Neutrons in strong magnetic fields and equation of state of neutron matter

Mohsen Bigdeli*
Department of Physics, University of Zanjan, 45371-38791 Zanjan, Iran
(Received 15 July 2016; revised manuscript received 10 October 2016; published 8 February 2017)

In the present work, I investigate the influence of neutron mass reduction due to magnetic field on the equation of state of neutron matter using the lowest-order constraint variational (LOCV) method with the AV_{18} potential.

DOI: 10.1103/PhysRevC.95.024309

I. INTRODUCTION

The study of the equilibrium state of neutron matter in a strong magnetic field can provide useful information on properties of the magnetized compact stars. It is well known that the soft γ-ray repeaters and anomalous x-ray pulsars have a strong surface magnetic field with a strength of 10^{14}–10^{15} G [1]. According to scalar virial theorem [2,3] the strength of the magnetic field at the core of neutron stars exceeds $B \gtrsim 10^{18}$ G. Moreover, an extremely large magnitude of the strength of magnetic fields could be produced in the heavy-ion collision experiments at CERN on the order of $B \approx 10^{17}$–10^{19} G [4].

Such super strong magnetic fields can affect the equation of state (EOS) in the following ways. First, by producing spin-polarized states, it modifies the matter contribution to the total energy and pressure. Second, the magnetic field is contributing to the total energy and pressure (Maxwell term). Third, and the most important one, the strong magnetic fields can change the internal structure of nucleons and reduce their masses [5]. For a magnetic field with a magnitude of approximately $B \simeq 1.99 \times 10^{18}$ Gauss, the first Landau radius, $r_B \simeq 1$, becomes comparable to the order of the nucleon size, and so the quarks degrees of freedom play a role. In this situation the mass spectra of nucleons, which are straightforwardly related to the quarks' energy levels, can be altered. The three-quark system with relativistic interaction in the external magnetic field has been investigated by Andreichikov *et al.* [5]. They have calculated the neutron mass dependency, $m_n(B)$, on the magnetic field and have shown that the neutron mass becomes small in strong magnetic fields due to color Coulomb and spin-spin interactions (see Fig. 1). Their results indicate that the neutron loses half of its mass at magnetic fields around $B \sim 1.28 \times 10^{19}$ G [5].

So far, many people have calculated the thermodynamic properties of nucleon matter in strong magnetic fields using different theoretical approaches at zero and finite temperature; here, some of their results are quoted [6–14]. In these studies the neutron has a constant mass. The calculations of cold neutron matter have predicted the antiparallel alignment of the neutron spins versus the magnetic field direction for the thermodynamically stable states [8,11]. The parallel ordering of the neutron spins with the magnetic field corresponding to some threshold densities have also been predicted in calculations like those of Isayev and Yang [8]. They have used the Skyrme effective interaction. A different point of view was also followed by Isayev and Yang [9], who used the BSk19, BSk20, and BSk21 interactions, showing that, due to the appearance of the longitudinal instability in a strong magnetic field, the partially induced ferromagnetic phase transition can occur for the neutron star matter in the magnetar core. In another study, a charged neutral homogeneous isospin asymmetric nuclear matter in β equilibrium at zero temperature under the effect of a magnetic field has been investigated by Angeles Perez-Garcia *et al.* [12]. They have employed a relativistic mean-field model and calculated the Landau Fermi-liquid parameters for this system. They have predicted that the proton (neutron and electron) spins tend to align parallel (antiparallel) with respect to the magnetic fields with strength higher than $B \approx 10^{16}$ G. In my previous study [13], I investigated asymmetric nuclear matter in the presence of strong external magnetic fields. According to my results, the induced phase transition to antiferromagnetic spin states occurs for the asymmetric nuclear matter in strong magnetic fields. However, my previous calculations have shown that the ground state of asymmetric nuclear matter is paramagnetic in the absence of magnetic field ($B = 0$) [15].

The aim of the present work is to investigate the influence of strong magnetic fields and the mass reduction of neutrons due to magnetic fields on the EOS of neutron matter. Here, I use the LOCV method and employ the modern AV_{18} [16] two-body potential. This potential is also expected to be modified by strong magnetic fields; however, it is not considered here.

II. FORMALISM

Consider neutron matter composed of A interacting neutrons and in the background of a uniform magnetic field in the z direction denoted by $\mathbf{H} = H\mathbf{k}$. In this work I assume, analogously to Refs. [7–10] that the contribution of induced magnetization to the magnetic field remains small and $B \approx H$. To obtain the magnetic properties and equilibrium state of this system at the external magnetic field, one should calculate the Helmholtz free energy, F. The free energy per particle at the external magnetic field is defined as [17]

$$F = E - BM_1, \tag{1}$$

where E is the total energy per particle of the spin-polarized neutron matter and the magnetization M_1 is

$$M_1 = \frac{1}{A}\int m dV, \quad m = \mu_n \rho \delta. \tag{2}$$

*m_ bigdeli@znu.ac.ir

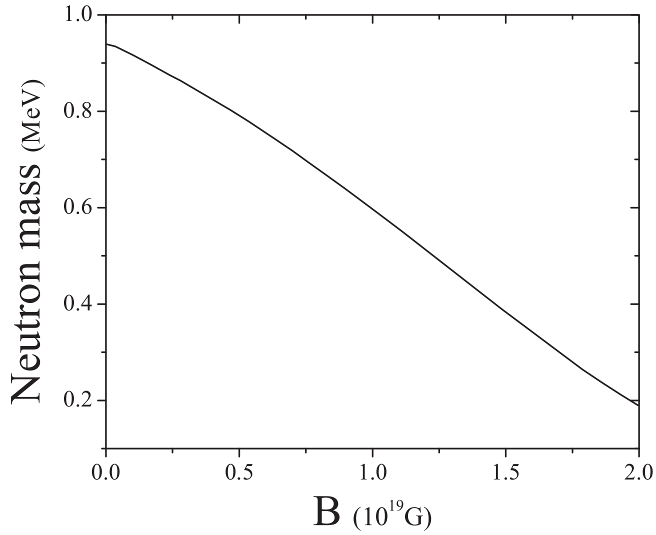

FIG. 1. Neutron mass vs magnetic field [5].

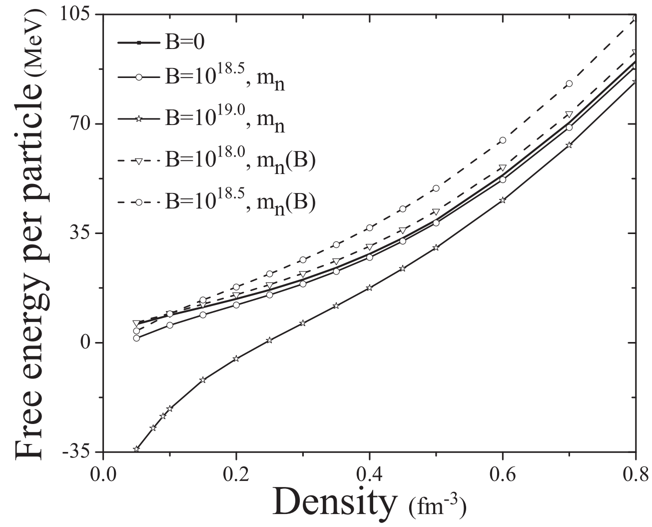

FIG. 2. The free energy per particle of the neutron matter versus density (ρ) for different magnitudes of magnetic field strength (B). The magnetic field strength is given in Gauss.

In this equation, $\mu_n = -6.030\,773\,8 \times 10^{-18}$ MeV/G is the neutron magnetic moment, $\delta = \frac{\rho^{(\uparrow)} - \rho^{(\downarrow)}}{\rho}$ is the spin polarization parameter and $\rho = \rho^{(\uparrow)} + \rho^{(\downarrow)}$ is the total density. Labels ($\uparrow$) and ($\downarrow$) are used for spins aligned parallel and antiparallel to the magnetic field, **B**, respectively.

The total energy per particle, E, is calculated with the LOCV method as follows. In this method, a trial many-body wave function of the form

$$\psi = \mathcal{F}\phi \tag{3}$$

is adopted, where ϕ is the uncorrelated ground-state wave function of A independent nucleons (simply the Slater determinant of the plane waves) and $\mathcal{F} = \mathcal{F}(1 \cdots A)$ is an appropriate A-body correlation operator which can be replaced by a Jastrow form, i.e.,

$$\mathcal{F} = \mathcal{S} \prod_{i>j} f(ij), \tag{4}$$

in which $\mathcal{S}$ is a symmetrizing operator.

Now, consider the cluster expansion of the energy functional up to the two-body term [18]:

$$E([f]) = \frac{1}{A} \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} = E_1 + E_2 \cdot \tag{5}$$

The smallness of the three-body cluster energy has been discussed in Ref. [19], where it is shown that cluster expansion converges reasonably and it is a good approximation to stop after the two-body energy term. For the nucleonic matter, the one-body term E_1 is

$$E_1 = \sum_{\sigma=\uparrow,\downarrow} \sum_{k \leqslant k_F^\sigma} \frac{\hbar^2 k^2}{2m_n(B)}, \tag{6}$$

where $k_F^\sigma = (6\pi^2 \rho^\sigma)^{1/3}$ is the Fermi momentum of each component of the system and $m_n(B)$ is the magnetic-field-dependent (MFD) neutron mass. The two-body energy E_2 is

$$E_2 = \frac{1}{2A} \sum_{ij} \langle ij | \nu(12) | ij - ji \rangle, \tag{7}$$

where

$$\nu(12) = -\frac{\hbar^2}{2m_n(B)} \left[f(12), \left[\nabla_{12}^2, f(12) \right] \right] + f(12) V(12) f(12). \tag{8}$$

Here, $f(12)$ and $V(12)$ are the two-body correlation and potential. Here, in this calculation, I use the AV_{18} two-body potential. For more detail see Refs. [13,15]. From the minimization of the two-body cluster energy one gets a set of coupled and uncoupled Euler-Lagrange differential equations [20]. The correlation functions can be calculated by numerically solving these differential equations, and then by using these correlation functions, the two-body energy is obtained.

Finally, the free energy [Eq. (1)] should be minimized with respect to the spin polarization parameter of the neutrons δ. By numerical minimization, one obtains this parameter at the minimum point of the energy corresponding to the equilibrium state of the system.

III. RESULTS AND DISCUSSION

In Fig. 2, the free energy per particle of the neutron matter has been shown as a function of total number density (ρ) for several values of the magnetic field strength, $B = 0$, $10^{18.0}$, $10^{18.5}$, and $10^{19.0}$ G. The curve corresponding to $B = 10^{19}$ G is given just for reference, because the longitudinal instability of the neutron matter unavoidably occurs at high strengths of a uniform magnetic field [9,21]. This figure includes the results of calculations using a constant neutron mass, m_n, as well as the magnetic-field-dependent mass, $m_n(B)$. Solid lines denote the case of a constant neutron mass and the MFD neutron mass

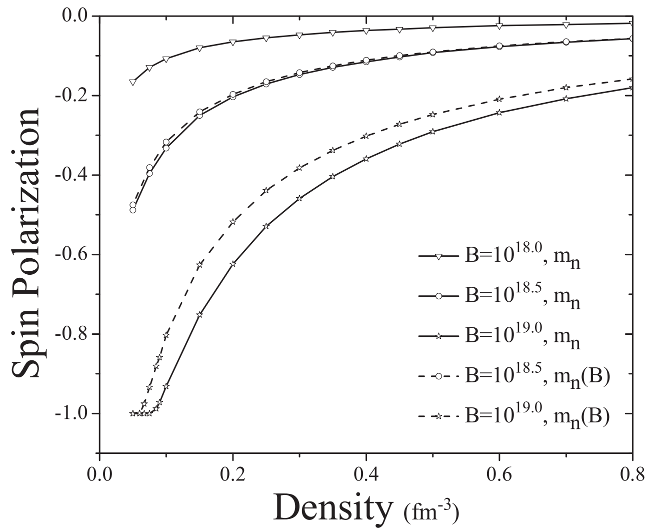

FIG. 3. The variation of the spin polarization parameter with magnetic field and density. The magnetic field strength is given in Gauss.

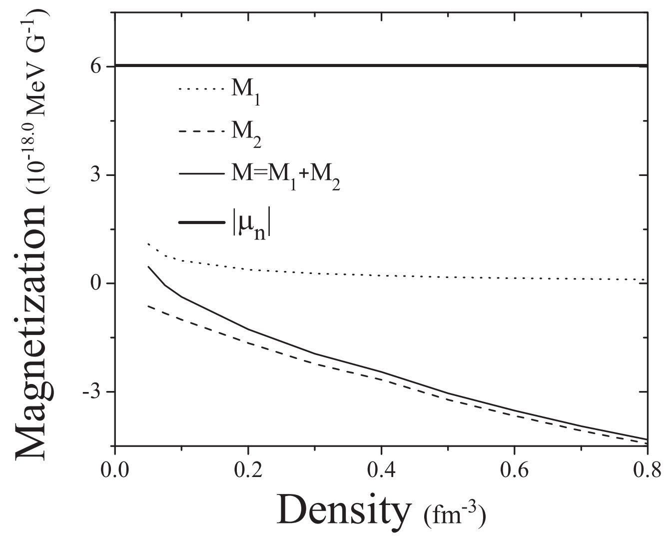

FIG. 4. Magnetization per particle of neutron matter versus density (ρ) at $B = 10^{18.0}$ G.

effects are ignored. The main general conclusion from Fig. 2 is that the free energy is extremely sensitive to the neutron mass. In the case of a constant neutron mass, the free energy decreases by increasing the magnetic field. The free energy reduction depends on the density; low density corresponds to more reduction, while at high densities the energy curves converge. At high densities, the contribution of the matter energy, E, to the total free energy is more important than that of the magnetic energy, BM_1.

When one uses the MFD mass, the free energy of the magnetized neutron matter shows different behaviors and the energy curves of the finite and zero fields cross each other at some critical density. The critical density increases by increasing the magnetic field strength. As can be seen, for densities lower than the critical density, the free energy is inversely proportional to the magnetic field, similar to the case of a constant neutron mass. However, at higher densities, the free energy per particle was shown to become repulsive as a result of increasing the magnetic field strength. Therefore, the inclusion of the MFD mass effects leads to distinct results, because even at fields of about $10^{18.0}$ G, the MFD mass plays an important role.

The spin polarization parameter (δ) corresponding to the equilibrium state of the neutron matter at the presence of external magnetic fields has been shown in Fig. 3. In this figure, the spin polarization has been presented as a function of the total number density for different values of the magnetic field strength. This figure shows that the spin of neutrons aligns antiparallel to the magnetic field direction. This figure also shows that the absolute value of the spin polarization parameter ($|\delta|$) increases by increasing the magnetic field strength for all densities. In addition, the dependence of the spin parameter of the neutron matter on the neutron mass has been considered and it has been found that it has different behaviors depending on the neutron mass. It is recognized that the absolute value of the spin polarization of the neutron matter decreases by taking the MFD neutron mass into account.

In what follows, the magnetization M, which is defined as the magnetic moment per particle, is considered and thermodynamically is given by [14]

$$M = -\left(\frac{dF}{dB}\right)_{\rho} = M_1 + M_2. \tag{9}$$

The first term, M_1, results from the explicit dependence of F on B [see Eq. (1)]. It can be written as $M_1 = \mu_n \delta$, and δ is the polarization of the ground state of the magnetized neutron matter (Fig. 3). The second term, M_2, coming from the implicit dependence through the field-dependent neutron mass $m_n(B)$, is

$$M_2 = -\left(\frac{\partial F}{\partial m_n}\right)_{\rho,\delta} \frac{dm_n}{dB}.$$

These quantities are shown versus the density at $B = 10^{18.0}$ G in Fig. 4. For comparison, the absolute value of the neutron magnetic moment is also included. Figure 4 displays that the value of M_2 is always negative and decreases with density. Compared with M_1, the magnetization M_2 is larger in magnitude at high densities. Therefore, the effects of the magnetization M_2 are more significant than those due to M_1 at these densities. For example, the free energy in the case of the MFD mass is increased above the constant mass diagram (Fig. 2).

The equation of state of the magnetized neutron matter, P, can be simply obtained by using

$$P = \rho^2 \frac{\partial F}{\partial \rho}. \tag{10}$$

Here, the Maxwell term $B^2/8\pi$ contribution from the pressure has been omitted. In Fig. 5, the pressure of the magnetized neutron matter has been presented as a function of density (ρ) for different values of the magnetic field strength (B). It can be seen that the pressure behaves as an increasing function of

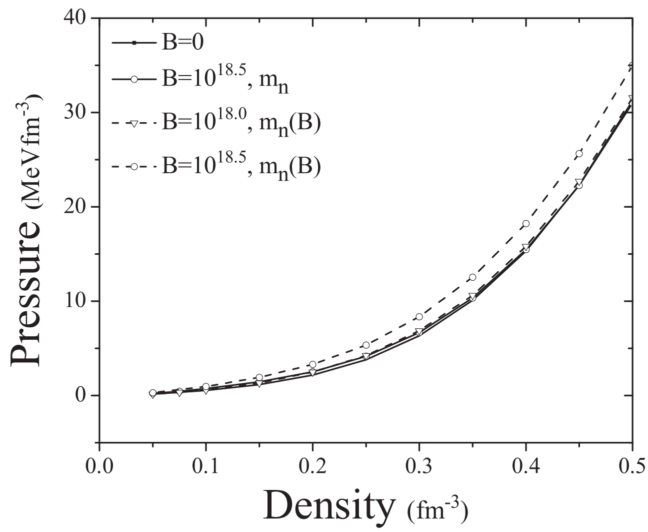

FIG. 5. Same as in Fig. 2 but for pressure.

the magnetic field strength. However, in the case of a constant mass the effects of the magnetic field on the pressure of the neutron matter is not considerable at a value of $B \leqslant 10^{18.5}$ G. For $B = 10^{18.5}$ G the pressure is indistinguishable from that of the field free case ($B = 0$), while the MFD neutron mass has appreciable effects on the equation of state. The pressure of the magnetized neutron matter increases considerably when $m_n(B)$ is used. This implies that the MFD neutron mass leads to a more stiff equation of state compared to the constant neutron mass.

IV. SUMMARY AND CONCLUSIONS

In this paper, the influence of neutron mass reduction due to the magnetic field on the equation of state and other properties of magnetized neutron matter have been studied. Here, the lowest-order constrained variational method has been used for calculating the free energy and the equation of state at different magnetic field and densities. It is seen that the decreasing of the neutron mass due to applying the magnetic field leads to a more stiff equation of state. It is also seen that the magnetic-field-dependent neutron mass has appreciable effects on the spin polarization and magnetization of the magnetized neutron matter.

ACKNOWLEDGMENT

I thank the University of Zanjan Research Council.

[1] R. C. Duncan and C. Thompson, Astrophys. J. **392**, L9 (1992).
[2] D. Lai and S. L. Shapiro, Astrophys. J. **383**, 745 (1991).
[3] A. Broderik, M. Prakash, and J. M. Lattimer, Astrophys. J. **537**, 351 (2000).
[4] V. V. Skokov, A. Y. Illarionov, and V. D. Toneev, Int. J. Mod. Phys. A **24**, 5925 (2009).
[5] M. A. Andreichikov, B. O. Kerbikov, V. D. Orlovsky, and Yu. A. Simonov, Phys. Rev. D **89**, 074033 (2014).
[6] C. Cardall, M. Prakash, and J. M. Lattimer, Astrophys. J. **554**, 322 (2001).
[7] M. Á. Pérez-García, Phys. Rev. C **77**, 065806 (2008).
[8] A. A. Isayev and J. Yang, Phys. Rev. C **80**, 065801 (2009).
[9] A. A. Isayev and J. Yang, Phys. Rev. C **84**, 065802 (2011).
[10] M. Á. Pérez-García, J. Navarro, and A. Polls, Phys. Rev. C **80**, 025802 (2009).
[11] G. H. Bordbar, Z. Rezaei, and A. Montakhab, Phys. Rev. C **83**, 044310 (2011).
[12] M. Á. Pérez-García, C. Providência, and A. Rabhi, Phys. Rev. C **84**, 045803 (2011).
[13] M. Bigdeli, Phys. Rev. C **85**, 034302 (2012).
[14] A. Rabhi, M. A. Pérez-García, C. Providência, and I. Vidaña, Phys. Rev. C **91**, 045803 (2015).
[15] M. Bigdeli, Phys. Rev. C **82**, 054312 (2010).
[16] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
[17] H. B. Callen, *Thermodynamics* (Wiley, New York, 1960).
[18] J. W. Clark and N. C. Chao, Lett. Nuovo Cimento **2**, 185 (1969).
[19] G. H. Bordbar and M. Modarres, J. Phys. G **23**, 1631 (1997).
[20] M. Bigdeli, G. H. Bordbar, and Z. Rezaei, Phys. Rev. C **80**, 034310 (2009).
[21] E. J. Ferrer, V. de la Incera, J. P. Keith, I. Portillo, and P. L. Springsteen, Phys. Rev. C **85**, 039802 (2012).