [6].

coupling constants. Another possibility is to rely on SU(3) and test its prescription.

Having built the matrix element for $\bar{p}+p\to\pi^-+\pi^+$, we may calculate the cross section of $\bar{p}+p\to K^-+K^+$, by applying a global factor, following Ref. [42]:

$$\sigma(\pi^-\pi^+):\sigma(K^-K^+)=1:\frac{4\lambda}{3},\quad \text{where } \lambda=0.4.$$

In the energy range considered, one set of data for the angular distribution exists at $\sqrt{s}=3.680$ GeV [6]. The results are reported in Fig. 7, showing a very good agreement with these data, without need of adjusting the parameters.

We may evaluate the integrated cross section, as $\sigma(\bar{p}+p\to K^++K^-)=2.1\pm0.8$ mb at $\sqrt{s}=3.680$ GeV.

V. CONCLUSIONS

A model, based on an effective meson Lagrangian, has been built in order to reproduce the existing data for two-charged-pion production in proton-antiproton annihilation at moderate energies. Form factors are implemented and parameters adjusted to the existing data for charged-pion pair production. Coupling constants are fixed from the known properties of the corresponding decay channels. The agreement with the existing data is satisfactory for the angular dependence as well as the energy dependence of the cross section. At large energies the model follows naturally the expected behavior from quark counting rules. A comparison with data from elastic $\pi^\pm p\to\pi^\pm p$, using crossing-symmetry prescriptions shows a good agreement and brings additional constraints at very backward angles. Discussions about the validity of crossing symmetry can be found in Refs. [7,8,38]. The present results verify that crossing symmetry works at least at backward angles, where one diagram is dominant. Moreover, applying SU(3) symmetry, one reproduces the existing data on the angular distribution for $\bar{p}+p\to K^++K^-$ in the corresponding energy range.

The logarithmic expression of the form factors implemented here is typically used for hadron electromagnetic form factors, in the annihilation region. In the scattering region it corresponds to the dipole form and follows quark counting rules [2,3], allowing the description of crossed reactions. In recent works it has been suggested that it enters into the early $\bar{q}-q$ pair formation from the quantum vacuum [43,44]. In the present work it is applied to fully hadronic reactions, providing an exhaustive description of the considered reactions in the FAIR-PANDA energy range.

At the PANDA experiment at FAIR, light mesons will be copiously produced in the considered energy range. PANDA will use an antiproton beam on a proton target. One important line of research will be the measurement of timelike form factors through $\bar{p}+p\to e^++e^-$ [11,12] or $\mu^++\mu^-$ [45]. Namely, the annihilation into $\pi^+\pi^-$ constitutes the largest background to lepton pair production [13,46]. The implementation of Monte Carlo simulations for predictions and optimization for the future experiments is planned.

This model can be extended to other binary channels; in particular to neutral pion and other light meson production.

These channels have smaller cross sections than charged pions, by at least a factor of ten.

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APPENDIX A: NEUTRON EXCHANGE

The relevant formulas for the amplitudes and their interferences are given below. The amplitude for nucleon exchange [see Fig. 1(a)] is written as

$$\mathcal{M}_N = \frac{g^2_{\pi NN}}{q_t^2 - M_N^2}\bar{v}(p_1)(-\hat{q}_t + M_N)u(p_2), \tag{A1}$$

where $u(p_2)$ [$\bar{v}(p_1)$] are the four-component spinors of the proton. The matrix element squared is written as

$$\begin{aligned}\overline{|\mathcal{M}_N|^2} &= \frac{g^4_{\pi NN}}{\left(q_t^2 - M_N^2\right)^2}\mathrm{Tr}[(\hat{p}_1 - M_N)(-\hat{q}_t + M_N) \\ &\quad \times (\hat{p}_2 + M_N)(-\hat{q}_t + M_N)] \\ &= -2\frac{g^4_{\pi NN}}{\left(t - M_N^2\right)^2}\left[m_\pi^4 + \left(M_N^2 - t\right)\left(M_N^2 - s - t + 2m_\pi\right)\right],\end{aligned} \tag{A2}$$

with $q_t = k_1 - p_1 = p_2 - k_2$, $q_t^2 = t$.

APPENDIX B: t-EXCHANGE OF Δ^0

The specific ingredients for Δ exchange [see Fig. 1(b)] are related to the spin $3/2$ nature of the Δ resonance. For the Δ-spin vector U_Δ, we take the expression from Refs. [47–49], where the density matrix is

$$\begin{aligned}P_{\mu\nu} &= U^\Delta_\mu(p_\Delta)\bar{U}^{*\Delta}_\nu(p_\Delta) \\ &= -g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{\gamma_\mu P_\nu - \gamma_\nu P_\mu}{3M_\Delta} + \frac{2}{3}\frac{P_\mu P_\nu}{M_\Delta^2}.\end{aligned} \tag{B1}$$

The Δ propagator is parametrized as

$$\frac{i}{(2\pi)^4}\frac{\hat{q}_t + M_\Delta}{q_t^2 - M_\Delta^2}P_{\mu\nu}, \tag{B2}$$

and the expression of the vertex $\Delta \to \pi N$ is

$$-i(2\pi)^4 g_{\Delta\pi N}k_1^\mu. \tag{B3}$$

$M_\Delta = 1232 \pm 2$ MeV is the weighted mass of the Δ resonance (i.e., the mass averaged over the Δ multiplet), and $g_{\Delta\pi N}$ is the coupling constant for the vertex $\Delta \to \pi N$.

The matrix element for the diagram in Fig. 1(b) is

$$\mathcal{M}_{\Delta^0} = -\frac{g^2_{\Delta\pi N}}{t - M_\Delta^2}\bar{v}(p_1)(\hat{q}_t + M_\Delta)P_{\mu\nu}(q_t)u(p_2)k_1^\mu k_2^\nu. \tag{B4}$$

Squaring the amplitude, one has

$$\begin{aligned}\overline{|\mathcal{M}_{\Delta^0}|^2} &= \frac{g^4_{\Delta\pi N}}{\left(t - M_\Delta^2\right)^2}k_1^\mu k_2^\nu k_1^\alpha k_2^\beta \mathrm{Tr}[(\hat{p}_1 - M_N)(\hat{q}_t + M_\Delta) \\ &\quad \times P_{\mu\nu}(q_t)(\hat{p}_2 + M_N)\tilde{P}_{\alpha\beta}(q_t)(\hat{q}_t + M_\Delta)].\end{aligned} \tag{B5}$$

Here we use the notation $\tilde{O} = \gamma_0 O^+ \gamma_0$. To find the value of $g_{\Delta N\pi}$ coupling constant we consider the decay width of Δ into nucleon and pion:

$$\Gamma_\Delta = \frac{3}{2}\frac{|\vec{p}_p|}{32\pi M_\Delta^2}|\mathcal{M}(\Delta \to N\pi)|^2, \tag{B6}$$

where p_p is the nucleon momentum in the system where the Δ is at rest. Using the experimental values of the decay width $\Gamma_\Delta = 117 \pm 3$ MeV [31], one can estimate $g_{\Delta N\pi} = 15.7 \pm 0.4\ \mathrm{GeV}^{-1}$.

APPENDIX C: u-EXCHANGE OF Δ^{++}

The diagram in Fig. 1(c) dominates π^- emission at backward angles. It involves the exchange of Δ^{++} and can be obtained from t exchange, Fig. 1(b), with the replacements: $t \leftrightarrow u$ and $k_1 \leftrightarrow k_2$.

APPENDIX D: INTERFERENCES WITH Δ

1. The Δ^0-N interference

The expression of the Δ^0-N interference is

$$2\mathrm{Re}\left[M_N^* M_\Delta^0\right] = 2\mathrm{Re}\left\{\frac{g^2_{\pi NN}g^2_{\Delta\pi N}}{\left(t - M_N^2\right)\left(t - M_\Delta^2\right)}\mathrm{Tr}[(\hat{p}_1 + M_N)(-\hat{q}_t + M_N)(\hat{p}_2 + M_N)\tilde{P}_{\mu\nu}(q_t)(\hat{q}_u + M_\Delta)]k_1^\mu k_2^\nu\right\}, \tag{D1}$$

with $q_u = k_2 - p_1 = p_2 - k_1$, $q_u^2 = u$.

2. The Δ^{++}-N interference

The expression of the Δ^{++}-N interference is

$$2\mathrm{Re}[M_N^* M_\Delta^{++}] = 2\mathrm{Re}\left\{\frac{g_{\pi NN}^2 g_{\Delta\pi N}^2}{\left(u-M_\Delta^2\right)\left(t-M_N^2\right)}\mathrm{Tr}[(\hat{p}_1+M_N)(-\hat{q}_t+M_N)(\hat{p}_2+M_N)\tilde{P}_{\mu\nu}(q_u)(\hat{q}_u+M_\Delta)]k_1^\nu k_2^\mu\right\}. \tag{D2}$$

3. The Δ^{++}-Δ^0 interference

The expression for the Δ^{++}-Δ^0 interference is

$$2\mathrm{Re}\left[M_\Delta^{*0} M_\Delta^{++}\right] = 2\mathrm{Re}\left\{\frac{g_{\Delta\pi N}^4}{\left(t-M_\Delta^2\right)\left(u-M_\Delta^2\right)}\mathrm{Tr}[(\hat{p}_1-M_N)(\hat{q}_t+M_N)P_{\mu\nu}(q_t)(\hat{p}_2+M_N)\tilde{P}_{\alpha\beta}(q_u)(\hat{q}_u+M_\Delta)]k_1^\mu k_2^\nu k_2^\alpha k_1^\beta\right\}. \tag{D3}$$

APPENDIX E: s-EXCHANGE OF ρ MESON

In the s channel, Fig. 1(d), we consider a ρ-meson contribution, with $\sim$100% branching ratio into two pions. We take for the ρ propagator:

$$-\frac{i}{(2\pi)^4}\left[\frac{g_{\mu\nu}-\left(q_s^\mu q_s^\nu\right)/m_\rho^2}{q_s^2-m_\rho^2+i\sqrt{q_s^2}\Gamma_\rho\left(q_s^2\right)}\right], \tag{E1}$$

and for the $\rho\pi\pi$ vertex:

$$-ig_{\rho\pi\pi}(k_1-k_2)^\nu(2\pi)^4, \tag{E2}$$

where $g_{\mu\nu}$ is the metric tensor, $q_s = p_1+p_2 = k_1+k_2$, and $q_s^2 = s$. The matrix element is written as

$$\mathcal{M}_\rho = \frac{g_{\rho pp}g_{\rho\pi\pi}}{\left[s-m_\rho^2+i\sqrt{s}\Gamma_\rho(s)\right]}[\bar{v}(p_1)\Gamma^\mu(q)u(p_2)](k_1-k_2)^\nu\left\{g_{\mu\nu}-\frac{q_\mu q_\nu}{m_\rho^2}\right\}. \tag{E3}$$

Squaring the amplitude one gets

$$\begin{aligned}\overline{|\mathcal{M}_\rho|^2} = {}& \frac{g_{\rho NN}^2 g_{\rho\pi\pi}^2}{\left[s-m_\rho^2+i\sqrt{s}\Gamma_\rho(s)\right]^2}(k_1-k_2)^\nu(k_1-k_2)^\beta\left(g_{\mu\nu}-\frac{(q_s)_\mu(q_s)_\nu}{m_\rho^2}\right)\left(g_{\alpha\beta}-\frac{(q_s)_\alpha(q_s)_\beta}{m_\rho^2}\right)\\ &\times\mathrm{Tr}[(\hat{p}_1-M_N)\Gamma^\mu(q_s)(\hat{p}_2+M_N)\Gamma^\alpha(q_s)].\end{aligned} \tag{E4}$$

The coupling constant $g_{\rho\to\pi\pi}$ is found from the the experimental value of the total width Γ for the decay $\rho\to\pi\pi$: $\Gamma(\rho) = 149.1 \pm 0.8$ MeV [31]. The branching ratio into two pions is $\approx$100%. The total width has the form

$$\Gamma = \frac{4}{3}\frac{g_{\rho\pi\pi}^2}{16\pi m_\rho^2}\left(m_\rho^2-4m_\pi^2\right)^{3/2}, \tag{E5}$$

where we added a factor 4/3 to take into account that there are three possible initial states of the ρ meson and four possible charged decays. Inverting Eq. (E5) and using the experimental value for the decay width one finds the following value of the coupling constant: $g_{\rho\pi\pi} = 5.175 \pm 0.017$.

APPENDIX F: INTERFERENCES WITH ρ

1. The N-ρ interference

The expression of the N-ρ interference is

$$\begin{aligned}2\mathrm{Re}[M_N^* M_\rho] = 2\mathrm{Re}\Bigg\{&\frac{g_{\pi NN}g_{\rho\pi\pi}g_{\rho NN}^2}{\left[s-m_\rho^2+i\sqrt{s}\Gamma_\rho(s)\right]\left(t-M_N^2\right)}\mathrm{Tr}[(\hat{p}_1-M_N)\Gamma^\mu(q_s)(\hat{p}_2+M_N)\tilde{P}_{\alpha\beta}(q_t)(-\hat{q}_t+M_N)]\\ &\times k_1^\alpha k_2^\beta(k_1-k_2)^\nu\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_\rho^2}\right)\Bigg\}.\end{aligned} \tag{F1}$$

2. The Δ^0-ρ interference

The expression of the Δ^0-ρ interference is

$$2\mathrm{Re}[M_{\Delta^0} m_\rho^*] = 2\mathrm{Re}\left\{\frac{g_{\rho NN} g_{\rho\pi\pi} g_{\Delta\rho N}^2}{\left[s - m_\rho^2 + i\sqrt{s}\Gamma_\rho(s)\right]\left(t - M_\Delta^2\right)} \mathrm{Tr}[(\hat{p}_1 - M_N)\Gamma^\mu(q_s)(\hat{p}_2 + M_N)\tilde{P}_{\alpha\beta}(q_t)(-\hat{q}_t + M_\Delta)] \times k_1^\alpha k_2^\beta (k_1 - k_2)^\nu \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_\rho^2}\right)\right\}. \tag{F2}$$

3. The Δ^{++}-ρ interference

The expression of the Δ^{++}-ρ interference is

$$2\mathrm{Re}[M_{\Delta^{++}} m_\rho^*] = 2\mathrm{Re}\left\{\frac{g_{\rho NN} g_{\rho\pi\pi} g_{\Delta\rho N}^2}{\left[s - m_\rho^2 + i\sqrt{s}\Gamma_\rho(s)\right]\left(u - M_\Delta^2\right)} \mathrm{Tr}[(\hat{p}_1 - M_N)\Gamma^\mu(q_s)(\hat{p}_2 + M_N)\tilde{P}_{\alpha\beta}(q_u)(-\hat{q}_u + M_\Delta)] \times k_1^\alpha k_2^\beta (k_1 - k_2)^\nu \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_\rho^2}\right)\right\}. \tag{F3}$$

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