Effect of the nuclear medium on α-cluster excitation in ^{6}Li

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The giant dipole resonance (GDR) in ^{6}Li was investigated via the ^{6}Li(γ,xn) reactions by using quasi-monoenergy γ rays in an energy range from 4.9 to 53.6 MeV. The γ rays were generated via Compton backscattering of Nd laser photons with relativistic energy electrons in an electron storage ring, NewSUBARU. The energy resolution in a full width at half maximum of γ ray was simulated to be 5% at 50 MeV. Photoneutrons were detected with a 4π-type neutron detector consisting of 41 ^{3}He-gas proportional counters. The (γ,n) cross sections were dominant, while the ($\gamma,2n$) and ($\gamma,3n$) cross sections were negligibly small. The energy integral of photoneutron cross sections up to 53.6 MeV was 59 MeV mb, which exhausted 65% of the Thomas-Reiche-Kuhn sum rule. The GDR in ^{6}Li was found to consist of mainly two components. The peak energy and the width for the low-energy component were $E_r = 12 \pm 1$ MeV and $\Gamma = 21 \pm 2$ MeV. Those for the high-energy component were $E_r = 33 \pm 2$ MeV and $\Gamma = 30 \pm 2$ MeV. The low-energy component corresponded to the GDR in ^{6}Li. The high-energy component was inferred to be the GDR owing to an α-cluster excitation in ^{6}Li. The existence of this component was recently proposed and was suggested by the experimental studies of the (p,p'), (^{3}He,t), and (^{7}Li,^{7}Be) reactions. The observed resonance shape of the high-energy component was well reproduced by modifying the GDR shape of a theoretical prediction for ^{4}He at $E_r = 26$ MeV with $\Gamma = 20$ MeV; with increasing the excitation energy by 7 MeV (Q value was more negative), widening the width by 1.5 ± 0.1 times, and decreasing a peak height by 0.29 ± 0.02 times. As a result, the magnitude of the energy integral of the cross sections for the high-energy component observed in the present work was 0.86 ± 0.06 times that in the theoretical prediction of the ^{4}He(γ,n) reaction. It is a well-known fact that a frequency of a vibrating system is inversely proportional to the size of the system. We suggest that in excitation of the α cluster in ^{6}Li, the mass of the α cluster increases by 7 ± 2 MeV, the size of the α cluster in ^{6}Li is smaller than that of the free ^{4}He by $\sim$20%, and the width of the GDR is broader than that of ^{4}He by 1.5 times owing to the nuclear medium effect.

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I. INTRODUCTION

The appearance of α clusters in light nuclei is a common phenomenon. Even in heavy nuclei, the existence of α decay indicates the importance of the α-clustering structure in nuclei. ^{6,7}Li are the typical examples of a well-developed α-clustering structure in light nuclei. Because an α cluster is weakly bound with other nucleons and is a spatially localized subsystem in atomic nuclei, an intrinsic excitation of the α cluster is expected. Indeed, Costa *et al.* suggested a possible α-cluster excitation in a photonuclear reaction on ^{6}Li [1]. Figure 1 shows their result. They claimed that there were two resonance components at the resonance energy of $E_r \approx 12$ MeV and $E_r \approx 26$ MeV. They interpreted that the resonance at $E_r \approx 12$ MeV was the isovector giant dipole resonance (GDR) in ^{6}Li, and the resonance at $E_r \approx 26$ MeV was the intrinsic excitation of the α cluster, because the GDR in free ^{4}He is well known to be located at $E_r = 26$ MeV. However, as shown in Fig. 1, other photonuclear-reaction cross sections for ^{6}Li reported in Refs. [2–5] are very different from those of Costa *et al.* Thus, evidence for the α-cluster excitation in photonuclear reactions was not commonly accepted.

Recently, Yamagata *et al.* carried out the experiments of the (p,p'), (^{3}He,t), and (^{7}Li,^{7}Be) reactions on ^{6,7}Li, and reported that two split dipole resonances were commonly excited in $A = 6$ nuclei of ^{6}Li, ^{6}Be, and ^{6}He, and $A = 7$ nuclei of ^{7}Li, ^{7}Be, and ^{7}He [6]. The Q values for the low-energy dipole resonances (LEDRs) in $A = 6$ nuclei were $Q \approx -12$ MeV and those for the high-energy dipole resonances (HEDRs) were $Q \approx -28$ MeV. However, those for the LEDRs and the HEDRs were $Q \approx -17$ MeV and $Q \approx -30$ MeV in $A = 7$ nuclei, respectively. Based on the comparison of the excitation energies, the resonance widths, the resonance shapes, and the excitation cross sections for these resonances with those of the GDR reported in ^{6,7}Li(γ,n) reaction [4], they concluded that the LEDRs corresponded to the GDRs and/or the isovector spin dipole resonances (SDRs) in ^{6,7}Li and their analogs [6]. Furthermore, Yamagata *et al.* also carried out the ^{4}He(p,p') experiment to determine the excitation energy, width, and resonance shape of the GDR/SDR in ^{4}He [7]. Detailed comparison of the excitation energies, the resonance widths, and the resonance shapes in ^{6,7}Li was done for the HEDRs with those of the GDR/SDR in ^{4}He. Yamagata *et al.* concluded that these HEDRs were the GDR/SDRs of the α clusters in ^{6,7}Li and their analogs.

In case of the nuclear reaction by using an incident particle with a spin, both the GDR and the SDR are possibly excited. We cannot distinguish them if the observable depending upon

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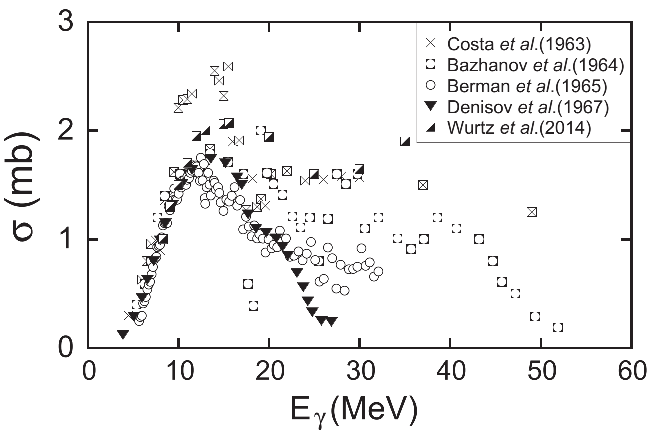

FIG. 1. The ^{6}Li(γ,n) cross sections reported by Costa *et al.* [1], Bazhnov *et al.* [2], Denisov *et al.* [3], Berman *et al.* [4], and Wurtz *et al.* [5]. For simplicity, error bars in the data are not shown.

the spin, e.g., the spin transfer, ΔS, is not experimentally determined.

Nakayama *et al.* studied the ^{6,7}Li(^{7}Li,^{7}Be) reactions to extract the spin transfers of $\Delta S = 0$ for the GDR and $\Delta S = 1$ for the SDR [8,9]. The SDR and GDR were found to be located at a similar excitation energy in ^{6}Li and ^{7}Li. The observed spectra were found to consist of two broad peaks of the LEDRs at $E_x \approx 14$ MeV and the HEDRs at $E_x \approx 30$ MeV in both ^{6,7}He. The LEDRs were understood to be the normal GDRs in ^{6,7}He, which were analogs of the GDRs in ^{6,7}Li. The excitation energies and the widths of the LEDRs are very similar to those of the resonance observed at ∼14 MeV, as shown in Fig. 1. However, the HEDRs do not clearly correspond to the resonances observed in photonuclear reactions.

Nakayama *et al.* also studied the ^{4}He(^{7}Li,^{7}Be) reaction and observed the analogs of the GDR/SDR in ^{4}He [10]. The GDR was found to distribute at the excitation energy higher by ∼4 MeV than that of the SDR in ^{4}He. They inferred that the HEDRs in ^{6,7}He were the analogs of the GDR/SDRs owing to the α-cluster excitation in ^{6,7}Li from the comparison of the excitation energy and the width with those of the GDR/SDR in ^{4}He [11].

The charged-particle decay modes from the GDR/SDRs owing to the α-cluster excitation in ^{6,7}He were studied using the ^{6,7}Li(^{7}Li,^{7}Be,x) reactions [12]. It was clearly shown that the decay proceeded as the analog of an α cluster; namely, a ^{4}H cluster was decaying. From these extended investigations on the HEDRs in $A = 6$ and 7 nuclei, it was concluded that these resonances were the GDR/SDRs owing to the α-cluster excitation and their analogs. At present, the existence of the GDR/SDR owing to the α-cluster excitation seems to be confirmed via the studies of these nuclear reactions. However, no apparent resonance component has been observed in photonuclear reactions in ^{6,7}Li, although the GDRs owing to the α-cluster excitation in ^{6,7}Li are expected to be observed as the HEDRs.

One unanswered question still remains: Can the α-cluster GDR be excited via photonuclear reactions? The α-cluster GDR should be observed as a strongly excited resonance in photonuclear reactions, if it exists. The physical nature of the α cluster in a nucleus might be different from that of a free ^{4}He owing to the nuclear medium effect. Thus, we have a good chance to detect modification effects of the physical observables in nuclear medium by observing the α-cluster excitation in photonuclear reactions.

In recent years, intense and quasimonoenergy γ rays are available via Compton backscattering of laser photons with relativistic energy electrons [13]. In this paper, we present the experimental result to observe the GDR via the ^{6}Li(γ,xn) reaction.

II. EXPERIMENT

A. Experimental setup

The experiment was carried out at the NewSUBARU, an electron storage-ring facility of the Laboratory of Advanced Science & Technology for Industry, University of Hyogo, Hyogo, Japan. Figure 2 shows the experimental setup. As shown in Fig. 2(a), Nd laser in the fundamental ($\lambda = 1064$ nm) or second harmonic modes ($\lambda = 532$ nm) at 20 kHz was injected into the concrete tunnel of the NewSUBARU. Reflecting with mirrors, laser photons were focused at a straight section of the NewSUBARU and were struck with relativistic energy (550–1300 MeV) electrons. A 10-cm-thick lead collimator with an aperture of 2 mm was located in a γ hutch-1 18.5 m downstream from the collision point to obtain the monochromatic backscattered photons (γ rays) [13].

Collimated γ rays were introduced into the GACKO hutch and bombarded a target positioned at the center of the neutron detector 7.5 m downstream from the collimator, as shown in Fig. 2(b). The intensity of γ rays passing through the target was measured by using a $6^\phi \times 5$-inch-thick NaI(Tl) detector located inside the concrete hole. Taking into account the γ ray attenuation effect through the target and the NaI(Tl) detector, the number of the incident γ rays was determined [14].

Neutrons emitted from the target were detected using 41 ^{3}He-gas proportional counters with 3 cm diameter and 40 cm length. These counters were embedded in a polyethylene moderator of a (60-cm)^3 cube arranged to be parallel to the incident γ-ray beam axis. The target was located at the center of the cube. The ^{3}He counters were located at concentric positions surrounding the target, making four rings from the target at 6 cm (3 counters, which we call as A ring), at 7.5 cm (9 as B ring), at 12 cm (14 as C ring), and at 18 cm (15 as D ring). The energy dependence of neutron detection efficiency is different for each ring because the distances from the target are different. Thus, to determine the neutron mean energies, we measured ring-counting fractions, which were ratios of each ring count to the total counts of all the ^{3}He counters. The moderator cube was surrounded by boron-doped polyethylene plates with a thickness of 12 cm to reduce neutron backgrounds.

The targets used were an enriched metallic ^{6}Li (96%) and a D_2O (99%), which were in a plastic cylinder with an inner diameter of 8 mm and a length of 9 cm sealed by 30-μm Mylar windows at both ends. Here "D" denotes a deuterium, ^{2}H. The target was located at the center of a 3-cm$^\phi$ hole of

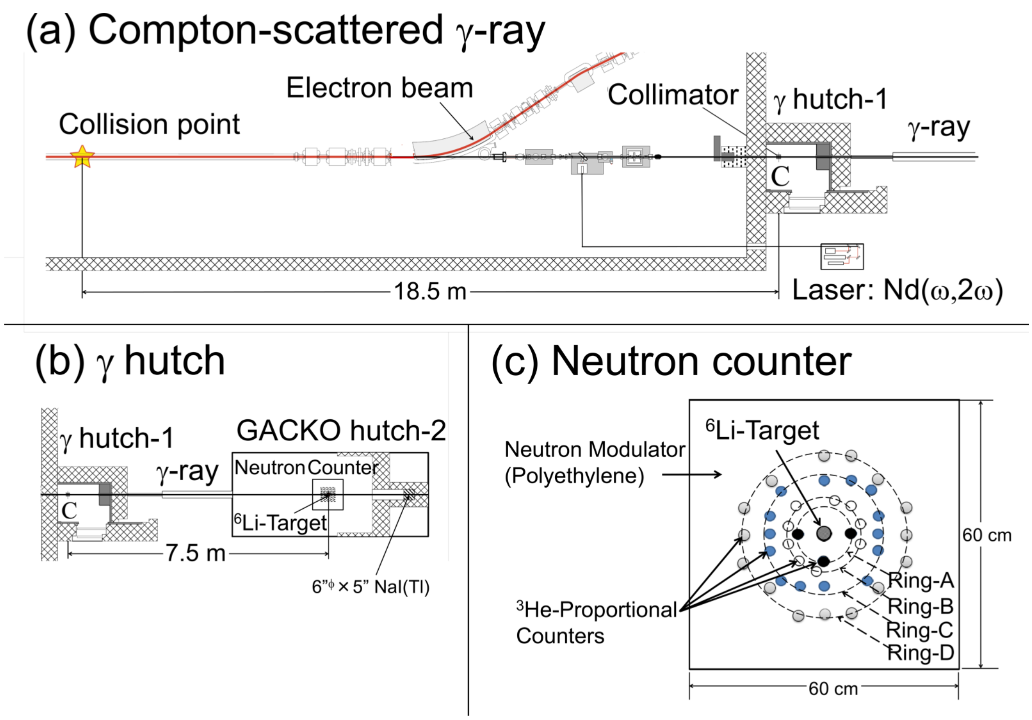

FIG. 2. Experimental setup. (a) The collision point of laser photons with an electron beam and a position of a collimator in a γ hutch-1. (b) The γ hutch-1 and the second hutch-2 GACKO, the ^{6}Li target, a neutron detector, and a NaI(Tl) detector to count the number of incident γ rays. (c) A front view of the neutron detector. 41 ^{3}He-gas proportional counters are embedded in a (60-cm)^3 cubic polyethylene moderator parallel to the γ-ray beam axis. The ^{3}He-gas proportional counters are arranged as a four-ring layer pattern centering the target.

the cubic moderator. The D_2O target was used to calibrate the mean energies of neutrons emitted from the ^{6}Li(γ,xn) photonuclear reaction and the efficiency of neutron detectors using the ring-counting fractions.

Figure 3 shows the incident γ-ray spectra simulated by using the Monte Carlo code, EGS4 [15] at $E_\gamma = 5$, 10, 20, and 40 MeV. The γ-ray energy is given as [13]

$$E_\gamma = \frac{4\eta^2\varepsilon_L}{1+(\eta\theta)^2+4\eta\varepsilon_L/mc^2}, \quad (1)$$

where ε_L, mc^2, and θ are the energy of a laser photon, the rest mass energy of an electron, and scattering angle of a laser photon with respect to the electron incident direction, respectively. The Lorentz factor of an electron is defined to be $\eta = E_e/mc^2$, where E_e is the total electron energy. The shapes of the γ-ray spectra have a top energy peak, which is originated by an exact 180° ($\theta = 0$) scattered photons. With decreasing the scattering angle from 180°, the γ-ray energy decreases in an exponential-like pattern, as shown in Fig. 3. The full width at half maximum (FWHM) value of γ rays, ΔE_γ, is determined by various parameters: the beam sizes of the electron and the laser photon, an aperture size of a collimator, etc. We estimated the FWHM value of the γ-ray energy to be about 5% in the EGS4 Monte Carlo simulation. In this paper, we define the energies of incident γ rays as $E_\gamma = E_{\rm top} - \Delta E_\gamma/2 = 0.975E_{\rm top}$. The spectra shown by dots indicate those measured with the NaI(Tl) detector. Spectra in gray show EGS4 simulation including the response of the NaI(Tl) detector. As shown in Fig. 3, the full energy peaks were not observed in the spectra measured with the NaI(Tl) detector, beyond $E_\gamma = 10$ MeV.

The nominal electron beam energies have been calibrated within the error of about 1% [16]. The error in nominal energy of the generated γ rays is two times larger than those of the electron beam [see Eq. (1)]. Therefore, the error of the γ-ray energies is about 2%, whose value is smaller than the value of the energy resolution ΔE_γ by about 0.4 times. Therefore, we do not take into account the error of the γ-ray energies in the present analysis.

Each signal from the ^{3}He-gas proportional counters was fed into an amplifier/discrimination (amp/discri) module. The logic signals from the amp/discri module were fed into scalers via fan-in modules to count the event number $N_{\rm A}$, $N_{\rm B}$, $N_{\rm C}$, and $N_{\rm D}$, of the A, B, C, and D ring, respectively. The ring-counting fractions, i.e., $N_{\rm A}/N_{\rm T}$, $N_{\rm B}/N_{\rm T}$, and so on, were derived from each scaler count, where $N_{\rm T}$ is total ring counts $N_{\rm T} = N_{\rm A} + N_{\rm B} + N_{\rm C} + N_{\rm D}$. Output signals from the fan-in modules were also used as the trigger signals for the Computer Automated Measurement And Control (CAMAC) system. The analog signals from the ^{3}He-gas proportional counter were analyzed using 41 CAMAC analog-to-digital converters (ADCs) via the amp/discri module with a 10-μs gate time. Not only singles events but also coincidence events of two or three ADCs were used to evaluate the (γ,n), (γ,2n), and (γ,3n) cross sections.

B. Experimental procedure

The Nd laser of the fundamental mode and the second harmonic mode was used to generate γ rays in the energy

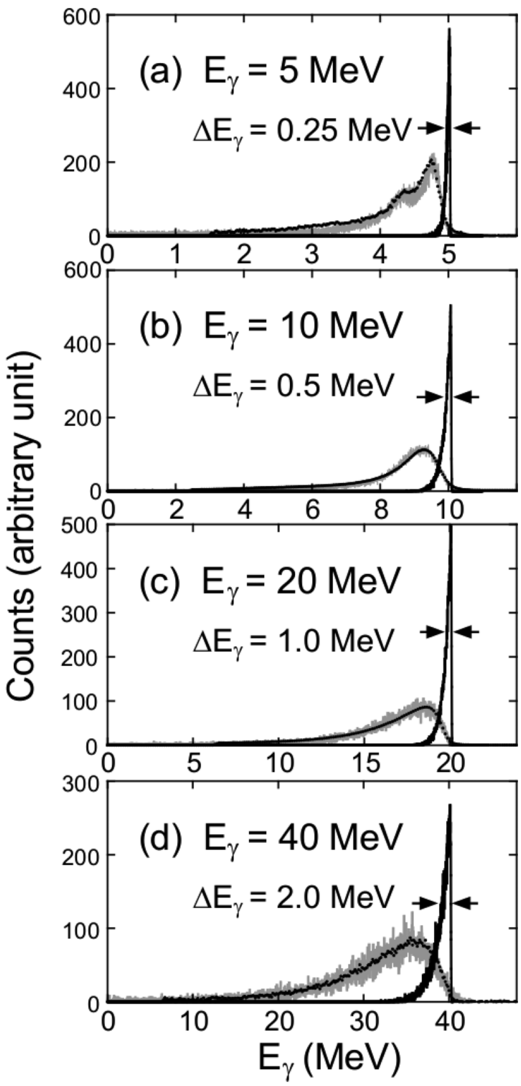

FIG. 3. Incident γ-ray spectra collimated with a diameter of 2 mm at the γ-ray energies of (a) 5 MeV, (b) 10 MeV, (c) 20 MeV, and (d) 40 MeV. The spectra in black show the results of the Monte Carlo simulations for incident γ rays using the code, EGS4 [15]. The spectra shown by dots were measured with the NaI(Tl) detector. Spectra in gray show the results of the EGS4 simulation including the response of the NaI(Tl) detector. The full width at half maximum (FWHM) of the incident γ-ray peaks was estimated to be $\Delta E_\gamma = 2$ MeV at $E_\gamma = 40$ MeV.

ranges of 4.9–30.0 and 20.0–53.6 MeV, respectively. To avoid mirror damage, the laser power was limited less than 4 W. Beam intensity of electrons injected from a linear accelerator into the NewSUBARU was 300 mA. The intensity of electrons in the NewSUBARU gradually decreased to 15 mA in about 10 h and then we reinjected the electron beam. A typical neutron counting rate was ~25 c/s. Data acquisition was repeated in every 100-ms interval, which was subdivided into 80 and 20 ms. Data were taken every 80 and 20 ms with the laser on and off, respectively. Events at the laser-off time were assumed to be the background (BG). The background free data were obtained by subtracting the BG data multiplied by 4 from the data at the laser-on time. The BG was mainly caused by bremsstrahlung photons generated in the NewSUBARU. The amounts of the BG were very small. We also measured BG neutrons coming from the plastic cylinder with Mylar windows of the target by using an empty target. The BG from the empty target was negligibly small, and we neglected this BG in data analysis.

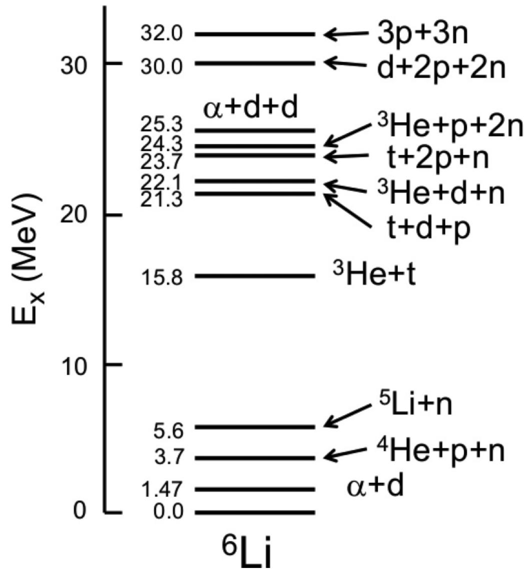

FIG. 4. Energy levels in ^{6}Li and their decay modes. The $1n$, $2n$, and $3n$ threshold energies are $E_x = 3.7$, 24, and 32 MeV, respectively [17].

1. Detection efficiency and neutron mean energy

Because most of the excitation levels in ^{6}Li have a multibody decay channel [17], as shown in Fig. 4, neutron detection efficiency ε as well as determination of the neutron energy E_n emitted in the photonuclear reaction is very important in the present study. The energy dependence of detection efficiency of the neutron detector $\varepsilon(E_n)$ was calculated using a Monte Carlo simulation with the code MCNP [18], as shown in Fig. 5. However, we measured the detection efficiency by using a ^{252}Cf neutron source (AEA Technology Nuclear Science, Oxfordshire, U.K.) to be 0.47 and normalized the calculated efficiency with this value at $E_n = 2.4$ MeV, which is the mean energy of neutrons from ^{252}Cf. Unfortunately, the error of the neutron flux from this source was not available. Because a simulated value with the MCNP was 0.61 at $E_n = 2.4$ MeV, we assumed that the difference of both values might be the maximum error, which was ±30%, though this error seemed to be too large for a standard source. Thus, the cross-section values reported in the present work may have ±30% uncertainties in the absolute values.

To determine the mean energy of neutrons following the ^{6}Li(γ,xn) reactions, we measured the ring-counting fractions as a function of the neutron energy. The ring-counting fractions depend only on the mean energy of neutrons emitted in the photonuclear reaction. The larger the ring-counting fraction is, the more sensitive the neutron detection is. The D(γ,n) reaction was carried out in an energy range from $E_\gamma = 5.0$ to 16.5 MeV,

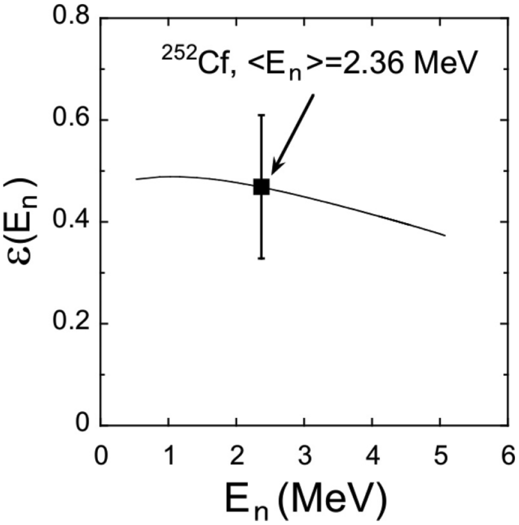

FIG. 5. The energy dependence of neutron detection efficiency $\varepsilon(E_n)$ simulated with the MCNP code [18]. The solid curve shows the result of the simulation. Note that the absolute value of the simulation is normalized with the measured value by using neutrons from ^{252}Cf at the mean energy of 2.36 MeV.

below the threshold energy of the ^{16}O(γ,n) reaction. In the D(γ,n) reaction, the neutron energy is definitely determined to be $E_n = (E_\gamma - 2.225)/2$ MeV, where 2.225 MeV is the binding energy of a deuteron. In Fig. 6(a), the ring-counting fractions measured in the D(γ,n) are shown as a function of the neutron energy. It is clearly seen that the ^{3}He-gas proportional counters at the rings A and B are sensitive for low-energy neutrons, while those at the rings C and D are sensitive for high-energy neutrons.

If we can measure the ring-counting fractions, we can derive the neutron energy based on the Fig. 6(a). However, the ring-counting fractions vary slowly according the change of the neutron energy; therefore, the derived neutron energy may have large uncertainty. To determine the mean energy of the detected neutron with small uncertainty using the ring-counting fractions in Fig. 6(a), we took the following specific method. We looked for the crossing points of the ring-counting fractions. Four crossing points of the ring-counting fractions are found in Fig. 6(a), as shown by the rectangles with number marks: (1) $N_\mathrm{A}/N_\mathrm{T} = N_\mathrm{C}/N_\mathrm{T} = 0.25$ at $E_n = 1.8$ MeV, (2) $N_\mathrm{B}/N_\mathrm{T} = (N_\mathrm{C} + N_\mathrm{D})/N_\mathrm{T} = 0.39$ at 2.3 MeV, (3) $N_A/N_\mathrm{T} = N_\mathrm{D}/N_\mathrm{T} = 0.18$ at 4.2 MeV, and (4) $(N_\mathrm{A} + N_\mathrm{B})/N_\mathrm{T} = (N_\mathrm{C} + N_\mathrm{D})/N_\mathrm{T} = 0.51$ at 5.1 MeV. From these crossing points, the neutron energy was determined as a function of the ring-counting fraction with rather small uncertainty.

Figure 6(b) shows the ring-counting fractions measured in the ^{6}Li(γ,xn) reaction as a function of E_γ. Crossing points similar to ones shown in Fig. 6(a) are clearly seen in Fig. 6(b). From these points the mean energies of neutrons, E_n, in the ^{6}Li(γ,xn) reaction were experimentally determined as a function of an incident γ-ray energy for the ^{6}Li target. The results are shown in Fig. 7(a). Because E_n were determined, the detection efficiency of neutrons $\varepsilon(E_n)$ could be determined

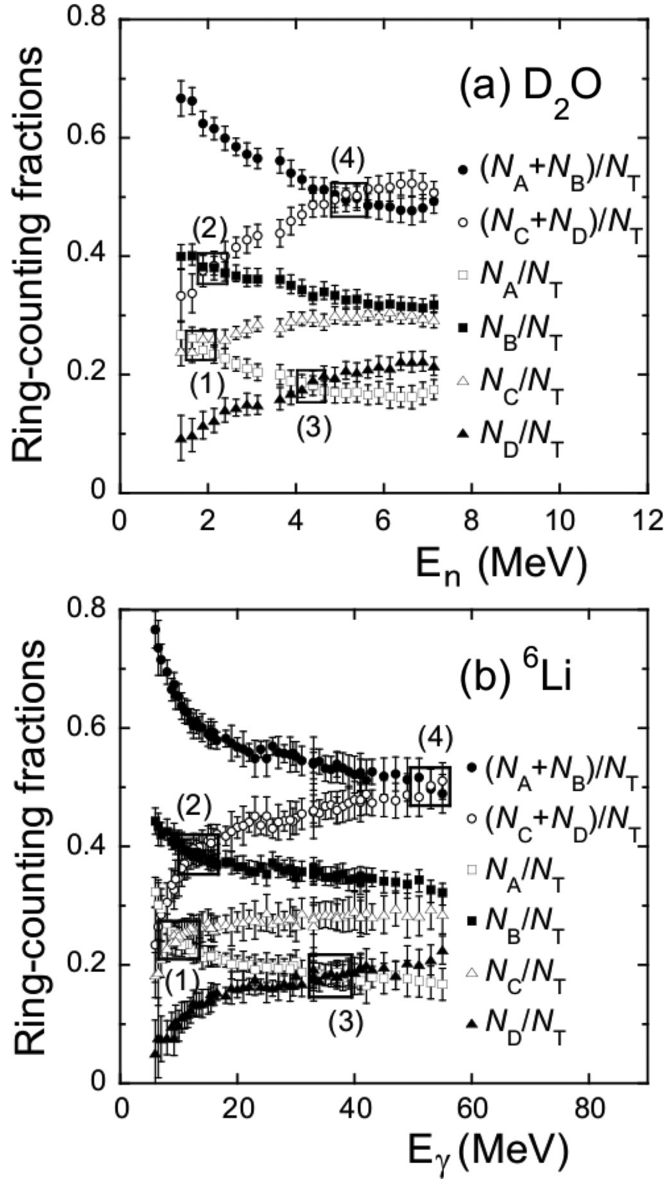

FIG. 6. (a) Ring-counting fractions for the D(γ,n) reaction as a function of E_n. Here "D" denotes a deuterium, ^{2}H. (b) Ring-counting fractions for the ^{6}Li(γ,xn) reaction as a function of E_γ. Here, N_A is a counting number of the A-ring counter, and so on. N_T is a total counting number with the A, B, C, and D rings; $N_\mathrm{T} = N_\mathrm{A} + N_\mathrm{B} + N_\mathrm{C} + N_\mathrm{D}$. The crossing points of the ring-counting fractions in (a) and (b) are enclosed within rectangles and labeled by the number marks.

by referring the energy dependence of the neutron detection efficiency in Fig. 5. The results are shown in Fig. 7(b). Because the ^{6}Li(γ,xn) reaction is followed by multibody decays, emitted neutron energies are low, at most 5 MeV. Accordingly, a range in the detection efficiency in the present experiment is restricted in the region of 0.38–0.48, as shown in Fig. 7(b).

2. *Determination of the (γ,n), (γ,2n), and (γ,3n) cross sections*

In the MCNP simulation, the moderation time of a neutron (a time interval between generation of a neutron by γ rays and

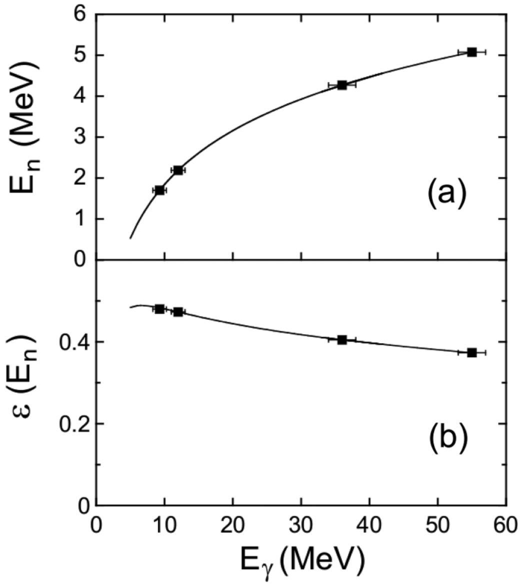

FIG. 7. (a) Four data points of the mean energies E_n of emitted neutrons in the ^{6}Li(γ,xn) reaction determined from the ring-counting fractions. To smoothly connect the data points, a least-squares fitting curve is shown by the solid line. (b) The detection efficiency of neutrons $\varepsilon(E_n)$ in the ^{6}Li(γ,xn) reaction.

disappearance of the neutron by absorption with the detector or escape from the detector cube) was estimated to be 77 μs. Because neutrons were observed as an exponential diffusion behavior with a type of detector [19] similar to the present one and the CAMAC gate time is 10 μs, coincidence efficiency ε_τ of two ADC's is expressed as

$$\varepsilon_\tau = \frac{\int_0^{10} e^{-t/77} dt}{\int_0^{\infty} e^{-t/77} dt} \approx 0.12. \tag{2}$$

In the present experiment, we measured the singles, the double coincidence, and the triple coincidence events to obtain the (γ,n), (γ,2n), and (γ,3n) cross sections. The singles event yield, Y_1, is expressed as

$$Y_1 = N_\gamma N_{\mathrm{Ta}} \varepsilon \{\sigma(\gamma,n) + 2\sigma(\gamma,2n) + 3\sigma(\gamma,3n)\}, \tag{3}$$

where N_γ, N_{Ta}, ε, $\sigma(\gamma,n)$, $\sigma(\gamma,2n)$, and $\sigma(\gamma,3n)$ are the number of the incident γ rays, number of the target atoms/cm^2, the neutron detection efficiency, and the (γ,n), (γ,2n), (γ,3n) cross sections, respectively. The coefficients 2 and 3 for $\sigma(\gamma,2n)$ and $\sigma(\gamma,3n)$ are the neutron multiplicities. Similarly, the double coincidence and the triple coincidence event yields, Y_2 and Y_3, are described as

$$Y_2 = N_\gamma N_{\mathrm{Ta}} \varepsilon^2 \varepsilon_\tau \{\sigma(\gamma,2n) + 3\sigma(\gamma,3n)\}, \tag{4}$$

$$Y_3 = N_\gamma N_{\mathrm{Ta}} \varepsilon^3 \varepsilon_\tau^2 \sigma(\gamma,3n). \tag{5}$$

By using these equations and taking into account a thick target correction [20], we derived the experimental values of the (γ,n), (γ,2n), and (γ,3n) cross sections from the measured yields Y_1, Y_2, and Y_3. Numerical cross-section values are listed in Table I. Only statistical uncertainties are shown.

TABLE I. Cross sections of the ^{6}Li(γ,n), (γ,2n), and (γ,3n) reactions.

E_γ (MeV)	$\sigma(\gamma,n)$ (mb)	$\sigma(\gamma,2n)$ (mb)	$\sigma(\gamma,3n)$ (mb)
4.9	0.07 ± 0.01		
5.9	0.28 ± 0.01		
6.3	0.43 ± 0.01		
6.8	0.59 ± 0.01		
7.8	0.87 ± 0.01		
8.6	1.06 ± 0.01		
9.0	1.11 ± 0.01		
9.3	1.16 ± 0.01		
9.8	1.35 ± 0.01		
10.2	1.31 ± 0.01		
10.7	1.35 ± 0.01		
11.2	1.41 ± 0.01		
11.7	1.56 ± 0.01		
12.2	1.49 ± 0.01		
12.7	1.45 ± 0.01		
13.2	1.49 ± 0.01		
13.7	1.49 ± 0.02		
15.1	1.46 ± 0.01		
15.6	1.46 ± 0.01		
16.1	1.62 ± 0.01		
17.6	1.60 ± 0.01		
18.5	1.45 ± 0.02		
19.5	1.42 ± 0.01		
20.5	1.39 ± 0.01		
21.5	1.27 ± 0.01		
22.4	1.34 ± 0.01		
23.4	1.24 ± 0.01	0.00 ± 0.01	
24.4	1.25 ± 0.02	0.00 ± 0.02	
25.4	1.25 ± 0.01	0.00 ± 0.01	
26.3	1.28 ± 0.02	0.01 ± 0.02	
27.3	1.32 ± 0.01	0.00 ± 0.01	
28.3	1.37 ± 0.03	0.00 ± 0.02	
28.8	1.31 ± 0.02	0.02 ± 0.01	
29.3	1.31 ± 0.03	0.05 ± 0.02	
30.2	1.42 ± 0.03	0.03 ± 0.02	
32.2	1.16 ± 0.06	0.18 ± 0.04	
33.2	1.34 ± 0.03	0.07 ± 0.02	
35.1	1.22 ± 0.03	0.07 ± 0.02	
36.1	1.19 ± 0.03	0.09 ± 0.02	
37.1	1.19 ± 0.02	0.06 ± 0.01	
38.0	1.24 ± 0.03	0.00 ± 0.02	0.03 ± 0.01
39.0	1.07 ± 0.03	0.09 ± 0.02	0.00 ± 0.01
40.0	1.13 ± 0.03	0.00 ± 0.02	0.04 ± 0.01
41.0	0.99 ± 0.03	0.08 ± 0.02	0.00 ± 0.01
41.9	0.89 ± 0.03	0.11 ± 0.02	0.03 ± 0.01
43.9	0.89 ± 0.04	0.02 ± 0.03	0.06 ± 0.01
45.8	0.93 ± 0.03	0.00 ± 0.02	0.10 ± 0.01
47.8	1.13 ± 0.04	0.00 ± 0.03	0.18 ± 0.01
49.7	0.69 ± 0.03	0.00 ± 0.02	0.05 ± 0.01
51.7	0.65 ± 0.03	0.05 ± 0.02	0.02 ± 0.01
53.6	0.66 ± 0.03	0.00 ± 0.02	0.06 ± 0.01

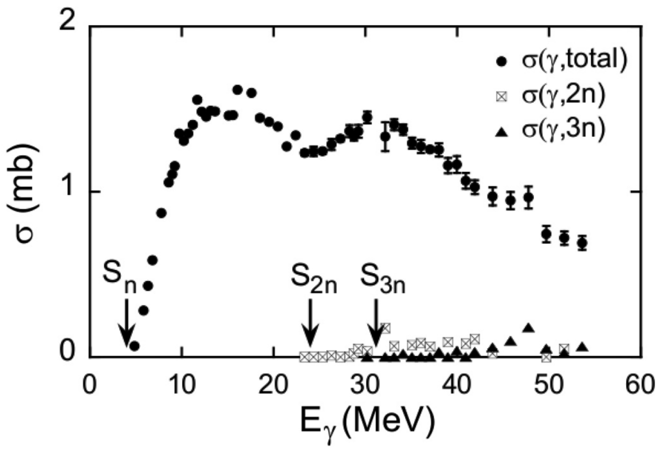

FIG. 8. The cross sections obtained in the present work for the ^{6}Li(γ,total), (γ,2n), and (γ,3n) reactions. The summation of the cross sections, $\sigma(\gamma,n)+\sigma(\gamma,2n)+\sigma(\gamma,3n)$, is represented as $\sigma(\gamma,\text{total})$. The arrows indicate the threshold energies for the (γ,n), (γ,2n), and (γ,3n) reactions.

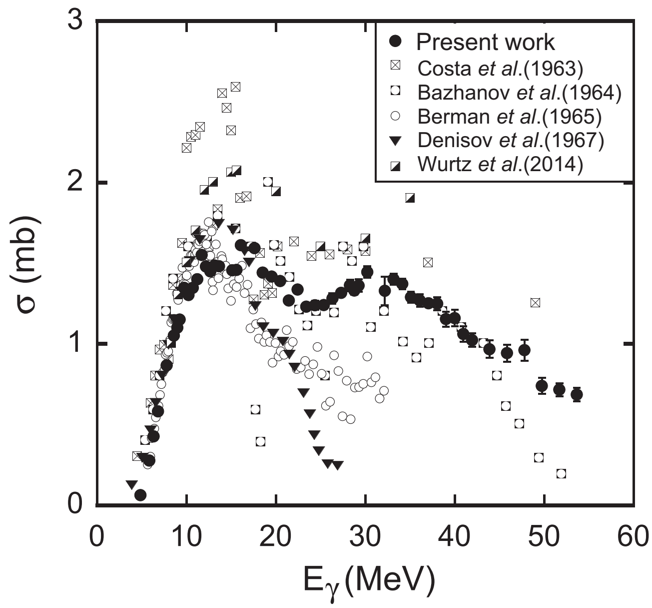

FIG. 9. Comparison of the present data with the ^{6}Li(γ,n) reaction data reported by Costa *et al.* [1], Bazhnov *et al.* [2], Denisov *et al.* [3], Berman *et al.* [4], and Wurtz *et al.* [5]. For simplicity, error bars in the previous data are not shown.

III. EXPERIMENTAL RESULTS AND DISCUSSION

A. Comparison with the previous data of photonuclear reactions

Figure 8 shows the cross sections for the ^{6}Li(γ,total), (γ,2n), and (γ,3n) reactions, i.e., $\sigma(\gamma,\text{total})$, $\sigma(\gamma,2n)$, and $\sigma(\gamma,3n)$, where $\sigma(\gamma,\text{total})=\sigma(\gamma,n)+\sigma(\gamma,2n)+\sigma(\gamma,3n)$. The $\sigma(\gamma,2n)$ and $\sigma(\gamma,3n)$ are found to be negligibly small compared with the $\sigma(\gamma,n)$. We can point out two specific features of the present results. First, the GDR in ^{6}Li is very broad. This is a strong contrast to the GDRs in other light and heavy nuclei [21]. Indeed, in the case of the GDR in ^{12}C, a typical light nucleus, the width is about 10 MeV. Second, the GDR consists of two resonances, as suggested from the previous results [6,7,9–12].

The present results are compared with the previous results obtained from the photonuclear reaction experiment in Fig. 9. The cross sections reported by Berman *et al.* and Denisov *et al.* show only a single peak at $E_r \approx 12$ MeV [3,4]. Costa *et al.*, Bazhanov *et al.*, and Wurtz *et al.* reported a broad GDR [1,2,5]. However, the cross sections by Bazhanov *et al.* show a very complicated shape for the GDR [2]. The shape of the GDR reported by Costa *et al.* and Wurtz *et al.* rather resembles the present results, though a two-resonance structure for the GDR is not visible in the case of their spectra [1,5]. We note that the data by Berman *et al* show a small resonance structure at $E_r \approx 16$ MeV [4]. In the present data, this structure is also observed at $E_r \approx 16$ MeV, though data points are only two. In the present experiment, there are significant advantages that the incident γ rays are quasimonoenergy with a good energy resolution of $\Delta E_\gamma/E_\gamma = 0.05$ and the neutron detector is simple. Furthermore, the statistical uncertainties are small. Therefore, the present improved ^{6}Li(γ,xn) experiment provides much reliable data to compare with the previous other data.

The energy integral of the cross section for the GDR is reported to be 27.8 MeV mb by Berman *et al.* [4]. This value exhausts 31% of the Thomas-Reiche-Kuhn (TRK) sum rule of $60NZ/A = 90$ MeV mb [21]. In the present work, the energy integral of the cross sections integrating from 4.9 to 53.6 MeV is 59 MeV mb, which exhausts 65% of the TRK sum rule.

B. Comparison with the nuclear reaction data

In Fig. 10, the photonuclear-reaction spectrum obtained in the present experiment is compared with the spectra of the ^{6}Li(p,p') reaction at 300 MeV [6], the ^{6}Li(^{3}He,t)^{6}Be reaction at 450 MeV [6], and the ^{6}Li(^{7}Li,^{7}Be)^{6}He reaction at 455 MeV [11]. The high-excitation-energy region of the spectra of the (^{7}Li,^{7}Be) reactions is lacking owing to the limit of the momentum acceptance of a magnetic spectrometer [22]. Because the horizontal scales of the (^{3}He,t) and the (^{7}Li,^{7}Be) reaction spectra are corrected for the Coulomb displacement energies, the horizontal axis of every spectrum in Fig. 10 is drawn in the same scale of the excitation energy in ^{6}Li. The peak energies for the GDR/SDRs of the α clusters observed via the ^{6}Li(p,p') and ^{6}Li(^{3}He,t) reactions are lower by ∼6 MeV than the peak energy observed via the photonuclear reaction. This is mainly attributable to the reason that the distribution of the SDR is shifted to the lower excitation energy by ∼5 MeV more than that of the GDR in ^{4}He, as shown in the ^{4}He(^{7}Li,^{7}Be) study [10].

In case of the (p,p') and the (^{3}He,t) reactions, the GDRs and the SDRs are not distinguished. Furthermore, the SDRs are more dominantly excited than the GDRs at these intermediate incident energies. The strength of nuclear effective interaction $V_{\sigma\tau}$, which associates the SDR excitation, is about 3.5 times larger than that of V_τ, which associates the GDR excitation. Therefore, observed cross sections of the SDR are expected to be about 10 times larger than those of the GDR [23]. In

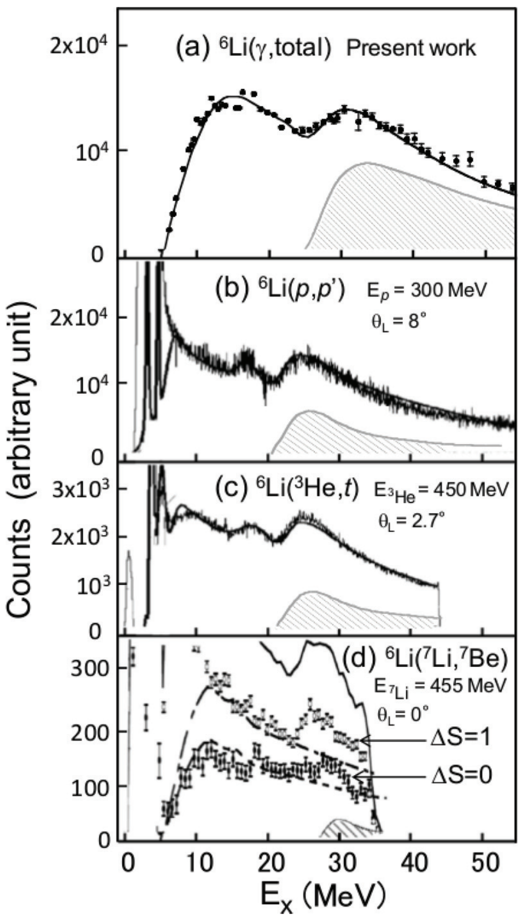

FIG. 10. Comparison of the present result with the results reported in the previous nuclear reactions on ^{6}Li. (a) The cross sections for the ^{6}Li(γ,total) reaction obtained in the present work. The solid curve is the fitting result. (b) The ^{6}Li(p,p') spectrum at $E_p = 300$ MeV [6]. (c) The ^{6}Li(^{3}He,t) spectrum at $E_{^3\mathrm{He}} = 450$ MeV [6]. (d) The ^{6}Li(^{7}Li,^{7}Be) spectrum at $E_{^7\mathrm{Li}} = 455$ MeV [11]. The broad bumps owing to the α-cluster excitation are inferred to exist in each spectrum. These resonances are marked by the oblique lines.

addition, the Coulomb excitation of the GDR in the (p,p') reaction is less effective for the light nuclei [24]. This seems to be the first reason why the GDR peak observed in the present work is not observed in Figs. 10(b) and 10(c). The second reason is that in the peak fitting of Figs. 10(b) and 10(c), the GDR shape reported by Berman *et al.* [4] was employed. However, their data were limited at $E_\gamma = 32$ MeV. In the peak fitting procedure beyond this energy, the GDR shape and the quasifree continuum were artificially assumed to be smooth and extended up to 50 MeV to fit the shape of the resonance tail [6].

However, in the (^{7}Li,^{7}Be) reaction, the spin transfers $\Delta S = 1$ (SDR) and $\Delta S = 0$ (GDR) are experimentally separated [11]. Indeed, as clearly shown in Fig. 10(d), the peak energy of the SDR is lower than that of the GDR by ∼5 MeV, though the statistical error is large [11]. However, the excitation energy of the GDR in the α clusters observed in the photonuclear reaction is well consistent with that observed in the $\Delta S = 0$ spectrum of the (^{7}Li,^{7}Be) reaction.

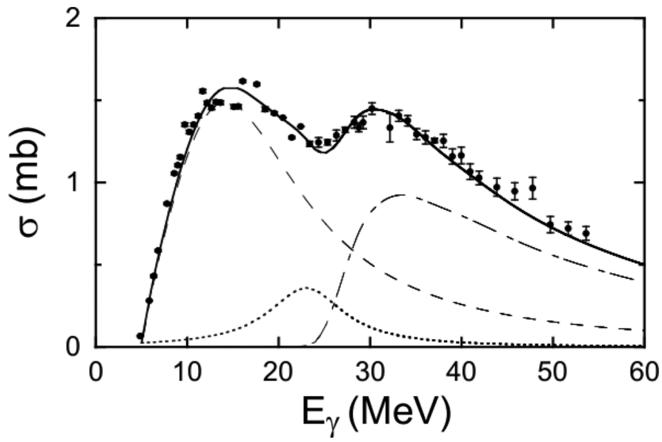

FIG. 11. Fitting of the present data assuming the LEDR at $E_r = 12$ MeV and $\Gamma = 21$ MeV (dashed curve) and the HEDR at $E_r = 33$ MeV and $\Gamma = 30$ MeV (dash-dotted curve). A small resonance was introduced at $E_r = 23$ MeV and $\Gamma = 10$ MeV (dotted curve). The solid curve is the fitting result. See text.

C. Peak fitting of the GDR in ^{6}Li

To quantitatively discuss the present photonuclear-reaction spectrum, we tried to fit the GDR shape by assuming mainly two components (the LEDR and the HEDR), as shown in Fig. 11. There might be nonresonant contribution in the GDR domain; however, we could not estimate this contribution. Therefore, we assume that the observed whole peak consists of only the GDR contribution. The shape of the LEDR is fitted with a Lorentz shape taking into account a threshold energy and an asymmetry shape effect,

$$\sigma(\mathrm{LEDR}) \propto \frac{E_\gamma - 5}{E_\gamma} \frac{\Gamma^2}{(E_\gamma - E_r)^2 + \Gamma^2/4}, \qquad (6)$$

where E_r and Γ are 12 ± 1 MeV and 21 ± 2 MeV, respectively. We employed a phenomenological function of $\frac{E_\gamma - 5}{E_\gamma}$ as a cutting function of the Lorentz shape owing to the neutron threshold energy of $S_n = 3.7$ MeV, only for simplicity. This fitting function well reproduces the shape of the LEDR.

Recently, Horiuchi *et al.* predicted theoretically the photonuclear-reaction cross sections (the GDR shape) for ^{4}He at $E_r = 26$ MeV and $\Gamma = 20$ MeV [25]. The predicted cross sections for the GDR consisted of the cross sections of the ^{4}He(γ,n) and the ^{4}He(γ,p) reactions, which were predicted to have almost the same values as each other. Schuster *et al.* also predicted the cross sections of the GDR in ^{4}He in a similar way [26]. Because the Horiuchi prediction agrees well with many previous experimental results, we modify the shape in the Horiuchi prediction of the photonuclear reaction to fit the shape of the HEDR observed in the present work: The peak energy is increased by 7 ± 2 MeV, namely, $E_r = 33 \pm 2$ MeV; the resonance width is widened by 1.5 ± 0.1 times, namely $\Gamma = 30 \pm 2$ MeV; and peak height is reduced by 0.29 ± 0.02 times. As a result, the energy integral of the HEDR is 0.86 ± 0.06 times that of the ^{4}He(γ,n) reaction in the Horiuchi prediction.

TABLE II. Physical quantities of the GDR in ^{4}He and in the α cluster in ^{6}Li.

	^{4}He[a]	α cluster in ^{6}Li
E_r (MeV)	26	33 ± 2[d]
		31.5 ± 2.0[e]
Γ (MeV)	20	30 ± 2
Size ratio	1[b]	$\sim$0.8[f]
$\int \sigma dE$ (MeV mb)	1[b,c]	0.86 ± 0.06

[a]Taken from Ref. [25].
[b]Normalized.
[c]Contribution from the ^{4}He(γ,n) reaction.
[d]Excitation energy.
[e]Excitation energy from the separation energy of $\alpha + d$.
[f]$E_r(^4\text{He})/E_r(\alpha$ cluster).

It is known that a small dipole resonance (SDR and/or GDR) exists at $E_x = 18$ MeV, as shown in Figs. 10(b) and Fig. 10(c) [6]. However, the nature of this resonance is not well known [6,11]. As mentioned in the previous section, we assume that this resonance observed in the (p,p') and $(^3\text{He},t)$ reactions is the SDR. Therefore, to obtain the better fitting of the $\sigma(\gamma,n)$ data, we introduce a small resonance of the GDR, which is a counterpart of this 18-MeV resonance (SDR), with a Lorentz shape of $E_r = 23.0 \pm 1.5$ MeV and $\Gamma = 10 \pm 2$ MeV.

The fitting result is shown by the solid curve in Fig. 11. Overall fitting of the spectrum is good. We conclude that the GDR of the α cluster in ^{6}Li seems to be observed at $E_r = 33 \pm 2$ MeV and with $\Gamma = 30 \pm 2$ MeV via photonuclear reaction. Thus, we suggest that the existence of the GDR owing to the α cluster in ^{6}Li is confirmed via both the experiments of the nuclear reaction and the photonuclear reaction.

In Table II, the physical quantities of the GDR in ^{4}He and in the α cluster in ^{6}Li are summarized. The quantities are significantly different between the excitation energies and the widths for the GDR in ^{4}He and the α cluster GDR in ^{6}Li. An increase of the excitation energy for the α-cluster GDR suggests that a size of the α cluster in ^{6}Li is smaller than that of ^{4}He by $\sim$0.2 times. Indeed, the GDR is split into low-excitation-energy and a high-excitation-energy components in deformed nuclei. The low- and the high-excitation-energy components are attributable to vibrations along the long and short axes of the deformed nucleus, respectively [27]. The energy integral of the cross sections of the α cluster GDR in ^{6}Li is nearly the same as that of the GDR in ^{4}He. An increase of the width of the α cluster GDR in ^{6}Li suggests that the relaxation time of the α-cluster GDR, which is a local excitation inside of the nucleus, is shorter than that of the GDR in free ^{4}He.

IV. CONCLUSIONS

We studied the ^{6}Li(γ,xn) reaction at an incident energy range of $E_\gamma = 4.9$ to 53.6 MeV to investigate the GDR in ^{6}Li using γ rays generated by Compton backscattering of laser photons with relativistic energy electrons. The incident γ-ray energy was quasimonochromatic energies with the energy resolution $\Delta E_\gamma / E_\gamma = 0.05$.

The energy integral of the cross sections was 59 MeV mb, which exhausted about 65% of the TRK sum rule. We found that the GDR consisted mainly of two resonances, the LEDR and the HEDR. The LEDR is inferred to be the GDR of ^{6}Li itself owing to the $1\hbar\omega$ single particle-hole excitation. However, the HEDR corresponds to the GDR owing to the α-cluster excitation in ^{6}Li.

Thus, we suggest that the existence of the HEDR in ^{6}Li is confirmed to be an intrinsic excitation of the α cluster in ^{6}Li via both the nuclear reaction and the photonuclear reaction. The resonance shape of the α-cluster GDR in ^{6}Li has been well fitted by modifying that of the GDR in the free ^{4}He predicted theoretically [25]. The excitation energy and the width of the α-cluster GDR in ^{6}Li are higher by 7 ± 2 MeV and wider by 1.5 times than those in the free ^{4}He, respectively. The energy integral of the cross sections for the α-cluster GDR is 0.86 ± 0.06 times that predicted for the ^{4}He(γ,n) reaction. The increase of the excitation energy of the GDR in the α cluster suggests that the mass increases by 7 ± 2 MeV and the size of the α cluster in ^{6}Li is smaller by about 20% than the free ^{4}He. The width increase of the α cluster GDR in ^{6}Li to compare with the GDR in the free ^{4}He may be caused by shortening of the relaxation time inside the nucleus owing to the nuclear medium effect.

We expect that the similar results obtained in the present work for ^{6}Li will be observed also in ^{7}Li, because the existence of the α-cluster excitation is suggested in the nuclear reactions [6,7,9–12]. Finally, we note here that the present study suggests that there exists the subnuclear excitation owing to the α cluster in a nucleus [28]. The hot-spot model in heavy-ion reaction proposed in 1975 should be reviewed again [29].

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