Isospin mixing of 2^+ states in ^{14}N

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I have investigated the possibility that the near vanishing of the proton strength of the 9.17-MeV state in ^{14}N could be due to isospin mixing. If the two levels involved are at 9.17 and 8.98 MeV, the mixing intensity is 0.41(9) with a T-mixing matrix element of 94(10) keV. Some consequences of such mixing are examined.

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I. INTRODUCTION

The 2^+ $T = 1$ states at 9.17 and 10.43 MeV in ^{14}N have long been known as analogs of the 7.01- and 8.32-MeV states in ^{14}C. Several decades ago, Warburton and Pinkston offered a simple description [1] of the structure of these two states, viz. an almost equal mixture of two basis states—a p-shell 2^+ state and the lowest $(sd)^2$ 2^+ state. This description has stood the test of time. Virtually all experimental information involving these two states is consistent with the simple picture. For example, in the ^{15}N(^{3}He,α) reaction the 9.17- and 10.43-MeV levels are populated almost equally [2]. However, as one can see below, a possible problem exists with the single-nucleon stripping spectroscopic factors S.

Properties of the two lowest 2^+ $T = 0$ and $T = 1$ states are listed in Table I [3–5]. The most striking feature of these states is the fact that the experimental proton width of the 9.17-MeV state is only 0.122(8) keV [3,6]. An earlier value was 0.135(8) keV [7]. In a simple potential model with a standard Woods-Saxon potential well, having $r_0, a, r_{0c} =$ 1.26, 0.60, 1.40 fm, the single-particle (sp) proton width for $\ell = 1$ decay is 520 keV. Thus, using the expression $C^2S = \Gamma_{\rm exp}/\Gamma_{\rm sp}$ (with $C^2 = 1/2$ here), S is only $4.7(3)\times 10^{-4}$—about 1% of the expected value [5]. The extreme smallness of this quantity has been known for a long time and has been attributed to a small (~4% [1] or 6% to 7% [8]) admixture of $(sd)^2$ in ^{13}C(g.s.) [where (g.s.) represents the ground state] plus destructive interference between the two major components of the first 2^+ $T = 1$ state. Two things are wrong with this explanation: (1) Because the 9.17-MeV state is the lower of the two mixed states, this relative phase should be constructive for it. (2) There is no evidence for a similar destructive interference in the parent state in ^{14}C. The two 2^+ states there have virtually identical cross sections and angular-distribution shapes in the reaction ^{13}C(d,p). In addition, the amount of core excitation needed in ^{13}C(g.s.) is significantly larger than theoretical estimates [9]. I think the explanation may lie in the area of isospin mixing (IM) as I now discuss.

Similar isospin mixing is well established in other light nuclei. For example, two high-lying 2^+ states of ^{8}Be are reasonably well described as having the structures ^{7}Li$+p$ (16.6 MeV) and ^{7}Be$+n$ (16.9 MeV) rather than $T = 0$ and 1 [10]. Isospin mixing of 1^+ and 3^+ states is significantly less. The 12.71- and 15.11-MeV 1^+ states in ^{12}C are also T mixed [11]. In ^{14}N, lower-lying 1^- states exhibit clear evidence of such mixing [12]. Various theoretical approaches have been used to estimate the magnitude of such T mixing. For ^{8}Be, Goldhammer [13] used wave functions from the Sussex interaction to compute mixing of the nuclear interaction and the Coulomb potential and reported excellent agreement. Wiringa *et al.* [14] used Green's function Monte Carlo calculations of isospin mixing (IM) matrix elements for the $2^+, 1^+$, and 3^+ $T = 0$ and 1 pairs of states at high excitation in ^{8}Be. They included the full electromagnetic interaction and several charge-symmetry-breaking terms of the strong force—both two and three body. They found that their calculation gave 85%–90% of the experimental IM value for the 2^+ doublet with about two-thirds coming from the Coulomb interaction. For T mixing of 1^- states in ^{14}N, Lie [15] assumed that T impurities were due to Coulomb forces only. He treated the Coulomb matrix elements connecting $T = 0$ and $T = 1$ eigenstates in first-order perturbation. He included only terms connecting basic states with the same configurations, stating that the others were smaller by a factor of about 1/100. His computed T mixing was somewhat smaller than experimental estimates.

II. CALCULATIONS AND RESULTS

A 2^+ $T = 0$ state exists at 8.980 MeV with a width of 8(2) keV [3]. The sp proton width at this energy is 360 keV, resulting in $S = 2\Gamma_{\rm exp}/\Gamma_{\rm sp} = 0.044(11)$. In the reaction ^{13}C(^{3}He,d), the limit on its strength is $(2J+1)\ C^2S < 0.2$, i.e., $S < 0.08$—in agreement with my S computed from the width. Shell-model calculations within the p shell [5] predict only one 2^+ $T = 0$ state anywhere near this energy region, and that is presumably the known 2^+ $T = 0$ state at 7.029 MeV. The p-shell energy prediction is 6.991 MeV. That state has $(2J+1)C^2S = 0.31$ in the reaction ^{13}C(^{3}He,d)—i.e., $S = 0.12$, to be compared with the p-shell prediction [5] of $S = 0.13$—very good agreement.

An $(sd)^2$ shell-model calculation puts the lowest 2^+ $T =$ 0 state about 0.7 MeV above the first 2^+ $T = 1$. Thus, mixing between the p-shell and $(sd)^2$ $T = 0$ states could be responsible for some of the proton strength observed for the 8.98-MeV state. However, even if the 2^+ $T = 0$ states do not mix, the 8.98-MeV state could acquire p strength from the lower 2^+ $T = 1$ state through isospin mixing. Because these two states are only 192 keV apart, only a small T-mixing matrix element would be required. And, in this case, the interference will be destructive for the upper (9.17-MeV) state so that the resulting single-nucleon strength could easily almost vanish.

I thus assume that the 8.98- and 9.17-MeV states arise from isospin mixing between a pure $T = 0$ and a $T = 1$ state. I take

TABLE I. Properties of 2^+ states in ^{14}N (energies in MeV and widths in keV).

T	E_x(exp)[a]	$S(^3\mathrm{He},d)$[b]	Γ_{exp}[a]	Γ_{sp}	S_Γ[c]	E_x(th)[d]	S_{th}[d]
0	7.029	0.12	Bound			6.991	0.13
0	8.980	<0.08	8(2)	360	0.044(11)		
1	9.172	<0.032	0.122(8)	520	$4.7(3)\times10^{-4}$	9.524	0.052
1	10.432	Not seen	33(3)	2300	0.029(3)		

[a]Reference [3].
[b]Reference [4].
[c]$S_\Gamma = 2\Gamma_{\mathrm{exp}}/\Gamma_{\mathrm{sp}}$.
[d]Reference [5].

TABLE II. Results of isospin-mixing analysis.

Quantity	Value
v^2	0.41(9)
A_0	0.148(33)
A_1	0.151[a]
V	94(10) keV

[a]From a shell-model calculation, assuming equal mixing of 2^+ $T = 1$ states before isospin mixing.

the $T = 1$ basis state to be the analog of the 7.01-MeV state in ^{14}C, whose structure is composed of approximately equal mixtures of a p-shell state and the lowest 2^+ $(sd)^2$ state. The theoretical spectroscopic factor for the pure p-shell state is 0.052 [5] or 0.0456 [9] of which about one-half will belong to the 7.01-MeV state and hence to the $T = 1$ basis state in ^{14}N. In the reaction ^{13}C(d,p) [16], the 7.01- and 8.32-MeV states have an approximately equal spectroscopic factor $S = 0.065$. Normalizing the data to distorted-wave calculations at forward angles (as is usually done) reduces these to 0.032 each.

For spectroscopic factors of the pure isospin states, I define $S_T = A_T^2$ so that A_1 is thus 0.161 or 0.151. I make no assumption about the nature of the $T = 0$ basis state and see what information emerges from the fitting.

For the physical states in ^{14}N, I write

$$|8.98\,\mathrm{MeV}\rangle = u|T=0\rangle + v|T=1\rangle;$$
$$|9.17\,\mathrm{MeV}\rangle = -v|T=0\rangle + u|T=1\rangle.$$

Then the spectroscopic amplitudes are $A(8.98) = uA_0 + vA_1, A(9.17) = -vA_0 + uA_1$, and the S's are the squares of these numbers. I then fit these expressions to the experimental strengths of the two physical states. If I take $A_1 = 0.151$ as given and use the relation $u^2 + v^2 = 1$, I have two experimental numbers with which to determine two parameters—A_0 and v^2, say. With $A_1 = 0.151$, $A(8.98) = 0.21(3)$, and $A(9.17) = 0.0217(7)$, I obtain the plot displayed as Fig. 1. The result is $v^2 = 0.41(9), A_0 = 0.148(33)$. The matrix element responsible for the isospin mixing is then 94(10) keV, considerably smaller than the corresponding result of 620 keV [12] for T mixing of 1^- states in ^{14}N. Results of the present analysis are presented in Table II. Long ago, Lie [15] had remarked "For positive-parity states no case exists where a particular particle state and a hole state couple to identical configurations with $T = 0$ and $T = 1$. The T impurities in the positive-parity states are accordingly supposed to be of minor importance and are not considered."

I now examine some of the consequences of such isospin mixing between the 2^+ states.

Before any isospin mixing, if the two 2^+ $T = 1$ states in ^{14}N are approximately equal mixtures of the two basis states discussed above and if ^{14}N(g.s.) is a pure p-shell state, the g.s. $M1$ strengths should be equal for the two 2^+ states. Electron inelastic scattering at 180° is a good measure of this quantity. Such an experiment [17] at 40.6 MeV reported cross sections of 13.19(77) and 13.17(107) nb/sr for the 9.17- and 10.43-MeV states, respectively (Table III). Their reported γ widths were 6.6(13) and 9.6(19) eV. Warburton and Pinkston [1] reported dimensionless $M1$ strengths of $\Lambda[M1(9.17)] = 4.1$ and $\Lambda[M1(10.43)] = 5.5$. The theoretical value for the pure p-shell 2^+ state was 12 [1]. Thus, in the electron-scattering experiment, the two $M1$'s are approximately equal, whereas in the γ experiment, the 9.17-MeV state is weaker. Other reported values [3,18–20] are also listed in Table III.

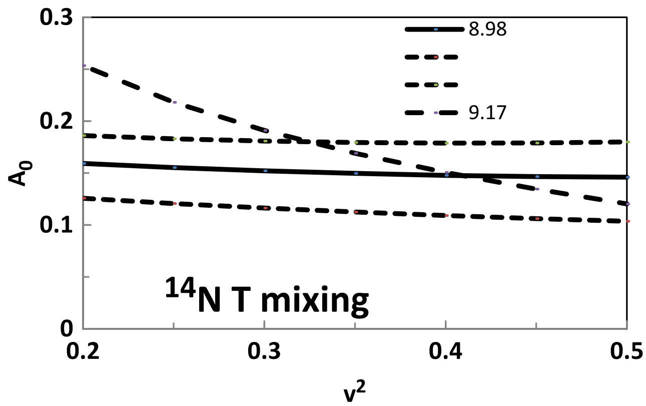

FIG. 1. Plot of the $T = 0$ spectroscopic amplitude A_0 vs the amount of T-mixing v^2 required to reproduce the experimental proton spectroscopic factors for the 8.98- and 9.17-MeV 2^+ states of ^{14}N.

TABLE III. Ground-state $M1$ strengths[a] of 9.17- and 10.43-MeV states of ^{14}N.

Process	Quantity	9.17 MeV	10.43 MeV	Reference
(e,e') 180°	Cross section	13.19(77) nb/sr	13.17(107) nb/sr	[17]
	$\Gamma(\gamma_0)$	6.6(13) eV	9.6(19) eV	[17]
^{13}C(p,γ)	$\Gamma(\gamma_0)$	7.7(9) eV	12.1(15) eV	[18]
	$\Gamma(\gamma_0)$	8.7(15) eV	17	[19,20]
	B($M1$)	0.53(9) W.u.	0.71 W.u.	[1,8]
	$\Lambda(M1)$	4.1	5.5	[1,8]
Compilation	$\Gamma(\gamma_0)$	6.2(3) eV	10.21(65) eV	[3]
	$B(M1)$	0.38(2) W.u.	0.43(3) W.u.	[3]

[a]W.u. is Weisskopf units.

TABLE IV. Cross sections for two-nucleon transfer to 2^+ $T = 1$ states in $A = 14$ nuclei.

^{12}C(t,p) ^{14}C[a]		^{12}C(^{3}He$,p$) ^{14}N[b]	
E_x	σ_{max}	E_x	σ (15°)
7.01 MeV	9.6 (mb/sr)	9.17 MeV	4.2 (mb/sr)
8.32 MeV	8.1 (mb/sr)	10.43 MeV	5.4 (mb/sr)
Ratio	1.18		0.78

[a]Reference [22].
[b]Reference [23].

If the postulated isospin mixing is indeed present, the 9.17-MeV state would lose some $M1$ strength to the 8.98-MeV state. However, the g.s. of ^{14}N is not purely of p-shell structure, and the $^{12}\mathrm{C}\times(sd)^2$ 1^+ component will have $M1$ strength from the $^{12}\mathrm{C}\times(sd)^2$ 2^+ component of the two 2^+ states. This contribution would be expected to be constructive for 9.17 and destructive for 10.43. In any case, the 8.98-MeV state should have some $M1$ strength for the g.s. In a very early experiment [21], it was observed to decay primarily to the g.s. with a reported γ width of only 0.10 eV. However, this resonance sits atop a wide resonance ($\Gamma \sim 400$ keV) corresponding to the 8.78-MeV 0^- state [3]. I have been unable to find any further information on the γ width of the 8.98-MeV state. Further research might be profitable.

In the (e,e') experiment, the 8.98- and 9.17-MeV states would not have been resolved. That might explain the observation of equal cross sections in (e,e') but a weaker $B(M1)$ for 9.17 MeV than for 10.43 MeV in the (p,γ) work.

Another consequence of the proposed isospin mixing would be a loss of two-nucleon transfer strength by the 9.17-MeV state and a gain of that strength by the 8.98-MeV state. Cross sections for the ^{12}C(t,p) reaction [22] to the parent states in ^{14}C and the ^{12}C(^{3}He$,p$) reaction [23] to the analogs in ^{14}N are listed in Table IV. With isospin conservation, the ratio of cross sections in the two reactions should be the same, and yet a large difference is observed. If the ratio in ^{14}N were to be the same as in ^{14}C, we would expect $\sigma(9.17) = (9.6/8.1)5.4 = 6.4$ mb/sr, to be compared to the observed value of 4.2 mb/sr. With the isospin-mixing intensity of 0.41(9) from above, the 9.17-MeV state would have lost 2.6(6) mb/sr to the 8.98-MeV state. Thus, the results of two-nucleon transfer are consistent with the proposed isospin mixing. The (^{3}He$,p$) cross section for the 8.98-MeV state is about 3 mb/sr, easily accommodating the cross section lost by the 9.17-MeV state.

The isospin mixing suggested here would allow some cross section for the 9.17-MeV state in a reaction that should populate only $T = 0$ states—such as ^{10}B(^{6}Li$,d$). In an early study of that reaction [24], the 9.17- and 9.13-MeV states would not have been resolved, and the latter was the second strongest state observed. (Only the 11.06-MeV state was stronger.) In a study of the same reaction in our laboratory [25], the resolution was about 45 keV, again insufficient to resolve the two states. I would expect a good resolution experiment should find a ratio of $\sigma(9.17)/\sigma(8.98) \sim 0.41/0.59$. The situation is complicated further by the fact that the 5^+ state at 8.96 MeV is very strong and not resolved from the 8.98-MeV state.

III. CONCLUSIONS

To summarize, I have investigated isospin mixing as the mechanism responsible for the nearly vanishing proton strength for the 9.17-MeV state in ^{14}N. If the 2^+ states at 8.98 and 9.17 MeV result from isospin mixing of $T = 0$ and 1 basis states, then the mixing intensity is found to be 0.41(9), and the T-mixing matrix element is 94(10) keV. Such mixing will influence the g.s. $M1$ strengths, decreasing it for 9.17 and increasing it for 8.98. The proposed mixing is in excellent agreement with the observed difference in $2n$ and np stripping strengths to the 2^+ states in ^{14}C and their analogs at 9.17 and 10.43 in ^{14}N. If the proposed mixing is correct, the 9.17-MeV state should be populated in reactions that should reach only $T = 0$ states—such as ^{10}B(^{6}Li$,d$).

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