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Luneburg-lens-like structural Pauli attractive core of the nuclear force at short distances

S. Ohkubo

Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan

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The nuclear force has been understood to have a repulsive core at short distances, similar to a molecular force, since Jastrow proposed it in 1951 [R. Jastrow, Phys. Rev. **81**, 165 (1951)]. The existence of the repulsion was experimentally confirmed from the proton-proton scattering $^1S_0$ phase shift, which becomes negative beyond 230 MeV. This repulsion is essential for preventing the nucleon-nucleon system from collapsing by attraction. The origin of the repulsion has been considered to be due to the Pauli principle, similar to the repulsion originally revealed in α-α scattering, in many studies including recent lattice QCD calculations. On the other hand, very recently it was shown that an internuclear potential including α-α interactions has a *Luneburg-lens-like attraction* at short distances rather than repulsion. I show that the nuclear force with an attractive potential at short distances that reproduces the experimental $^1S_0$ phase shifts well has a Luneburg-lens-like *structural Pauli attractive core* at short distances and acts as apparent *repulsion*. The apparent repulsion is caused by the deeply embedded unobservable Pauli forbidden state similar to nucleus-nucleus potentials.

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I. INTRODUCTION

In this paper it is shown that a nuclear force with an attractive potential at short distances that reproduces the experimental $^1S_0$ phase shifts well has a Luneburg-lens-like *structuralPauli attractive core* (SPAC) similar to the nucleus-nucleus potential and acts as an *apparent repulsion*. This study was inspired by the recent discovery of the Luneburg-lens-like *structural Pauli attraction* in nucleus-nucleus potentials [1].

The nuclear force is essential for the existence of nuclei [2]. It binds nucleons, which allows the stable existence of atoms and matter, therefore life. The origin of the nuclear force was theoretically revealed by Yukawa [3]. The nuclear force was extensively studied by the Japanese nuclear force group [4–7] based on the three-stage theory of Taketani [4,5]. Jastrow proposed the existence of short-range repulsion at short distances [8], which was supported by the negative $^1S_0$ phase shift observed by 310 MeV proton-proton scattering [9]. As shown in Fig. 1, a tremendous number of studies [5,8–17] show that the nuclear force has a repulsive core (hard or soft) at short distances in the innermost region (region III) and is attractive in the intermediate range region (region II) and in the outermost one-pion-exchange potential (OPEP) region (region I). Phenomenological potential models proposed in the 1960s include the Hamada-Johnston (HJ) potential with a hard core [10], the Reid soft core potential [11], and Tamagaki's Gaussian 3 range soft (G3RS) core potential [12]. The modern high-precision potentials fitting many NN data [17] include Argonne V18 [13], CD-Bonn [14], Reid93 [15], and ESC04 [16], in which a repulsive core is introduced phenomenologically. The origin of the repulsive core has remained a challenging subject. It has been ascribed to heavy meson exchanges [18] and the Pauli principle due to the substructure of the nucleon [19–22].

After QCD was established, new light was shed on the origin of the repulsive core from the quark model [23–31]. Neudatchin *et al.* [23] argued that the repulsive core in the S wave can arise from the Pauli forbidden state of the orbital symmetry [42]. References [25,26] showed that the color-magnetic quark-quark force favors the mixed symmetry state [42] acting attractively and disfavors the completely symmetric orbital state [6] acting repulsively. The two states can be almost degenerate [30], which means that in S-wave scattering the mixed symmetry state can contribute almost equally as the symmetric orbital state in the inner region (region III). Reference [27] showed that the repulsive core of the equivalent local potentials of the resonating group method (RGM), which were derived using quark forces that cause different admixtures of the mixed symmetry and WKB method, largely originates from the color-magnetic exchange kernel. Recent lattice QCD calculations [33] reported that the repulsive core is due to the Pauli principle [19–23].

The idea that the repulsive core at short distances comes from the Pauli principle [19–22] was originally inspired by analogy with the origin of the phenomenological repulsive core potential in $\alpha + \alpha$ scattering. It was shown in Ref. [34] that the *repulsive core* in $\alpha + \alpha$ scattering, which is followed by an angular-momentum (L)-dependent *shallow attraction* in the outer region, is a potential representation of the damped inner oscillations in the relative wave function caused by the Pauli principle [35–37]. On the other hand, it was also shown later that not only $\alpha + \alpha$ scattering but also $\alpha + {}^{16}$O scattering can be well reproduced by an L-independent local *deep attractive* potential *without* a repulsive core in which the Pauli forbidden states of the RGM are embedded [38–42].

Very recently it has been shown [1] that the Pauli principle causes a Luneburg-lens-like structural *Pauli attraction* in the internal region of the nucleus-nucleus deep potential in contrast to the traditional understanding that it causes a *repulsive core* [34–37]. This was demonstrated from the systematic study of nuclear rainbow scattering, prerainbows, anomalous large -angle scattering (ALAS), molecular structure, and cluster structure [1]. In a naive potential picture, the existence of repulsion at short distances seems generally indispensable to prevent a system collapse by attraction, for example, for two-atom molecules such as H-H. Historically, the observation of the S-wave negative phase shifts in $\alpha + \alpha$ scattering [32]

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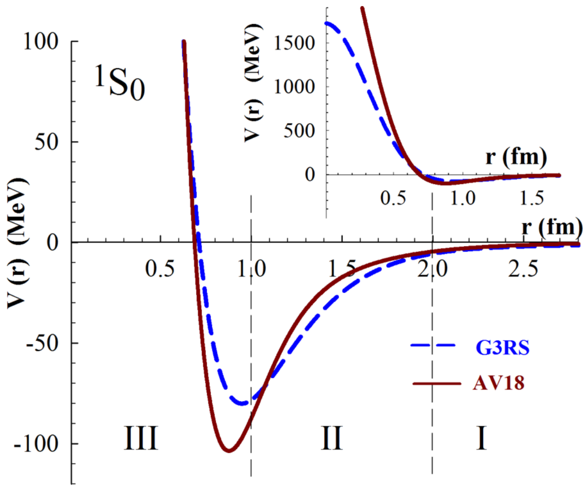

FIG. 1. Typical nuclear potentials for the $^1S_0$ channel, the G3RS potential (dashed line) [12], and the Argonne V18 potential (solid line) [13].

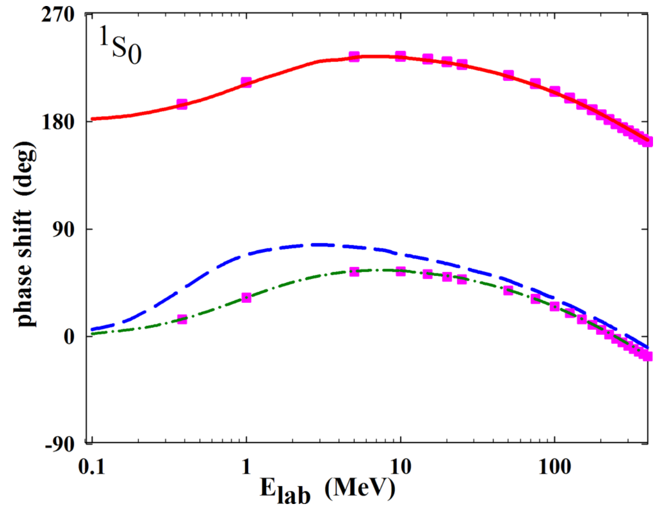

FIG. 2. The proton-proton scattering $^1S_0$ phase shifts calculated with the SPAC potential of Eq. (2) (solid line), the G3RS potential of Eq. (1) (dashed line), and the Reid93 potential [15] (dash-dotted line) are displayed. The experimental data (squares) are from Ref. [45].

and in proton-proton scattering [9] in the 1950s led naturally to a repulsive core potential at short distances based on quantum scattering theory of *structureless* particles. However, the recent finding of Ref. [1] urges one to ask whether a similar *Pauli attractive* core persists at short distances in a nucleon-nucleon potential given that the nucleon is composed of fermions.

From the quark model viewpoint the Moscow group [28–30,43] have been involved in developing a model for such a nucleon-nucleon potential that has an attractive core due to the Pauli forbidden states. They treated region III and region II on the same footing phenomenologically representing them either by a single Woods-Saxon potential, a single Gaussian potential, or a single Yukawa potential. The apparent core radius of the nuclear potential of Refs. [28–30] is rather large, extending to near 1 fm [30]. Also underbinding of triton was pointed out [44]. It is important to separate region III and the established [31] regions II and I.

II. NUCLEAR FORCE WITH STRUCTURAL PAULI ATTRACTIVE CORE

I investigate $^1S_0$ nucleon-nucleon scattering where the complications due to the spin and angular-momentum-dependent forces such as a tensor force are absent. The basic components of the modern high-precision potentials, which have 40 (AV18) or a similar number of adjustable parameters, are all present in the potentials of HJ [10], Reid [11], and G3RS [12]. I take the G3RS potential (set ^1E-1) [12], which was modeled to reproduce the experimental phase shifts at $E_{\text{lab}} = 25-660$ MeV by using a Gaussian function for the three regions, as follows:

$$V(r) = -5e^{-(r/2.5)^2} - 270e^{-(r/0.942)^2} + 2000e^{-(r/0.447)^2}. \quad (1)$$

The strength of the potential is in MeV and the range parameter is in fm. The phase shifts calculated using Eq. (1) are displayed in Fig. 2 by the dashed line. My philosophy and prescription to find a *deep* potential is as follows. According to Ref. [1], the $^1S_0$ phase shifts would be equally well reproduced by replacing the repulsive core potential in Eq. (1) by a SPAC, namely, by changing the sign of the strength of the third core term (region III) of Eq. (1). The attractive first term (region I) and the second term (region II) of the SPAC potential in Eq. (2) correspond exactly to the first term (OPEP) and the second term (one-boson-exchange potential, OBEP) of Eq. (1), respectively, which are based on the established sound meson theoretical foundation [12]. The third term of the core (region III), repulsive in Eq. (1) and attractive in Eq. (2), is based on the theoretical foundation due to the Pauli principle:

$$V(r) = -5e^{-(r/2.5)^2} - 270e^{-(r/0.942)^2} - 1850e^{-(r/0.447)^2}. \quad (2)$$

It is surprising that a good fit is easily obtained by a slight adjustment to -1850 MeV. The phase shifts calculated by the SPAC potential of Eq. (2) are displayed in Fig. 2 by the solid line. Because of the generalized Levinson theorem, the phase shift starts from 180° at $E_{\text{lab}} = 0$ MeV. The quality of fits to the experimental phase shifts is even better than the results with the G3RS potential, which cannot reproduce a virtual state near threshold without reducing the height of the core. The SPAC potential is almost phase shift equivalent to Eq. (1).

I investigate whether the attractive core at short distances is similar in nature to a Luneburg-lens-like potential. A Luneburg lens [46] is an aberration-free, spherically symmetric gradient-index lens, which decreases radially from the center to the outer surface $r = R$ and refracts all the parallel incident trajectories to the focus $r = R_f$ ($<R$). For such a lens the refractive index n is given by

$$n^2(r \leqslant R) = \left(R_f^2 - r^2 + R^2\right)/R_f^2, \quad n(r > R) = 1. \quad (3)$$

The potential having this property [47] is

$$V(r \leqslant R) = V_0(r^2/R^2 - 1), \quad V(r > R) = 0, \quad (4)$$

where $V_0 = E(R/R_f)^2$ is the depth at $r = 0$ with E being the energy of a material particle moving in a potential $V(r)$. This is a harmonic oscillator (HO) potential truncated at

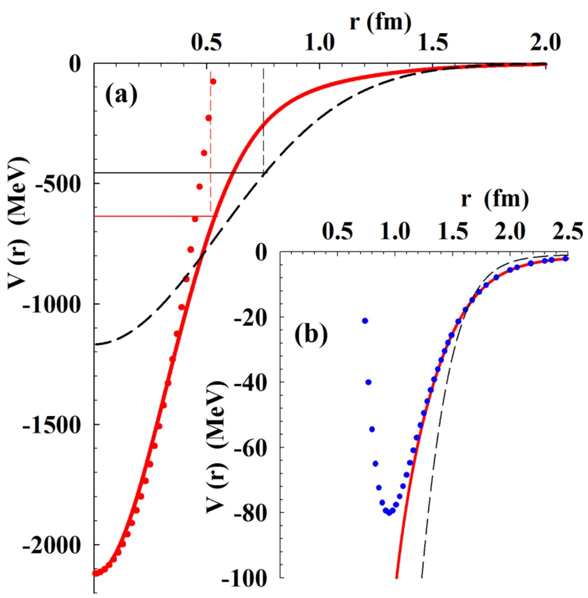

FIG. 3. (a) The SPAC nuclear force with the Pauli attractive core at the short distances (red solid line) of Eq. (2), the Moscow potential (dashed line) of Ref. [28], and the Luneburg-lens potential (red dotted line) are compared. The horizontal lines in panel (a) indicate the energy of the unobservable Pauli forbidden $0s$ state. The vertical dashed lines are to guide the eye. (b) A magnified SPAC and Moscow potential in panel (a) are compared with the G3RS potential (blue dotted line).

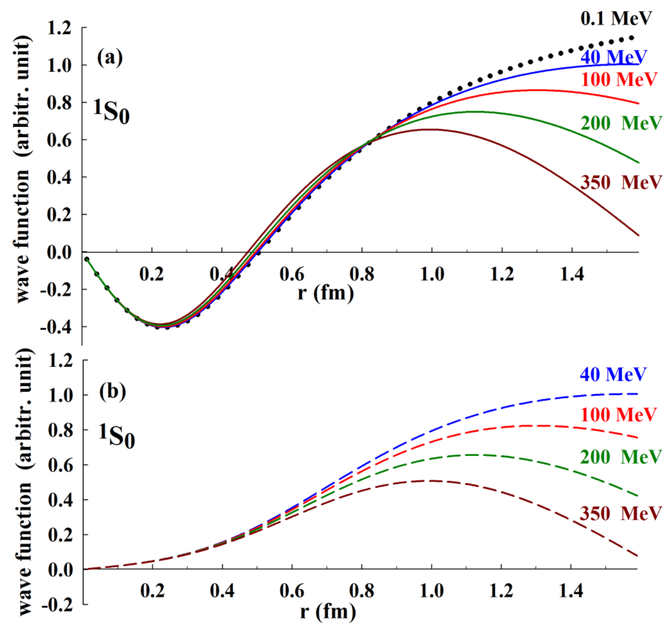

FIG. 4. The calculated $^1S_0$ wave functions of proton-proton scattering at $E_{\text{lab}} = 0.1-350$ MeV using (a) the SPAC nuclear force potential with the Pauli attractive core at the short distances and (b) the G3RS potential with the repulsive core at the short distances. The difference of wave functions in panels (a) and (b) are seen in the core region $r < 0.5$.

$r = R$. In Fig. 3 the SPAC potential is shown in comparison the Luneburg-lens-like potential together with the G3RS potential and the Moscow potential of Ref. [28]. The deeply bound unphysical Pauli forbidden $0s$ state is indicated by the horizontal line in Fig. 3(a). We see in Fig. 3(a) that the short-distance region of the nuclear potential resembles the Luneburg lens with $V_0 = 2120$ MeV and $R = 0.54$ fm. The attraction in the intermediate region (region II) and the outermost region (region I) corresponds to the diffuse tail part of the potential, which causes aberration [1]. The Luneburg-lens-like nature of the nuclear force with the structural attractive core at short distances originates from the third term of Eq. (2). The effect of the potential of the first and second terms of Eq. (2) scarcely changes the Luneburg-lens-like origin of the core. This can be understood analytically by the Taylor expansion of Eq. (2) to the first order, which leads to $V(r) = 2125[(r/0.47)^2 - 1]$. The 2125 MeV and 0.47 fm are close to the values of the above Luneburg-lens parameters. The third term of Eq. (2) alone is well simulated by a Luneburg lens with $V_0 = 1850$ MeV and $R = 0.48$ fm, which are close to the values $V_0 = 1850$ and $R = 0.447$ fm derived from its Taylor expansion. The Moscow potential is considerably "shallower" than the SPAC potential in the core region, thus bringing a shallower Pauli forbidden state and a larger core radius.

In Fig. 4 wave functions for proton-proton scattering calculated using Eqs. (1) and (2) are displayed. One sees in Fig. 4(a) that the wave functions have a node at around $r = 0.5$ fm for any incident energies. This shows that the S waves are forced to be orthogonal to the Pauli forbidden $0s$ state deeply embedded in the potential indicated in Fig. 5(a).

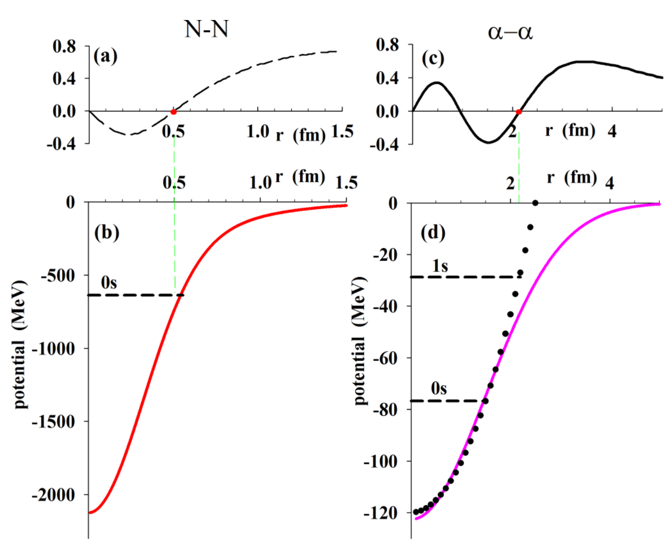

FIG. 5. The potentials and the wave functions (in arbitrary units) for the nucleon-nucleon and α-α systems are compared. The SPAC $^1S_0$ NN potential (b) with the SPAC at short distances and the wave function (a) are displayed in comparison with the α-α deep potential with a structural Pauli attraction of Ref. [1] (d) and the Pauli-allowed s-wave function (c). The horizontal dashed lines in panels (b) and (d) indicate the energy of the unobservable Pauli forbidden states embedded in the NN and α-α potentials. The wave function of $1s$ in panel (a) and $2s$ in panel (d) have been calculated using the bound-state approximation.

One also notices that the amplitudes of the wave functions at the short distances are damped. The node plays the role of preventing penetration of the wave functions into the region $r < 0.5$ fm, namely, collapsing of the system. This role is similar to the repulsive core at short distances in Fig. 1. Now it is clear that the Luneburg-lens-like structural Pauli attractive core plays the role of apparent repulsion for any energy via the embedded Pauli forbidden state. From Figs. 4(a) and 5(a), one sees that the Luneburg-lens radius R corresponds well to the energy-independent nodal position $r \approx 0.5$ fm. In Fig. 4(b) the wave functions for proton-proton scattering calculated using the G3RS potential are displayed. As far as the asymptotic behavior is concerned, the two wave functions calculated with the G3RS repulsive core potential and the SPAC potential are "phase shift equivalent." However, while the wave functions are strongly damped at short distances for the repulsive core potential, for the SPAC potential they survive with nonvanishing significant amplitudes of the *inner oscillations* at short distances. One sees that the energy-independent nodal position in Fig. 4(a) corresponds well to the the repulsive core radius at around $r = 0.5$ fm in Fig. 4(b).

In Fig. 5 the SPAC nuclear force potential is compared with the α-α potential. Similar to the nucleon composed of three quarks, the α particle is composed of four tightly bound fermions and the interaction is well described by a deep potential between structureless point particles. In Fig. 5(d), the deep potential at short distances closely resembles the Luneburg-lens truncated HO potential indicated by the dotted lines. The overlap of the calculated deeply bound 0s and 1s states with the HO wave functions of the Pauli forbidden states of the RGM is 1 as was shown in Ref. [1]. Therefore the deeply bound states embedded in this potential play the role of the Pauli forbidden states. The physical 0^+ state is forced to be orthogonal to them, by which the wave function has two nodes as seen in Fig. 5(c). The outermost node at around $r = 2$ fm, which arises due to the orthogonality, corresponds to the repulsive core radius of the shallow α-α potential. The situation of the NN system is very similar to α-α. The solved eigenfunction of the deeply embedded bound 0s state at about -637 MeV, which is indicated in Fig. 5(b), is also very similar to the 0s wave function of the HO potential. This means that the three quarks are likely to be confined in a harmonic oscillator potential. The deeply bound 0s state plays the role of the Pauli forbidden state of the RGM, similar to the α-α system. In agreement with experiment, neither a physical bound state nor a resonant state appear in the $^1S_0$ channel. In Fig. 5(a) the wave function displayed is a virtual state obtained in the bound-state approximation to show that the node appears at around $r = 0.5$ fm by the orthogonality to the 0s Pauli forbidden state. It was demonstrated mathematically in Ref. [1] that the Luneburg-lens-like attractive potential is a manifestation of the Pauli principle.

III. DISCUSSION

It is well known in nucleus-nucleus potentials that an L-independent deep potential and an L-dependent shallow potential with a repulsive core are interrelated. The latter is derived phenomenologically [48,49] or mathematically by supersymmetry theory [50] from the former but not vice versa. The widely used Ali-Bodmer L-dependent shallow α-α potential with a repulsive core [51] is an approximate supersymmetry partner of the L-independent deep α-α potential in which the Pauli forbidden states are embedded [50]. Similarly the shallow nucleon-nucleon potentials with a repulsive core can be considered to be an approximate supersymmetric partner of the deep SPAC potential [49]. One is reminded that in the nucleus-nucleus potential case the differences between the shallow and the deep potentials are clearly distinguished physically in the observations such as ALAS and nuclear rainbow, by which shallow potentials could not survive. In the present case, the difference between the wave functions at $r < 0.5$ in Figs. 4(a) and 4(b) may be seen in physical quantities such as the binding energies in few-body systems. The underbinding problem for tritons using a wide variety of modern NN interaction models with a repulsive core is well known [52], which has been ascribed to three-body forces. It is also to be noted that any high-precision nuclear forces with a repulsive core cannot explain the existence of the recently observed tetraneutron [53] without inconsistent modifications such as to introduce a remarkably attractive three-body [54]. The nonvanishing amplitudes of the inner oscillations are expected to give a significant energy gain for the binding of three- and four-nucleon systems.

Although a shallow nucleus-nucleus potential has prevailed in the past decades [55], it is now definitely agreed [56] that a nuclear potential is deeply attractive at short distances [1,42,57–59], which is due to the Pauli principle [1,57]. On the other hand, the concept of baryon-baryon interaction with an attractive deep potential at short distances is unfamiliar probably because the fundamental nuclear model and theory were developed using a shallow potential with a repulsive core [2,60]. Reference [33] reports that a deep attractive potential appears in the $\{\mathbf{1}\}$ representation $^1S_0$ channel of SU(3) $\mathbf{8} \times \mathbf{8}$ of the flavor octet baryon with spin-$\frac{1}{2}$. Oka and Yazaki reported that the Δ-Δ potential is attractive at short distances [24]. As for the ω-meson theory of the core, a recent holographic model using a D4-D8 brane configuration [61] reports that the core originates from extra spatial dimension and that the one-boson-exchange potential of an ω exchange captures merely a part of the towers of massive mesons.

IV. SUMMARY

To summarize, it was shown that the nuclear force with an attractive potential at short distances that reproduces the experimental $^1S_0$ phase-shift well has a Luneburg-lens-like SPAC similar to the nucleus-nucleus potential [1]. The attractive core is as deep as -1850 MeV so that the embedded unobservable deeply bound 0s state is closely similar to the Pauli forbidden state. The SPAC strongly prevents penetration of the wave function into the core region, thus playing the role of apparent *repulsion*. The energy-independent node at around $r = 0.5$ fm with damped inner oscillations in the wave function corresponds to the core radius and the Luneburg-lens radius R. The wave function can penetrate into the core region significantly with the inner oscillation in contrast to the repulsive core potential. The nuclear forces with a repulsive core can be considered to be an approximate supersymmetric shallow potential partner of the SPAC potential like the α-α system.

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