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Return of backbending in ^{169}Tm and the effect of the $N = 98$ deformed shell gap

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The high spin excited states of ^{169}Tm have been studied via the ^{169}Tm(^{32}S, ^{32}S$'$)^{169}Tm* reaction at a beam energy of 164 MeV. The γ rays were detected using an array of 19 Compton-suppressed clover HPGe detectors. The band based on the $[411]1/2^+$ Nilsson orbital has been extended to observe the band crossing for the first time in this nucleus. A sharp backbending, similar to ^{165}Tm, is observed in this nucleus which is in complete contrast with its immediate neighbor ^{167}Tm. The data were interpreted in the cranked shell model approach by calculating the total Routhian surfaces, crossing frequencies, and the interaction strengths for these nuclei which are in excellent agreement with the experimental data. It is suggested that the $N = 98$ deformed shell gap has larger effect in determining the nature of alignment than its shape driving effect.

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The high spin structures of odd-A Tm ($Z = 69$) nuclei show interesting features. As they are situated in the deformed rare-earth region, these nuclei are well deformed. The lighter nuclei are reported to be triaxial and the heavier ones ($A > 163$) are axially deformed [1–11]. For the heavier Tm isotopes near the stability line, the neutron Fermi level lies near the deformed shell gap at the neutron number $N = 98$ which may play important role in determining the shape of the heavier isotopes. However, rotational bands observed in ^{157}Tm [1] indicate that the deformation in Tm isotopes set in even at neutron number far away from $N = 98$. For the heavier odd-A Tm isotopes around $N = 98$, band structures built on different single-particle proton orbitals have been observed [7–11]. However, the band crossing and the band interaction phenomena for these isotopes are not well studied due to lack of data above the band crossing except for a few of those isotopes. The band crossing in Tm nuclei are due to the alignment of a pair of neutrons and it depends on the shape of the nucleus as well as on the position of the neutron Fermi level. The band crossings, of the $\pi[411]1/2^+$ bands, due to the $i_{13/2}$ neutron pair alignment reported for ^{165}Tm and ^{167}Tm show contrasting behavior. In case of ^{165}Tm, a sharp back bending and in ^{167}Tm a smooth or gradual up-bending are reported [7,8]. However, the gain in alignment in these two isotopes are very similar, indicating that the neutron pair alignments are taking place at the same orbitals. A systematic investigation of the effect of the neutron Fermi level on the alignment process could not be extended to the heavier axially deformed isotopes as the high spin data on ^{169}Tm [9,10] and heavier isotopes are limited below the band crossing region. In the present work, we have extended the level scheme of ^{169}Tm to observe, for the first time, the band crossing in its $\pi[411]1/2^+$ band to shed some light on this issue.

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The excited states in ^{169}Tm were produced by the reaction ^{169}Tm(^{32}S, ^{32}S$'$)^{169}Tm*. The 164-MeV ^{32}S beam, from the 14-UD BARC-TIFR Pelletron, Mumbai, was bombarded on a self-supporting thick (8 mg/cm^2) foil of ^{169}Tm. The γ rays were detected using 19 Compton-suppressed clover HPGe detectors of INGA (Indian National Gamma Array) which were arranged at six different angles with three clovers each at $\pm 40^\circ$, $\pm 65^\circ$, and -23° while four clovers were at 90°. For energy and relative efficiency calibration, ^{133}Ba and ^{152}Eu radioactive sources were used. A digital data acquisition (DDAQ) system, based on Pixie-16 modules [12,13], was used for the data collection with a sampling rate of 100 MHz. Time-stamped γ-γ coincidence data were collected when at least two clover detectors were fired. Data sorting was done by Multi-pARameter time-stamped based COincidence Search (MARCOS) code, developed at TIFR, Mumbai. The γ-γ matrix and γ-γ-γ cube were formed and analyzed by RADWARE software packages [14]. A time window of 500 ns was chosen for the above matrix and cube formation.

The γ-γ coincidence and intensity relations of the γ rays were used for constructing the level scheme of ^{169}Tm. The intensity of each γ ray has been obtained by putting a single gate on a strong transition from the matrix. Double gated spectra with gates put on several known γ rays in ^{169}Tm are shown in Fig. 1. All the known γ rays in the two signature partners of the $1/2^+$ band in ^{169}Tm [10] could be observed along with a few new γ lines. A new and improved level scheme of ^{169}Tm is proposed and is presented in Fig. 2. In this level scheme the γ lines observed for the first time in this work are marked by an asterisk (*). The placement of the new lines in the level scheme is based on the coincidence relations as depicted in Fig. 1. It can be seen that all the new lines are clearly observed in Fig. 1(b) which is for $\alpha = -1/2$ signature partner ($3/2^+$, $7/2^+$, $11/2^+, \ldots$ sequence), where as the 359-, 431-, and 437-keV γ rays are not observed in Fig. 1(a) which is for the $\alpha = +1/2$ signature partner ($1/2^+$, $5/2^+$, $9/2^+, \ldots$ sequence). This is in accordance with their

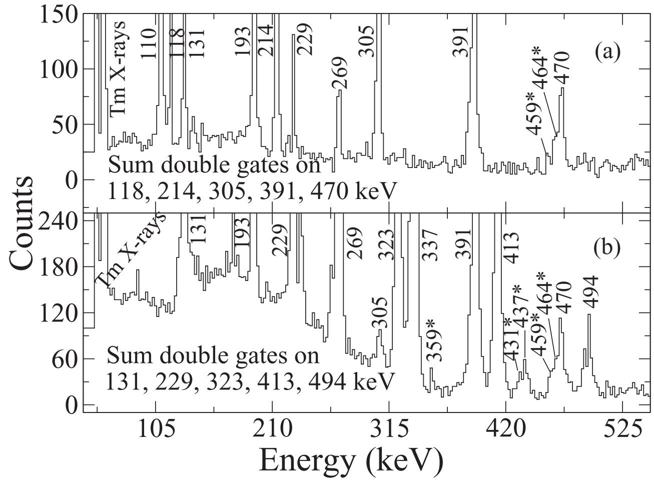

FIG. 1. γ-ray spectra of ^{169}Tm obtained by sum of double gates. The new γ peaks are marked by asterisks (*).

placement in the level scheme. No γ rays above the 2421-keV or 2466-keV states are observed possibly because of the fact that almost the limit of high spin that can be excited by the inelastic excitation, adopted in this work, has been reached.

The spin and parity of the levels in ^{169}Tm were known from the previous work [10] using γ-ray angular correlation studies. In this work, we have obtained directional correlation from oriented states (DCO) ratio and integrated polarization directional correlation (IPDCO) ratio of the γ transitions from our data in order to assign spin and parity of the states. For DCO ratio measurement, an asymmetry γ-γ matrix was formed from the coincidence events in the -23° and the 90° detectors. The IPDCO ratios were obtained following the prescription of Refs. [15,16]. Data from the 90° detectors only were used to measure the perpendicular ($N_\perp$) and the parallel ($N_\parallel$) scattered (with respect to the reaction plane) counts of

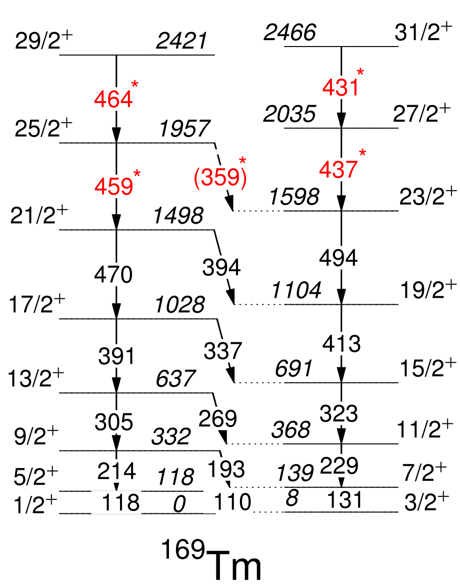

FIG. 2. Proposed level scheme of ^{169}Tm in the present work. The new γ rays are marked by asterisks (*).

a γ transition to obtain IPDCO ratio (Δ_{IPDCO}). Positive or negative values of Δ_{IPDCO} indicate that a transition is electric or magnetic type, respectively. However, no disagreement has been found, in the present work, to the adopted spin-parities of the levels assigned from the earlier work.

Table I shows the intensity, DCO ratio, IPDCO ratio and the other parameters of all the γ rays observed in the present work which are placed in the proposed level scheme in Fig. 2. In the present work, the level scheme has been extended up to excitation energy of $\sim$2.5 MeV and spin of $31/2\,\hbar$.

The band shown in Fig. 2 was known to be $\pi[411]1/2^+$ band in ^{169}Tm [10]. This band has been compared in the three isotopes ^{165}Tm, ^{167}Tm, and ^{169}Tm. The aligned angular momenta (i_x) for this band are plotted as a function of rotational frequency $\hbar\omega$ in Fig. 3. It can be seen that with the placement of the new γ rays in the present work, the particle (neutron) alignment could be well observed in ^{169}Tm for the first time. The alignment of ^{169}Tm can thus be compared with the other isotopes.

It can be seen in Fig. 3 that a sharp back-bending is observed in ^{169}Tm, similar to ^{165}Tm and in contrast to the smooth up-bending in its immediate neighbor ^{167}Tm. It can also be seen from this plot that the alignment frequency ($\hbar\omega_c$) decreases with the increase in neutron number in these nuclei. The band crossing phenomenon in a nucleus is interpreted as the crossing of two bands, the ground state (gs) band and the s band. The s band is resulted from the alignment of a pair of particles, which is by breaking of a neutron pair in this case. The alignment or crossing frequency depends on the orbitals involved in the alignment and on the deformation of the nucleus. The gain in alignment (Δi_x) can be clearly estimated in ^{165}Tm to be about $10\,\hbar$ and in ^{167,169}Tm it seems to approach this value, suggesting that the neutron pair alignments are taking place at $i_{13/2}$ orbital for all these three isotopes.

The nature of the band crossing depends on the interaction strength between the gs and the s bands. For smaller interaction strength, sharp back-bending occurs where as gradual alignment indicates a larger interaction strength. It is, therefore, evident that, in the case of Tm isotopes around the stability line, the interaction strength between the one- and the three-quasiparticle bands is larger in case of $N = 98$ nucleus ^{167}Tm and decreases on either side of neutron number in ^{165}Tm and ^{169}Tm.

In order to understand the band crossing frequency and the difference in the nature of band crossings of the $[411]1/2^+$ band in the ^{165,167,169}Tm isotopes, theoretical calculations were carried out using the principal axis cranking model. In these calculations, the quasiparticle energies have been calculated using deformed Woods-Saxon potential and BCS pairing. Coriolis term ωj_x has been introduced to include the cranking, where ω is the rotational frequency of the nucleus and j_x is the projection of the single particle total angular momentum on the rotation axis. The calculated single particle Routhians as a function of rotational frequency gives the idea about the alignment frequency as well as the interaction strengths between the gs and the s bands.

The total Routhian surface (TRS) calculations were also performed using the above model for the nuclei in this region, using the Strutinsky shell correction method to get the

TABLE I. Energy (E_γ) and intensity (I_γ) of the γ rays along with the other relevant quantities concerning the levels in ^{169}Tm as measured in the present work. The uncertainties in E_γ are 0.3 keV and 0.6 keV for $I_\gamma \geqslant 100$ and otherwise, respectively.

E_γ(keV)	E_i(keV)	$J_i^\pi \rightarrow J_f^\pi$	I_γ [a]	R_{DCO}	Δ_{IPDCO}	Deduced multipolarity
109.6	117.8	$5/2^+ \rightarrow 3/2^+$	1000.0(9)	0.86(1) [b]		$M1+E2$
117.8	117.8	$5/2^+ \rightarrow 1/2^+$	101.29(30)	0.94(5) [b]		$E2$
130.6	138.5	$7/2^+ \rightarrow 3/2^+$	726.48(75)	0.95(3) [b]		$E2$
192.9	331.1	$9/2^+ \rightarrow 7/2^+$	614.85(69)	0.88(1) [c]	−0.026(1)	$M1+E2$
213.6	331.1	$9/2^+ \rightarrow 5/2^+$	255.49(45)	0.96(1) [b]	0.046(3)	$E2$
228.6	366.9	$11/2^+ \rightarrow 7/2^+$	543.59(65)	0.99(1) [c]	0.038(1)	$E2$
269.4	636.0	$13/2^+ \rightarrow 11/2^+$	250.09(43)	0.64(1) [c]	−0.034(1)	$M1+E2$
304.9	636.0	$13/2^+ \rightarrow 9/2^+$	208.79(41)	1.06(1) [c]	0.089(2)	$E2$
322.9	689.8	$15/2^+ \rightarrow 11/2^+$	168.71(37)	1.01(1) [c]	0.081(2)	$E2$
336.8	1026.6	$17/2^+ \rightarrow 15/2^+$	45.53(21)	0.45(2) [c]	−0.037(1)	$M1+E2$
359.0	1956.0	$25/2^+ \rightarrow 23/2^+$				$(M1+E2)$
390.7	1026.6	$17/2^+ \rightarrow 13/2^+$	94.89(29)	1.02(2) [b]	0.093(3)	$E2$
393.7	1496.2	$21/2^+ \rightarrow 19/2^+$	11.23(12)	0.47(3) [d]	−0.136(15)	$M1+E2$
412.7	1102.5	$19/2^+ \rightarrow 15/2^+$	48.94(19)	1.02(5) [c]	0.098(3)	$E2$
431.4	2465.5	$31/2^+ \rightarrow 27/2^+$	0.18(2)			$(E2)$
436.9	2033.9	$27/2^+ \rightarrow 23/2^+$	0.53(1)			$(E2)$
459.4	1957.0	$25/2^+ \rightarrow 21/2^+$	0.30(1)			$(E2)$
464.2	2421.2	$29/2^+ \rightarrow 25/2^+$	0.47(1)			$(E2)$
469.9	1496.5	$21/2^+ \rightarrow 17/2^+$	21.77(19)	1.01(5) [b]	0.059(6)	$E2$
494.5	1597.0	$23/2^+ \rightarrow 19/2^+$	1.15(21)			$(E2)$

[a] Relative γ-ray intensities are estimated from prompt spectra and normalized to 1000 for the total intensity of 109.6 keV γ ray.
[b] From 304.9 keV ($E2$) DCO gate.
[c] From 130.6 keV ($E2$) DCO gate.
[d] From 322.9 keV ($E2$) DCO gate.

quantitative idea about the shape of a nucleus in a particular configuration. The TRS code of Nazarewicz *et al.* [17,18] was used for the calculations. The procedure has been outlined in Refs. [19,20]. The TRSs were calculated in the β_2-γ deformation mesh points, with minimization in the hexadecapole deformation β_4, at different rotational frequencies $\hbar\omega$, where β_2 and γ are the quadrupole deformation parameters. We have used the convention $\gamma = 0^\circ$ and $\gamma = -60^\circ$ to correspond to prolate and oblate shape, respectively.

The contour plots of the TRS calculations for ^{165}Tm, ^{167}Tm, and ^{169}Tm isotopes in the $[411]1/2^+$ configuration are shown in Figs. 4(a), 4(b) and 4(c), respectively. These surfaces were calculated at $\hbar\omega = 0.2$ MeV which is just below the crossing frequencies. As seen in Fig. 4, the minimum of the TRS comes out to be at prolate shape with $\gamma \sim 0^\circ$ for all the three isotopes. The calculated deformation parameters are presented in Table II which shows a small increase of the quadrupole deformation β_2 with neutron number in these isotopes and there is a relatively large variation in the values of β_4.

The deformed shapes are realized in these nuclei as the proton Fermi surface lies in the midshell region and also there would be deformation driving effect of the deformed shell gap at neutron number $N = 98$. However, the observation of rotational bands in all the lighter Tm isotopes up to ^{157}Tm [1] indicates that the deformed shape persists even up to $N = 88$ [1]. Deformed shape is also obtained in our TRS calculations for the negative parity band in lighter Tm isotopes up to ^{155}Tm. It suggests that the structure in Tm nuclei, away from $N = 98$, are still deformed mostly because the proton Fermi level lies in the midshell.

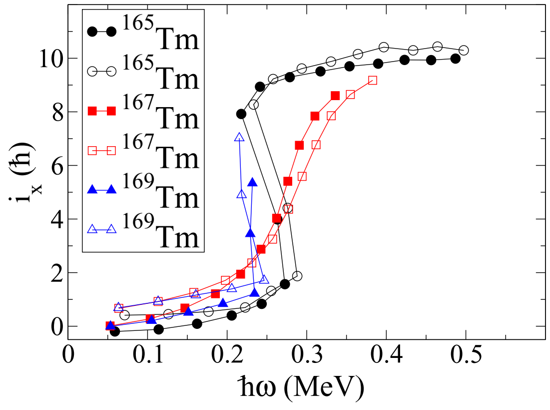

FIG. 3. Quasiparticle aligned angular momenta (i_x) as a function of rotational frequency ($\hbar\omega$) for odd-A Tm isotopes. Haris parameters $J_0 = 35\,\hbar^2\,\text{MeV}^{-1}$ and $J_1 = 43\,\hbar^4\,\text{MeV}^{-3}$ are taken.

TABLE II. Deformation parameters β_2, β_4, and γ obtained from TRS calculations for the $[411]1/2^+$ band in Tm isotopes.

A	β_2	β_4	γ
165	0.274	0.035	0.5
167	0.284	0.005	1.1
169	0.292	−0.095	0.8

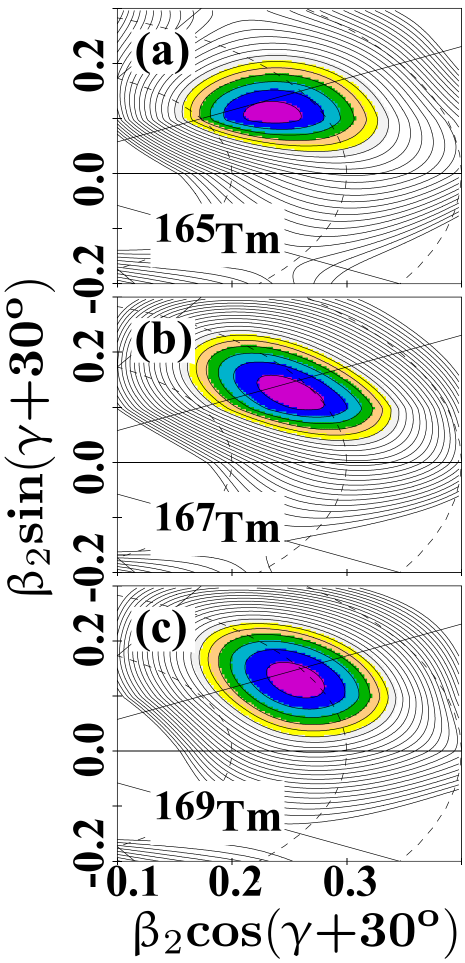

FIG. 4. TRS calculations of ^{165}Tm (a), ^{167}Tm (b), and ^{169}Tm (c) nuclei at $\hbar\omega = 0.2$ MeV. The contours are 250 keV apart.

The single particle Routhians are also obtained in this work from the cranked shell model (CSM) calculation. In case of the $\pi[411]1/2^+$ band in Tm isotopes the first band crossing is due to the alignment of a pair of neutrons in the $i_{13/2}$ orbital. The calculated quasiparticle energies for neutron levels for $N = 96, 98$, and 100 corresponding to ^{165}Tm, ^{167}Tm, and ^{169}Tm nuclei are shown in Figs. 5(a), 5(b), and 5(c), respectively, as a function of rotational frequency $\hbar\omega$. In these

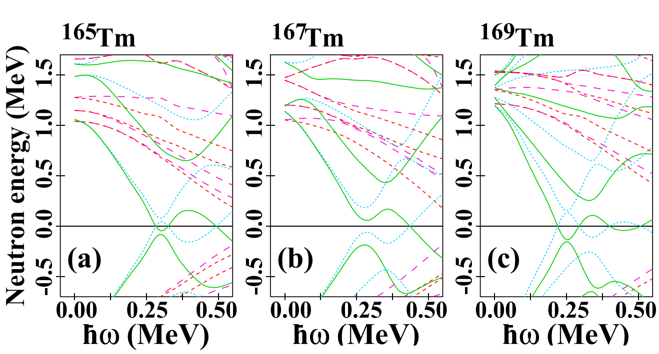

FIG. 5. Calculated quasineutron energy levels for $N = 96, 98$, and 100 corresponding to ^{165}Tm (a), ^{167}Tm (b), and ^{169}Tm (c) nuclei.

TABLE III. Experimental and calculated crossing frequencies (ω_c) and calculated interaction strengths (V) at the band crossings for the $[411]1/2^+$ band in Tm isotopes.

A	ω_c(exp) (MeV)	ω_c(cal) (MeV)	V (keV)
165	0.29	0.30	20
167	~ 0.27	0.28	125
169	0.24	0.25	10

calculations, the deformation parameters obtained from the minimum of TRS calculations as shown in Table II are taken. In this model, the crossing frequency, $\omega_c(cal)$, corresponds to the value of rotational frequency at which the Routhians from above and below the Fermi level come close together and the interaction strength at the crossing corresponds to half the distance of closest approach of the Routhians. The calculated values of ω_c(cal) and the interaction strength (V) are presented in Table III along with the experimental values of crossing frequencies, ω_c(exp), for the three isotopes. It can be seen from Table III that the calculated crossing frequencies are in excellent agreement with the measured ones.

The differences in the nature of the alignments as depicted in Fig. 3 can also be understood from the CSM calculations. A much larger interaction strength ($V = 125$ keV) is obtained for ^{167}Tm compared to the other two isotopes (see Table III) which clearly supports the observed smooth up-bend in this $N = 98$ isotope in contrast to the back-bend in ^{165}Tm and ^{169}Tm. A larger interaction strength is realized for a larger energy gap in the single particle diagram. As mentioned earlier, in the Nilsson diagram, there is a gap at $N = 98$ for deformation $\epsilon \sim 0.27$. It is, therefore, suggested that the smooth up-bend in ^{167}Tm may be because of this deformed gap. This is also supported by the observed smooth up-bend in the neighboring $N = 98$ isotones of even-even ^{166}Er [21] and odd-A ^{169}Lu [22].

The calculations are extended for ^{171}Tm ($N = 102$) and an interaction strength $V = 75$ keV was obtained, which is smaller than in ^{167}Tm. This is, possibly, due to a smaller deformed shell gap at $N = 102$ than the one at $N = 98$. This indicates that an alignment, similar to ^{167}Tm, with less smooth up-bend is expected in ^{171}Tm. Experimental data, however, are not available yet for this nucleus to test this prediction.

To summarize, the excited states in ^{169}Tm have been investigated by the reaction ^{169}Tm(^{32}S, ^{32}S$'$)^{169}Tm* at the beam energy of 164 MeV and using Indian National Gamma Array (INGA) with 19 clover HPGe detectors. The level scheme of ^{169}Tm has been extended to observe the neutron pair alignment in this nucleus for the first time in $\pi[411]1/2^+$ band and the same has been compared with those in its neighboring isotopes around the $N = 98$ deformed shell gap. Apart from a slightly different crossing frequency, a sharp back-bending, similar to ^{165}Tm, has been observed in ^{169}Tm which is in sharp contrast to its immediate neighbor ^{167}Tm in which smooth up-bending has been observed for the same configuration. The crossing frequencies and the band crossing behavior have been interpreted in the cranked shell model approach in which the single particle Routhians and the total Routhian surfaces were calculated. The observed crossing frequencies in the three isotopes could be very well reproduced by the calculations with similar deformation β_2 but different values of β_4 obtained from the TRS calculations. The difference in the alignment behavior of ^{167}Tm compared to ^{165,169}Tm is well understood from the large interaction strength between the gs and the s bands, calculated for $N = 98$. The calculations predict that if one moves towards $N = 82$ spherical shell closure, the deformed shape persists for neutron number as low as $N = 86$ in ^{155}Tm. Therefore, it may be concluded that the shapes of the Tm nuclei are mostly determined by the proton Fermi level and has only limited effect of the $N = 98$ deformed shell gap. However, this deformed shell gap significantly affects the nature of the alignment of the nuclei in the $A \sim 170$ region. It is also predicted that the interaction strength for ^{171}Tm is less than that in ^{167}Tm but still large enough to show up-bending due to neutron alignment.

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