Elliptic flow as a probe for the *ψ***(2***S***) production mechanism in relativistic heavy ion collisions**

Baoyi Chen*

Department of Physics, Tianjin University, Tianjin 300352, China (Received 15 August 2016; revised manuscript received 10 December 2016; published 20 March 2017)

I discuss the elliptic flows of $\psi(2S)$ with different production mechanisms in $\sqrt{s_{NN}} = 2.76$ TeV Pb-Pb collisions. If the final $\psi(2S)$ s are mainly from the recombination of uncorrelated charm and anticharm quarks at $T \approx T_c$, charm and anticharm quarks will carry large collective flows of the bulk medium, which will be inherited by the regenerated $\psi(2S)$ s. This indicates a larger elliptic flow of $\psi(2S)$ than that of J/ψ which can be regenerated at $T \ge T_c$, $v_2^{\psi(2S)} > v_2^{J/\psi}$. However, if the final $\psi(2S)$ s are mainly from the transitions of *J/ψ* → *ψ*(2*S*) caused by the color screening of quark-gluon plasma its elliptic flow should be close to the elliptic flow of J/ψ , $v_2^{\psi(2S)} \sim v_2^{J/\psi}$. Therefore, $\psi(2S)$ elliptic flow is a sensitive probe for its production mechanisms in relativistic heavy ion collisions.

DOI: [10.1103/PhysRevC.95.034908](https://doi.org/10.1103/PhysRevC.95.034908)

I. INTRODUCTION

A J/ψ consists of a charm and an anticharm quark with a large binding energy. Its abnormal suppression by a deconfined matter has been considered as a signal of the existence of the quark-gluon plasma (QGP) produced in heavy ion collisions [\[1\]](#page-4-0). Charmonium can be dissociated by the color screening $[2-4]$ and the inelastic scatterings $[5-10]$ with partons in QGP. Also, the final yields of charmonium can be enhanced by the recombination of a charm and an anticharm quark during the evolution of QGP $[11–17]$. This mechanism is called the "regeneration." It even dominates the total yield of *J/ψ* at the available colliding energies of the Large Hadron Collider (LHC) [\[12,17\]](#page-4-0). Cold nuclear matter effects, such as the shadowing effect $[18-20]$ and Cronin effect $[8,21-23]$, can also change the spatial and momentum distributions of the primordial charmonium produced in nucleus-nucleus collisions. Different theoretical models [\[8,11,13,24–29\]](#page-4-0) have been built to explain the experimental data of the nuclear modification factor R_{AA} , the mean transverse momentum squared $\langle p_T^2 \rangle$, and the elliptic flow v_2 of J/ψ .

Recently, some experimental data of $\psi(2S)$ have been published. Different from the ground state J/ψ , $\psi(2S)$ is a loosely bound state with a small binding energy. Its dissociation temperature is close to the critical temperature of the hadronization transition, $T_d(\psi(2S)) \approx T_c$ [\[2\]](#page-4-0), which means *ψ*(2*S*) eigenstate can barely survive in QGP. The CMS Collaboration published the data of prompt $\frac{R_{AA}[\psi(2S)]}{R_{AA}(J/\psi)}$ in $\sqrt{s_{NN}}$ = 2.76 TeV Pb-Pb collisions [\[30\]](#page-4-0). Different mechanisms have been proposed for the $\psi(2S)$ prompt and inclusive yields [\[31–33\]](#page-4-0). These mechanisms include the primordial production at the nucleus colliding time, the recombination of a charm and an anticharm quark (or D and \bar{D} mesons) in the later stage of the hot medium evolution and decays from *B* hadrons. Recent studies indicate that the formation time of charmonium eigenstates can be delayed by the hot medium in heavy ion collisions [\[34\]](#page-4-0). $\psi(2S)$ may suffer less suppression if they are formed later in the gradually cooling

QGP. With the formation process, a *cc*¯ dipole produced in the nucleus-nucleus collisions may exist as a combination of different eigenstates. The internal evolution of the $c\bar{c}$ dipole wave function is affected by the hot medium. This changes the fractions of J/ψ and $\psi(2S)$ in the $c\bar{c}$ dipole and the double ratio $\frac{R_{AA}(1/\psi(2S))}{R_{AA}(J/\psi)}$ [\[33\]](#page-4-0). On the other hand, J/ψ and $\psi(2S)$ can be regenerated at different stage of QGP evolution, and they will carry different collective flows of the bulk medium. The elliptic flows v_2 of $\psi(2S)$ from coalescence at $T \approx T_c$ and transitions of $J/\psi \rightarrow \psi(2S)$ should be different from each other. Elliptic flow should be a sensitive probe to distinguish which production mechanism dominates the $\psi(2S)$ final yield.

The article is organized as follows. In Sec. II, I introduce the Langevin equation for the charm quark evolution and the hydrodynamic equations for the QGP evolution. In Sec. [III,](#page-1-0) different mechanisms of the *ψ*(2*S*) production are discussed in detail. In Sec. [IV,](#page-2-0) I fit the parameters in the Langevin equation to explain the experimental data of *D* mesons, and then give the elliptic flows of charmonium. Section V is devoted to the summary.

II. DYNAMICS OF HEAVY QUARKS IN HEAVY ION COLLISIONS

In this work, I focus on the charmonium regeneration in heavy ion collisions. They are mainly from the recombination of charm and anticharm quarks in the low transverse momentum bin, where multi-elastic scatterings dominate the energy loss of charm quarks [\[35–39\]](#page-4-0), and the medium-induced gluon radiation [\[40–42\]](#page-4-0) is less important. In the limit of small momentum transfer, multi-quasi-elastic scatterings of heavy quarks in QGP can be treated as a Brownian motion and is usually described by the Langevin equation [\[43–45\]](#page-4-0)

$$
\frac{d\vec{p}}{dt} = -\eta_D(p)\vec{p} + \vec{\xi}.
$$
 (1)

The first term on the right-hand side is the drag force with the momentum dependence. The second term is the random force. Assuming *ξ* is independent of the momentum of each particle, this noise term satisfies the correlation relation

$$
\langle \xi^i(t)\xi^j(t')\rangle = \kappa \delta^{ij}\delta(t-t'),\tag{2}
$$

*baoyi.chen@tju.edu.cn

κ represents the momentum space diffusion coefficient of heavy quarks. The fluctuation-dissipation relation indicates [\[39,44\]](#page-4-0)

$$
\eta_D(p) = \frac{\kappa}{2TE},\tag{3}
$$

where *T* is the temperature of fluid cells in QGP, *E* is the energy of charm quarks. The spatial diffusion coefficient *D* of heavy quarks is connected with the momentum space diffusion coefficient by

$$
D = \frac{2T^2}{\kappa}.\tag{4}
$$

I follow Ref. [\[39\]](#page-4-0) and take $D = C/(2\pi T)$. The value of the parameter*C* can be fixed by the experimental data of *D* mesons in $\sqrt{s_{NN}}$ = 2.76 TeV Pb-Pb collisions.

For numerical evolutions, the Langevin equation can be discretized as [\[44,45\]](#page-4-0)

$$
\vec{p}(t + \Delta t) = \vec{p}(t) - \eta_D(p)\vec{p}\Delta t + \hat{\xi}\Delta t,
$$

$$
\langle \xi^i(t)\xi^j(t - n\Delta t) \rangle = \frac{\kappa}{\Delta t} \delta^{ij} \delta^{0n}.
$$
 (5)

Here Δt is the time step of the numerical evolution. The noise term in Eq. (5) is taken to be the Gaussian distribution with the width $\sqrt{\frac{\kappa}{\Delta t}}$.

At the time of nucleus collisions, charm pairs are produced from the parton fusions with a large momentum transfer. The number of $c\bar{c}$ pairs is proportional to the number of binary collisions. Without cold nuclear matter effects, the spatial distribution of charm quarks is proportional to the function

$$
\frac{dN_{\rm PbPb}}{d\vec{x}_T} \propto \frac{T_{\rm Pb}(\vec{x}_T - \vec{b}/2)T_{\rm Pb}(\vec{x}_T + \vec{b}/2)}{T_{\rm Pb}(0)T_{\rm Pb}(0)}.
$$
(6)

Here $T_{\text{Pb}}(\vec{x}_T) = \int dz \rho_{\text{Pb}}(\vec{x}_T, z)$ is the thickness function of Pb. $\rho_{\text{Pb}}(\vec{x}_T, z)$ is the nucleon density, which is taken to be the Woods-Saxon distribution. The denominator is for a normalization. For the cold nuclear matter effects such as the shadowing effect, I employ the EPS09s LO results $r_i^A(x, Q^2, \vec{x}_T)$ which already include the spatial dependence in a nucleus [\[46\]](#page-4-0). Here, $x = \frac{m_T}{\sqrt{s_{NN}}}$ exp($\pm y$) and $Q^2 = m_T^2$ [\[47,48\]](#page-4-0). *y* and $m_T = \sqrt{m_{c\bar{c}}^2 + p_T^2}$ are the rapidity and the transverse energy, respectively. \vec{x}_T is the transverse coordinate. The momentum distribution of charm quarks can be generated by PYTHIA. The shadowing effect is included by multiplying charm p_T spectra from PYTHIA by the shadowing factor $r_i^A(x, Q^2, \vec{x}_T)$.

The QGP evolutions in heavy ion collisions can be described with $(2 + 1)$ -dimensional ideal hydrodynamics

$$
\partial_{\mu}T^{\mu\nu} = 0,\tag{7}
$$

where $T^{\mu\nu} = (e + p)u^{\mu}u^{\nu} - g^{\mu\nu}p$ is the energy-momentum tensor, u^{μ} is the velocity of fluid cells, and *e* and *p* are the energy density and the pressure. For the equation of state of the medium, the deconfined phase is an ideal gas of massless *u* and *d* quarks, 150-MeV massed *s* quarks and gluons [\[49\]](#page-4-0). The hadron phase is an ideal gas of all known hadrons and resonances with mass up to 2 GeV [\[50\]](#page-4-0). With the charged multiplicity at the midrapidity $dN_{ch}/dy = 1600$ [\[51,52\]](#page-4-0), the maximum temperature of QGP at the initial time $\tau_0^{\text{QGP}} =$

0.6 fm/c is initialized to be 484 MeV [\[52\]](#page-4-0). τ_0^{QGP} is the time of the medium reaching local equilibrium.

After charm quarks are generated in the spatial and the momentum space with cold nuclear matter effects, their evolutions in QGP can be simulated by the Langevin equation Eq. [\(1\)](#page-0-0). After the evolutions, one can obtain the nuclear modification factor $R_{AA}(p_T)$ and the elliptic flow $v_2(p_T)$ of *D* mesons.

III. DIFFERENT MECHANISMS OF THE *ψ***(2***S***) PRODUCTION**

The dissociation temperature of $\psi(2S)$ eigenstate is close to the critical temperature of deconfined phase transition. Sequential regeneration model indicates that the final prompt *ψ*(2*S*)s are mainly from the recombination of uncorrelated charm and anticharm quarks at the hadronization (and *D* and *D* in hadron phase) [\[32\]](#page-4-0). To calculate J/ψ and $\psi(2S)$ regeneration, I employ the Langevin equation for charm quark evolutions in QGP and coalescence model for their recombination at a certain temperature. The Wigner function for charm quark recombination is taken as a Gaussian function

$$
f(r,q) = A_0 \exp(-r^2/\sigma^2) \exp(-q^2 \sigma^2),
$$
 (8)

*A*⁰ is the normalization factor for $\int f(r,q)r^2 dr q^2 dq = 1$. The Gaussian width is related to the mean-square-radius by $\sigma^2 = 8 \langle r_{\Psi}^2 \rangle / 3$ [\[53\]](#page-4-0). For a charm and an anticharm quarks with a relative distance r and relative momentum q , they have a probability $P(r,q) = r^2q^2f(r,q)$ to recombine into a charmonium bound state. I employ the Monte Carlo method to simulate the coalescence process. If the probability $P(r,q)$ is larger than a random number between 0 and 1, then these charm and anticharm quarks can recombine into a new charmonium. Considering the regenerated charmonium are mainly from the uncorrelated charm pairs, charm and anticharm quarks are generated in nucleus collisions in uncorrelated initial coordinates $(\vec{x}_{c0}, \vec{q}_{c0})$ and $(\vec{x}_{\bar{c}0}, \vec{q}_{\bar{c}0})$. As this work focus on the effect of QGP collective flows on charmonium production, I neglect the the difference between Wigner functions of different charmonium eigenstates, and take $\langle r_{\Psi}^2 \rangle = 0.5^2$ fm² from potential model. The additional hot medium suppression on regenerated charmonium is also neglected.

The prompt yield of *ψ*(2*S*) may also come from correlated $c\bar{c}$ pairs. Correlated *c* and \bar{c} are produced with a small separation in the spatial space, and need some time to evolve into a certain charmonium eigenstate [\[28,34\]](#page-4-0). The dipole with a small size is not likely to be dissociated at the early stage of QGP, which can enhance the final production of J/ψ and/or $\psi(2S)$. The color screening on heavy quark potential affects the internal evolutions of $c\bar{c}$ dipoles, which corresponds to the transitions between different eigenstates. Employing the time-dependent Schrödinger equation for the $c\bar{c}$ dipole internal evolutions in deconfined matter, one can evolve the wave function of $c\bar{c}$ dipoles, and obtain the fractions of charmonium eigenstates by projecting the $c\bar{c}$ dipole wave function to a certain eigenstate. The heavy quark potential at finite temperature is taken to be the free energy *F* from Lattice results [\[54\]](#page-4-0). The initial wave function is taken as a Gaussian function, and the Gaussian width is fitted to satisfy the ratio of direct J/ψ and $\psi(2S)$ yields in proton-proton collisions. In

FIG. 1. The time evolution of J/ψ and ψ (2*S*) fractions in a $c\bar{c}$ dipole in the static medium with a constant temperature $T = 1.5T_c$. The initial wave function of $c\bar{c}$ dipole is taken as a Gaussian function with the width $\sigma_0^{c\bar{c}} = 0.23$ fm. The heavy quark potential is taken to be the free energy $V = F$ from Lattice results.

Fig. 1, both fractions of J/ψ and $\psi(2S)$ and the ratio of their yields in the $c\bar{c}$ dipole changes with time.

Both of the above mechanisms contribute to the $\psi(2S)$ prompt production. It would be interesting to find an observable which can distinguish the different production mechanisms of $\psi(2S)$. Here, I propose the elliptic flow v_2 as a probe for the $\psi(2S)$ production. For the final prompt $\psi(2S)$, if most of them are from the regeneration, they should be produced at the later stage of the QGP evolution. The elliptic flow of $\psi(2S)$ will be much larger than the elliptic flow of J/ψ , see Fig. [4](#page-3-0) in Sec. IV. However, if most of the prompt $\psi(2S)$ are from the correlated $c\bar{c}$ dipoles with the formation process, then the elliptic flow of $\psi(2S)v_2^{\psi(2S)}(p_T)$ should be similar to $v_2^{J/\psi}(p_T)$. The detailed discussions are given in Sec. IV.

IV. OBSERVABLES OF THE CHARM FLAVOR

The evolutions of heavy quarks in the hot medium can be described by the Langevin equation. Different drag coefficients are employed in different models [\[55\]](#page-4-0). I fit the experimental data of *D* mesons with different values of the parameter *C*, see Figs. 2 to 3. At the critical temperature T_c , the deconfined matter is transformed into the hadron gas. Charm quarks are transformed to *D* mesons with coalescence and fragmentation [\[45,56,57\]](#page-4-0). The process of hadronization can shift the $v_2(p_T)$ by about a 20–25% upward [\[58\]](#page-4-0). Both collective flows of the bulk medium and *D* mesons are mainly developed in the deconfined phase. In this work, my intent is to employ a reasonable drag coefficient inspired by the experimental data of *D* mesons, and show the big difference between *ψ*(2*S*) elliptic flows with different production mechanisms. Therefore, I neglect the process of charm quarks transforming to *D* mesons and the evolutions of *D* mesons in hadron gas. These simplifications should not change the conclusions about the elliptic flows of charmonium in Secs. IV and [V.](#page-3-0)

FIG. 2. The nuclear modification factor R_{AA} of D mesons as a function of the transverse momentum p_T in $\sqrt{s_{NN}} = 2.76$ TeV Pb-Pb collisions with different diffusion coefficients. The thick and thin solid lines correspond to the situations of $C = 4.0$ and $C = 1.0$, respectively. The experimental data is from the ALICE Collaboration [\[59\]](#page-4-0).

When heavy quarks move in the QGP, they lose energy and carry collective flows of the bulk medium. It seems difficult to explain $R_{AA}(p_T)$ and $v_2(p_T)$ of *D* mesons at the same time at $\sqrt{s_{NN}}$ = 2.76 TeV Pb-Pb collisions [\[55\]](#page-4-0). With smaller value of the parameter*C*, heavy quarks are easier to be thermalized in the QGP. This will result in a stronger suppression of $R_{AA}(p_T)$ in the high p_T bin (thin line in Fig. 2) and a stronger elliptic flow of charm quarks (dotted line in Fig. 3). In Fig. 3, lines and data points are for the charm quark and *D* meson elliptic flows, respectively. Considering the additional hadronization process and the hadron phase effects will shift the lines upward, the value of $C = 2$ is employed for the prediction of $\psi(2S)$ elliptic flows in Fig. [4.](#page-3-0)

FIG. 3. The elliptic flow of *D* mesons as a function of the transverse momentum p_T in $\sqrt{s_{NN}} = 2.76$ TeV Pb-Pb collisions with different diffusion coefficients. The solid, dashed, and dotted lines correspond to the situation of $C = 4,2,1$, respectively. The experimental data are from the ALICE Collaboration [\[60\]](#page-4-0).

FIG. 4. The elliptic flow of $\Psi = [J/\psi, \psi(2S)]$ as a function of the transverse momentum p_T in $\sqrt{s_{NN}} = 2.76$ TeV Pb-Pb collisions. The solid-circle, solid-square, and hollow-square lines correspond to the situations that Ψ are regenerated from the recombination of charm and anticharm quarks at $T_{\Psi} = (1.5, 1.2, 1.0)T_c$, respectively. Data are from the ALICE Collaboration [\[62\]](#page-4-0).

The final prompt charmonium consists of three parts: primordial production at the nucleus colliding time, the regeneration from the recombination of *c* and \bar{c} (or *D* and \bar{D}) during the evolution of the hot medium, and the transitions from other charmonium eigenstates. With a realistic description of charm quark evolution, one can obtain the distributions of the regenerated charmonium. Compared to J/ψ , the regeneration of $\psi(2S)$ can only happen at the later stage of the QGP evolution due to its small binding energy. At that time, the collective flows of QGP are stronger. Therefore, the elliptic flow of $\psi(2S)$ should be much larger than the elliptic flow of *J/ψ* which can be regenerated in a relatively earlier time of the QGP evolution.

After charmonium is produced, their elliptic flows are almost not changed anymore. (for example, the elliptic flow of the primordially produced J/ψ is close to zero) [\[61\]](#page-4-0). As the binding energy of J/ψ is large, they can be regenerated at $T \geq T_c$. In Fig. 4, let us assume that a certain eigenstate Ψ is regenerated at $(1.5, 1.2, 1.0)T_c$, respectively. Its elliptic flow can be obtained (see solid-circle, solid-square, and hollowsquare lines in Fig. 4). The dissociation temperature of J/ψ is around $T_d^{J/\psi} = (1.5 - 2.0)T_c$ [\[2\]](#page-4-0). Therefore the regeneration

of *J/* ψ happens during $T_c \le T^{\text{QGP}} < T_d^{J/\psi}$. The elliptic flow of the situation $T_{\Psi} = 1.2T_c$ is close to the experimental data of inclusive J/ψ [\[62\]](#page-4-0). For the final prompt $\psi(2S)$ s, if they are from the transitions of J/ψ , $v_2^{\psi(2S)}(p_T)$ should be close to the dashed line. If the final prompt *ψ*(2*S*)s are mainly from the regeneration, they should be regenerated at $T^{QGP} \approx T_c$. And the elliptic flow of $\psi(2S)$ should be close to the dotted line. Different mechanisms result in very different elliptic flows of $\psi(2S)$, which makes $v_2^{\psi(2S)}(p_T)$ a sensitive probe for the *ψ*(2*S*) production mechanism.

The elliptic flows in Fig. 4 only include the regenerated charmonium. After including the primordially produced charmonium, the lines at $p_T > 3$ GeV/c will be shifted downward a little and approach zero at very high p_T bin. But it does not change the relation between three lines in Fig. 4. With different forms of the drag coefficient, as long as the regeneration dominates the final yield, the elliptic flow of $\psi(2S)$ should be larger than that of J/ψ . In the other situation, they should be similar to each other. If the drag coefficient is larger at a lower temperature (see the parametrization in Ref. [\[55\]](#page-4-0)), the difference between elliptic flows of regenerated J/ψ and $\psi(2S)$ will be even larger. In a more realistic situation, charmonium should be regenerated in a temperature region, not at a certain temperature T_{Ψ} . Different choices of heavy quark potential at finite temperature also affect the regeneration process. Both of these effects can be approximated by employing different values of T_{Ψ} in Fig. 4. These will be treated more seriously in the future works.

V. CONCLUSION

In summary, I employ the Langevin equation to describe the charm quark evolutions and Wigner function for charmonium regeneration in QGP. Different production mechanisms are discussed for the $\psi(2S)$ prompt production in $\sqrt{s_{NN}} = 2.76$ TeV Pb-Pb collisions. The elliptic flow of $\psi(2S)$ is proposed as a sensitive probe to distinguish the different production mechanisms. If the final prompt $\psi(2S)$ are mainly from the *correlated cc*̄ dipoles, the elliptic flow of $\psi(2S)$ $v_2^{\psi(2S)}(p_T)$ should be close to $v_2^{J/\psi}(p_T)$. The prompt $\psi(2S)$ may also come from the recombination of *uncorrelated c* and \bar{c} , which happens at the later stage of the QGP evolution. In this situation, charm quarks carry large collective flows, which will be inherited by the regenerated $\psi(2S)$ s. Therefore the elliptic flow of $\psi(2S)$ is much larger than that of J/ψ . The relation between $v_2^{\psi(2S)}(p_T)$ and $v_2^{J/\psi}(p_T)$ is sensitive to the production mechanisms of $\psi(2S)$. With different drag coefficients in the Langevin equation, the conclusions about the relations between J/ψ and ψ (2*S*) elliptic flows do not change. This makes the elliptic flow a sensitive and robust probe for the *ψ*(2*S*) production mechanism.

ACKNOWLEDGMENTS

The author thanks Dr. Y. Liu, M. He, P. Zhuang and R. Rapp for helpful discussions, and J. Zhao for proof reading the manuscript. The work is supported by the NSFC under Grant No. 11547043.

- [1] T. Matsui and H. Satz, [Phys. Lett. B](https://doi.org/10.1016/0370-2693(86)91404-8) **[178](https://doi.org/10.1016/0370-2693(86)91404-8)**, [416](https://doi.org/10.1016/0370-2693(86)91404-8) [\(1986\)](https://doi.org/10.1016/0370-2693(86)91404-8).
- [2] H. Satz, [J. Phys. G](https://doi.org/10.1088/0954-3899/32/3/R01) **[32](https://doi.org/10.1088/0954-3899/32/3/R01)**, [R25](https://doi.org/10.1088/0954-3899/32/3/R01) [\(2006\)](https://doi.org/10.1088/0954-3899/32/3/R01).
- [3] X. Zhao and R. Rapp, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.82.064905) **[82](https://doi.org/10.1103/PhysRevC.82.064905)**, [064905](https://doi.org/10.1103/PhysRevC.82.064905) [\(2010\)](https://doi.org/10.1103/PhysRevC.82.064905).
- [4] B. Chen, T. Guo, Y. Liu, and P. Zhuang, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2016.12.021) **[765](https://doi.org/10.1016/j.physletb.2016.12.021)**, [323](https://doi.org/10.1016/j.physletb.2016.12.021) [\(2017\)](https://doi.org/10.1016/j.physletb.2016.12.021).
- [5] M. E. Peskin, [Nucl. Phys. B](https://doi.org/10.1016/0550-3213(79)90199-8) **[156](https://doi.org/10.1016/0550-3213(79)90199-8)**, [365](https://doi.org/10.1016/0550-3213(79)90199-8) [\(1979\)](https://doi.org/10.1016/0550-3213(79)90199-8).
- [6] G. Bhanot and M. E. Peskin, [Nucl. Phys. B](https://doi.org/10.1016/0550-3213(79)90200-1) **[156](https://doi.org/10.1016/0550-3213(79)90200-1)**, [391](https://doi.org/10.1016/0550-3213(79)90200-1) [\(1979\)](https://doi.org/10.1016/0550-3213(79)90200-1).
- [7] L. Grandchamp and R. Rapp, [Phys. Lett. B](https://doi.org/10.1016/S0370-2693(01)01311-9) **[523](https://doi.org/10.1016/S0370-2693(01)01311-9)**, [60](https://doi.org/10.1016/S0370-2693(01)01311-9) [\(2001\)](https://doi.org/10.1016/S0370-2693(01)01311-9).
- [8] X. L. Zhu, P. F. Zhuang, and N. Xu, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2004.12.023) **[607](https://doi.org/10.1016/j.physletb.2004.12.023)**, [107](https://doi.org/10.1016/j.physletb.2004.12.023) [\(2005\)](https://doi.org/10.1016/j.physletb.2004.12.023).
- [9] R. L. Thews and M. L. Mangano, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.73.014904) **[73](https://doi.org/10.1103/PhysRevC.73.014904)**, [014904](https://doi.org/10.1103/PhysRevC.73.014904) [\(2006\)](https://doi.org/10.1103/PhysRevC.73.014904).
- [10] B. Chen, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.93.044917) **[93](https://doi.org/10.1103/PhysRevC.93.044917)**, [044917](https://doi.org/10.1103/PhysRevC.93.044917) [\(2016\)](https://doi.org/10.1103/PhysRevC.93.044917).
- [11] R. L. Thews, M. Schroedter, and J. Rafelski, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.63.054905) **[63](https://doi.org/10.1103/PhysRevC.63.054905)**, [054905](https://doi.org/10.1103/PhysRevC.63.054905) [\(2001\)](https://doi.org/10.1103/PhysRevC.63.054905).
- [12] X. Zhao and R. Rapp, [Nucl. Phys. A](https://doi.org/10.1016/j.nuclphysa.2011.05.001) **[859](https://doi.org/10.1016/j.nuclphysa.2011.05.001)**, [114](https://doi.org/10.1016/j.nuclphysa.2011.05.001) [\(2011\)](https://doi.org/10.1016/j.nuclphysa.2011.05.001).
- [13] A. Andronic, P. Braun-Munzinger, K. Redlich, and J. Stachel, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2003.07.066) **[571](https://doi.org/10.1016/j.physletb.2003.07.066)**, [36](https://doi.org/10.1016/j.physletb.2003.07.066) [\(2003\)](https://doi.org/10.1016/j.physletb.2003.07.066).
- [14] A. Andronic, P. Braun-Munzinger, K. Redlich, and J. Stachel, [Nucl. Phys. A](https://doi.org/10.1016/j.nuclphysa.2007.02.013) **[789](https://doi.org/10.1016/j.nuclphysa.2007.02.013)**, [334](https://doi.org/10.1016/j.nuclphysa.2007.02.013) [\(2007\)](https://doi.org/10.1016/j.nuclphysa.2007.02.013).
- [15] L. Yan, P. Zhuang, and N. Xu, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.97.232301) **[97](https://doi.org/10.1103/PhysRevLett.97.232301)**, [232301](https://doi.org/10.1103/PhysRevLett.97.232301) [\(2006\)](https://doi.org/10.1103/PhysRevLett.97.232301).
- [16] B. Chen, K. Zhou, and P. Zhuang, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.86.034906) **[86](https://doi.org/10.1103/PhysRevC.86.034906)**, [034906](https://doi.org/10.1103/PhysRevC.86.034906) [\(2012\)](https://doi.org/10.1103/PhysRevC.86.034906).
- [17] B. Chen, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.93.054905) **[93](https://doi.org/10.1103/PhysRevC.93.054905)**, [054905](https://doi.org/10.1103/PhysRevC.93.054905) [\(2016\)](https://doi.org/10.1103/PhysRevC.93.054905).
- [18] K. J. Eskola, V. J. Kolhinen, and C. A. Salgado, [Eur. Phys. J. C](https://doi.org/10.1007/s100529900005) **[9](https://doi.org/10.1007/s100529900005)**, [61](https://doi.org/10.1007/s100529900005) [\(1999\)](https://doi.org/10.1007/s100529900005).
- [19] D. de Florian and R. Sassot, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.69.074028) **[69](https://doi.org/10.1103/PhysRevD.69.074028)**, [074028](https://doi.org/10.1103/PhysRevD.69.074028) [\(2004\)](https://doi.org/10.1103/PhysRevD.69.074028).
- [20] [K. J. Eskola, H. Paukkunen, and C. A. Salgado,](https://doi.org/10.1088/1126-6708/2009/04/065) J. High Energy Phys. [04](https://doi.org/10.1088/1126-6708/2009/04/065) [\(2009\)](https://doi.org/10.1088/1126-6708/2009/04/065) [065.](https://doi.org/10.1088/1126-6708/2009/04/065)
- [21] X. Zhao and R. Rapp, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2008.03.068) **[664](https://doi.org/10.1016/j.physletb.2008.03.068)**, [253](https://doi.org/10.1016/j.physletb.2008.03.068) [\(2008\)](https://doi.org/10.1016/j.physletb.2008.03.068).
- [22] M. C. Abreu *et al.* (NA50 Collaboration), [Phys. Lett. B](https://doi.org/10.1016/S0370-2693(01)00019-3) **[499](https://doi.org/10.1016/S0370-2693(01)00019-3)**, [85](https://doi.org/10.1016/S0370-2693(01)00019-3) [\(2001\)](https://doi.org/10.1016/S0370-2693(01)00019-3).
- [23] A. Adare *et al.* (PHENIX Collaboration), [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.98.232002) **[98](https://doi.org/10.1103/PhysRevLett.98.232002)**, [232002](https://doi.org/10.1103/PhysRevLett.98.232002) [\(2007\)](https://doi.org/10.1103/PhysRevLett.98.232002).
- [24] L. Grandchamp and R. Rapp, [Nucl. Phys. A](https://doi.org/10.1016/S0375-9474(02)01027-8) **[709](https://doi.org/10.1016/S0375-9474(02)01027-8)**, [415](https://doi.org/10.1016/S0375-9474(02)01027-8) [\(2002\)](https://doi.org/10.1016/S0375-9474(02)01027-8).
- [25] [Y. Park, K. I. Kim, T. Song, S. H. Lee, and C. Y. Wong,](https://doi.org/10.1103/PhysRevC.76.044907) Phys. Rev. C **[76](https://doi.org/10.1103/PhysRevC.76.044907)**, [044907](https://doi.org/10.1103/PhysRevC.76.044907) [\(2007\)](https://doi.org/10.1103/PhysRevC.76.044907).
- [26] T. Song, W. Park, and S. H. Lee, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.81.034914) **[81](https://doi.org/10.1103/PhysRevC.81.034914)**, [034914](https://doi.org/10.1103/PhysRevC.81.034914) [\(2010\)](https://doi.org/10.1103/PhysRevC.81.034914).
- [27] T. Song, W. Park, and S. H. Lee, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.84.054903) **[84](https://doi.org/10.1103/PhysRevC.84.054903)**, [054903](https://doi.org/10.1103/PhysRevC.84.054903) [\(2011\)](https://doi.org/10.1103/PhysRevC.84.054903).
- [28] B. Z. Kopeliovich, I. K. Potashnikova, I. Schmidt, and M. Siddikov, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.91.024911) **[91](https://doi.org/10.1103/PhysRevC.91.024911)**, [024911](https://doi.org/10.1103/PhysRevC.91.024911) [\(2015\)](https://doi.org/10.1103/PhysRevC.91.024911).
- [29] S. Cho, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.91.054914) **[91](https://doi.org/10.1103/PhysRevC.91.054914)**, [054914](https://doi.org/10.1103/PhysRevC.91.054914) [\(2015\)](https://doi.org/10.1103/PhysRevC.91.054914).
- [30] V. Khachatryan *et al.* (CMS Collaboration), [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.113.262301) **[113](https://doi.org/10.1103/PhysRevLett.113.262301)**, [262301](https://doi.org/10.1103/PhysRevLett.113.262301) [\(2014\)](https://doi.org/10.1103/PhysRevLett.113.262301).
- [31] B. Chen, Y. Liu, K. Zhou, and P. Zhuang, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2013.09.036) **[726](https://doi.org/10.1016/j.physletb.2013.09.036)**, [725](https://doi.org/10.1016/j.physletb.2013.09.036) [\(2013\)](https://doi.org/10.1016/j.physletb.2013.09.036).
- [32] X. Du and R. Rapp, [Nucl. Phys. A](https://doi.org/10.1016/j.nuclphysa.2015.09.006) **[943](https://doi.org/10.1016/j.nuclphysa.2015.09.006)**, [147](https://doi.org/10.1016/j.nuclphysa.2015.09.006) [\(2015\)](https://doi.org/10.1016/j.nuclphysa.2015.09.006).
- [33] B. Chen, X. Du, and R. Rapp, $arXiv:1612.02089$.
- [34] T. Song, C. M. Ko, and S. H. Lee, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.91.044909) **[91](https://doi.org/10.1103/PhysRevC.91.044909)**, [044909](https://doi.org/10.1103/PhysRevC.91.044909) [\(2015\)](https://doi.org/10.1103/PhysRevC.91.044909).
- [35] E. Braaten and M. H. Thoma, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.44.R2625) **[44](https://doi.org/10.1103/PhysRevD.44.R2625)**, [R2625](https://doi.org/10.1103/PhysRevD.44.R2625) [\(1991\)](https://doi.org/10.1103/PhysRevD.44.R2625).
- [36] H. van Hees, V. Greco, and R. Rapp, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.73.034913) **[73](https://doi.org/10.1103/PhysRevC.73.034913)**, [034913](https://doi.org/10.1103/PhysRevC.73.034913) [\(2006\)](https://doi.org/10.1103/PhysRevC.73.034913).
- [37] G. Y. Qin, J. Ruppert, C. Gale, S. Jeon, G. D. Moore, and M. G. Mustafa, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.100.072301) **[100](https://doi.org/10.1103/PhysRevLett.100.072301)**, [072301](https://doi.org/10.1103/PhysRevLett.100.072301) [\(2008\)](https://doi.org/10.1103/PhysRevLett.100.072301).
- [38] S. Cao, G. Y. Qin, S. A. Bass, and B. Mller, [Nucl. Phys. A](https://doi.org/10.1016/j.nuclphysa.2013.02.100) **[904-905](https://doi.org/10.1016/j.nuclphysa.2013.02.100)**, [653c](https://doi.org/10.1016/j.nuclphysa.2013.02.100) [\(2013\)](https://doi.org/10.1016/j.nuclphysa.2013.02.100).
- [39] S. Cao, G. Y. Qin, and S. A. Bass, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.92.054909) **[92](https://doi.org/10.1103/PhysRevC.92.054909)**, [054909](https://doi.org/10.1103/PhysRevC.92.054909) [\(2015\)](https://doi.org/10.1103/PhysRevC.92.054909).
- [40] X. N. Wang and M. Gyulassy, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.68.1480) **[68](https://doi.org/10.1103/PhysRevLett.68.1480)**, [1480](https://doi.org/10.1103/PhysRevLett.68.1480) [\(1992\)](https://doi.org/10.1103/PhysRevLett.68.1480).
- [41] M. Gyulassy and X. N. Wang, [Nucl. Phys. B](https://doi.org/10.1016/0550-3213(94)90079-5) **[420](https://doi.org/10.1016/0550-3213(94)90079-5)**, [583](https://doi.org/10.1016/0550-3213(94)90079-5) [\(1994\)](https://doi.org/10.1016/0550-3213(94)90079-5).
- [42] R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne, and D. Schiff, [Nucl. Phys. B](https://doi.org/10.1016/S0550-3213(96)00553-6) **[483](https://doi.org/10.1016/S0550-3213(96)00553-6)**, [291](https://doi.org/10.1016/S0550-3213(96)00553-6) [\(1997\)](https://doi.org/10.1016/S0550-3213(96)00553-6).
- [43] M. He, R. J. Fries, and R. Rapp, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.86.014903) **[86](https://doi.org/10.1103/PhysRevC.86.014903)**, [014903](https://doi.org/10.1103/PhysRevC.86.014903) [\(2012\)](https://doi.org/10.1103/PhysRevC.86.014903).
- [44] S. Cao, G. Y. Qin, and S. A. Bass, [J. Phys. G](https://doi.org/10.1088/0954-3899/40/8/085103) **[40](https://doi.org/10.1088/0954-3899/40/8/085103)**, [085103](https://doi.org/10.1088/0954-3899/40/8/085103) [\(2013\)](https://doi.org/10.1088/0954-3899/40/8/085103).
- [45] S. Cao, G. Y. Qin, and S. A. Bass, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.88.044907) **[88](https://doi.org/10.1103/PhysRevC.88.044907)**, [044907](https://doi.org/10.1103/PhysRevC.88.044907) [\(2013\)](https://doi.org/10.1103/PhysRevC.88.044907).
- [46] I. Helenius, K. J. Eskola, H. Honkanen, and C. A. Salgado, [J. High Energy Phys. 07](https://doi.org/10.1007/JHEP07(2012)073) [\(2012\)](https://doi.org/10.1007/JHEP07(2012)073) [073.](https://doi.org/10.1007/JHEP07(2012)073)
- [47] R. Vogt, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.71.054902) **[71](https://doi.org/10.1103/PhysRevC.71.054902)**, [054902](https://doi.org/10.1103/PhysRevC.71.054902) [\(2005\)](https://doi.org/10.1103/PhysRevC.71.054902).
- [48] R. Vogt, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.81.044903) **[81](https://doi.org/10.1103/PhysRevC.81.044903)**, [044903](https://doi.org/10.1103/PhysRevC.81.044903) [\(2010\)](https://doi.org/10.1103/PhysRevC.81.044903).
- [49] J. Sollfrank, P. Huovinen, M. Kataja, P. V. Ruuskanen, M. Prakash, and R. Venugopalan, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.55.392) **[55](https://doi.org/10.1103/PhysRevC.55.392)**, [392](https://doi.org/10.1103/PhysRevC.55.392) [\(1997\)](https://doi.org/10.1103/PhysRevC.55.392).
- [50] K. Hagiwara et al. [\(Particle Data Group Collaboration\),](https://doi.org/10.1103/PhysRevD.66.010001) *Phys.* Rev. D **[66](https://doi.org/10.1103/PhysRevD.66.010001)**, [010001](https://doi.org/10.1103/PhysRevD.66.010001) [\(2002\)](https://doi.org/10.1103/PhysRevD.66.010001).
- [51] K. Gulbrandsen (ALICE Collaboration),[J. Phys. Conf. Ser.](https://doi.org/10.1088/1742-6596/446/1/012027) **[446](https://doi.org/10.1088/1742-6596/446/1/012027)**, [012027](https://doi.org/10.1088/1742-6596/446/1/012027) [\(2013\)](https://doi.org/10.1088/1742-6596/446/1/012027).
- [52] C. Shen and U. Heinz, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.85.054902) **[85](https://doi.org/10.1103/PhysRevC.85.054902)**, [054902](https://doi.org/10.1103/PhysRevC.85.054902) [\(2012\)](https://doi.org/10.1103/PhysRevC.85.054902); **[86](https://doi.org/10.1103/PhysRevC.86.049903)**, [049903\(E\)](https://doi.org/10.1103/PhysRevC.86.049903) [\(2012\)](https://doi.org/10.1103/PhysRevC.86.049903).
- [53] V. Greco, C. M. Ko, and R. Rapp, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2004.06.064) **[595](https://doi.org/10.1016/j.physletb.2004.06.064)**, [202](https://doi.org/10.1016/j.physletb.2004.06.064) [\(2004\)](https://doi.org/10.1016/j.physletb.2004.06.064).
- [54] S. Digal, O. Kaczmarek, F. Karsch, and H. Satz, [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s2005-02309-7) **[43](https://doi.org/10.1140/epjc/s2005-02309-7)**, [71](https://doi.org/10.1140/epjc/s2005-02309-7) [\(2005\)](https://doi.org/10.1140/epjc/s2005-02309-7).
- [55] S. K. Das, F. Scardina, S. Plumari, and V. Greco, *[Phys. Lett. B](https://doi.org/10.1016/j.physletb.2015.06.003)* **[747](https://doi.org/10.1016/j.physletb.2015.06.003)**, [260](https://doi.org/10.1016/j.physletb.2015.06.003) [\(2015\)](https://doi.org/10.1016/j.physletb.2015.06.003).
- [56] L. Ravagli and R. Rapp, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2007.07.043) **[655](https://doi.org/10.1016/j.physletb.2007.07.043)**, [126](https://doi.org/10.1016/j.physletb.2007.07.043) [\(2007\)](https://doi.org/10.1016/j.physletb.2007.07.043).
- [57] M. He, R. J. Fries, and R. Rapp, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.82.034907) **[82](https://doi.org/10.1103/PhysRevC.82.034907)**, [034907](https://doi.org/10.1103/PhysRevC.82.034907) [\(2010\)](https://doi.org/10.1103/PhysRevC.82.034907).
- [58] R. J. Fries, M. He, and R. Rapp, [J. Phys. G](https://doi.org/10.1088/0954-3899/38/12/124068) **[38](https://doi.org/10.1088/0954-3899/38/12/124068)**, [124068](https://doi.org/10.1088/0954-3899/38/12/124068) [\(2011\)](https://doi.org/10.1088/0954-3899/38/12/124068).
- [59] B. Abelev *et al.* (ALICE Collaboration), [J. High Energy Phys.](https://doi.org/10.1007/JHEP09(2012)112) [09](https://doi.org/10.1007/JHEP09(2012)112) [\(2012\)](https://doi.org/10.1007/JHEP09(2012)112) [112.](https://doi.org/10.1007/JHEP09(2012)112)
- [60] B. Abelev *et al.* (ALICE Collaboration), [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.111.102301) **[111](https://doi.org/10.1103/PhysRevLett.111.102301)**, [102301](https://doi.org/10.1103/PhysRevLett.111.102301) [\(2013\)](https://doi.org/10.1103/PhysRevLett.111.102301).
- [61] K. Zhou, N. Xu, Z. Xu and P. Zhuang, [Phys. Rev. C](https://doi.org/10.1103/PhysRevC.89.054911) **[89](https://doi.org/10.1103/PhysRevC.89.054911)**, [054911](https://doi.org/10.1103/PhysRevC.89.054911) [\(2014\)](https://doi.org/10.1103/PhysRevC.89.054911).
- [62] H. Yang (ALICE Collaboration), [Nucl. Phys. A](https://doi.org/10.1016/j.nuclphysa.2013.02.105) **[904-905](https://doi.org/10.1016/j.nuclphysa.2013.02.105)**, [673c](https://doi.org/10.1016/j.nuclphysa.2013.02.105) [\(2013\)](https://doi.org/10.1016/j.nuclphysa.2013.02.105).