Examining possible neutron-halo nuclei heavier than ^{37}Mg

Ikuko Hamamoto
Riken Nishina Center, Wako, Saitama 351-0198, Japan
and Division of Mathematical Physics, Lund Institute of Technology at the University of Lund, Lund, Sweden
(Received 8 November 2016; published 26 April 2017)

The even-Z odd-N neutron-halo nuclei, which are the lightest neutron-halo nuclei heavier than ^{37}Mg, are explored by studying the shell structure unique in weakly bound neutrons for spherical or deformed shapes. It is pointed out that due to the narrowed $N = 50$ spherical energy gap and a few resulting close-lying neutron one-particle levels, $1g_{9/2}$, $3s_{1/2}$, and $2d_{5/2}$, for spherical shape, nuclei with some weakly bound neutrons filling in those levels may be deformed and have a good chance to show deformed s-wave halos. Promising candidates are $^{71}_{24}$Cr$_{47}$, $^{73}_{24}$Cr$_{49}$, $^{75}_{24}$Cr$_{51}$, and $^{77}_{26}$Fe$_{51}$ in the case that those nuclei lie inside the neutron drip line. An interesting possibility of the deformed p-wave or s-wave halos is suggested also for the nucleus $^{53}_{18}$Ar$_{35}$.

DOI: 10.1103/PhysRevC.95.044325

I. INTRODUCTION

Halo phenomena, which were first investigated in Ref. [1], so far observed have been mostly in lighter nuclei close to neutron drip line and are some of the most interesting phenomena in nuclei away from the stability line. In order to detect the occurrence of neutron halos, either the measurement of the larger mean square radius of the matter distribution or the detection of an appreciable amount of the probability of (one or several) weakly bound neutrons far away from the well-bound core nuclei is most efficient. The typical experiment done so far to measure the former quantity is the measurement of reaction cross sections (σ_R) [2], while the typical experiment to efficiently detect the latter one is to observe Coulomb breakup reactions [3].

To my knowledge, the nucleus $^{37}_{12}$Mg$_{25}$ is so far the heaviest nucleus which showed (deformed p-wave) halo properties [4]. One may wonder which nucleus heavier than ^{37}Mg may be expected to show a (spherical or deformed) halo. In heavier nuclei, it is getting more difficult to clearly identify halo phenomena from the values of root-mean-square radius, because the ratio of the number of halo particles to that of nucleons in a well-bound core nucleus gets smaller. Furthermore, some amount of increase of the matter radius may occur also by the change from spherical to deformed shape, and the rate of the increase of the matter radius due to possible deformation may not generally decrease in heavier nuclei. On the other hand, methods such as Coulomb break-up reactions can be efficient in discovering possible halos also in heavier nuclei.

It is noted that the analysis of the data on Coulomb and nuclear breakup reactions of ^{37}Mg so far seems to show that the experimental data can be well understood in terms of particles without introducing the effects of pair correlations [4,5]. The question is where we may find any neutron-halo nuclei heavier than ^{37}Mg. The Coulomb and nuclear breakup data on the nucleus ^{37}Mg were interpreted as the one-neutron deformed p-wave halo [4] with the neutron separation energy (S_n) of $0.22^{+0.12}_{-0.09}$ MeV. In the case of a neutron p-wave halo, the S_n values should be much smaller than, say, 1 MeV in order to clearly detect the characteristic feature of a halo. In contrast, in the case of a neutron s-wave halo, the presence of a neutron halo can be detected for S_n values appreciably larger than 1 MeV. Since S_n values of possible halo nuclei not yet found in the region of neutron drip line cannot be estimated precisely and thus a successful prediction of neutron (spherical or deformed) p-wave halos must have very good luck due to the narrow range of S_n values for the realization of halo phenomena, in the following I try mainly to look for possible spherical or deformed s-wave halo nuclei heavier than ^{37}Mg. In the present work, only axially symmetric quadrupole deformation is considered as deformation.

If I limit myself to spherical nuclei, the $\ell = 0$ halo can occur only around the nuclei with very particular neutron numbers, of which the neutron Fermi level is placed around neutron $s_{1/2}$ orbits. One-particle $s_{1/2}$ orbit appears only once in every even-N_{ho} major shell, where N_{ho} expresses the principal quantum number of the harmonic oscillator (ho) potential. The appearance may occur in nuclei around the neutron number $N = 51$ where the $3s_{1/2}$ orbit should be filled (see Fig. 3 of Ref. [6]) by the 51st neutron in nuclei close to neutron drip line. In contrast, if nuclei are deformed and the angular-momentum component of the particle along the symmetry axis is expressed by Ω, all $\Omega^\pi = 1/2^+$ levels have, in principle, a chance to become $\ell = 0$ halo [7]. This is because all $\Omega^\pi = 1/2^+$ orbits acquire $\ell = 0$ components induced by the deformation and especially because the probability of the $\ell = 0$ component in the bound one-particle wave functions of neutron $\Omega^\pi = 1/2^+$ orbits increases strongly as the one-particle energy $\varepsilon_{\Omega^\pi=1/2^+}$ approaches zero [8,9].

It is noted that the possibility of some isotopes in a certain region of the neutron number to be deformed depends strongly on the proton number. For example, when the evolution of 2_1^+ energies in Ti, Cr, and Fe isotopes for the neutron-number varying from 34 towards 50 is examined, it is seen that $_{24}$Cr or $_{26}$Fe isotopes are easy to be deformed while $_{22}$Ti isotopes do not seem to be so. For example, see Fig. 9 of Ref. [10].

Since for a given proton number Z the neutron separation energy of odd-N nuclei approaches zero before even-N nuclei reach the neutron drip line, mainly due to the pairing-blocking effect in odd-N nuclei, it will be experimentally easier to find neutron-halo nuclei among odd-N isotopes. For example, while $^{37}\mathrm{Mg}_{25}$ is found to be a deformed p-wave halo nucleus and $^{39}\mathrm{Mg}_{27}$ lies outside the neutron drip line, both $^{38}\mathrm{Mg}_{26}$ and $^{40}\mathrm{Mg}_{28}$ lie inside the neutron drip line. Moreover, generally speaking, the scheme of low-lying levels in odd-N nuclei would directly show the underlying neutron shell structure. Thus, in the present work I limit myself to the phenomena of one-neutron halo in nuclei with even-Z and odd-N in the region heavier than ^{37}Mg, keeping the nuclei as light as possible. Partly because no quantitative information on the size of pair correlation in nuclei around the relevant neutron drip line is available, in the following I present the analysis of the wave function of the last-odd neutron in odd-N nuclei which is obtained by neglecting the pair correlation. Even in the presence of neutron pair correlation in neighboring even-N nuclei, the wave function of the last-odd neutron in the ground state of odd-N nuclei can be studied in this way, at least in the first approximation.

In Sec. II the model used and the result of search for even-Z odd-N nuclei with possible neutron halo are presented, while conclusions and discussions are given in Sec. III.

II. MODEL AND CANDIDATES FOR EVEN-Z ODD-N NUCLEI WITH POSSIBLE NEUTRON HALO

The basic points of the model are very similar to those used in Refs. [6,11]. I use Woods-Saxon potentials with the standard parameters, which are described on p. 239 of Ref. [12], unless I mention specifically some parameters. While both weakly bound and resonant one-particle energies of neutrons in neutron-rich nuclei are examined in the present work, the major part of the nuclear potentials for those neutrons is produced by protons, which are deeply bound in the case of neutron-rich nuclei. Therefore, it may not be a surprise that the parameters of the Woods-Saxon potential which were adjusted to various data on stable nuclei in Ref. [12] could satisfactorily work in the analysis of the data on neutrons in neutron-rich nuclei [2–4].

The way of calculating both bound and resonant one-particle energies in a spherical potential is straightforward and can be found in various textbooks. In a deformed potential, the calculation of energies of bound neutrons is an eigenvalue problem by solving in coordinate system the coupled differential equations obtained from the Schrödinger equation, together with the asymptotic behavior of bound wave functions in respective (ℓ,j) channels for $r \to \infty$. On the other hand, one-particle resonant energies are calculated by solving the coupled differential equations in coordinate space using the asymptotic behavior of scattering states in respective (ℓ,j) channels and are defined as the energies, at which one of the eigenphases increases through $\pi/2$ as the one-particle energy increases [13]. One-particle resonance is absent if none of the eigenphases increase through $\pi/2$ as the energy increases. In the limit of $\beta \to 0$, the one-particle resonances defined for deformed potentials in terms of eigenphase, of course, agree with the one-particle resonances in spherical potentials defined in terms of phase shift.

The mean deformation of relevant nuclei is not microscopically calculated in the present article, because a reliable effective interaction to be used in Hartree-Fock (HF) calculations around neutron drip line is not yet available. Instead, I use the knowledge that a large energy gap in one-particle spectra for some deformation as a function of deformation (Nilsson diagram) indicates the stability of the system for the deformation and the particle number. Furthermore, I present the results only for prolate shape, because the number of medium-heavy even-even nuclei, of which the ground state is observed to be oblate, is so few, though the overwhelming prolate dominance is so far not really understood. It is also noted that there is no trivial reason why neutron-drip-line nuclei would prefer the oblate shape, compared with nuclei around the stability line.

In the present work, I must explore the probabilities of the $\ell = 0$ or $\ell = 1$ component in the wave function of the last-odd neutron in the laboratory system. When the even-even core configuration in a deformed odd-N nucleus is restricted to the ground state of the neighboring deformed even-even nucleus which has $I = K = 0$, the probabilities of (ℓ, j) components of the wave function of the last-odd neutron in the deformed odd-N nucleus are obtained, in a good approximation, from the angular-momentum projection of one-particle wave functions of the odd-neutron in corresponding deformed potentials. The good approximation is well known from the analysis of one-nucleon transfer reactions such as (d,p) and (p,d) on the deformed ground state of even-even nuclei, when proper kinematic factors unique in respective reactions are taken into account. See, for example, Chapter 5-3 of Ref. [14]. In reality, the even-even core configurations other than the ground-state configuration may contribute to the ground-state wave functions of odd-N nuclei. However, the contributions by those other configurations, for example, the lowest-lying $(K = 0, I = 2)$ configuration or other excited core configurations mixed by rotational perturbation are pretty small in the ground and bandhead state of the neighboring odd-N nuclei. Therefore, in the following when I look for possible deformed halo nuclei, I am satisfied by finding one-particle levels of the last-odd neutron, of which the wave functions in deformed potentials contain an appreciable amount of $\ell = 0$ or $\ell = 1$ components. In contrast, in spherical vibrational nuclei, this approximation of the ground state of odd-A nuclei consisting of the one-particle picture of the last-odd particle and the ground state of the neighboring even-even nuclei can be a much poorer approximation.

A. Even-Z odd-N nuclei around N = 50

First, I figure out the level scheme of neutron one-particle levels as a function of axially symmetric quadrupole deformation for the nucleus, which can be one of good candidates for deformed s-wave halo in the region of the neutron number 50. Then, I try to find the lightest $\ell = 0$ neutron-halo nuclei heavier than ^{37}Mg, using the data on S_n values of even-Z odd-N nuclei listed in Ref. [15]. Some S_n values in Ref. [15] are evaluated values, which I also use, though those evaluated S_n values have certainly an ambiguity that is difficult to be estimated. Using those evaluated S_n values is the best which I can do for the moment. In the following, I mark those evaluated S_n values by asterisks (*).

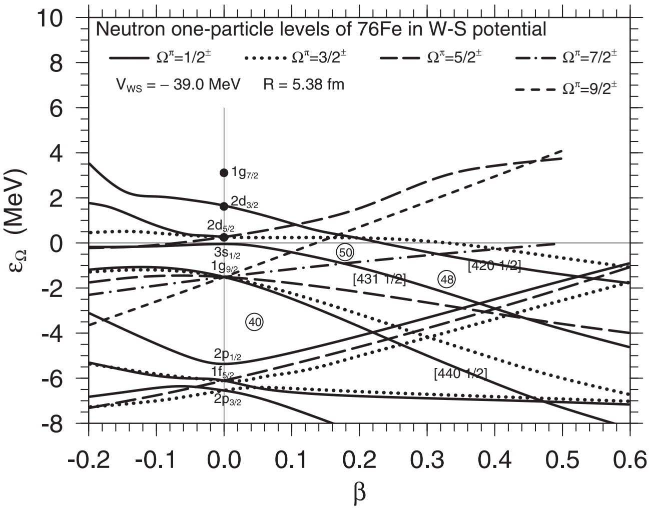

FIG. 1. Calculated one-particle energies for neutrons in the potential given by the core nucleus ${}^{76}_{26}Fe_{50}$ as a function of axially symmetric quadrupole deformation parameter β. Bound one-particle energies at $\beta = 0$ are -6.56, -6.14, -5.39, -1.53, and -0.05 MeV for the $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, $1g_{9/2}$, and $3s_{1/2}$ levels, respectively, while one-particle resonant $2d_{5/2}$, $2d_{3/2}$, and $1g_{7/2}$ levels are obtained at 0.25, 1.62, and 3.12 MeV (denoted by filled circles) with the widths of 0.024, 1.92, and 0.13 MeV, respectively. The $\Omega^\pi = 1/2^+$ resonant level connected to the $2d_{5/2}$ level at $\beta = 0$ cannot be obtained as a one-particle resonant level for $\beta > 0.007$. Three bound $\Omega^\pi = 1/2^+$ levels are denoted by the asymptotic quantum numbers on the prolate side [14], $[N n_z \Lambda \Omega]$ = [440 1/2], [431 1/2], and [420 1/2], although the wave functions of those levels plotted in this figure are quite different from those expressed by the asymptotic quantum numbers. The neutron numbers 40, 48, and 50, which are obtained by filling all lower lying levels, are indicated with circles, for reference. For simplicity, calculated widths of one-particle resonant levels are not shown. The parity of levels can be seen from the ℓ values denoted at $\beta = 0$; $\pi = (-1)^\ell$. One-particle resonant energies for $\beta \neq 0$ are not always plotted if they are not important for the present discussions.

First of all, it is clear that when I limit myself to the spherical shape the possible $\ell = 0$ neutron halo which I look for must have a considerable amount of the neutron $3s_{1/2}$ one-particle component. In the shell structure of spherical stable nuclei, the $3s_{1/2}$ level is known to appear in the second half of the major shell consisting of the neutron number $N = 51$–82. In contrast, in the case of very weakly bound neutrons, the $3s_{1/2}$ level can be the lowest one-particle level among the j levels in the major shell consisting of $N = 51$–82 [6]. In Fig. 1, the neutron one-particle levels calculated in the Woods-Saxon potential, namely both eigenenergies of bound neutrons for $\varepsilon_\Omega < 0$ and one-particle resonant energies for $\varepsilon_\Omega > 0$, are shown as a function of axially symmetric quadrupole-deformation parameter β. The parameters of the potential are taken from those for the nucleus ${}^{76}_{26}Fe_{50}$ except for the depth of the Woods-Saxon potential, which is reduced by 1.6 MeV, and the strength of the spin-orbit potential, which is about 80% of the standard value [12]. The reduction of the depth by 1.6 MeV is made in order to make the $3s_{1/2}$ level at $\beta = 0$ bound, while the 80% spin-orbit strength is chosen in order just to avoid the complicated level crossing around $\beta \approx 0$. The resulting shell structure for $\beta > 0$ to be discussed in the following is not really changed by these changes of the potential parameters. The lowest-lying $\Omega^\pi = 1/2^+$ level in Fig. 1 coming from the spherical $N_{ho} = 4$ shell, which is denoted by [440 1/2] and is connected to the $1g_{9/2}$ level at $\beta = 0$, does not contain an appreciable amount of $\ell = 0$ component for realistic values of quadrupole deformation $\beta < 0.5$, simply because of the vanishing matrix element of the Y_{20} operator between the $1g_{9/2}$ and $3s_{1/2}$ orbits. The quadrupole coupling between the two orbits becomes effective mainly via the neighboring $2d_{5/2}$ orbit, which couples with both the $1g_{9/2}$ and $3s_{1/2}$ orbits by the Y_{20} operator. For example, the probability of the $\ell = 0$ component in the wave function of the lowest-lying $\Omega^\pi = 1/2^+$ level in Fig. 1 is 0.014, 0.035, and 0.041 for $\beta = 0.2$, 0.4, and 0.6, respectively.

Thus, except for the case that one-particle energy is extremely small, among the levels coming from the $N_{ho} = 4$ major shell in Fig. 1 the lowest-lying $\Omega^\pi = 1/2^+$ neutron level, which for realistic values of $\beta(<0.5)$ contains an appreciable amount of the $\ell = 0$ component, is the second (or third) lowest $\Omega^\pi = 1/2^+$ level indicated as the [431 1/2] (or [420 1/2]) level. Limiting myself to spherical or prolately deformed shapes, the neutron numbers of which the Fermi level may be placed on the second or third lowest $\Omega^\pi = 1/2^+$ level in Fig. 1 are $N = 47$ with the [431 1/2] level for $0.18 < \beta < 0.38$, $N = 49$ with the [431 1/2] level for $0.10 < \beta < 0.18$ and with the [420 1/2] level for $0.32 < \beta < 0.52$, and $N = 51$ with the [431 1/2] level for $0 \leqslant \beta < 0.10$ and with the [420 1/2] level for $0.19 < \beta < 0.32$. In all the bands constructed based on those $\Omega^\pi = 1/2^+$ one-particle levels, the decoupling parameters a are found to be in the range of $-1 < a < +4$ for the relevant deformations and, thus, the bandhead states in the laboratory system have the spin parity $1/2^+$.

For Ni isotopes, the S_n values of ${}^{71}_{28}Ni_{43}$, ${}^{73}_{28}Ni_{45}$, ${}^{75}_{28}Ni_{47}$, and ${}^{77}_{28}Ni_{49}$ are 4.3, 4.0, 3.9*, and 3.2* MeV, respectively. Thus, though ${}^{79}_{28}Ni_{51}$ would lie inside the neutron drip line and be presumably spherical, the S_n value would not be small enough to expect that the nucleus may show a possible halo. For Ti isotopes, the S_n values of ${}^{55}_{22}Ti_{33}$, ${}^{57}_{22}Ti_{35}$, ${}^{59}_{22}Ti_{37}$, ${}^{61}_{22}Ti_{39}$, and ${}^{63}_{22}Ti_{41}$ are 4.1, 2.7, 2.6*, 2.1*, and 1.3* MeV, respectively. Therefore, the odd-N nucleus ${}^{69}_{22}Ti_{47}$ may be expected to lie outside the neutron drip line. Then, by examining available data on nuclei towards the possible neutron drip line, promising candidates for s-wave neutron-halo nuclei are ${}^{71}_{24}Cr_{47}$, ${}^{73}_{24}Cr_{49}$, ${}^{75}_{24}Cr_{51}$, and ${}^{77}_{26}Fe_{51}$, in the case that those nuclei lie inside the neutron drip line. (The two neutron separation energies S_{2n} of those nuclei are expected to be positive so that the nuclei are stable against $2n$ emission.) The reason is the following: The heaviest member of Cr isotopes so far found was ${}^{70}_{24}Cr_{46}$ [16] while S_n values of ${}^{61}_{24}Cr_{37}$, ${}^{63}_{24}Cr_{39}$, ${}^{65}_{24}Cr_{41}$, and ${}^{67}_{24}Cr_{43}$ are 4.0, 2.9, 2.6*, and 2.0* MeV, respectively. For Fe isotopes the presence of ${}^{75}_{26}Fe_{49}$ was observed at MSU [16], while S_n values of ${}^{65}_{26}Fe_{39}$, ${}^{67}_{26}Fe_{41}$, ${}^{69}_{26}Fe_{43}$, and ${}^{71}_{26}Fe_{45}$ are 4.3, 4.0, 3.3*, and 2.8* MeV, respectively. In short, the S_n values of ${}^{71}Cr$, ${}^{73}Cr$, ${}^{75}Cr$, and ${}^{77}Fe$ may be less than 1 MeV, in the case that they are not negative.

It is noted that the shell structure for $\beta > 0$ consisting of both weakly bound levels and low-lying one-particle resonances in Fig. 1 remains almost quantitatively unchanged when the depth of the Woods-Saxon potential is changed by, say, less than 1.5 MeV so that some of one-particle levels are changed from bound states to one-particle resonances or vice versa. Therefore, Fig. 1, which is for the system [^{76}Fe + n], is used to explore the possibility of deformed s-wave halo in nuclei with $N = 47$, 49, and 51, which have weakly bound neutrons.

The similarity of the neutron shell structure above $N = 40$ to that above $N = 20$ for respective neutron-rich nuclei is sometimes pointed out; for example, see Ref. [10]. The latter leads to the region of deformed nuclei called the island of inversion. For nuclei with weakly bound neutrons, the spherical energy gap at $N = 50$ becomes smaller as seen in Fig. 1 and, moreover, four one-particle levels, $1g_{9/2}$, $3s_{1/2}$, $2d_{5/2}$, and $2d_{3/2}$, occur within a few MeV at $\beta = 0$. Consequently, the ground states of those nuclei with $N \approx 41$–51 may be deformed, if the proton numbers of respective nuclei allow the deformation. The neutron number 51 is the sum of $N = 40$ and a half of the neutron number 22, which can be accommodated in the close-lying shells consisting of $1g_{9/2}$, $3s_{1/2}$, $2d_{5/2}$, and $2d_{3/2}$. In this respect, it is interesting to note that in the recent shell-model calculations in Ref. [17] the nuclei $^{74}_{24}$Cr$_{50}$ and $^{76}_{26}$Fe$_{50}$ are predicted to be deformed, though in the calculation of Ref. [17] harmonic-oscillator wave functions and effective single-particle energies (ESPEs), which are adjusted based on the knowledge of single-particle energies in nuclei far away from the neutron drip line, are used. That means that the important effect of very weak bindings of neutrons on the neutron single-particle energies is not taken into account in Ref. [17].

B. Even-Z odd-N nuclei around $N = 34$

In this subsection, I describe a possibility of halo phenomena related to the nucleus $^{53}_{18}$Ar$_{35}$. The presence of ^{53}Ar inside the neutron drip line was reported in MSU [18]. The proton number $Z = 18$ does not seem to particularly favor prolate deformation, though some Ar isotopes such as $^{48}_{18}$Ar$_{30}$ can be interpreted as being deformed.

The heaviest member of Ar isotopes so far found is ^{53}Ar, while S_n values of $^{45}_{18}$Ar$_{27}$, $^{47}_{18}$Ar$_{29}$, $^{49}_{18}$Ar$_{31}$, and $^{51}_{18}$Ar$_{33}$ are 5.17(0.002), 3.55(0.08), 3.60(1.41) from Ref. [19], and 1.0* MeV, respectively. Thus, the S_n value of ^{53}Ar is expected to be small. If the nucleus is spherical or slightly oblate, the spin parity of the ground state is likely to be $5/2^-$. However, in the following it is shown that if the nucleus is prolately deformed and the S_n value is smaller than, say several hundred keV, there may be a chance for the ground state to show deformed s-wave or p-wave halo phenomena.

As seen in Fig. 1, the possible lowest-lying $\Omega^\pi = 1/2^+$ one-particle level on the prolate side coming from the $N_{ho} = 4$ major shell is the one connected to the $1g_{9/2}$ level at $\beta = 0$. As described in the previous subsection, this lowest-lying $\Omega^\pi = 1/2^+$ level contains usually only less than a few percent probability of $\ell = 0$ component in the range of realistic quadrupole deformation, say, $\beta < 0.5$. The only exception is the case that this $\Omega^\pi = 1/2^+$ level has a very small binding energy, say, $|\varepsilon_\Omega|$ smaller than a few hundred keV. As described in Refs. [8,9], the probability of $\ell = 0$ component in the one-particle neutron wave function with $\Omega^\pi = 1/2^+$ approaches unity in the limit of $|\varepsilon_\Omega| \to 0$. The value of $|\varepsilon_\Omega|$, at which the probability of $\ell = 0$ component starts to strongly increase, depends on respective $\Omega^\pi = 1/2^+$ orbits.

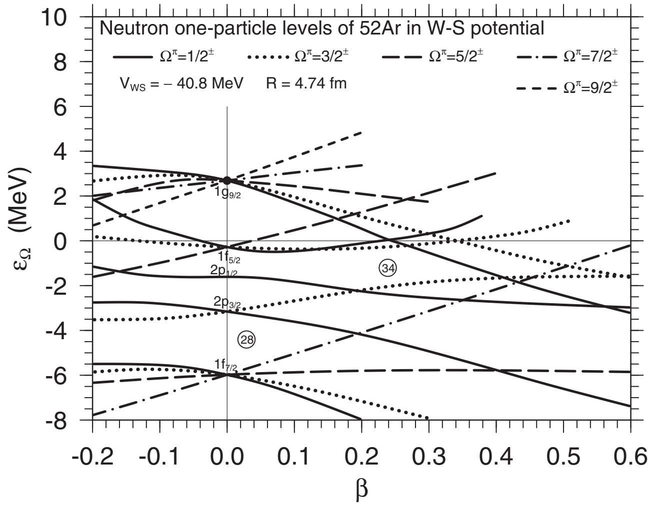

FIG. 2. Calculated one-particle energies for neutrons in the potential given by the core nucleus $^{52}_{18}$Ar$_{34}$ as a function of axially symmetric quadrupole deformation. Bound one-particle energies at $\beta = 0$ are -5.99, -3.18, -1.63, and -0.31 MeV for the $1f_{7/2}$, $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ levels, respectively, while one-particle resonant $1g_{9/2}$ level is obtained at 2.69 MeV with the width of 0.061 MeV. The $2d_{5/2}$ level is not obtained as a one-particle resonant level for this potential. The neutron numbers 28 and 34, which are obtained by filling all lower lying levels, are indicated with circles. One-particle resonant energies for $\beta \neq 0$ are not always plotted if they are not important for the present discussions.

In Fig. 2, neutron one-particle energies are plotted as a function of axially symmetric quadrupole deformation parameter β, taking the standard parameters of Ref. [12] for the system of [$^{52}_{18}$Ar$_{34}$ + n]. Each orbit is doubly degenerate. Thus, if I take literally this figure, the 35th neutron in ^{53}Ar occupies the orbit with $\Omega^\pi = 5/2^-$ for $-0.16 < \beta < 0$, $\Omega^\pi = 1/2^-$ for $0 < \beta < 0.13$, $\Omega^\pi = 3/2^-$ for $0.13 < \beta < 0.27$, and $\Omega^\pi = 1/2^+$ for $0.27 < \beta < 0.41$. When I examine the wave functions of neutrons, I find that for $\beta < 0.05$ the main component of the $\Omega^\pi = 1/2^-$ orbit will be $f_{5/2}$, while that of the $\Omega^\pi = 3/2^-$ orbit will be $f_{5/2}$ for $\beta < 0.18$. Therefore, deformed p-wave halo may not be obtained for respective β values. On the other hand, if the S_n value is larger than about 400 keV, the probability of an $\ell = 0$ component in the wave function of the lowest-lying $\Omega^\pi = 1/2^+$ orbit is less than 0.10 and the main components of the one-neutron wave function become $g_{9/2}$ and $d_{5/2}$. Both the calculated decoupling parameter of the $\Omega^\pi = 1/2^-$ level for $0 < \beta < 0.3$ and that of the $\Omega^\pi = 1/2^+$ level for $0.24 < \beta < 0.5$ are in the range of $-1 < a < +4$. Thus, in the absence of band mixing, the spin parity of the bandhead states based on those one-particle orbits with $\Omega = 1/2$ would be $I^\pi = 1/2^-$ and $I^\pi = 1/2^+$ in the range of respective deformations. On the other hand, for $0 < \beta < 0.2$ the one-particle energies $|\varepsilon_\Omega|$ of the very weakly bound levels with $\Omega^\pi = 1/2^-$ and $3/2^-$ are so close that the two configurations constructed based on the two respective one-particle levels may be strongly mixed in observed states. It is rather difficult to say something more in the present stage since no spectroscopic information on the nucleus ^{53}Ar is available. Therefore, I would state that the condition literally obtained from Fig. 2 is the following: The nucleus ^{53}Ar is expected to show a deformed p-wave halo if it has a moderate deformation of $\beta \approx 0.20$–0.25, while it will show a deformed s-wave halo for $\beta \approx 0.25$–0.40 if it has an S_n value smaller than a few hundred keV.

I add just one comment on the decoupling parameters of very weakly bound $\Omega = 1/2$ one-particle levels. In the lowest-lying $\Omega^\pi = 1/2^+$ level in Fig. 2 connected to the $1g_{9/2}$ level at $\beta = 0$, the decoupling parameter generally increases as $\beta(>0) \to 0$ and approaches +5, which is the value of the decoupling parameter for a $j = 9/2$ particle. However, as $\beta(>0.24) \to 0.24$ in Fig. 2, the value of $|\varepsilon_\Omega(<0)|$ approaches zero and the probability of the $\ell = 0$ component in the one-particle wave function steeply increases and approaches unity. Then, the decoupling parameter, of course, approaches the value of +1, which is the value for a $j = 1/2$ particle.

Though in order for the ground state of ^{53}Ar to show halo phenomena the conditions described above must be satisfied, it would be very interesting if one can find the halo phenomena in the nucleus because it may exhibit exciting quantum-mechanical features such as deformation, deformed shell structure of weakly bound neutrons, the possible strong increase of $\ell = 0$ component in the wave functions of very weakly bound $\Omega^\pi = 1/2^+$ neutrons, and so on. Since the presence and presumably small S_n value of the nucleus ^{53}Ar are already guaranteed, I hope a proper experiment will be carried out in the near future.

III. CONCLUSIONS AND DISCUSSIONS

Recognizing that the nucleus ^{37}Mg is so far the heaviest halo nucleus which was experimentally found, one may wonder whether the halo phenomena can be found also in very heavy nuclei. Then, a question is which nucleus heavier than ^{37}Mg will be next found to show the halo property. Using the shell structure unique in nuclei with weakly bound neutrons, I have tried to look for possible neutron-halo nuclei heavier than ^{37}Mg to be next studied. Candidates for such deformed s-wave halo nuclei are those with the neutron numbers $N = 47$, 49, and 51. Examining available S_n values of even-Z odd-N isotopes, promising nuclei are $^{71}_{24}\mathrm{Cr}_{47}$, $^{73}_{24}\mathrm{Cr}_{49}$, $^{75}_{24}\mathrm{Cr}_{51}$, and $^{77}_{26}\mathrm{Fe}_{51}$. The $\ell = 0$ components in the wave functions of those possible halo neutrons come mainly from the $3s_{1/2}$ level. A one-neutron level which contains an appreciable amount of the $\ell = 0$ component is, except for the very weakly binding case of the lowest lying $\Omega^\pi = 1/2^+$ level, the second (or third) lowest $\Omega^\pi = 1/2^+$ neutron level among the levels coming from the spherical $N_{ho} = 4$ major shell. Those promising candidates for halo nuclei are expected to be deformed because in the spherical limit the $N = 50$ energy gap is narrowed and a set of close-lying one-particle levels, $1g_{9/2}$, $3s_{1/2}$, $2d_{5/2}$, and $2d_{3/2}$, appear.

In other words, the possible observation of an s-wave halo in the above $N = 47$, 49, and/or 51 nuclei directly suggests the shell structure unique in very weakly bound neutrons, such as the disappearance of $N = 50$ magic number and the very lowering of the $3s_{1/2}$ level inside the major shell $50 < N < 82$.

Mean-field calculations such as HF calculations to obtain nuclear shape have not properly been carried out for those neutron-rich nuclei with weakly bound neutrons, mainly because of the lack of the proper HF two-body interaction which has been established for those nuclei. Under these circumstances, I have used the fact that the presence of a few close-lying one-particle levels in the spherical limit is known to lead to the prolately deformed shape of nuclei when neutrons start to fill the first half of the close-lying levels (Jahn-Teller effect), though the possible presence of pair correlation which acts against the tendency of deformation may delay the occurrence of deformation in the beginning of the first half. Examples are the deformation of stable rare-earth nuclei with $90 \leqslant N \leqslant 112$ [14] and the deformation of neutron-rich Mg isotopes [5]. By the way, to my knowledge the reason why very few oblately deformed nuclei with neutrons filling in the second half of a major shell (or close-lying shells) are realized has not yet really been pinned down.

In the present work, I had to use the evaluated (or extrapolated) S_n values in Ref. [15] for some neutron-rich nuclei, which are often crucial to predict possible halo nuclei. There is no other more reliable way; therefore, the resulting conclusion contains the corresponding ambiguity.

The basis to expect that the nucleus ^{53}Ar is prolately deformed is certainly weaker; however, the interpretation of the observed prolate deformation of nuclei such as $^{31}_{12}\mathrm{Mg}_{19}$ [20] or $^{29}_{10}\mathrm{Ne}_{19}$ [21] in the region of island of inversion encourages the expectation. Namely, as $\beta(>0) \to$ large, one-particle levels with $\Omega^\pi = 1/2^+$ and $\Omega^\pi = 3/2^+$ originating from the high-j ($1g_{9/2}$ in the present case) shell energetically come down steeply in Fig. 2, and the filling of those one-particle levels may help for the relevant nuclei to acquire prolate deformation.

To my knowledge, in heavier stable nuclei the pair correlation becomes important almost whenever deformation is likely to occur. In the present work, one-neutron wave functions in the absence of pair correlation are used since the neutron-halo phenomena in the ground state of odd-N nuclei are explored. It may be interesting to find how halo phenomena in heavier odd-N nuclei are affected by a possible strong pair correlation in the even-even core nuclei.

ACKNOWLEDGMENT

The author expresses her thanks to Professor T. Nakamura for his comments.

[1] I. Tanihata *et al.*, Phys. Rev. Lett. **55**, 2676 (1985).
[2] For example, M. Takechi *et al.*, Phys. Rev. C **90**, 061305 (2014).
[3] T. Nakamura *et al.*, Phys. Rev. Lett. **103**, 262501 (2009); **112**, 142501 (2014).

[4] N. Kobayashi *et al.*, Phys. Rev. Lett. **112**, 242501 (2014).
[5] I. Hamamoto, Phys. Rev. C **76**, 054319 (2007).
[6] I. Hamamoto, Phys. Rev. C **85**, 064329 (2012).
[7] I. Hamamoto and B. R. Mottelson, C. R. Phys. **4**, 433 (2003).
[8] T. Misu, W. Nazarewicz, and S. Åberg, Nucl. Phys. A **614**, 44 (1997).
[9] I. Hamamoto, Phys. Rev. C **69**, 041306(R) (2004).
[10] A. Gade and B. Sherrill, Phys. Scr. **91**, 053003 (2016).
[11] I. Hamamoto, Phys. Rev. C **89**, 057301 (2014).
[12] A. Bohr and B. R. Mottelson, in *Nuclear Structure* (Benjamin, Reading, MA, 1969), Vol. I, p. 239.
[13] I. Hamamoto, Phys. Rev. C **72**, 024301 (2005); **73**, 064308 (2006).
[14] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, Reading, MA, 1975), Vol. II.
[15] National Nuclear Data Center, http://www.nndc.bnl.gov
[16] O. B. Tarasov *et al.*, Phys. Rev. C **87**, 054612 (2013).
[17] F. Nowacki, A. Poves, E. Caurier, and B. Bounthong, Phys. Rev. Lett. **117**, 272501 (2016).
[18] O. B. Tarasov *et al.*, Phys. Rev. Lett. **102**, 142501 (2009).
[19] Z. Meisel *et al.*, Phys. Rev. Lett. **114**, 022501 (2015).
[20] G. Neyens *et al.*, Phys. Rev. Lett. **94**, 022501 (2005).
[21] N. Kobayashi *et al.*, Phys. Rev. C **93**, 014613 (2016).