Directed flow in asymmetric nucleus-nucleus collisions and the inverse Landau-Pomeranchuk-Migdal effect

V. D. Toneev,[1] V. Voronyuk,[1,2] E. E. Kolomeitsev,[3] and W. Cassing[4]
[1]*Joint Institute for Nuclear Research, Dubna, Russia*
[2]*Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine*
[3]*Matej Bel University, Faculty of Natural Sciences, Banska Bystrica, Slovakia*
[4]*Institute for Theoretical Physics, University of Giessen, Giessen, Germany*
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It is proposed to identify a strong electric field—created during relativistic collisions of asymmetric nuclei—via the observation of pseudorapidity and transverse momentum distributions of hadrons with the same mass but opposite charge. The results of detailed calculations within the parton-hadron string dynamics (PHSD) approach for the charge-dependent directed flow v_1 are presented for semicentral Cu+Au collision at $\sqrt{s_{NN}} =$ 200 GeV incorporating the inverse Landau-Pomeranchuk-Migdal (iLPM) effect, which accounts for a delay in the electromagnetic interaction with the charged degrees of freedom. By including the iLPM effect, we achieve a reasonable agreement of the PHSD results for the charge splitting in $v_1(p_T)$ in line with the recent measurements by the STAR Collaboration for Cu+Au collisions at $\sqrt{s_{NN}} = 200$ GeV while an instant appearance and coupling of electric charges at the hard collision vertex overestimates the splitting by about a factor of 10. We predict that the iLPM effect should practically disappear at energies of $\sqrt{s_{NN}} \approx 9$ GeV, which should lead to a significantly larger charge splitting of v_1 at the future FAIR/NICA facilities.

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I. INTRODUCTION

The properties of the very initial degrees of freedom in ultrarelativistic heavy-ion collisions during the passage time of the impinging nuclei is presently unknown and the ideas vary from a color-glass condensate (CGC) [1,2] to a gluon-dominated plasma [3] or a longitudinal color field that decays to strongly interacting partons [4]. Various suggestions have been made to distinguish between such scenarios [5–7]; however, a clear discrimination has not been achieved yet [8]. It was proposed in Refs. [9,10] that a strong electric field—produced early by the spectator charges—could help to clarify the problem by investigating the charge splitting of the directed flow of particles with equal mass and opposite electric charge as a function of rapidity and transverse momentum.

Indeed, it has been demonstrated early in Ref. [11] that the collective motion of spectator charges in relativistic heavy-ion collisions can produce extremely strong electromagnetic fields. Particularly, in peripheral Au-Au collisions at the center-of-mass energy $\sqrt{s_{NN}} = 200$ GeV the magnetic field in the very initial interaction state can be as high as $|eB_y| \sim 5m_\pi^2 c^3/\hbar^2 e \sim 5 \times 10^{18}$ Gauss, which is the largest value reachable at terrestrial conditions and even larger than magnetic fields in magnetars. However, the subsequent analysis of Au+Au collisions in the energy range up to the top RHIC energies revealed no visible effect of strong electromagnetic interactions on global characteristics and, in particular, on sensitive quantities such as the directed or elliptic flow. The reason for that is not the very short interaction time of the electromagnetic field with the charges of the partonic system, as one might expect naively, but rather a compensation of electric and magnetic forces in symmetric systems as found in Ref. [12]. However, it has been argued that in asymmetric collisions this compensation effect is largely suppressed due to the different number of protons in the colliding nuclei [9,10]. Since the strength of the induced electric field is strongly asymmetric inside the overlap region, one may expect to observe an asymmetry in the momentum distributions of produced charged hadrons. In particular, in Cu+Au collisions the directed flow, i.e., the first flow harmonic $v_1 = \langle p_x/p_T \rangle$ (p_x denoting the momentum projection on the reaction plane while p_T is the transverse momentum), exhibits a dependence on the charge of partonic or hadronic particles. This has been shown explicitly in Ref. [9] within microscopic calculations in the framework of the parton-hadron-string dynamics (PHSD) approach [13,14] for Cu+Au collisions at $\sqrt{s_{NN}} = 200$ GeV (cf. Fig. 2 in Ref. [9]), where one finds a strong electric field in the central region of the overlap area which is directed from the Au nucleus to the Cu nucleus.

Detailed calculations of the directed flow v_1 for $\pi^\pm$, $K^\pm$, p, and $\bar{p}$ at the energy $\sqrt{s} = 200$ GeV have been carried out in Ref. [9], taking into account the influence of the retarded electromagnetic field created by spectators on the particle trajectories. The PHSD calculations have been performed also for Cu+Au collisions for the NICA energies of $\sqrt{s_{NN}} = 9$ and 5 GeV. Here the charge-dependent separation effect may be observed also at 9 GeV as clearly as at 200 GeV; however, it becomes much weaker for $\sqrt{s_{NN}} = 5$ GeV [15].

Although two years have passed since the start of the data collection for Cu+Au collisions at $\sqrt{s}_{NN} = 200$ GeV, the first preliminary data on the charge-dependent anisotropic flow have been reported only recently [16,17]. The comparison between the PHSD results and the data shows that our calculations overestimate the measured splitting in the directed flow of charged particles,

$$\Delta v_1 = v_1(h^+) - v_1(h^-),$$

by a factor of about ten in the scenario which assumes the presence of electric charges already at the space-time point

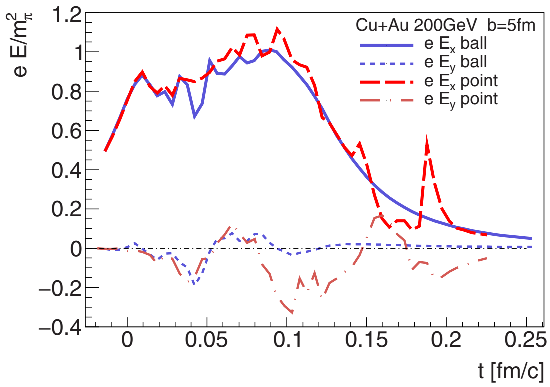

FIG. 1. Time dependence of the electric field components E_x and E_y as generated by pointlike charges (dash-dotted line) in the central point of the overlap region $(x,y,z) = (0,0,1\text{ fm})$ for Cu + Au at $b = 5$ fm and $\sqrt{s_{NN}} = 200$ GeV. The solid and dotted lines correspond to the fields from the Lorentz contracted ball-like charge distributions (see text).

of the initial hard vertices. Thus, it is necessary to discuss possible mechanisms which can reduce Δv_1 within the PHSD model; this is the aim of the present study.

II. ELECTROMAGNETIC FIELDS AND CHARGE SPLITTING FROM PHSD

For our microscopic investigations we employ the standard PHSD model that also has been used in Refs. [9,15]. For a review on the various PHSD results we refer the reader to Ref. [4].

As noted in Ref. [12], the electromagnetic field (EMF) is formed predominantly by charged spectators at the early stage of the collision during the passage time of the two colliding nuclei. Since the number of spectator nucleons decreases with decreasing impact parameter b, the electromagnetic fields should also decrease gradually with increasing centrality. However, as found in Ref. [12] the strength of the average E_x component of the electric field does not change much in the interval of $b = 3$–7 fm.

As seen from the time evolution of the electric field for Cu + Au at $b = 5$ fm and $\sqrt{s_{NN}} = 200$ GeV in Fig. 1, the average strength of the dominant component $\langle E_x \rangle$ fields reaches maximal values of $\langle eE_x \rangle \approx 1.0\,\text{m}_\pi^2\, c^3/\hbar\ \text{GeV}^2$ for a time of $\sim$0.15 fm/c, which is about the passage time of the two nuclei. The other components are practically negligible.

To investigate the influence of the EMF we have calculated within the PHSD approach various characteristics of the asymmetric Cu+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. In Fig. 2 the directed flow v_1 is presented as a function of pseudorapidity η for charged pions and kaons. We see that within the pseudorapidity window $|\eta| < 3$ the η distributions for $\pi^+(K^+)$ and $\pi^-(K^-)$ are very close to each other when discarding the EMF in the dynamics. We recall that the difference increases for larger rapidities and becomes sizable only for forward or backward rapidities $|\eta| > 3$ which can be attributed to a difference in the production mechanism of these mesons [9]. The inclusion of the EMF, however, leads to a sizable separation of these distributions for opposite charges. The recent experimental data [16,17] indeed show the predicted charge separation effect; however, its magnitude is reduced by about a factor of 10.

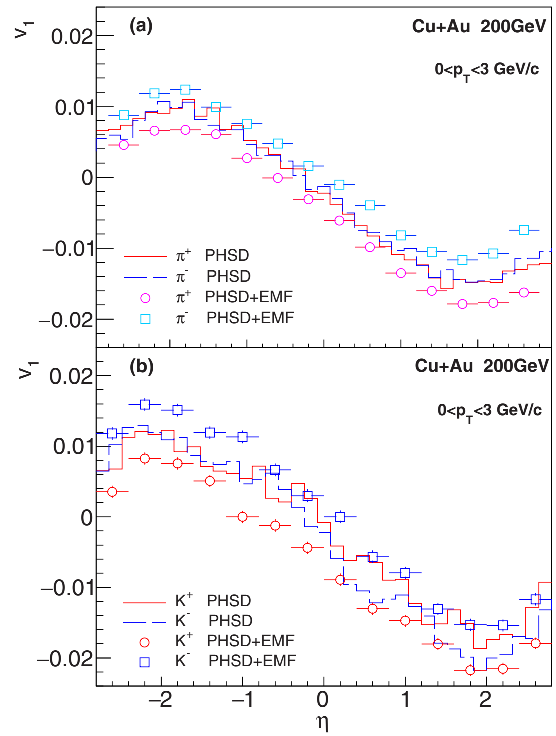

FIG. 2. Pseudorapidity distributions within the pseudorapidity interval $|\eta| < 3$ for positive and negative pions (a) and kaons (b) created in Cu+Au collisions at $\sqrt{s_{NN}} = 200$ GeV in the impact parameter interval 4.4–9.5 fm. The histograms are the result of the standard PHSD transport approach without EMF, and the symbols are obtained when the electromagnetic force is additionally included.

III. POSSIBLE MECHANISMS FOR THE REDUCTION OF THE CHARGE SPLITTING

In this section we briefly discuss possible mechanisms that might have lead to an overestimation of the charge splitting in the directed flow within the PHSD calculations.

(i) One should note first that the EM field variation with time could be too fast such the classical treatment of the EMF is not allowed. For example, according to Ref. [18] the amplitude of the electric field eE should by larger than the critical field $e\,E_{\text{crit}} = e\sqrt{\hbar c}/(c\Delta t)^2$, where Δt is a typical time of the field variation. Since EMFs are described by the Liénard-Wiechert potentials we can estimate the field variation time for ultrarelativistic collisions as $\Delta t = E/\dot{E} \sim \gamma\langle b\rangle/c$, where $\langle b\rangle$ the average impact parameter of the collision; then we get

$$e\,E_{\text{crit}} = 0.17\gamma^2\, m_\pi^2/c^3\hbar/(b/\text{fm})^2. \qquad (1)$$

Therefore, for the typical impact parameter range considered, the field strength $\gtrsim 1 m_\pi^2$ (as shown in Fig. 1) is enough to treat the EMF classically.

(ii) In our treatment we have considered the charged particles as pointlike and, therefore, the Coulomb interaction becomes singular at the charge location. Generally, the interplay of the Coulomb and hadronic interactions should be taken into account. The problem of the parton cloud structure of hadrons at ultrahigh energy has been widely discussed in this respect [19–21]. A slow growth of the parton cloud radius with increasing energy points towards a glue-ball origin of the parton clouds [22,23]; the glue-ball origin of the parton clouds then leads to the universality of all hadron total cross sections. Thus we have recalculated the EM fields in Cu+Au collisions assuming that the spectator charges have the shape of a Lorentz-contracted ball. The corresponding results are shown in Fig. 1 and we find that the event-averaged field strengths do not change much. Hence, this modification cannot explain the observed discrepancy.

(iii) The analysis of ultrarelativistic elastic pp scattering has revealed an interesting structure [20]. The parametrization of the NN scattering data at LHC energies indicate a hole in the inelasticity profile $n_{in}(b)$, i.e., a dip at the origin while at the energy $\sqrt{s} \lesssim$ 70 GeV there is a maximum in the origin. In other words, there is less absorption for head-on collisions ($b = 0$) than at a nonzero b [19,21]. This certainly may influence the generated electromagnetic field but the scale of this effect is about the same as the change of the pointlike charge distribution by the ball-like charge distribution as discussed above.

(iv) A large electric conductivity σ and large chiral magnetic conductivity σ_χ might have some impact on the EM fields. However, as shown in Ref. [24] this also should have a small effect on the retarded electric and magnetic fields created in heavy-ion collisions. Anyhow, the electric conductivity is expected to be rather low in the strong QGP [25]. Nevertheless, as is shown in Ref. [26], the magnetic field—created by spectators in a collision—is not modified by the presence of matter for reasonable parameters. One should emphasize that, as demonstrated in Ref. [12], in nuclear collisions the force components of the electric and magnetic fields compensate for each other. However, in the case of external fields, as realized for example in stars or astrophysics, there is no compensation effect and electromagnetic fields may achieve rather large values.

IV. THE INVERSE LPM EFFECT

Some stronger effects on the v_1 splitting might be expected from changes in the interaction of charges with the electric and magnetic fields. It is well known that the radiation of photons by high-energy electrons passing through matter is suppressed for photons with a wavelength larger than the electron mean-free path. For such wavelengths a transition occurs from an incoherent radiation of photons in each electron interaction in matter to a coherent radiation from many interactions. This is the Landau-Pomeranchuk-Migdal (LPM) effect predicted first in Ref. [27] and described in a fully quantum-mechanical manner in Ref. [28]. In terms of nonequilibrium Green's function the LPM effect has been reconsidered in Refs. [29,30]. The radiative spectrum from a collision of a fast charged particle on a vast number of static scatterers was calculated in Ref. [31]. This effect can be interpreted as a time delay for an electron after a collision before it can fully participate in the electromagnetic interactions again [32]. In applications to hadron physics the same arguments were used first by Pomeranchuk and Feinberg in Refs. [33,34]. Later, Feinberg in Ref. [35] argued that after a hard interaction a charged particle seemingly shakes off its field and stays in a state in which its subsequent interactions differ from the normal one for some time delay until the field is reestablished. We note that the suppression of soft photon production in relativistic heavy-ion collisions also has been analyzed in Ref. [36] and the LPM effect has been parameterized in terms of the inverse interaction rate.

To provide some relation to the hadron formation time concept we mention that the LPM effect for hadrons may be considered as the decay of a virtual hadron into an on-shell hadron and photon $h^* \to h + \gamma$. Then, the coherence time for the emission of a photon with energy ω by a fast hadron with energy E_h (in the laboratory frame), as appearing in the derivation of the LPM effect,

$$\tau_{\rm LPM} = \frac{E_h^2}{(m_h c^2)^2}\frac{1}{\omega} = \frac{E_h}{m_h c^2}\frac{1}{\omega_0} = \gamma_h t_f^{(\gamma)} \quad (2)$$

can be viewed as a Lorentz-dilated formation time of the photon with the frequency ω_0, $t_f^{(\gamma)} = 1/\omega_0$, in the rest frame of the hadron. If successive collisions are separated by times shorter than τ_{LPM}, they do not increase the number of produced photons.

Similarly, the hadronic formation time can be written as $\tau_f \simeq \gamma_h t_{\rm f}^{(h)}$ where the hadron formation time in its rest frame is $t_{\rm f}^{(h)} \simeq \hbar/m_{h,T}c^3$, where $m_{h,T}$ is the transverse mass of the hadron. The formal derivation of the hadron formation time in line with the derivation of the LPM effect has been carried out in Ref. [37]. The concept of the hadronic formation time is inherent in the Lund string model [38], which incorporates a simple anzatz for the formation time $\tau_f = \hbar E_h/m_T^2 c^5$ for quark-antiquark pairs with transverse mass m_T and energy E_h

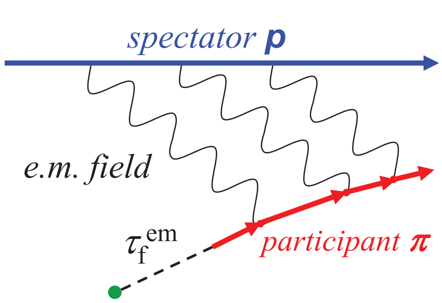

FIG. 3. Illustration of the inverse LPM effect. The dashed line displays the insensitivity to the electromagnetic field during the formation time for τ_f^{em} of a participant π; the wavy lines denote the electric field.

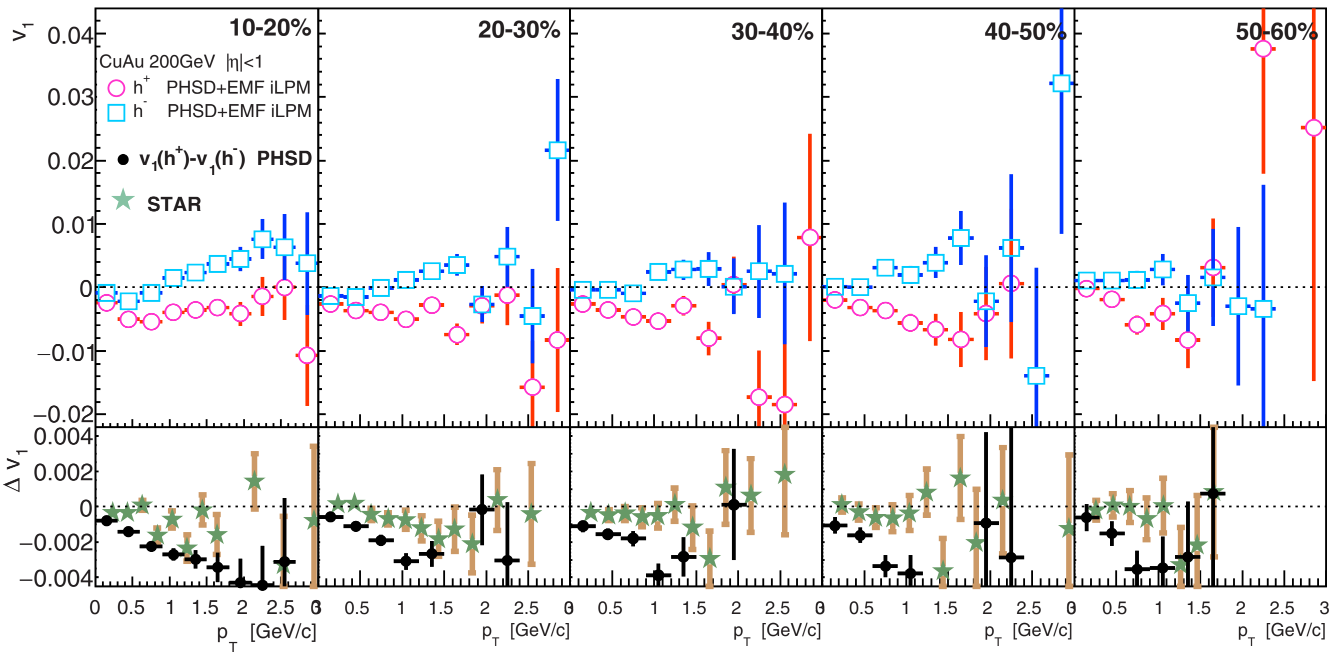

FIG. 4. Charge-dependent p_T distributions of positive and negative hadrons (top) and their difference Δv_1 (bottom) from asymmetric Cu+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and various centralities including the inverse LPM effect with $\tau_f^{em} = \tau_f/10$. The experimental data (stars) are taken from Ref. [16].

as well as for the formation of new hadrons while disregarding a formation time for leading particles (cf. the review [39] where the formation-time concept for hadrons and the physics of the LPM effect are considered on the same ground in their different applications). We recall that this formation concept is also employed in the PHSD approach with a hadronic formation time $\tau_0 \approx 0.8\,\mathrm{fm}/c$ (in the hadron rest frame), which allows for a good description of the hadron multiplicities in heavy-ion collisions in the large energy range from $\sqrt{s_{NN}} = 3$ GeV to 5 TeV [4].

V. PHSD CALCULATIONS FOR CU+AU COLLISIONS

In Ref. [12] the PHSD approach has been generalized to take into account the coupling of a moving charged particle with the generated electric and magnetic fields. The formation time concept was taken into account in the particle dynamics such that the generation of electromagnetic fields only occurs from formed particles, dominantly spectator protons.

Then, this radiation is traced in space-time towards a point where it meets a participant charged particle. This particle may be formed or not yet (the latter case is shown by the dashed line in Fig. 3). *A priori*, it is not evident how the particle will respond to the field under such conditions. In our early calculations [9] we assumed that the EMF acts in the same way on both formed and preformed charged particles, i.e., $\tau_f^{em} = 0$. This assumption leads to the maximal charge splitting shown in Fig. 2. As noted above, these calculations strongly overestimate the charge splitting of v_1 compared to the measured data.

In the opposite limiting case, when there is no influence of the EMF on a preformed propagating electric charge (shown by histograms in Fig. 2), no v_1 splitting is seen for particles with opposite electric charges. (This result was obtained on the statistical level of about 10^6 events and thus is robust.)

The suppression of the coupling of the electromagnetic field to the conserved charge of a particle in the preformed state in processes like $h^*(p) + \gamma \to h(p')$, where the preformed hadron changes its momentum from p to p' can be called as *the inverse LPM effect*. This intermediate case is presented in Fig. 4. Here it is assumed that the electric field starts to act on the preformed electric charge with a delay of $\tau_f^{em} = \tau_f/10$. We mention that by relating $\tau_f^{em} \sim \tau_f$ we do not imply that both times are due to the same physics, but only use the fact that both the times are dilated by the Lorentz γ factor. In physical terms one should consider τ_f as a natural scale in the dynamics and other time scales can be taken in units of τ_f. As seen in Fig. 4, in the latter case the charge splitting Δv_1 is in a reasonable agreement with the experimental data.[1] No free normalization factor is used here, which implies that preformed charged particles appear to sense the electromagnetic field long before being completely formed, i.e., for times $t > \tau_f/10$.

The transverse momentum (p_T) dependencies of the directed flow v_1 of pions (created in Cu+Au at $\sqrt{s_{NN}} = 200$ GeV) are shown in Figs. 5(a) and 5(c). The shape of the p_T spectra in the forward ($\eta > 0$) (c) and backward ($\eta < 0$) (a) directions are noticeably different. Without the EMF effect the $v_1(p_T)$ dependence varies between 0.5% and 1% in the absolute magnitude (solid lines in Fig. 5). The inclusion of the EMF splits the distributions, pushing the $v_1(\pi^+)$ upward and $v_1(\pi^-)$ downward with respect to the case without EMF. The

[1] In different publications the directions of the bombarding Au or Cu nuclei are inverted.

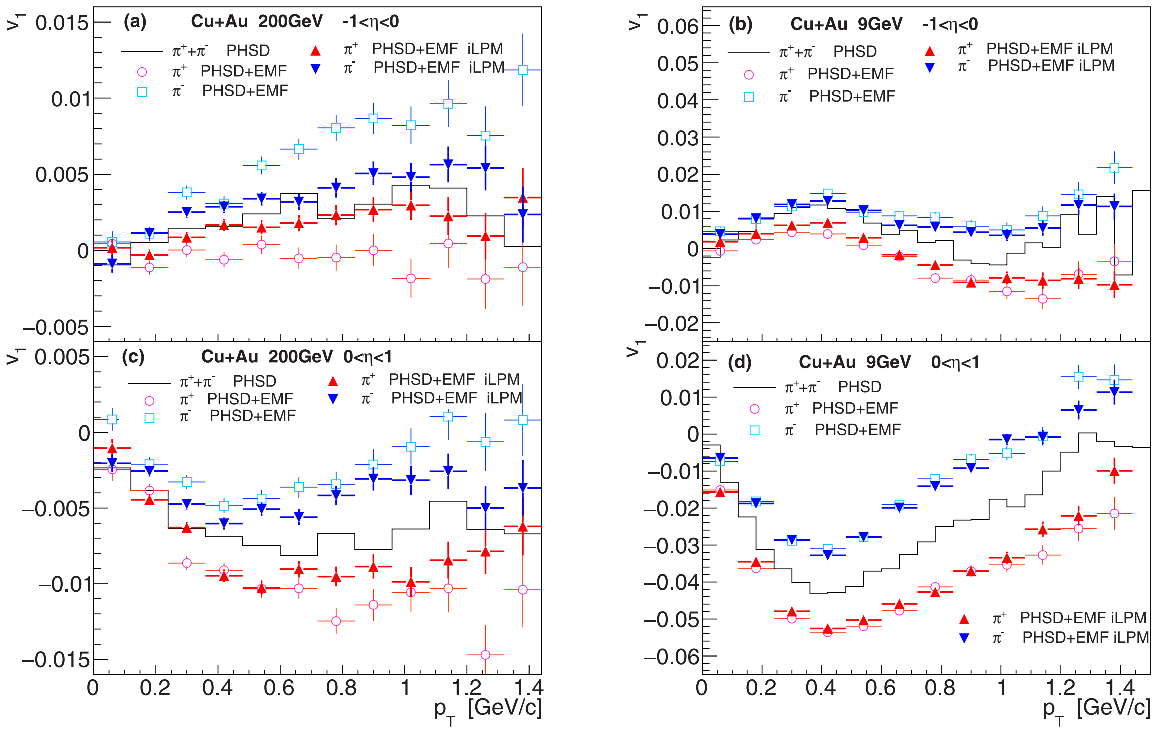

FIG. 5. Charge-dependent transverse momentum p_T dependence of the directed flow for pions from asymmetric Cu+Au collisions at $\sqrt{s_{NN}} = 200$ GeV (left) and $\sqrt{s_{NN}} = 9$ GeV (right). The backward and forward emitted pions are plotted in panels (a) and (c) for 200 GeV and panels (b) and (d) for 9 GeV, respectively. The inverse LPM effect is taken into account with $\tau_f^{em} = (1/10)\tau_f$ and shown by full symbols. Other parameters employed and the notation are as in Fig. 2.

charge splitting Δv_1 becomes larger with increasing transverse momentum p_T. We note that an additional implementation of the iLPM effect at the top RHIC energy strongly suppresses the directed flow in the backward direction but only moderately influences the forward component.

We now consider the NICA energy range where the particle creation occurs at a high baryon density or a large baryonic chemical potential μ_B. The maximal average energy density reached in a central cylinder with radius $R = 2$ fm and length $|z| < 2.5/\gamma$ fm (where $\gamma \approx \sqrt{s_{NN}}/2m_N$ is the Lorentz factor of the colliding nuclei) is about 1.6 GeV/fm^3 for a collision at $\sqrt{s_{NN}} = 9$ GeV, which implies that a sizable volume gets converted to partonic degrees of freedom during the collision. In addition to μ_B, the electric charge chemical potential μ_e is also important since we are interested in hadrons with opposite electric charges.

The transverse momentum (p_T) dependence of the directed flow v_1 of pions (created in Cu+Au at $\sqrt{s_{NN}} = 9$ GeV) is shown for forward [Fig. 5(d)] and backward [Fig. 5(b)] rapidities. As in case of collisions at the top RHIC energy the created EM field produces an essential charge splitting of $v_1(p_T)$; however, an extra inclusion of the iLPM practically does not effect the $v_1(p_T)$ distributions due to a much longer passage time of the nuclei and the low delay time τ_f^{em} due to the iLPM effect. Note also that the magnitude of the directed flow at 9 GeV is much higher than at 200 GeV, which opens up interesting perspectives for lower beam energies (at FAIR/NICA).

VI. SUMMARY

From the present study we conclude that the experimental observation of a charge-dependent splitting of pseudorapidity and transverse momentum distributions in the directed flow v_1 provides experimental evidence for the early creation of strong electromagnetic fields in relativistic heavy-ion collisions. When accounting for the inverse LPM (iLPM) effect the coupling of unformed charged partons becomes slightly delayed and suppresses the charge splitting Δv_1 in asymmetric nuclear collisions. The decoherence time ($\tau_f^{em} \sim \tau_f/10$) allows us to reconcile the PHSD results with the preliminary experimental observations at the top RHIC energy by the STAR Collaboration. We predict that the inverse LPM effect should practically disappear at energies of $\sqrt{s_{NN}} \approx 9$ GeV, which leads to a significantly larger charge splitting of v_1 at energies in the BESII program at RHIC and at the future FAIR and NICA facilities.

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