

# Neutrino-nucleon scattering in supernova matter from the virial expansion

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We extend our virial approach to study the neutral-current neutrino response of nuclear matter at low densities. In the long-wavelength limit, the virial expansion makes model-independent predictions for neutrino-nucleon scattering rates and the density  $S_V$  and spin  $S_A$  responses. We find that  $S_A$  is significantly reduced from one even at low densities. We provide a simple fit  $S_A^f(n, T, Y_p)$  of the axial response as a function of density  $n$ , temperature  $T$ , and proton fraction  $Y_p$ , which can be incorporated into supernova simulations in a straightforward manner. This fit reproduces our virial results at low densities and the Burrows and Sawyer random-phase approximation (RPA) model calculations at high densities. Preliminary one-dimensional supernova simulations suggest that the virial reduction in the axial response may enhance neutrino heating rates in the gain region during the accretion phase of a core-collapse supernovae.

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## I. INTRODUCTION

Neutrinos radiate 99% of the energy and play a crucial role in core-collapse supernovae [1–3]. The scattering of neutrinos and their transport of energy to the shock region are sensitive to the physics of low-density nucleonic matter, which is a complex problem due to bound nuclei and the strong correlations induced by nuclear forces. A recent three-dimensional supernova simulation was sensitive to modest changes in neutral-current neutrino-nucleon interactions and exploded when strange-quark contributions were included [4]. However, these strange-quark contributions were probably taken to be unrealistically large [5]. In this paper, we explore if similar reductions in neutral-current interactions can arise, not from strange-quark contributions, but from correlations in low-density nucleonic matter. The physics of neutrino-matter interactions is a broad and active field, where many interesting studies of neutrino-matter interactions have been performed recently [6–20].

For low densities and high temperatures, the virial expansion provides a model-independent approach. In previous works, we have presented the virial equation of state of low-density nuclear matter [21] and of pure neutron matter [22]. In particular, the virial expansion can be used to describe matter in thermal equilibrium around the neutrinosphere in supernovae. The temperature of the neutrinosphere is roughly  $T \sim 4$  MeV from about 20 neutrinos detected in SN1987a [23,24] and the mass density is  $\rho \sim 10^{11} - 10^{12}$  g/cm<sup>3</sup>. For pure neutron matter, the virial expansion in terms of the fugacity  $z = e^{\mu/T}$  is valid for

$$\rho = \frac{2m}{\lambda^3} z + O(z^2) \lesssim 4 \times 10^{11} (T/\text{MeV})^{3/2} \text{g/cm}^3, \quad (1)$$

where  $m$  is the nucleon mass and  $\lambda = (2\pi/mT)^{1/2}$  denotes the thermal wavelength. A conservative validity range of the virial equation of state is given by  $z < 1/2$ , which gives the limiting density in Eq. (1). Therefore, the virial approach is applicable for the conditions of the neutrinosphere. Following our virial equation of state, we have generalized the approach to study spin-polarized neutron matter and the consistent neutrino response of neutron matter at low densities [25].

In this paper, we use the virial expansion to describe how neutrinos interact with low-density nuclear matter composed of protons and neutrons. We neglect alpha particles and other light nuclei [26,27]. These will be included in later work. In Sec. II, we present our formalism. Our results for the axial response and preliminary one-dimensional supernova simulations are presented in Sec. III. Finally, we conclude in Sec. IV.

## II. NEUTRINO RESPONSE

In this section, we use the virial expansion to describe how neutrinos interact with low-density nuclear matter. We focus on neutral-current neutrino interactions. We expect similar results for charged-current reactions; however, we leave these to later work. We calculate the neutrino cross section per unit volume. The virial expansion provides model-independent results in the limit of low momentum transfer  $q \rightarrow 0$ .

The free cross section for neutrino-nucleon neutral-current scattering is

$$\frac{d\sigma_0}{d\Omega_{vN}} = \frac{G_F^2 E_\nu^2}{4\pi^2} [C_{a,N}^2 (3 - \cos\theta) + C_{v,N}^2 (1 + \cos\theta)], \quad (2)$$

where  $G_F$  is the Fermi constant,  $E_\nu$  is the neutrino energy, and  $\theta$  is the scattering angle. The axial coupling up to strange-quark corrections is  $|C_{a,N}| = |g_a|/2 = 0.63$  where  $g_a$  is the axial

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charge of the nucleon. The weak vector charge is  $C_{v,n} = -1/2$  for scattering from a neutron  $n$  and  $C_{v,p} = 1/2 - 2 \sin^2 \theta_W \approx 0$  for scattering from a proton  $p$ . Here  $\theta_W$  is the weak mixing angle. The cross section in Eq. (2) neglects corrections of order  $E_v/m$  from weak magnetism and other effects; for details, see Ref. [28].

The free cross section per unit volume for scattering from a mixture of neutrons and protons is then given by

$$\begin{aligned} \frac{1}{V} \frac{d\sigma_0}{d\Omega} &= n_n \frac{d\sigma_0}{d\Omega_{vn}} + n_p \frac{d\sigma_0}{d\Omega_{vp}}, \\ &= \frac{G_F^2 E_v^2}{16\pi^2} [g_a^2(3 - \cos\theta)(n_n + n_p) + (1 + \cos\theta)n_n]. \end{aligned} \quad (3)$$

$$(4)$$

In the medium this cross section is modified by the density (vector)  $S_V$  and the spin (axial)  $S_A$  response. The response of the system to density fluctuations is described by  $S_V$ , while  $S_A$  describes the response of the system to spin fluctuations. The response functions are normalized to unity in the low-density limit  $S_V, S_A \rightarrow 1$  as  $n \rightarrow 0$ . The cross section per unit volume in the medium is then given by

$$\begin{aligned} \frac{1}{V} \frac{d\sigma}{d\Omega} &= \frac{G_F^2 E_v^2}{16\pi^2} [g_a^2(3 - \cos\theta)(n_n + n_p)S_A \\ &\quad + (1 + \cos\theta)n_n S_V]. \end{aligned} \quad (5)$$

Note that  $d\sigma/d\Omega$  reduces to the free cross section  $d\sigma_0/d\Omega$  as  $S_A, S_V \rightarrow 1$ . In general both  $S_V$  and  $S_A$  depend on momentum transfer  $q$ . However, in the limit  $q \rightarrow 0$  we can derive model-independent virial results.

### A. Virial equation of state

Next, we briefly review the virial equation of state for a system with neutrons and protons [21]. We use this to calculate  $S_V$  and  $S_A$ . The pressure  $P$  is expanded to second order in the fugacities of neutrons,  $z_n$ , and protons,  $z_p$ ,

$$\frac{P}{T} = \frac{\ln Q}{V} = \frac{2}{\lambda^3} [z_n + z_p + (z_n^2 + z_p^2)b_n + 2z_p z_n b_{pn}]. \quad (6)$$

Here  $T$  is the temperature,  $V$  is the volume of the system, and  $Q$  is the grand-canonical partition function. The fugacities are related to the neutron  $\mu_n$  and proton  $\mu_p$  chemical potentials by  $z_n = e^{\mu_n/T}$  and  $z_p = e^{\mu_p/T}$ . Finally, the second virial coefficients  $b_n$  and  $b_{pn}$  are calculated from nucleon-nucleon elastic-scattering phase shifts. These are tabulated in Ref. [21].

The neutron  $n_n$  and proton  $n_p$  densities follow from derivatives of  $\ln Q$ ,

$$n_i = z_i \frac{\partial}{\partial z_i} \left( \frac{\ln Q}{V} \right) \Big|_{V,T}. \quad (7)$$

This gives

$$n_n = \frac{2}{\lambda^3} (z_n + 2z_n^2 b_n + 2z_p z_n b_{pn}), \quad (8)$$

$$n_p = \frac{2}{\lambda^3} (z_p + 2z_p^2 b_n + 2z_p z_n b_{pn}). \quad (9)$$

### B. Vector response

The vector response  $S_V$  is equal to the static structure factor  $S_q$ ; see, for example, Refs. [25,29]. For a single-component system

$$S_V(q=0) = \frac{T}{(\partial P/\partial n)_T}. \quad (10)$$

By using the virial equation of state this can be rewritten with  $dP/dn = n/(Tz)(dz/dn)$  as

$$S_V = \frac{1}{n} z \frac{\partial}{\partial z} n. \quad (11)$$

Following Ref. [7], we generalize this result to a mixture of neutrons and protons:

$$S_V = \frac{C_v^{n2} S_{nn} + 2C_v^n C_v^p S_{np} + C_v^{p2} S_{pp}}{C_v^{n2} n_n + C_v^{p2} n_p}, \quad (12)$$

where

$$S_{nn} = z_n \frac{\partial}{\partial z_n} n_n = n_n + \frac{4}{\lambda^3} z_n^2 b_n, \quad (13)$$

$$S_{np} = z_p \frac{\partial}{\partial z_p} n_n = \frac{4}{\lambda^3} z_p z_n b_{pn}, \quad (14)$$

$$S_{pp} = z_p \frac{\partial}{\partial z_p} n_p = n_p + \frac{4}{\lambda^3} z_p^2 b_n. \quad (15)$$

By using Eqs. (13)–(15), we have for  $S_V$

$$S_V = 1 + \frac{4}{\lambda^3} \frac{C_v^{n2} z_n^2 b_n + 2C_v^n C_v^p z_n z_p b_{pn} + C_v^{p2} z_p^2 b_n}{C_v^{n2} n_n + C_v^{p2} n_p}. \quad (16)$$

In the limit  $C_v^p \approx 0$  this reduces to the neutron-matter result [25]

$$S_V = 1 + \frac{4}{\lambda^3} \frac{z_n^2 b_n}{n_n}. \quad (17)$$

Here the impact of protons is to somewhat modify the neutron fugacity  $z_n$  because of the  $b_{pn}$  term in the neutron density, Eq. (8). The virial coefficient  $b_n \approx 0.32$  is small and positive. As a result, the vector response is slightly enhanced (larger than one) as shown in Fig. 1. Attractive nucleon-nucleon interactions increase the probability to find nucleons close together. These density fluctuations increase the (local) weak charge and produce a vector response  $S_V > 1$ .

### C. Axial response

To calculate the axial response  $S_A$  we generalize our virial equation of state to describe spin-polarized nuclear matter. Let  $z_p^+, z_n^+$  be the fugacities for spin-up  $p$  and  $n$ , and  $z_p^-, z_n^-$  be the spin-down fugacities. Generalizing the results of Ref. [25], we have for the density of spin-up neutrons  $n_n^+$ ,

$$n_n^+ = \frac{1}{\lambda^3} [z_n^+ + 2b_+ z_n^{+2} + 2z_n^+ (b_- z_n^- + b_{pn}^+ z_p^+ + b_{pn}^- z_p^-)]. \quad (18)$$

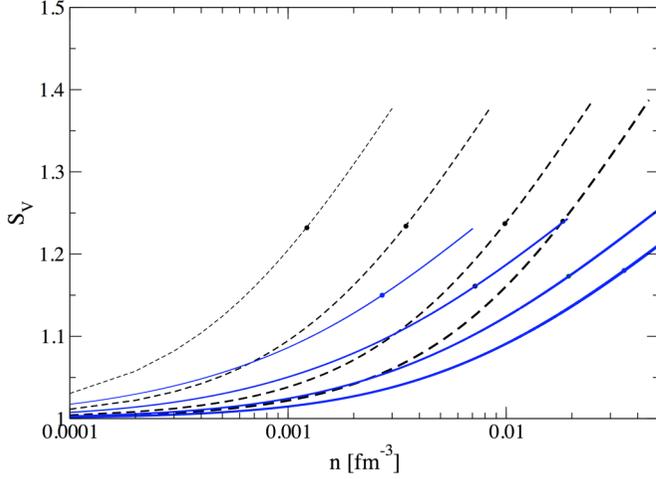


FIG. 1. Vector response  $S_V$  versus density  $n$  for proton fractions of  $Y_p = 0$  (dashed lines) and 0.3 (solid lines). The curves are for temperatures of (from left to right with increasing thickness) 2.5, 5, 10, and 15 MeV. The solid circles show where  $z_n = 0.5$ . The virial expansion is most valid to the left of these points.

We discuss the spin virial coefficients  $b_+$ ,  $b_-$ ,  $b_{pn}^+$ , and  $b_{pn}^-$  in Sec. II D. Likewise the density of spin-down neutrons  $n_n^-$  is

$$n_n^- = \frac{1}{\lambda^3} [z_n^- + 2b_+ z_n^{-2} + 2z_n^- (b_- z_n^+ + b_{pn}^+ z_p^- + b_{pn}^- z_p^+)]. \quad (19)$$

Similarly, the density of spin-up protons  $n_p^+$  is

$$n_p^+ = \frac{1}{\lambda^3} [z_p^+ + 2b_+ z_p^{+2} + 2z_p^+ (b_- z_p^- + b_{pn}^+ z_n^+ + b_{pn}^- z_n^-)], \quad (20)$$

while the density of spin-down protons  $n_p^-$  is

$$n_p^- = \frac{1}{\lambda^3} [z_p^- + 2b_+ z_p^{-2} + 2z_p^- (b_- z_p^+ + b_{pn}^+ z_n^- + b_{pn}^- z_n^+)]. \quad (21)$$

We define axial or spin fugacities  $z_n^a = (z_n^+ / z_n^-)^{1/2}$  and  $z_p^a = (z_p^+ / z_p^-)^{1/2}$  and following Ref. [7] write the axial response as

$$S_A = \frac{S_{pp}^A + S_{nn}^A - 2S_{np}^A}{n_n + n_p}, \quad (22)$$

with

$$S_{nn}^A = z_n^a \frac{\partial}{\partial z_n^a} (n_n^+ - n_n^-) \Big|_{z_n^a=1} = n_n + \frac{4}{\lambda^3} (b_+ - b_-) z_n^2, \quad (23)$$

$$S_{np}^A = z_p^a \frac{\partial}{\partial z_p^a} (n_n^+ - n_n^-) \Big|_{z_p^a=1} = \frac{4}{\lambda^3} z_n z_p (b_{pn}^+ - b_{pn}^-), \quad (24)$$

$$S_{pp}^A = z_p^a \frac{\partial}{\partial z_p^a} (n_p^+ - n_p^-) \Big|_{z_p^a=1} = n_p + \frac{4}{\lambda^3} (b_+ - b_-) z_p^2. \quad (25)$$

Note that the minus sign for the  $S_{np}^A$  term in Eq. (22) is because the axial charge of a neutron is opposite to that of

a proton. To clean up the notation, we define the axial virial coefficients

$$b_a = b_+ - b_-, \quad (26)$$

$$b_{pn}^a = b_{pn}^+ - b_{pn}^-. \quad (27)$$

The final result for  $S_A$  can then be written as

$$S_A = 1 + \frac{4}{\lambda^3} \frac{(z_p^2 + z_n^2) b_a - 2z_p z_n b_{pn}^a}{n_n + n_p}. \quad (28)$$

To lowest order in the density one has  $z_p \approx \lambda^3 n_p / 2$  and  $z_n \approx \lambda^3 n_n / 2$  so that

$$S_A \approx 1 + \lambda^3 \frac{(n_n^2 + n_p^2) b_a - 2n_n n_p b_{pn}^a}{n_p + n_n}. \quad (29)$$

Note that we use the full Eq. (28) for results in the next section. Because the spin virial coefficient  $b_a \approx -0.6$  (see below), the axial response is reduced  $S_A < 1$ . This is because two neutrons or two protons are likely to be correlated in a  ${}^1S_0$  state because of the Pauli principle and this spin-zero state reduces the spin response.

We define the total response  $S_{\text{tot}}$  as the ratio of the in-medium transport cross section to the free one,

$$S_{\text{tot}} = \frac{\int d\Omega \frac{d\sigma}{d\Omega} (1 - \cos \theta)}{\int d\Omega \frac{d\sigma_0}{d\Omega} (1 - \cos \theta)}. \quad (30)$$

From Eqs. (4) and (5), we thus have

$$S_{\text{tot}} = \frac{5g_a^2 S_A + (1 - Y_p) S_V}{5g_a^2 + 1 - Y_p}, \quad (31)$$

where  $Y_p$  is the proton fraction. The total response depends on both  $S_V$  and  $S_A$ . However, in general  $S_A$  is the most important because of the large factor  $5g_a^2$ . We present results for  $S_A$  in Sec. III. However, first we discuss the spin virial coefficients.

#### D. Spin virial coefficients

The virial coefficient  $b_a = b_+ - b_-$  is discussed in Ref. [25] and describes spin interactions between two protons or two neutrons. We now discuss  $b_{pn}^a = b_{pn}^+ - b_{pn}^-$  that involves interactions between protons and neutrons. The virial coefficient  $b_{pn}^+$  describes interactions between a  $p$  and a  $n$  with like-spin projections, while  $b_{pn}^-$  describes interactions between nucleons with unlike spins. We therefore have

$$b_{pn}^a = \frac{1}{2^{1/2}} (e^{E_d/T} - 1) + \frac{2^{1/2}}{\pi T} \int_0^\infty dE e^{-E/2T} [\delta_{pn}^+(E) - \delta_{pn}^-(E)], \quad (32)$$

with

$$\delta_{pn}^+(E) = \frac{1}{2} \delta_3 S_1 + \frac{1}{6} \delta_3 P_0 + \frac{1}{2} \delta_3 P_1 + \frac{5}{6} \delta_3 P_2 + \frac{1}{2} \delta_3 D_1 + \frac{5}{6} \delta_3 D_2 + \frac{7}{6} \delta_3 D_3 + \dots \quad (33)$$

Here  $E_d$  is the deuteron binding energy and the factor in front of each phase shift is  $(2J + 1)/[2(2S + 1)]$  where the factor of 1/2 is from the average over isospin 1 and 0 states. In

TABLE I. Spin virial coefficients  $b_a$  and  $b_{pn}^a$  based on nucleon-nucleon phase shifts.

$T$ (MeV)	$b_a$	$b_{pn}^a$
1	-0.638	6.18
2	-0.653	1.74
3	-0.651	1.05
4	-0.648	0.785
5	-0.643	0.640
6	-0.637	0.561
7	-0.631	0.504
8	-0.625	0.463
9	-0.620	0.432
10	-0.615	0.408
12	-0.605	0.374
14	-0.597	0.352
16	-0.589	0.336
18	-0.583	0.324
20	-0.577	0.315

our calculation, we neglected states with  $L > 2$ . Similarly, we have

$$\delta_{pn}^-(E) = \frac{1}{4}\delta_{1S_0} + \frac{1}{4}\delta_{3S_1} + \frac{3}{4}\delta_{1P_1} + \frac{1}{12}\delta_{3P_0} + \frac{1}{4}\delta_{3P_1} + \frac{5}{12}\delta_{3P_2} + \frac{5}{4}\delta_{1D_2} + \frac{1}{4}\delta_{3D_1} + \frac{5}{12}\delta_{3D_2} + \frac{7}{12}\delta_{3D_3} + \dots \quad (34)$$

Now the factor for each phase shift is  $(2J+1)/[4(2S+1)]$ , where the  $1/4$  is from an average over both isospin 1 and 0 and spin 1 and 0 states. We calculate the spin virial coefficients based on the Nijmegen partial-wave analysis of nucleon-nucleon scattering [30]. Our results for  $b_a$  and  $b_{pn}^a$  are collected in Table I. The other virial coefficients  $b_n$  and  $b_{pn}$  needed to calculate the neutrino responses have already been tabulated in Ref. [21].

In contrast to  $b_a$ ,  $b_{pn}^a$  is positive because a proton and a neutron can be correlated into the spin-one  ${}^3S_1$  state (deuteron like), enhancing the spin response. However the axial charge of a proton is opposite to that of a neutron. This leads to a minus sign in Eq. (28) for the  $b_{pn}^a$  term. As a result, both  $b_a$  and  $b_{pn}^a$  reduce the total axial response and lead to  $S_A < 1$ .

### E. Combining correlations, strange-quark, weak magnetism, and recoil corrections

We end this formalism section by describing the combination of correlation, strange-quark, weak magnetism, and recoil corrections. To a good approximation all of these effects can be combined in a straightforward way that avoids double counting. We write for the neutrino cross section per unit volume [see Eq. (5)],

$$\frac{1}{V} \frac{d\sigma}{d\Omega} \approx \frac{G_F^2 E_\nu^2}{16\pi^2} \{ [(g_a + g_a^s)^2 n_n + (g_a - g_a^s)^2 n_p] \times (3 - \cos\theta) S_A + (1 + \cos\theta) n_n S_V \} R(E_\nu/m). \quad (35)$$

Effects from nucleon-nucleon correlations in the medium are described by the vector  $S_V$  and axial  $S_A$  response functions;

see Eqs. (17) and (28), respectively. The vector response is slightly greater than one and the axial response is significantly less than one. As a result, correlations reduce the cross section for both neutrino-proton and neutrino-neutron scattering.

Strange-quark contributions to the nucleon spin are described by the parameter  $g_a^s$  (note  $g_a = 1.26$ ). Melson *et al.* [4] consider  $g_a^s = -0.2$  and this value reduces neutrino-neutron and increases neutrino-proton scattering cross sections. For neutron-rich conditions this leads to a net reduction in the neutrino-scattering opacity. Therefore, both correlation effects and strange quarks, if present (with  $g_a^s < 0$ ), reduce the opacity, and the two effects add. Note that, in Ref. [4], strange-quark contributions were probably taken to be unrealistically large [5]. On the one side, strange-quark contributions to the vector current have been measured by several parity-violating electron-scattering experiments to be small [31], but direct experimental limits on strange-quark contributions to the nucleon spin are relatively poor and are based on an old Brookhaven neutrino-scattering experiment [32]. Therefore, it would be very useful to have a better laboratory limit on  $g_a^s$  from a modern neutrino-nucleon scattering experiment.

Finally, recoil and weak magnetism corrections can be approximately described by a factor  $R(E_\nu/m)$ . This is discussed in Ref. [28] and reduces antineutrino-nucleon scattering cross sections, while having only a modest effect on neutrino-nucleon cross sections.

## III. RESULTS FOR AXIAL RESPONSE

In this section, we focus on results for the axial response  $S_A$ , and not on the vector response  $S_V$ , for two reasons. First, the axial response is more important for neutrino-transport cross sections because of a large  $5g_a^2$  factor; see, Eq. (35). Second, we have not included alpha particles or other light nuclei. Preliminary calculations suggest that spin-zero alpha particles can significantly enhance  $S_V$ , but do not strongly impact  $S_A$ . Therefore we postpone a full discussion of  $S_V$  to later work, where we will explicitly include alpha particles and other light nuclei. For the present, a reasonable approximation is to simply set  $S_V = 1$  in Eq. (31).

In Fig. 2 we show  $S_A$  for temperatures of  $T = 2.5$  to 15 MeV. Our virial results (red dashed lines) are valid at low densities. To evaluate  $S_A$  for higher densities, where  $z_n > 0.5$ , one presently needs to employ a model-dependent calculation. We consider the random-phase approximation (RPA) calculations of Burrows and Sawyer [7], because they are simple, well known, and have been employed in supernova simulations. We caution that these calculations may have a number of limitations. First, they predict that the vector response is less than one  $S_V < 1$  while Fig. 1 shows  $S_V > 1$ . Second, the calculations use a Landau parameter for the effective interaction that is appropriate for symmetric nuclear matter. A Landau parameter appropriate for pure neutron matter could lead to a smaller  $S_A$ . See also the discussion in Ref. [25]. In future work we will revisit the behavior of  $S_A$  at high densities, but for now we consider the Burrows and Sawyer results. The green dotted lines in Fig. 2 show the Burrows and Sawyer RPA calculation [7]. Note that the RPA

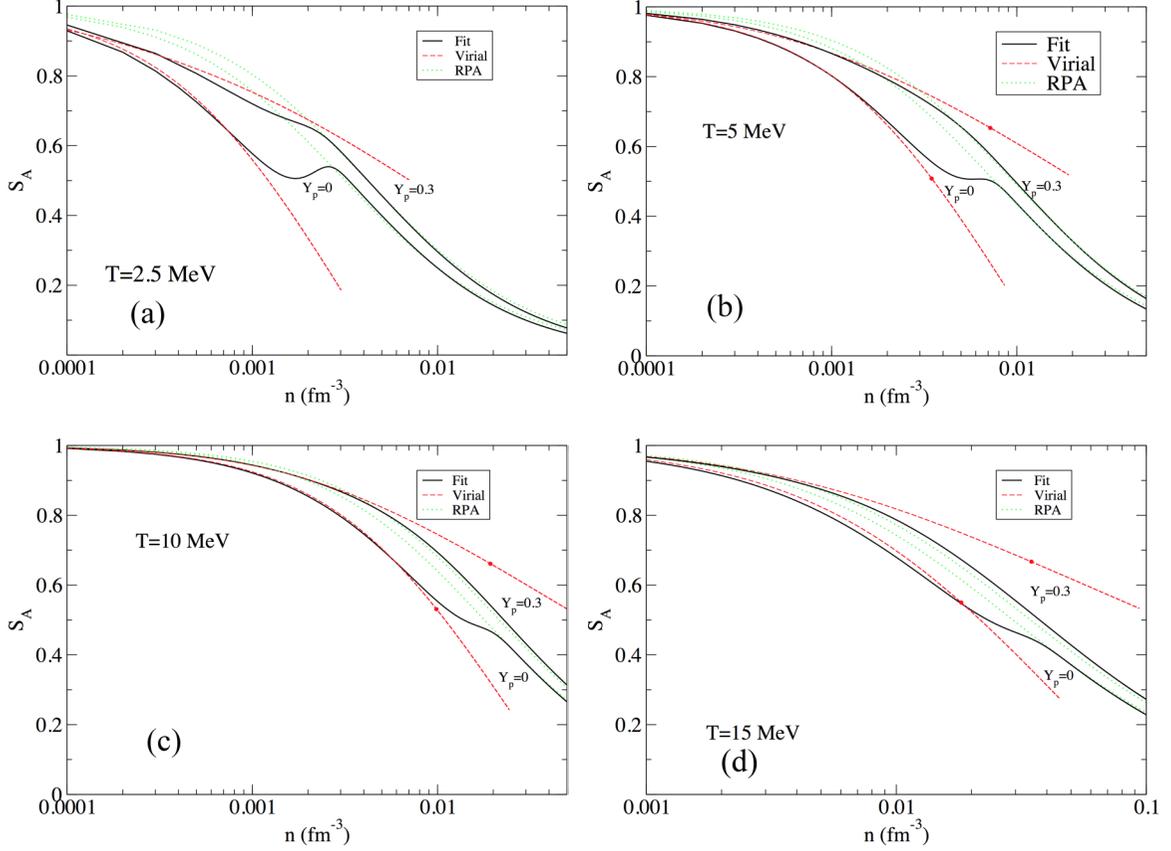


FIG. 2. Axial response  $S_A$  versus density  $n$  for temperatures of (a)  $T = 2.5$  MeV, (b) 5 MeV, (c) 10 MeV, (d) and 15 MeV. The red dashed lines are our virial expansion results, Eq. (28), for the indicated proton fractions. The solid red dots indicate where  $z_n = 0.5$ . The virial expansion is most valid to the left of these points. The green dotted lines show the original Burrows and Sawyer RPA results [7]. Finally, the solid black lines show the interpolating fit  $S_A^f$ , Eq. (36).

depends very weakly on the momentum transfer  $q$  and we use  $q = 3T$ .

Finally, we fit the virial results for  $S_A$  at low densities and the RPA results at high densities with an interpolating function  $S_A^f(n, T, Y_p)$  that is a simple function of density  $n$ , temperature  $T$ , and proton fraction  $Y_p$ :

$$S_A^f(n, T, Y_p) = \frac{1}{1 + A(1 + Be^{-C})}, \quad (36)$$

where the functions  $A$ ,  $B$ , and  $C$  are given by

$$A(n, T, Y_p) = A_0 \frac{n(1 - Y_p + Y_p^2)}{T^{1.22}}, \quad (37)$$

$$B(T) = \frac{B_0}{T^{0.75}}, \quad (38)$$

$$C(n, T, Y_p) = C_0 \frac{nY_p(1 - Y_p)}{T^{0.5}} + D_0 \frac{n^4}{T^6}. \quad (39)$$

The fit parameters  $A_0$ ,  $B_0$ ,  $C_0$ , and  $D_0$  are collected in Table II for  $n$  in  $\text{fm}^{-3}$  and  $T$  in MeV. This fit is most accurate for  $5 < T < 10$  MeV,  $Y_p \leq 0.3$ , and  $n < 0.05 \text{ fm}^{-3}$ , but yields reasonable values outside this range. We make an implementation of this fitting function available online in NuLib [33,34]. Note that if one sets  $B_0 = 0$  in Eq. (38), Eq. (36) will approximately reproduce the original Burrows

and Sawyer RPA results at low density. We see that  $S_A$  is significantly reduced from 1 even at relatively low densities. This is especially the case at low  $Y_p$ .

We briefly explore the effect of the reduced axial response arising from the virial expansion on the heating rate obtained in simulations of the accretion phase of a core-collapse supernovae. To compare with Ref. [4], we use the Lattimer and Swesty [35] equation of state (with  $K_0 = 220$  MeV) and the  $20 M_\odot$  progenitor star from Ref. [36]. We use the one-dimensional code GRID [33], available online [37]. For the sake of comparison with Ref. [4], we perform simulations with and without strange-quark contributions. We show the heating rate realized in the gain region in Fig. 3. The solid lines denote

TABLE II. Fit parameters for  $S_A^f$  fitting function; see Eqs. (36)–(39). These assume  $n$  in  $\text{fm}^{-3}$  and  $T$  in MeV.

Fit parameter	Value
$A_0$	920
$B_0$	3.05
$C_0$	6140
$D_0$	$1.5 \times 10^{13}$

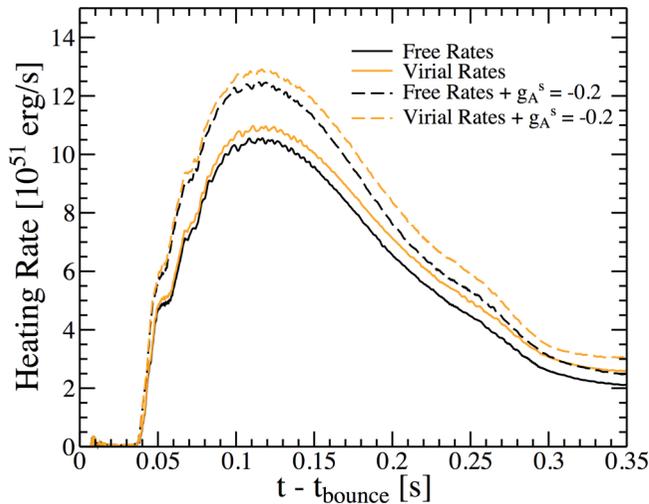


FIG. 3. Heating rate in the gain region for one-dimensional simulations of the accretion phase of core-collapse supernovae for various assumptions on the neutral-current neutrino-nucleon scattering cross section. The solid lines represent simulations that ignore the contribution to the rates from strange quarks, whereas dashed lines denote simulations that include strange-quark contributions, assuming  $g_a^s = -0.2$  for comparison with Ref. [4]. The black lines show the heating rate for simulations that assume  $S_A = 1$ , while orange lines show the larger heating rates with  $S_A^f$  from Eq. (36).

simulations using no strange-quark corrections, while the dashed lines denote simulations using  $g_a^s = -0.2$ . The black lines are for simulations that use free particle rates, while the orange lines denote simulations using the virial corrected rates. Similar to Ref. [4], the assumed strange-quark contribution raises the heating rate by  $\sim 20\%$  around 100 ms after bounce. We find that the virial rates increase the heating by  $\sim 5\%$  around 100 ms after bounce and higher at later times. We expect the reduction in  $S_A$  to play an even larger role at later times when the neutrinos decouple from regions with higher matter densities.

Our preliminary two-dimensional supernova simulations, with a reduced axial response  $S_A^f$ , Eq. (36), explode

100–150 ms earlier than simulations with  $S_A = 1$ , for 15, 20, and  $25M_\odot$  stars. For a  $12M_\odot$  star,  $S_A^f$  leads to an explosion at late times while a simulation with  $S_A = 1$  fails to explode. Further details of these simulations will be provided in a later publication. Very recent results by Burrows *et al.* [38], based on an earlier version of this paper, find similar but perhaps somewhat larger effects of the reduced axial response  $S_A^f$ .

#### IV. SUMMARY AND CONCLUSIONS

Supernova simulations may be sensitive to the neutral-current interactions of mu and tau neutrinos at low densities near the neutrinosphere. In this paper, we calculated the axial or spin response  $S_A$  of nuclear matter in a virial expansion that is model-independent at low densities and high temperatures. We find  $S_A$  to be significantly reduced. Our results can be incorporated into supernova simulations by multiplying both the neutrino-proton and neutrino-neutron neutral current scattering rates by  $S_{\text{tot}}$  given by Eq. (31) with  $S_V = 1$  and  $S_A$  given by our fit function  $S_A^f$  from Eqs. (36)–(39). Preliminary one-dimensional supernova simulations suggest that the reduction in the axial response may enhance the neutrino heating rates in the gain region during the accretion phase of a core-collapse supernova. In future work, we will extend the calculation to the vector response  $S_V$  in a virial expansion and study the impact of light nuclei.

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