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Local mass relations and the N_pN_n scheme

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In this paper we show that $ip - jn$ relations for nuclear masses and charge radii of four neighboring nuclei reconcile with Casten's N_pN_n scheme.

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Atomic nuclei are very complex systems that display a rich and fascinating variety of phenomena. To describe these features, one has to resort to theoretical models, e.g., the shell model and collective models, to study their structure, including the properties of ground states and excited states. On the other hand, the evolution of nuclear structure with changing numbers of nucleons has also offered important insight into nuclear structure, phase transitions, predictions of properties of newly discovered nuclei, signatures of structure, and underlying shell structure. For example, the N_pN_n scheme (the product of valence proton number and valence neutron number), suggested by Casten and studied extensively by Casten and Zamfir, provides a simple, yet powerful, guide to understanding and predicting the systematic behavior of nuclear properties [1,2].

In recent years there have been many studies of local nuclear mass relations, such as those of Garvey and Kelson [3], as well as several others now generically referred to as $ip - jn$ relations. Such relations were summarized in Ref. [4], and were generalized to study α decay energies [5] and nuclear charge radii [6,7]. A unified form for these $ip - jn$ (for $1 \leqslant i,\ j \leqslant 2$) relations [4] is given by

$$\delta F \equiv F_{Z,N} + F_{Z+i,N+j} - F_{Z+i,N} - F_{Z,N+j} \simeq 0. \quad (1)$$

Here the quantity F may refer to the separation energies of i protons and j neutrons S_{ip-jn}, as in Ref. [4], or to α decay energies Q_α, as in Ref. [5], or to nuclear charge radii R, as in Refs. [6,7]. In this paper we show that these properties can also be described in terms of the N_pN_n scheme, and correspondingly that both $B(E2)$ values and the $E_{2_1^+}$ energies of even-even nuclei, two properties that are typically described by the N_pN_n scheme [8], can be represented in terms of the $ip - jn$ relations of Eq. (1). In other words, the $ip - jn$ relations, as presented in Eq. (1), seem to be compatible with the N_pN_n scheme.

We begin by discussing a quartet of neighboring nuclei, with proton and neutron numbers (Z,N), $(Z+i,N)$, $(Z,N+j)$, and $(Z+i,N+j)$. These four nuclei correspond precisely to those involved in Eq. (1). We assume that their valence proton and neutron numbers are (N_p,N_n), (N_p+i,N_n), (N_p,N_n+j), and (N_p+i,N_n+j), respectively, namely, that both the valence protons and valence neutrons are particle-like. For this quartet of neighboring nuclei, the products of valence proton number and valence neutron number are, respectively, N_pN_n, $N_pN_n + iN_n$, $N_pN_n + jN_p$, and $N_pN_n + jN_p + iN_n + ij$. Since the sum of the product of valence nucleon numbers for two nuclei, one with proton and neutron numbers (Z,N) and the other with proton and neutron numbers $(Z+i,N+j)$, is equal to $2N_pN_n + jN_p + iN_n + ij \simeq 2N_pN_n + jN_p + iN_n$ ($ij \sim 0$ because $1 \leqslant i,\ j \leqslant 2$), which is the sum of the N_pN_n values for the two residual nuclei among the above four neighboring nuclei. Therefore, if a physical quantity F forms a compact trajectory in the N_pN_n scheme, one would expect that this quantity also satisfies Eq. (1).

Well-known examples in which the N_pN_n scheme has been used successfully are for the energies of the lowest 2^+ state, $E_{2_1^+}$, and for $B(E2)$ values, in both cases for even-even nuclei [8]. Therefore we first investigate whether the δF relation defined in Eq. (1) is also satisfied for these two quantities, as a function of neutron number N. In this case $i = j = 2$, since both quantities are for even-even nuclei. The results of $\delta F/\overline{F}$ versus neutron number N are plotted in Fig. 1, where $\overline{F}$ is the average of the four F values involved in Eq. (1). One sees that the values of both $\delta E_{2_1^+}/\overline{E}_{2_1^+}$ and $\delta B(E2)/\overline{B}(E2)$ are very small, except in those cases of small mass numbers or very close to the magic numbers. We thus conclude that Eq. (1) indeed works well for both of these quantities.

We next ask whether those quantities which are known to follow Eq. (1) correspondingly show compact trajectories in the N_pN_n scheme. In Fig. 2(a) we plot the charge radii R with $82 < Z \leqslant 104$ and $126 < N \leqslant 155$ versus N_pN_n. In this case, as noted earlier, $\delta R \simeq 0$ for $1 \leqslant i,j \leqslant 2$ [6,7]. As is evident from the figure, the N_pN_n correlation for R values is very good.

In Refs. [9,10], the quantity δF, where F is the binding energy B, was discussed. This quantity (denoted by δV_{ip-jn} in previous studies [9,10], with $i,j = 2$ in Ref. [9] and $1 \leqslant i,j \leqslant 2$ in Ref. [10]) reflects the interaction between the last i valence protons and the last j valence neutrons of a nucleus. As shown in Fig. 1(a) of Ref. [10], δB with $i = j = 1$ is typically a few hundred keV for medium and heavy nuclei, and is rather small in comparison with the total binding energy of the nucleus. Thus we may assume that in this case $\delta B \simeq 0$. In Fig. 2(b) we plot the corresponding binding energies B of nuclei with $82 < Z \leqslant 104$ and $126 < N \leqslant 155$ versus N_pN_n. One sees that in this region the correlation between binding energy B and N_pN_n is in general remarkable.

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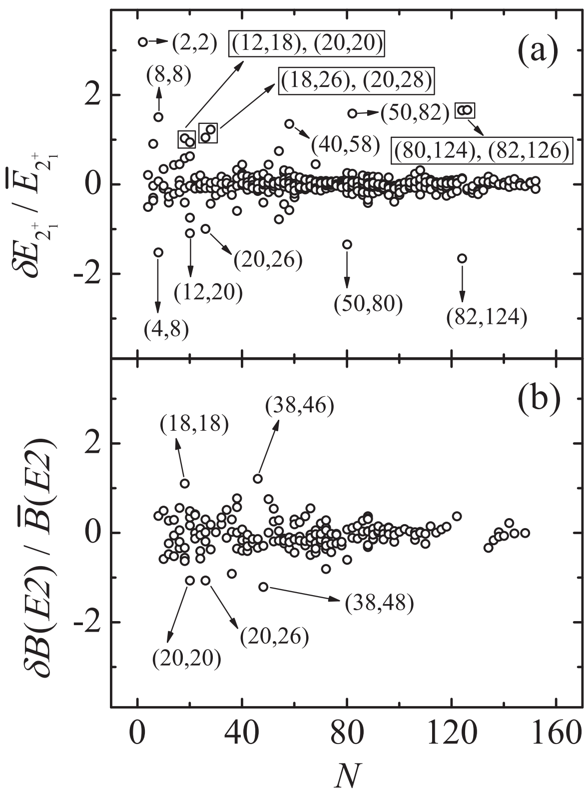

FIG. 1. $\delta F/\overline{F}$ vs neutron number N, for $i = j = 2$. Here $\overline{F}$ is the average of F for corresponding four nuclei involved in Eq. (1). (a) F corresponds to experimental values of $E_{2_1^+}$, taken from Ref. [11]. (b) F corresponds to experimental values of $B(E2)$ [$E2$ transition rates], from Ref. [11]. One sees that both $\delta F/\overline{F}$ here are small, with a few exceptions for nuclei with small mass numbers or very close to the magic numbers.

We should note, however, that the $ip - jn$ relations are not equivalent to the N_pN_n scheme, as one does not always lead to the other. As an example, let us consider α-decay energies Q_α. As shown in Ref. [5], Eq. (1) works remarkably well for $F = Q_\alpha$. However, as can be seen in Fig. 3, Q_α does not form a compact trajectory in the N_pN_n scheme.

To summarize, in this paper we have shown that the simple $ip - jn$ local relations for nuclear masses and charge radii of four neighboring nuclei, in the form of Eq. (1) of this paper, reconcile with Casten's N_pN_n scheme [1,2]. Any quantity which forms a compact trajectory in the N_pN_n scheme seems to follow the $ip - jn$ relations of Eq. (1). Good examples are the 2_1^+ energies and $B(E2)$ values of even-even nuclei, both of which were known to follow the N_pN_n scheme [8] and which we have now shown also satisfy the $ip - jn$ relations. We have now seen that charge radii and binding energies, both known earlier to satisfy Eq. (1), also form compact trajectories in the N_pN_n scheme. On the other hand, we have seen that there are some nuclear properties, such as α decay energies, for which Eq. (1) works very accurately [5], but which do not follow the N_pN_n scheme. This raises the question of whether the local $ip - jn$ relations of Refs. [4,6], Eq. (1) in this paper, might perhaps provide a generalization of the N_pN_n scheme. Further work to clarify this issue is needed.

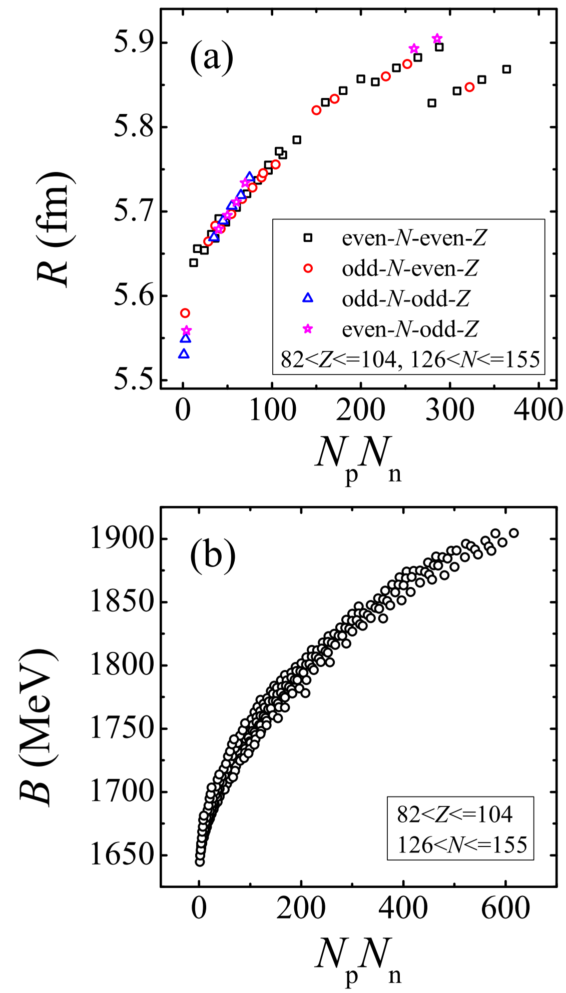

FIG. 2. F vs N_pN_n for nuclei with $82 < Z \leqslant 104$ and $126 < N \leqslant 155$. (a) F corresponds to R, experimental values of nuclear charge radii, from Ref. [12]. (b) F corresponds to B, nuclear binding energy, from Ref. [13]. Both F in this figure exhibit good correlations in the N_pN_n scheme.

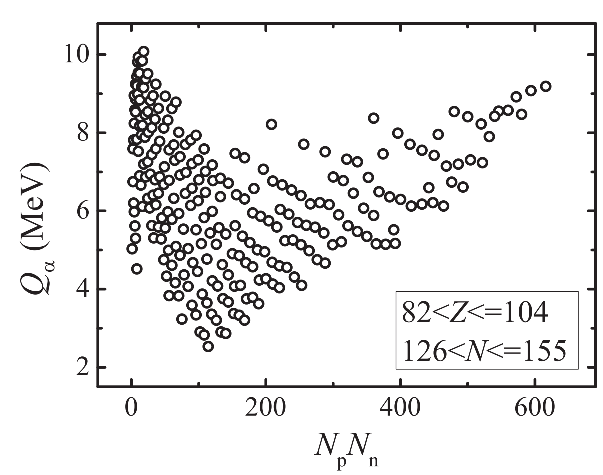

FIG. 3. Experimental values of α decay energy Q_α (from Ref. [13]) vs N_pN_n for nuclei with $82 < Z \leqslant 104$ and $126 < N \leqslant 155$.

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