PHYSICAL REVIEW C **95**, 014901 (2017)

Multiplicity fluctuation and correlation of identified baryons in a quark combination model

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(Received 13 October 2016; revised manuscript received 2 December 2016; published 3 January 2017)

The dynamical multiplicity fluctuations and correlations of identified baryons and antibaryons produced by the hadronization of a bulk quark system are systematically studied in a quark combination model. Starting from the most basic dynamics of the quark combination which is necessary for multiplicity study, we analyze moments (variance, skewness, and kurtosis) of inclusive multiplicity distributions of identified baryons, two-baryon multiplicity correlations, and baryon-antibaryon multiplicity correlations after the hadronization of a quark system with given quark number and antiquark number. We obtain a series of interesting results, e.g., binomial behavior of multiplicity moments, coinciding flavor-dependent two-baryon correlation, and universal baryon-antibaryon correlation, which can be regarded as general features of the quark combination. We further take into account correlations and fluctuations of quark numbers before hadronization and study their influence on multiple production of baryons and antibaryons. We find that quark number fluctuations and flavor conservation lead to a series of important results such as the negative $p\bar{\Omega}^+$ multiplicity correlation and universal two-baryon correlations. We also study the influence of resonance decays in order to compare our results with future experimental data in ultrarelativistic heavy ion collisions at the Large Hadron Collider.

DOI: 10.1103/PhysRevC.95.014901

I. INTRODUCTION

In ultrarelativistic heavy ion collisions, a new state of matter—quark gluon plasma (QGP)—is created at the early stage of collisions. The produced QGP expands, cools, and changes into a hadronic system at a critical energy density [1]. Because of the nonperturbative difficulty of quantum chromodynamics, the transition from QGP to hadrons (i.e., hadronization) can only be described currently by phenomenological models such as statistical hadronization models [2,3] and quark (re)combination/coalescence models [4–10]. These models have been tested against the available experimental data of hadronic yields, momentum spectra, and flows.

Dynamical correlations and fluctuations of multihadron production carry more sophisticated hadronization dynamics. They are quantified by various covariances and moments on multiplicities or momenta of identified hadrons, and are measured in experiments via an event-by-event method. Their studies can further test those existing phenomenological models of hadron production at hadronization and yield deep insights into the dynamics of a realistic hadronization process. We can also obtain information on the correlations and fluctuations of quarks and antiquarks just before hadronization by studying their projection on hadronic observables. On the other hand, study of identified hadrons is also helpful for the investigation of correlations and fluctuations of conservative charges, which has been a hot topic in both experimental and theoretical studies recently [11–15]. There, one should know how the conservative charges populate in various identified hadrons, which depends on their coherent abundances and thus is directly related to their multiple production dynamics at hadronization.

In the past few years, only data on fluctuations of the pion, kaon, and proton have been reported [16–19], and the available theoretical studies are usually based on statistical models [20–26]. With the improvement of statistics and experimental measurement precision, observation of more hadron species such as Λ, Ξ^-, and Ω^- can be expected in the near future. Therefore, the corresponding theoretical predictions by different hadron production models are necessary, in order to guide the experimental data analysis, reveal the underlying dynamics of the observation, and test these models.

In this paper, we study the multiplicity fluctuations and correlations of various identified baryons and antibaryons produced directly by hadronization. We focus on the $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons in the flavor SU(3) ground state, with particular emphasis on various strange baryons. There are obvious advantages in measuring these baryons: (1) The baryon is a sensitive probe of the hadron production mechanism at hadronization. (2) The rapidity shift in baryon productions and resonance decays is small, which is suitable for experimental observation at finite rapidity window size.

We use the quark combination mechanism (QCM) to describe the production of hadrons at quark system hadronization. QCM has been used to reproduce lots of low and intermediate transverse momentum data at the Relativistic Heavy Ion Collder (RHIC) and the Large Hadron Collider (LHC), in particular the data of yields and rapidity distributions [10,27–29]. The related entropy and pion production issues have been extensively addressed in the literature [30–33]. Explaining fluctuations and/or correlations of hadron production is very intuitive in QCM. When a quark hadronizes, it can come into either a baryon or a meson, which leads to the fluctuation of global baryon multiplicity; it can come into either a specific baryon (e.g., a proton for a u quark hadronization) or another specific baryon (e.g., a Δ^+), which leads to the multiplicity fluctuations of the proton and Δ^+ and also an

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anticorrelation between two baryons. In addition, correlations and fluctuations of quarks and antiquarks will pass to hadrons after hadronization.

Concretely, we calculate various moments of inclusive multiplicity distributions of baryons, e.g., variance, skewness and kurtosis, the correlations between two baryons, and correlations between baryons and antibaryons. We analyze the dominant dynamics among these correlations and fluctuations and give predictions of QCM which can be tested by future experimental data. This paper mainly discusses baryon production at zero baryon number density at LHC, and the extension to RHIC energies and the meson sector is the goal of future work.

The paper is organized as follows. In Sec. II, we introduce a working model which includes the necessary dynamics of QCM for multiplicity study and discuss the dynamical sources of the multiplicity correlations and fluctuations in baryon production. In Sec. III, we study multiplicity fluctuations and correlations of baryons and antibaryons which are produced from a quark system with given numbers of quarks and antiquarks. In Sec. IV, we take into account fluctuations and correlations of quark numbers before hadronization to study their influence on baryon and antibaryon production. In Sec. V, we further take into account effects of resonance decays. A summary and discussion are given in Sec. VI.

II. A WORKING MODEL

Due to the difficulty of nonperturbative QCD, a widely accepted theoretical framework of QCM has not yet been established that can self-consistently describe the whole picture of hadronization dynamics. In this paper, we need a working model which includes the necessary dynamics of QCM for multiplicity study and to obtain correlations and fluctuations of the produced hadrons. We will present the assumptions and/or inputs explicitly whenever necessary and make the study as independent of the particular model as possible. Because there are no relevant works in the literature, the purpose of this paper is to focus on results of the most basic QCM dynamics, which will serve as a preliminary test of the model using future experimental data and a baseline for sophisticated hadronization dynamics.

We consider a system consisting of various quarks and antiquarks with constituent masses, corresponding to the "dressed" quarks and antiquarks in the nonperturbative QCD regime. We denote the number of quarks of flavor q_i in the system by N_{q_i} and that of antiquarks by $N_{\bar{q}_i}$. Three flavors, up, down, and strange, are considered in this paper. As the system hadronizes, these quarks and antiquarks combine with each other to form color singlet hadrons. Finally, the system produces, in an event, various hadrons with numbers $\{N_{h_i}\}$ where $i=\pi,K,\rho,K^*,\ldots,p,\Lambda,\Xi,\Omega^-$ up to all included hadron species. Here, we consider only the ground state $J^P=0^-$ and 1^- mesons and $J^P=\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons in the flavor SU(3) group. The numbers of these hadrons are varied event-by-event around their average values and follow a certain distribution $\mathcal{P}(\{N_{h_i}\};\{N_{q_j},N_{\bar{q}_j}\})$ which is governed by hadronization dynamics.

The precise form of $\mathcal{P}(\{N_{h_i}\};\{N_{q_j},N_{\bar{q}_j}\})$ depends on the full knowledge of hadronization dynamics. Among all the available QCM models, few can give specific solutions of $\mathcal{P}$. In addition, the high dimensionality feature of $\mathcal{P}$ makes the analytic solution quite difficult to get. In this paper, we generalize the quark combination simulation in SDQCM [10] to focus only on multiplicity properties of the produced hadrons and obtain $\mathcal{P}(\{N_{h_i}\};\{N_{q_j},N_{\bar{q}_j}\})$, considering that this model has reproduced lots of experimental data of multiplicities of various hadrons in relativistic heavy ion collisions at different energies [10,27–29].

The main idea of the quark combination simulation in SDQCM is as follows: (1) Assign all quarks and antiquarks in the system into an abstract one-dimensional sequence. The relative distance between any two quarks and/or antiquarks in the sequence represents their map in realistic phase space. (2) Combine these quarks and antiquarks in the sequence into hadrons according to a quark combination rule (QCR). A schematic example is as follows:

$$\begin{aligned}&q_1\bar{q}_2\bar{q}_3\bar{q}_4\bar{q}_5q_6\bar{q}_7q_8q_9q_{10}\bar{q}_{11}q_{12}q_{13}q_{14}\bar{q}_{15}q_{16}q_{17}\bar{q}_{18}\bar{q}_{19}\bar{q}_{20}\\&\overset{\text{QCR}}{\to} M(q_1\bar{q}_2)\,\overline{B}(\bar{q}_3\bar{q}_4\bar{q}_5)\,M(q_6\bar{q}_7)\,M(q_8\bar{q}_{11})\,B(q_9q_{10}q_{12})\\&\quad\times M(q_{13}\bar{q}_{15})\,B(q_{14}q_{16}q_{17})\,\overline{B}(\bar{q}_{18}\bar{q}_{19}\bar{q}_{20}).\end{aligned}\tag{1}$$

QCR depends on the combination dynamics. As shown directly by the above example, QCR should first satisfy two basic dynamics: (1) Baryon formation is by the combination of three quarks which are close to each other in phase space, and meson formation is by a quark and an antiquark. Therefore, a neighboring or next-nearest neighboring quark combination in the sequence is needed. (2) After hadronization, there are no free quarks and antiquarks left.

Considering the fact that the produced baryons are much fewer than mesons after hadronization, the key content of QCR is how to describe the production of baryons relative to that of mesons for a given quark configuration. We adopt the following procedure. For the local quark populations such as $q\bar{q}$ and $qq\bar{q}$, we can assign $q\bar{q}\to M$ and $qq\bar{q}\to M+q$ with relative probability 1. When the case of possible baryon production qqq occurs, we give a probability or conditional criterion. If the nearest neighbor of qqq is still a quark, the opportunity of baryon formation should be significantly increased, and we can assign $qqqq\to B+q$ with relative probability 1. In contrast, if the nearest neighbor of qqq is an antiquark $\bar{q}$, then this $\bar{q}$ can have the chance of capturing one q to form a meson and two quarks are left to combine with other quarks and antiquarks. We denote the probability of this channel by $P_{qqq\bar{q}\to M+qq}\equiv P_0$. The baryon formation probability in the $qqq\bar{q}$ configuration is then $P_1\equiv P_{qqq\bar{q}\to B+\bar{q}}=1-P_0$.

A naive analysis gives $P_0/P_1\sim(3\times\frac{1}{9})/(1\times\frac{1}{27})=9$ where the factor 3 is the number of the possible combinations for meson formation in the $qqq\bar{q}$ configuration, and factor 1 is for baryon formation. Factors $\frac{1}{9}$ and $\frac{1}{27}$ are the color weights of forming a color singlet meson and baryon in the stochastically colored quark combination, respectively. Therefore, baryon formation probability P_1 in the $qqq\bar{q}$ case should be a small value $\sim$0.1. In practice, a value of about 0.04 for P_1 can well explain the observed baryon yields in relativistic heavy ion collisions.

The above consideration of baryon formation is one kind of nonisolation approximation for the quark combination process; i.e., baryon formation is nontrivially influenced by the environment (the surrounding quarks and antiquarks). It is different from those (re)combination/coalescence models which were popular in early RHIC experiments [5–9]. They apply the sudden hadronization (i.e., isolation) approximation for the combination probability by the overlap between quark wave functions and the hadron they form.

The remaining quarks and antiquarks in $qq\bar{q} \to M + q$, $qqq\bar{q} \to M + qq$, and $qqq\bar{q} \to B + \bar{q}$ processes will subsequently combine with the following quarks and antiquarks in the sequence to form hadrons, until at last all quarks and antiquarks are combined into hadrons. This procedure reflects, to a certain extent, the spread of the hadronization in space-time.

Another point of QCR is the order of the combination. As long as the quark number is large, different orders such as from left to right, from right to left, and from middle to sides are equivalent and give the same result.

Based on the above discussions, we give the following combination algorithm for the hadronization of a quark system:

(i) Start from the first parton (q or $\bar{q}$) in the sequence.
(ii) If the first and second partons are either $\bar{q}q$ or $q\bar{q}$, they combine into a meson and are removed from the sequence, then go back to (i). If the first two are qq or $\bar{q}\bar{q}$, then go to the next.
(iii) Look at the third parton. If three partons are $qq\bar{q}$ or $\bar{q}\bar{q}q$, and the first and third partons combine into a meson and are removed from the sequence, then go back to (i). If three partons are qqq or $\bar{q}\bar{q}\bar{q}$, then go to the next.
(iv) Look at the fourth parton. If four partons are $qqqq$ or $\bar{q}\bar{q}\bar{q}\bar{q}$, the first three partons combine into a baryon or an antibaryon and are removed from the sequence, then go back to (i). If four partons are $qqq\bar{q}$ or $\bar{q}\bar{q}\bar{q}q$, there are two choices: (a) The first and fourth partons combine into a meson with probability P_0 and are removed from the sequence, then go back to (ii). (b) The first three partons combine into a baryon or an antibaryon with probability P_1 and are removed from the sequence; then go back to (i).

The above algorithm does not differentiate quark flavors in consideration of the flavor blind of strong interactions. Compared with the combination rule in Ref. [10], this algorithm addresses more explicitly the baryon production by the addition of step (iv) to better tune baryon meson production competition. In essence, it can be regarded as the generalization of the combination rule in Ref. [10] in a multiplicity description of the produced baryons.

For a given $q_1\bar{q}_2$ which is known to form a meson by the above combination algorithm, it can form either a $J^p = 1^-$ vector (V) meson or a $J^p = 0^-$ pseudoscalar (PS) meson. Similarly, a $q_1q_2q_3$ (except for three identical qqq case) can form either a $J^p = (\frac{1}{2})^+$ baryon or a $J^p = (\frac{3}{2})^+$ baryon. Following previous works [10,28], we use the parameter $R_{V/P}$ to denote the relative production ratio of vector mesons to pseudoscalar mesons and $R_{O/D}$ the ratio of octet baryons to decuplet baryons. Then we get the branch ratio of each hadronization channel for a $q_1\bar{q}_2$ combination,

$$C_{M_j} = \begin{cases} 1/(1+R_{V/P}) & \text{for } J^P = 0^- \text{ mesons,} \\ R_{V/P}/(1+R_{V/P}) & \text{for } J^P = 1^- \text{ mesons,} \end{cases}$$

and for a $q_1q_2q_3$ combination,

$$C_{B_j} = \begin{cases} R_{O/D}/(1+R_{O/D}) & \text{for } J^P = (1/2)^+ \text{ baryons,} \\ 1/(1+R_{O/D}) & \text{for } J^P = (3/2)^+ \text{ baryons.} \end{cases}$$

As in previous works, we can apply the above combination algorithm to relativistic heavy ion collisions by considering some properties of the produced quark system. It is observed that (1) the longitudinal expansion is predominant both in momentum space and in spatial space; (2) the longitudinal velocity of quarks is closely correlated to their spatial position; (3) the rapidity density of quark numbers is very large and is relatively slowly varied. Therefore, we can sort all quarks and antiquarks according to their rapidities into a one-dimensional sequence, and then combine neighboring quarks and antiquarks into hadrons. In the transverse direction, the transverse momentum (p_T) distribution of quarks is exponential decreased. Therefore, one cannot directly combine neighboring quarks because their relative intervals Δp_T exponentially increase with p_T of quarks/antiquarks. So we use the statistical combination approach; i.e., the p_T distribution of a hadron is the convolution of quark p_T distributions and combination kernel, where the combination kernel is mainly dependent on Δp_T between two quarks/antiquarks. It is thus similar to those inclusive recombination/coalescence approaches using the hadron wave function [5–9]. But our model is different from those inclusive methods in the proper treatment of unitarity in hadronization and the ability to well explain hadronic yield and longitudinal rapidity distributions observed in relativistic heavy ion collisions [10,27–29].

Let us summarize the origin of correlations and fluctuations of the produced baryons and antibaryons. First, local qqq aggregation in phase space is stochastic for a system consisting of free quarks and antiquarks. Second, the $qqq \to B$ process is probabilistic under the surrounding noise (i.e., stochastic populated quarks and antiquarks in the neighborhood). Together with the branch ratio of a given $q_1q_2q_3$ to a specific hadron state, they lead to multiplicity fluctuations of the produced identified baryons. The conservation of baryon number in the quark combination process constrains the global production of baryons and antibaryons and also the production of identified baryons and their antiparticles. The production correlation between two baryons mainly comes from a so-called "exclusion" effect; i.e., once a quark enters into a B_i at hadronization it is consumed and therefore cannot be recombined into B_j. These effects lead to a nontrivial and complex multihadron multiplicity distribution $\mathcal{P}(\{N_{h_i}\};\{N_{q_j},N_{\bar{q}_j}\})$.

III. BARYON PRODUCTION FROM A GIVEN QUARK SYSTEM

In this section, we study fluctuations and correlations of baryons and antibaryons which are produced from a quark system with a given number of quarks and antiquarks. This enables us to learn more clearly the properties of baryon production from the quark combination process itself. Analytical results of various moments (mean, variance, skewness, kurtosis) of the inclusive multiplicity distributions of identified baryons are given first, according to the basic dynamics of the quark combination discussed in previous section. Then two-baryon multiplicity correlations, baryon-antibaryon correlations, and multibaryon multiplicity correlations are studied systematically.

A. Moments of multiplicity distributions of baryons

First, we discuss properties of inclusive multiplicity distributions of various identified baryons calculated from the above combination algorithm. As a demonstration, Fig. 1 shows multiplicity distribution of total baryons and those of identified p, Λ, Ξ^0, as a quark system with $N_q = N_{\bar{q}} = 500$ hadronizes. Here, the relative ratios of different quark flavors are set to be $N_u : N_d : N_s = 1 : 1 : 0.43$. We see that the distribution of total baryons is close to a Gaussian distribution while those of identified baryons are close to a Poisson distribution to a certain extent. In the following text, we study the production property of these identified baryons by analyzing moments of their multiplicity distributions.

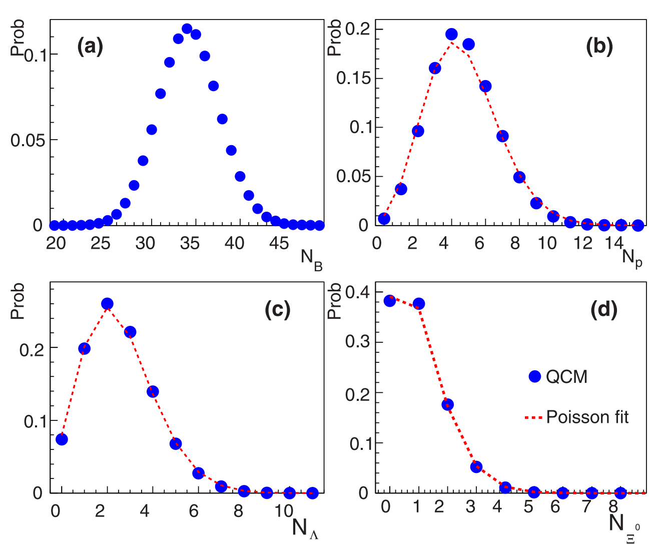

FIG. 1. Normalized multiplicity distribution of total baryons (a) and those of identified baryons p (b), Λ (c), Ξ^0 (d) produced by hadronization of a quark system with $N_q = N_{\bar{q}} = 500$. Here, the relative ratios of different quark flavors are set to be $N_u : N_d : N_s = 1 : 1 : 0.43$. Symbols are numerical results of the QCM algorithm in Sec. II and the dashed line is a Poisson distribution.

For the average multiplicity of identified baryons,

$$\overline{N}_{B_j} = \sum_{\{N_{h_i}\}} N_{B_j} \mathcal{P}(\{N_{h_i}\}; \{N_{q_i}, N_{\bar{q}_i}\}), \tag{2}$$

we have obtained the empirical solution in previous studies [10,28]

$$\overline{N}_{B_j} = P_{B_j} \overline{N}_B, \tag{3}$$

where $\overline{N}_B = \sum_i \overline{N}_{B_i}$ is the average number of total baryons and P_{B_j} denotes the production weight of B_j in all baryons. P_{B_j} can be decomposed to $C_{B_j} P_{q_1q_2q_3,B}$ where $P_{q_1q_2q_3,B}$ is the probability that, as a baryon is known to be produced, the flavor content of this baryon is $q_1q_2q_3$. Considering that every q_1, q_2, and q_3 in the system can have the chance of entering into B_j at hadronization, we get $P_{q_1q_2q_3,B} = N_{B_j}^{(q)}/N_{qqq}$. $N_{qqq} = N_q(N_q - 1)(N_q - 2)$ is the possible total number of three-quark combinations where $N_q = \sum_f N_f$ is the total quark number in the system. $N_{B_j}^{(q)} = N_{\text{iter}} \prod_f \prod_{i=1}^{n_{f,B_j}} (N_f - i + 1)$ is the possible number of $q_1q_2q_3$ combinations, where n_{f,B_j} is the number of valance quarks f contained in hadron B_j. Here index f runs over all quark flavors. N_{iter} is the iteration factor, taken to be 1, 3, and 6 for the case of three identical flavors, two different flavors, and three different flavors contained in a baryon, respectively.

We have used Eq. (3) to reproduce the experimental data of yields and yield ratios of various identified baryons in relativistic heavy ion collisions at different collision energies [10,27–29]. For detailed discussions of the average yield formula of identified baryons as well as those of antibaryons we refer readers to Refs. [27–29]. We argue that, just based on the good performance of the combination algorithm in Sec. II on the event-average yields, we make a further test of fluctuations and correlations in this paper.

We further study the variance, skewness, and kurtosis of the multiplicity distribution for various identified baryons. Their definitions are

$$\begin{aligned}\overline{\sigma}^2_{B_j} &= \overline{\delta N^2_{B_j}} = \overline{(N_{B_j} - \overline{N}_{B_j})^2} \\ &= \sum_{\{N_{h_i}\}} (N_{B_j} - \overline{N}_{B_j})^2 \mathcal{P}(\{N_{h_i}\}; \{N_{q_j}, N_{\bar{q}_j}\})\end{aligned} \tag{4}$$

and similarly

$$\overline{S}_{B_j} = \frac{\overline{\delta N^3_{B_j}}}{\overline{\sigma}^3_{B_j}}, \quad \overline{K}_{B_j} = \frac{\overline{\delta N^4_{B_j}}}{\overline{\sigma}^4_{B_j}} - 3. \tag{5}$$

Note that we always use an *overline* to denote the average hadronic quantities by hadronization of a given quark system.

To analyze their properties, we have to consider joint production of multibaryons. Taking variance, for example, two-B_j pair production is given by

$$\overline{N_{B_j}(N_{B_j} - 1)} = P_{2B_j} \overline{N_B(N_B - 1)}, \tag{6}$$

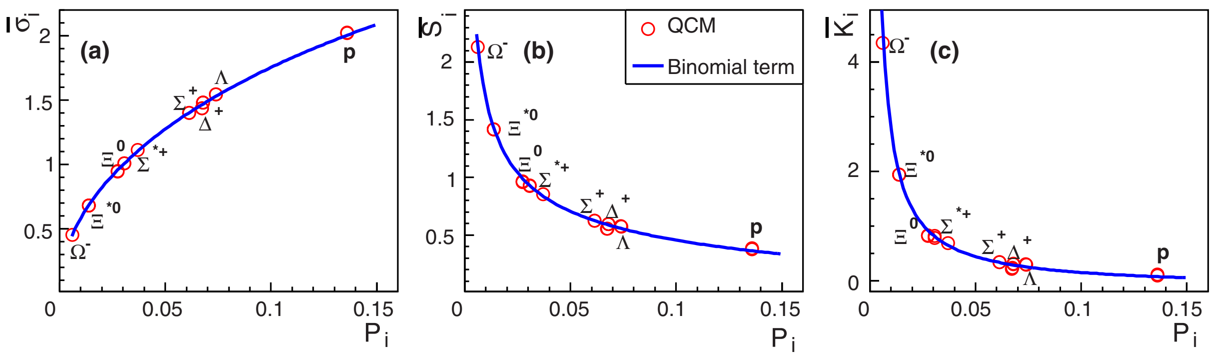

FIG. 2. The square root of variance, skewness, and kurtosis of the multiplicity distributions of various identified baryons with respect to their production weights $P_i = \overline{N}_{B_i}/\overline{N}_B$. The size of the quark system before hadronization is chosen to be $N_q = N_{\bar{q}} = 500$ and the relative ratios of different quark flavors are set to be $N_u : N_d : N_s = 1 : 1 : 0.43$. Symbols are full results and lines are leading terms of full results which have the form of a binomial distribution with parameters $(\overline{N}_B, P_i)$.

where the production probability of a two-B_j pair can be evaluated by $P_{2B_j} = C^2_{B_j} N^{(q)}_{2B_j}/N_{6q}$, with the number of six-quark clusters possible for two-B_j pair production $N^{(q)}_{2B_j} = N^2_{\text{iter}} \prod_f \prod_{i=1}^{2n_{f,B_j}} (N_f - i + 1)$ and that for any two-baryon pair production $N_{6q} = \prod_{i=1}^{6}(N_q - i + 1)$. Rewriting $P_{2B_j} = P^2_{B_j}(1 - A_1)$, we finally have

$$\overline{\sigma}^2_{B_j} = \overline{N}_B P_{B_j}(1-P_{B_j}) + P^2_{B_j}\left[(1-A_1)\overline{\sigma}^2_B - A_1 \overline{N}_B(\overline{N}_B - 1)\right] \tag{7}$$

where the first term in the right-hand side of the equation is the dominant part. Similarly, we have

$$\begin{aligned}\overline{S}_{B_j} = \frac{1}{\overline{\sigma}^3_{B_j}} \big\{ &\overline{N}_B P_{B_j}(1 - P_{B_j})(1 - 2P_{B_j}) + 3P^2_{B_j}\left[(1 - A_1)\overline{\sigma}^2_B - A_1\overline{N}_B(\overline{N}_B - 1)\right] \\ &+ P^3_{B_j}\left[(1 - A_2)\overline{S}_B\overline{\sigma}^3_B + 3(A_1 - A_2)\overline{N}_B\overline{\sigma}^2_B - 3(1 - A_2)\overline{\sigma}^2_B + \overline{N}_B(\overline{N}_B - 1)[(3A_1 - A_2)\overline{N}_B + 2A_2]\right]\big\},\end{aligned} \tag{8}$$

and

$$\begin{aligned}\overline{K}_{B_j} + 3 = \frac{1}{\overline{\sigma}^4_{B_j}} \big\{ &\overline{N}_B P_{B_j}(1 - P_{B_j})\left[1 - 6P_{B_j}(1 - P_{B_j}) + 3\overline{N}_B P_{B_j}(1 - P_{B_j})\right] + 7P^2_{B_j}\left[(1 - A_1)\overline{\sigma}^2_B - A_1\overline{N}_B(\overline{N}_B - 1)\right] \\ &+ 6P^3_{B_j}\left[(1 - A_2)\overline{S}_B\overline{\sigma}^3_B + \left[(1 - 3A_2 + 2A_1)\overline{N}_B - 3(1 - A_2)\right]\overline{\sigma}^2_B + \overline{N}_B(\overline{N}_B - 1)[(2A_1 - A_2)\overline{N}_B + 2A_2]\right] \\ &+ P^4_{B_j}\big[(1 - A_3)(K_B + 3)\overline{\sigma}^4_B + \left[4(A_2 - A_3)\overline{N}_B - 6(1 - A_3)\right]S_B\overline{\sigma}^3_B \\ &+ \left[(12A_2 - 6A_3 - 6A_1)\overline{N}^2_B + (18A_3 - 12A_2 - 6)\overline{N}_B + 11(1 - A_3)\right]\overline{\sigma}^2_B \\ &+ (4A_2 - 6A_1 - A_3)\overline{N}^4_B + (6A_1 + 6A_3 - 12A_2)\overline{N}^3_B + (8A_2 - 11A_3)\overline{N}^2_B + 6A_3\overline{N}_B\big]\big\},\end{aligned} \tag{9}$$

where three coefficients A_1, A_2, and A_3 are

$$A_L = 1 - \prod_{k=1}^{L}\left(\frac{\prod_f \prod_{i=1}^{n_{f,B_j}}\left(1 - k\frac{n_{f,B_j}}{N_f - i + 1}\right)}{\prod_{m=1}^{3}\left(1 - k\frac{3}{N_q - m + 1}\right)}\right) \tag{10}$$

with $L = 1,2,3$.

In the above formulas of variance, skewness, and kurtosis, the first term in right-hand side of the equation is always the dominant part, and we find that it is just the result of a binomial distribution with parameters $(\overline{N}_B, P_{B_j})$. In Fig. 2, we plot $\overline{\sigma}_{B_j}$, $\overline{S}_{B_j}$, and $\overline{K}_{B_j}$ of various identified baryons as functions of their production weights P_{B_j}. Symbols are full results and lines are binomial distributions as leading approximations. The size of the quark system here is chosen to be $N_q = N_{\bar{q}} = 500$ and the relative ratios of different quark flavors are set to be $N_u : N_d : N_s = 1 : 1 : 0.43$. In addition, at large $\overline{N}_B$ and small P_{B_i}, the binomial distribution converges toward a Poisson distribution. For multistrange hyperons such as Ω and Ξ^*, their multiplicity distributions are well approximated by a Poisson distribution because of quite small production weights $\sim$0.01. However, multiplicity distributions of the proton and Λ cannot be well approximated by a Poisson distribution because of their relatively large production weights $\sim$0.1.

The multiplicity distribution of total baryons shows some slightly different properties from those of identified baryons. The variance of the total baryon multiplicity is proportional to system size via $\overline{\sigma}^2_B/\overline{N}_B \approx 0.35$ at current baryon-meson competition, and skewness is inversely proportional to system size via $\overline{S}_B\overline{N}^{1/2}_B \approx 0.37$. These properties are general expectations of a stochastic combination process. But proportional coefficients cannot to be explained in terms of the binomial distribution. This is easily understood. The number of quarks consumed by total baryon formation is about 20% of the total quark number in the system. This fact causes the deviation from the independent and stochastic feature of the binomial trial in each baryon production.

B. Two-baryon correlations

Production of two different kinds of baryons is usually anti-associated in the hadronization of quark system with fixed quark numbers, characterized by the negative covariances of their multiplicities. The multiplicity covariance is defined as

$$\overline{C}_{B_iB_j} = \overline{\delta N_{B_i}\delta N_{B_j}} = \overline{N_{B_i}N_{B_j}} - \overline{N_{B_i}}\,\overline{N_{B_j}}. \tag{11}$$

We consider two-baryon joint production,

$$\overline{N_{B_i}N_{B_j}} = P_{B_iB_j}\,\overline{N_B(N_B-1)}, \tag{12}$$

where the joint production probability of B_iB_j pair can be evaluated by $P_{B_iB_j} = N^{(q)}_{B_iB_j}/N_{6q}$ with the number of six-quark clusters possible for B_iB_j pair production $N^{(q)}_{B_iB_j} = N^i_{\text{iter}}N^j_{\text{iter}}\prod_f\prod_{i=1}^{n_{f,B_i}+n_{f,B_j}}(N_f-i+1)$ and that for any two-baryon pair production $N_{6q} = \prod_{i=1}^{6}(N_q-i+1)$.

Substituting Eqs. (2) and (12) into the covariance of two baryons, we get

$$\frac{\overline{C}_{B_iB_j}}{\overline{N}_{B_i}\overline{N}_{B_j}} = \frac{P_{B_iB_j}}{P_{B_i}P_{B_j}}\frac{\overline{N_B(N_B-1)}}{\overline{N}_B^2} - 1, \tag{13}$$

in which

$$\frac{P_{B_iB_j}}{P_{B_i}P_{B_j}} = \frac{\prod_f\prod_{k=1}^{n_{f,B_i}}\left(1-\frac{n_{f,B_j}}{N_f-k+1}\right)}{\prod_{m=1}^{3}\left(1-\frac{3}{N_q-m+1}\right)}$$
$$= 1-\sum_f n_{f,B_i}n_{f,B_j}\frac{1}{N_f}+\frac{9}{N_q}+O\left(N_q^{-2}\right), \tag{14}$$

where the product and summation of index f runs over all quark flavors and n_{f,B_i} is the number of valance quarks f contained in hadron B_i. Finally, we have

$$\frac{\overline{C}_{B_iB_j}}{\overline{N}_{B_i}\overline{N}_{B_j}} = -\sum_f\frac{n_{f,B_i}n_{f,B_j}}{N_f} - \left(\frac{1}{\overline{N}_B} - \frac{\overline{\sigma}_B^2}{\overline{N}_B^2} - \frac{9}{N_q}\right)$$
$$+O\left(N_q^{-2}\right). \tag{15}$$

The first part in the right-hand side of the equation is the leading order contribution. It essentially originates from the fact that, at hadronization, once a quark enters into a B_i it is consumed and therefore cannot recombine into B_j. This part is inversely proportional to the quark number of the coinciding flavor in two baryons, so the relative anticorrelation among strange baryons is usually greater than those of light flavor baryons. The part in parentheses is the next-to-leading order contribution, which is usually a few percent of the first part. It is negligible in correlations for most baryon pairs with coinciding valance quark content but becomes important for correlations

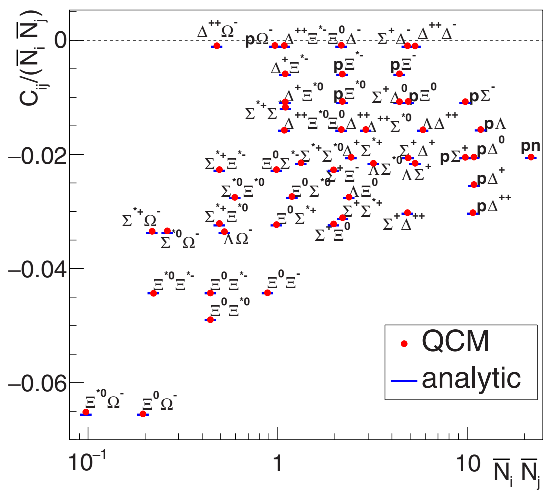

FIG. 3. Multiplicity covariance between two identified baryons. Different two-baryon pairs are distinguished in the horizontal axis by their multiplicity products. The size of the quark system before hadronization is chosen to be $N_q = N_{\bar{q}} = 500$ and the relative ratios of different quark flavors are set to be $N_u : N_d : N_s = 1 : 1 : 0.43$. Symbols are numerical results of the QCM algorithm in Sec. II and short solid lines are analytic results in Eq. (15).

between baryon pairs with totally different quark flavors, such as $C_{p\Omega^-}$, $C_{\Delta^{++}\Delta^-}$, etc.

In Fig. 3, we show results of the relative covariance $\overline{C}_{B_iB_j}/(\overline{N}_{B_i}\overline{N}_{B_j})$ of identified baryons produced from quark system hadronization with $N_q = N_{\bar{q}} = 500$. Here, the relative ratios of different quark flavors are set to be $N_u : N_d : N_s = 1 : 1 : 0.43$. Results of different two-baryon pairs are distinguished in the horizontal axis by their multiplicity products. Symbols are numerical results of the QCM algorithm in Sec. II and the short solid lines are analytic results in Eq. (15). As discussed above, we see that the production of $\Xi\Omega^-$ and other hyperon pairs which share more strange content is the most anti-associated while those containing coinciding light flavors are less anti-associated, such as pn. For $p\Omega^-$, $\Delta\Omega^-$, $\Xi^0\Delta^-$ etc, there is no coinciding flavor between two baryons but their productions are still anti-associated, although quite weakly. This is due to the second term in the right-hand side of Eq. (15); the physical origin is that the successive baryon production in the combination process is suppressed by the baryon number conservation.

C. Baryon-antibaryon correlations

It is generally expected that baryons and antibaryons are associated in their production, characterized by the positive covariance $\overline{C}_{B_i\bar{B}_j} = \overline{\delta N_{B_i}\delta N_{\bar{B}_j}} = \overline{N_{B_i}N_{\bar{B}_j}} - \overline{N_{B_i}}\,\overline{N_{\bar{B}_j}}$ of their multiplicities. One main reason for this association comes from the global baryon number conservation in hadronization which is denoted by the quark number conservation $N_B - N_{\bar{B}} = \frac{1}{3}(N_q - N_{\bar{q}})$ in the combination process. This causes the following correlation between baryon and antibaryon:

$$\overline{C}_{B_i\bar{B}_j} = p_{B_i}p_{\bar{B}_j}\,\overline{\delta N_B\,\delta N_{\bar{B}}} = p_{B_i}p_{\bar{B}_j}\,\overline{\sigma}_B^2, \tag{16}$$

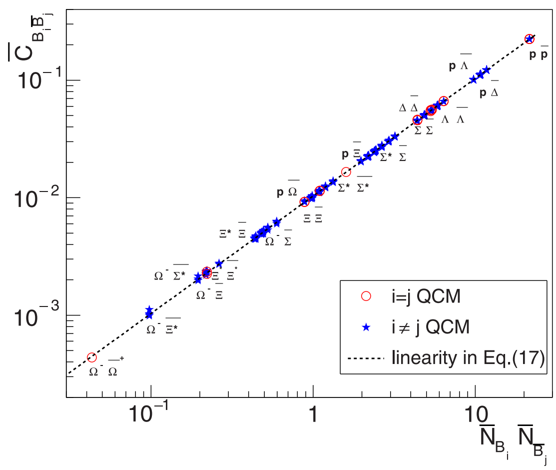

FIG. 4. Multiplicity covariance between identified baryons and antibaryons. Different baryon-antibaryon pairs are distinguished in the horizontal axis by their multiplicity products. The size of the quark system before hadronization is chosen to be $N_q = N_{\bar{q}} = 500$ and the relative ratios of different quark flavors are set to be $N_u : N_d : N_s = 1 : 1 : 0.43$. Symbols are numerical results of the QCM algorithm in Sec. II and the dashed line is the scaling given by Eq. (17).

where we use $\overline{\delta N_B\, \delta N_{\bar{B}}} = \bar{\sigma}^2_B = \bar{\sigma}^2_{\bar{B}}$ at fixed quark numbers. Inserting posterior production weight $p_{B_i} = \langle N_{B_i}\rangle/\langle N_B\rangle$, we get a scaling property

$$\frac{\overline{C}_{B_i \bar{B}_j}}{\overline{N}_{B_i}\overline{N}_{\bar{B}_j}} = \frac{\bar{\sigma}^2_B}{\overline{N}_B \overline{N}_{\bar{B}}} \tag{17}$$

for baryon-antibaryon multiplicity correlations.

In Fig. 4, we compare the above formula with numerical results obtained from the algorithm in Sec. II with the quark system $N_q = N_{\bar{q}} = 500$, in which the relative ratios of different quark flavors are set to be $N_u : N_d : N_s = 1 : 1 : 0.43$. The good agreement suggests that global baryon number conservation is the dominant reason for the production correlation between identified baryons and antibaryons. One interesting result is that both $\overline{C}_{B_i \bar{B}_i}$ and $\overline{C}_{B_i \bar{B}_j}$ $(i \neq j)$ follow the same scaling line, which indicates that the production of baryon-antibyaron pairs does not have a greater constraint than that of two different baryons. This is reasonable in the case of free combination of quarks and antiquarks. Using $\overline{N}_B/\overline{N}_M \approx 1/12$ and $N_s/N_u \approx 0.43$, which reproduce yield data in relativistic heavy ion collisions, we can estimate $\overline{N}_B \approx \frac{1}{6} N_u \approx \frac{1}{3} N_s$, which means the baryon number conservation is the strongest constraint in baryon-antibaryon joint production. The production of identified baryon B_i and antibaryon $\bar{B}_j$ consumes only a small fraction of total quarks and antiquarks and thus does not reach the conservation threshold of specific quark flavors.

D. Multibody correlations

Following a similar procedure, we also get multibaryon correlations due to the exclusion effect of successive baryon production discussed in Sec. III B and baryon number conservation in baryon-antibaryon production in Sec. III C. The three-baryon correlation is

$$\begin{aligned}\overline{C}_{\alpha\beta\gamma} &= \overline{\delta N_\alpha \delta N_\beta \delta N_\gamma}\\ &= \overline{N_\alpha N_\beta N_\gamma} - \overline{N}_\alpha \overline{N}_\beta \overline{N}_\gamma - \overline{N}_\alpha \overline{C}_{\beta\gamma} - \overline{N}_\beta \overline{C}_{\alpha\gamma} - \overline{N}_\gamma \overline{C}_{\alpha\beta}\end{aligned} \tag{18}$$

and the four-baryon correlation is

$$\begin{aligned}\overline{C}_{\alpha\beta\gamma\epsilon} &= \overline{\delta N_\alpha \delta N_\beta \delta N_\gamma \delta N_\epsilon}\\ &= \overline{N_\alpha N_\beta N_\gamma N_\epsilon} - \overline{N}_\alpha \overline{N}_\beta \overline{N}_\gamma \overline{N}_\epsilon - \overline{N}_\alpha \overline{C}_{\beta\gamma\epsilon} - \overline{N}_\beta \overline{C}_{\alpha\gamma\epsilon}\\ &\quad - \overline{N}_\gamma \overline{C}_{\alpha\beta\epsilon} - \overline{N}_\epsilon \overline{C}_{\alpha\beta\gamma} - \overline{N}_\alpha \overline{N}_\beta \overline{C}_{\gamma\epsilon} - \overline{N}_\alpha \overline{N}_\gamma \overline{C}_{\beta\epsilon}\\ &\quad - \overline{N}_\alpha \overline{N}_\epsilon \overline{C}_{\beta\gamma} - \overline{N}_\beta \overline{N}_\gamma \overline{C}_{\alpha\epsilon} - \overline{N}_\beta \overline{N}_\epsilon \overline{C}_{\alpha\gamma} - \overline{N}_\gamma \overline{N}_\epsilon \overline{C}_{\alpha\beta}.\end{aligned} \tag{19}$$

The average multiplicity product of three baryons ($\alpha\beta\gamma \in$ baryon) can be written as

$$\begin{aligned}\overline{N_\alpha N_\beta N_\gamma} = &\ (1 - A_{\alpha\beta\gamma})\, \overline{N}_\alpha \overline{N}_\beta \overline{N}_\gamma\\ &\times \frac{\overline{\delta N^3_B} + 3\bar{\sigma}^2_B(\overline{N}_B - 1) + \overline{N}_B(\overline{N}_B - 1)(\overline{N}_B - 2)}{\overline{N}^3_B}\\ &+ \delta_{\alpha\beta}(1 - \delta_{\alpha\gamma})(\overline{C}_{\alpha\gamma} + \overline{N}_\alpha \overline{N}_\gamma)\\ &+ (\delta_{\alpha\gamma} + \delta_{\beta\gamma})(1 - \delta_{\alpha\beta})(\overline{C}_{\alpha\beta} + \overline{N}_\alpha \overline{N}_\beta)\\ &+ \delta_{\alpha\beta}\, \delta_{\alpha\gamma}\left[3\bar{\sigma}^2_\alpha + 3\overline{N}_\alpha(\overline{N}_\alpha - 1) + \overline{N}_\alpha\right],\end{aligned} \tag{20}$$

where $\delta_{\alpha\beta}$ is Kronecker delta function and $\overline{\delta N^3_B} \equiv \overline{S}_B \bar{\sigma}^3_B$ is the third moment of total baryons. For that of two baryons and one antibaryon, e.g., $\alpha\beta \in$ baryon, $\bar{\gamma} \in$ antibaryon, we have

$$\overline{N_\alpha N_\beta N_{\bar{\gamma}}} = (1 - A_{\alpha\beta})\, \overline{N}_\alpha \overline{N}_\beta \overline{N}_{\bar{\gamma}} \times \frac{\overline{\delta N^3_B} + \bar{\sigma}^2_B(3\overline{N}_B + 2c - 1) + \overline{N}_B(\overline{N}_B - 1)\overline{N}_{\bar{B}}}{\overline{N}^2_B \overline{N}_{\bar{B}}} + \delta_{\alpha\beta}(\overline{C}_{\alpha\bar{\gamma}} + \overline{N}_\alpha \overline{N}_{\bar{\gamma}}). \tag{21}$$

Here, $c = \overline{N}_B - \overline{N}_{\bar{B}}$ is the number of net baryons and was taken to be zero at LHC.

The average multiplicity product of four baryons ($\alpha\beta\gamma\epsilon \in$ baryon) can be written as

$$\begin{aligned}\overline{N_\alpha N_\beta N_\gamma N_\epsilon} = &\ (1 - A_{\alpha\beta\gamma\epsilon})\, \overline{N}_\alpha \overline{N}_\beta \overline{N}_\gamma \overline{N}_\epsilon \frac{1}{\overline{N}^4_B} \times \left\{\overline{\delta N^4_B} + (4\overline{N}_B - 6)\overline{\delta N^3_B} + \bar{\sigma}^2_B\left(6\overline{N}^2_B - 18\overline{N}_B + 11\right)\right.\\ &\left. + \overline{N}_B(\overline{N}_B - 1)(\overline{N}_B - 2)(\overline{N}_B - 3)\right\} + \delta_{\alpha\beta}\delta_{\alpha\gamma}\delta_{\alpha\epsilon}\left(6\overline{N^3_\alpha} - 11\overline{N^2_\alpha} + 6\overline{N}_\alpha\right) + \delta_{\alpha\beta}\delta_{\alpha\gamma}(1 - \delta_{\alpha\epsilon})\left(3\overline{N^2_\alpha N_\epsilon} - 2\overline{N_\alpha N_\epsilon}\right)\\ &+ \delta_{\alpha\beta}\delta_{\alpha\epsilon}(1 - \delta_{\alpha\gamma})\left(3\overline{N^2_\alpha N_\gamma} - 2\overline{N_\alpha N_\gamma}\right) + (\delta_{\alpha\gamma}\delta_{\alpha\epsilon} + \delta_{\beta\gamma}\delta_{\beta\epsilon})(1 - \delta_{\alpha\beta})\left(3\overline{N^2_\alpha N_\beta} - 2\overline{N_\alpha N_\beta}\right)\end{aligned}$$

$$+\delta_{\alpha\beta}\delta_{\gamma\epsilon}(1-\delta_{\alpha\gamma})\big(\overline{N_\alpha^2 N_\gamma}+\overline{N_\alpha N_\gamma^2}-\overline{N_\alpha N_\gamma}\big)+(\delta_{\alpha\gamma}\delta_{\beta\epsilon}+\delta_{\alpha\epsilon}\delta_{\beta\gamma})(1-\delta_{\alpha\beta})\big(\overline{N_\alpha^2 N_\beta}+\overline{N_\alpha N_\beta^2}-\overline{N_\alpha N_\beta}\big)$$
$$+\delta_{\alpha\beta}(1-\delta_{\alpha\gamma})(1-\delta_{\alpha\epsilon})(1-\delta_{\gamma\epsilon})\overline{N_\alpha N_\gamma N_\epsilon}+(\delta_{\alpha\gamma}+\delta_{\beta\gamma})(1-\delta_{\alpha\beta})(1-\delta_{\alpha\epsilon})(1-\delta_{\beta\epsilon})\overline{N_\alpha N_\beta N_\epsilon}$$
$$+(\delta_{\alpha\epsilon}+\delta_{\beta\epsilon}+\delta_{\gamma\epsilon})(1-\delta_{\alpha\beta})(1-\delta_{\alpha\gamma})(1-\delta_{\beta\gamma})\overline{N_\alpha N_\beta N_\gamma}, \tag{22}$$

where $\overline{\delta N_B^4}\equiv(\overline{K}_B+3)\overline{\sigma}_B^4$ is the fourth moment of total baryons. The average multiplicity product of three or two baryons can be read from Eqs. (20) and (12). For that of three baryons and one antibaryon, e.g., $\alpha\beta\gamma\in$ baryon, $\bar\epsilon\in$ antibaryon, we have

$$\overline{N_\alpha N_\beta N_\gamma N_{\bar\epsilon}}=(1-A_{\alpha\beta\gamma})\,\overline{N}_\alpha\overline{N}_\beta\overline{N}_\gamma\overline{N}_{\bar\epsilon}\,\frac{1}{\overline{N}_B^3\overline{N}_{\bar B}}\times\big\{\overline{\delta N_B^4}+(4\overline{N}_{\bar B}+3c-3)\overline{\delta N_B^3}+\overline{\sigma}_B^2\big[6\overline{N}_{\bar B}^2+9(c-1)\overline{N}_{\bar B}+3c^2-6c+2\big]$$
$$+\overline{N}_B(\overline{N}_B-1)(\overline{N}_B-2)\overline{N}_{\bar B}\big\}+\delta_{\alpha\beta}\delta_{\alpha\gamma}\big(3\overline{N_\alpha^2 N_{\bar\epsilon}}-2\overline{N_\alpha N_{\bar\epsilon}}\big)+\delta_{\alpha\beta}(1-\delta_{\alpha\gamma})\overline{N_\alpha N_\gamma N_{\bar\epsilon}}+(\delta_{\alpha\gamma}+\delta_{\beta\gamma})(1-\delta_{\alpha\beta})\overline{N_\alpha N_\beta N_{\bar\epsilon}}, \tag{23}$$

and for that of two baryons and two antibaryons, e.g., $\alpha\beta\in$ baryon, $\bar\gamma\bar\epsilon\in$ antibaryon, we have

$$\overline{N_\alpha N_\beta N_{\bar\gamma} N_{\bar\epsilon}}=(1-A_{\alpha\beta})(1-A_{\bar\gamma\bar\epsilon})\overline{N}_\alpha\overline{N}_\beta\overline{N}_{\bar\gamma}\overline{N}_{\bar\epsilon}\,\frac{1}{\overline{N}_B^2\overline{N}_{\bar B}^2}\times\big\{\overline{\delta N_B^4}+(4\overline{N}_{\bar B}+2c-2)\overline{\delta N_B^3}+\overline{\sigma}_B^2\big[6\overline{N}_{\bar B}^2+6(c-1)\overline{N}_{\bar B}+c^2-3c+1\big]$$
$$+\overline{N}_B(\overline{N}_B-1)\overline{N}_{\bar B}(\overline{N}_{\bar B}-1)\big\}+\delta_{\alpha\beta}\delta_{\bar\gamma\bar\epsilon}\big(\overline{N_\alpha^2 N_{\bar\gamma}}+\overline{N_\alpha N_{\bar\gamma}^2}+\overline{N_\alpha N_{\bar\gamma}}\big)+\delta_{\alpha\beta}(1-\delta_{\bar\gamma\bar\epsilon})\,\overline{N_\alpha N_{\bar\gamma} N_{\bar\epsilon}}+\delta_{\bar\gamma\bar\epsilon}\,(1-\delta_{\alpha\beta})\,\overline{N_\alpha N_\beta N_{\bar\gamma}}. \tag{24}$$

Coefficients $A_{\alpha\beta}$, $A_{\alpha\beta\gamma}$, and $A_{\alpha\beta\gamma\epsilon}$ are extensions of Eq.(10),

$$A_{\alpha\beta\gamma\epsilon}=1-\frac{\prod_f\prod_{h=\beta}^{\epsilon}\prod_{i=1}^{n_{f,h}}\big(1-\frac{\sum_{h'=\alpha}^{h-1}n_{f,h'}}{N_f-i+1}\big)}{\prod_{k=1}^{n_h-1}\prod_{m=1}^{3}\big(1-k\frac{3}{N_q-m+1}\big)}. \tag{25}$$

Here n_h in the denominator denotes the number of involved baryons, i.e., $n_h=4$ for $\alpha\beta\gamma\epsilon$ and 3 for $\alpha\beta\gamma$. $n_{f,h}$ is the number of valance quarks of flavor f contained in hadron h. $h-1$ in the numerator denotes the hadron before h in combination $\alpha\beta\gamma\epsilon$. Taking the charge conjugation operation, we get coefficients of antibaryons.

We can check that the following normalization is satisfied:

$$\sum_{\alpha\beta\gamma\in B}\overline{C}_{\alpha\beta\gamma}=\overline{\delta N_B^3},$$
$$\sum_{\alpha\beta\gamma\epsilon\in B}\overline{C}_{\alpha\beta\gamma\epsilon}=\overline{\delta N_B^4}, \tag{26}$$

and

$$\sum_{\alpha\beta\gamma\in B,\bar B}(-1)^m\overline{C}_{\alpha\beta\gamma}=\overline{\delta(N_B-N_{\bar B})^3}=0,$$
$$\sum_{\alpha\beta\gamma\epsilon\in B,\bar B}(-1)^m\overline{C}_{\alpha\beta\gamma\epsilon}=\overline{\delta(N_B-N_{\bar B})^4}=0, \tag{27}$$

where m denotes the number of antibaryons in $\alpha\beta\gamma$ and $\alpha\beta\gamma\epsilon$ combinations.

IV. BARYON PRODUCTION FROM THE QUARK SYSTEM WITH VARIATIONAL QUARK NUMBERS

In this section, we take into account effects of fluctuations and correlations of quark numbers in the system before hadronization on multiple production of baryons and antibaryons. We first give the general procedure for including quark number fluctuations and correlations in hadronic observables and then show the specific formulas for moments and two-body correlations of baryons and antibaryons. Then we discuss properties of quark number fluctuations and correlations in the context of ultrarelativistic heavy ion collisions, and we show numerical results of baryon moments, two-baryon correlations, and baryon-antibaryon correlations.

A. General formulas for including variational quark numbers

The produced quark system in heavy ion collisions at a specific collision energy is always varied in size event-by-event, and the number of quarks and that of antiquarks in the system at hadronization should follow a certain distribution $\mathcal{P}(\{N_{q_i},N_{\bar q_i}\};\{\langle N_{q_i}\rangle,\langle N_{\bar q_i}\rangle\})$ around the event-average quark numbers $\langle N_{q_i}\rangle$ and antiquark numbers $\langle N_{\bar q_i}\rangle$, where $q_i=u$, d, s are considered in this paper. In QCM, the distribution includes also the possible contribution of small-amount dynamical production of newborn quarks and antiquarks during the hadronization process due to the requirement of exact energy conservation and entropy increase [33]. The event average of a hadronic physical quantity A_h is

$$\langle A_h\rangle=\sum_{\{N_{h_j}\}}A_h\mathcal{P}(\{N_{h_j}\};\{\langle N_{q_i}\rangle,\langle N_{\bar q_i}\rangle\})=\sum_{\{N_{q_i},N_{\bar q_i}\}}\sum_{\{N_{h_j}\}}A_h\,\mathcal{P}(\{N_{h_j}\};\{N_{q_i},N_{\bar q_i}\})\mathcal{P}(\{N_{q_i},N_{\bar q_i}\};\{\langle N_{q_i}\rangle,\langle N_{\bar q_i}\rangle\})$$
$$=\sum_{\{N_{q_i},N_{\bar q_i}\}}\overline{A}_h\,\mathcal{P}(\{N_{q_i},N_{\bar q_i}\};\{\langle N_{q_i}\rangle,\langle N_{\bar q_i}\rangle\}). \tag{28}$$

If $\overline{A}_h$ is known already, we can expand it as a Taylor series at the event average of quark numbers $\{\langle N_{q_i}\rangle,\langle N_{\bar{q}_i}\rangle\}$:

$$\overline{A}_h = \overline{A}_h\Big|_{\substack{\langle N_{q_i}\rangle\\ \langle N_{\bar{q}_i}\rangle}} + \sum_{f_1} \frac{\partial \overline{A}_h}{\partial N_{f_1}}\Bigg|_{\substack{\langle N_{q_i}\rangle\\ \langle N_{\bar{q}_i}\rangle}} \delta N_{f_1} + \frac{1}{2}\sum_{f_1,f_2} \frac{\partial^2 \overline{A}_h}{\partial N_{f_1}\partial N_{f_2}}\Bigg|_{\substack{\langle N_{q_i}\rangle\\ \langle N_{\bar{q}_i}\rangle}} \delta N_{f_1}\delta N_{f_2} + \frac{1}{3!}\sum_{f_1,f_2,f_3} \frac{\partial^3 \overline{A}_h}{\partial N_{f_1}\partial N_{f_2}\partial N_{f_3}}\Bigg|_{\substack{\langle N_{q_i}\rangle\\ \langle N_{\bar{q}_i}\rangle}} \delta N_{f_1}\delta N_{f_2}\delta N_{f_3} + O(\delta^4), \tag{29}$$

where indexes f_1, f_2, and f_3 run over all quark and antiquark flavors and $\delta N_{f_1} = N_{f_1} - \langle N_{f_1}\rangle$. The subscript $\langle N_{q_i}\rangle,\langle N_{\bar{q}_i}\rangle$ denotes the evaluation at the event average point. Substituting it into Eq. (28), we get

$$\langle A_h\rangle = \overline{A}_h\Big|_{\substack{\langle N_{q_i}\rangle\\ \langle N_{\bar{q}_i}\rangle}} + \frac{1}{2}\sum_{f_1,f_2} \frac{\partial^2 \overline{A}_h}{\partial N_{f_1}\partial N_{f_2}}\Bigg|_{\substack{\langle N_{q_i}\rangle\\ \langle N_{\bar{q}_i}\rangle}} C_{f_1f_2} + \frac{1}{3!}\sum_{f_1,f_2,f_3} \frac{\partial^3 \overline{A}_h}{\partial N_{f_1}\partial N_{f_2}\partial N_{f_3}}\Bigg|_{\substack{\langle N_{q_i}\rangle\\ \langle N_{\bar{q}_i}\rangle}} C_{f_1f_2f_3} + O(\delta^4), \tag{30}$$

where $C_{f_1f_2} = \langle \delta N_{f_1}\delta N_{f_2}\rangle$ and $C_{f_1f_2f_3} = \langle \delta N_{f_1}\delta N_{f_2}\delta N_{f_3}\rangle$ are two-body and three-body correlation functions of quarks and antiquarks, respectively. Then the influence of quark number distribution on hadronic quantities can be taken into account by the mean, two-body, and multibody correlations of quark numbers. In the following equations we drop the subscript $\langle N_{q_i}\rangle,\langle N_{\bar{q}_i}\rangle$ for convenience.

B. Formulas of identified baryons

Using Eq. (30), we first get the event average of baryon multiplicity

$$\langle N_{B_i}\rangle = \overline{N}_{B_i} + \frac{1}{2}\sum_{f_1,f_2}\frac{\partial^2 \overline{N}_{B_i}}{\partial N_{f_1}\partial N_{f_2}} C_{f_1f_2} + O\big(N_f^{-2}\big). \tag{31}$$

The effect of two-quark correlations on baryon multiplicity is the order of magnitude of $1/\langle N_f\rangle$, which is only a few percent of the leading term due to the large quark number (i.e., hundreds of quarks and antiquarks per unit rapidity at RHIC and LHC energies). The influence of three-body and four-body correlations of quarks and antiquarks is suppressed further by $1/N_f^2$. Therefore, effects of quark number correlations and fluctuations can be safely neglected in studies of inclusive multiplicities of identified hadrons in relativistic heavy ion collisions, as we did in previous works.

For moments of multiplicity distributions of identified hadrons, we have

$$\sigma^2_{B_i} = \overline{\sigma}^2_{B_i} + \sum_{f_1,f_2}\left(\partial_1\overline{N}_{B_i}\partial_2\overline{N}_{B_i} + \frac{1}{2}\partial_{12}\overline{\sigma}^2_{B_i}\right)C_{f_1,f_2} + O\big(N_f^{-2}\big), \tag{32}$$

$$S_{B_i} = \overline{S}_{B_i}\left\{1 + \sum_{f_1,f_2}\left[\frac{\partial_{12}\overline{\delta N^3_{B_i}} + 3\partial_1\overline{N}_{B_i}\partial_2\overline{\sigma}^2_{B_i} + 3\partial_2\overline{N}_{B_i}\partial_1\overline{\sigma}^2_{B_i}}{2\overline{\delta N^3_{B_i}}} - 3\frac{(\partial_1\overline{N}_{B_i})(\partial_2\overline{N}_{B_i}) + \frac{1}{2}\partial_{12}\overline{\sigma}^2_{B_i}}{2\overline{\sigma}^2_{B_i}}\right]C_{f_1f_2} + O\big(N_f^{-2}\big)\right\}, \tag{33}$$

$$K_{B_i} = \overline{K}_{B_i} + (\overline{K}_{B_i}+3)\left\{\sum_{f_1,f_2}\left[\frac{\partial_{12}\overline{\delta N^4_{B_i}} + 8\partial_1\overline{\delta N^3_{B_i}}\partial_2\overline{N}_{B_i} + 12\overline{\sigma}^2_{B_i}\partial_1\overline{N}_{B_i}\partial_2\overline{N}_{B_i}}{2\overline{\delta N^4_{B_i}}} - 2\frac{\partial_1\overline{N}_{B_i}\partial_2\overline{N}_{B_i} + \frac{1}{2}\partial_{12}\overline{\sigma}^2_{B_i}}{\overline{\sigma}^2_{B_i}}\right]C_{f_1f_2} + O\big(N_f^{-2}\big)\right\}. \tag{34}$$

Here, we have used $\partial_1 \equiv \frac{\partial}{\partial N_{f_1}}$ and $\partial_{12} \equiv \frac{\partial^2}{\partial N_{f_1}\partial N_{f_2}}$ for abbreviation. Because higher order contributions of quark correlations and fluctuations are usually suppressed by the factor $1/\langle N_f\rangle$, here we only show effects of second-order correlations and fluctuations of quark numbers on the directly produced baryons.

For two-body correlations of baryons and antibaryons, we have

$$C_{\alpha\beta} = \overline{C}_{\alpha\beta} + \frac{1}{2}\sum_{f_1,f_2}[2\partial_1\overline{N}_\alpha\partial_2\overline{N}_\beta + \partial_{12}\overline{C}_{\alpha\beta}]C_{f_1f_2} + O\big(N_f^{-2}\big). \tag{35}$$

Here, the contribution of second-order quark correlations is the same order as $\overline{C}_{\alpha\beta}$, and they might cancel with each other significantly. The influence of higher order contributions of quark correlations is about few percent at LHC and is neglected here. As $\alpha = \beta$, we obtain Eq. (32), which is also hardly influenced by higher order quark correlations.

In the Appendix, we supplement the procedure of obtaining the full expression of Eqs. (32)–(35) up to the four-body quark correlations for the readers' convenience and for decay calculations in the next section.

C. Quark number correlations and fluctuations

We first determine the size of the quark system before hadronization, which is consistent with that produced in relativistic heavy ion collisions at LHC energy. By fitting the rapidity density of hadronic yield in central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, we obtain $\langle N_q\rangle = \langle N_{\bar{q}}\rangle = 1710$ and the strangeness content $\langle N_s\rangle/\langle N_u\rangle = \langle N_s\rangle/\langle N_d\rangle = 0.43$ for the quark system in unit rapidity window $y_w = 1$ in the central rapidity region. We note that the obtained strangeness suppression factor $\lambda_s \equiv \langle N_s\rangle/\langle N_u\rangle = \langle N_s\rangle/\langle N_d\rangle = 0.43$ is in agreement with the Wroblewski parameter calculated by lattice QCD [34,35]. In the following sections, we use it as the default size of the quark system. If a different y_w is selected, quark numbers in the system are multiplied by factor y_w because we always focus on the central rapidity plateau region $y_w < 1.5$ where the rapidity distribution of quark numbers is uniform.

For two-body correlation $C_{f_1 f_2}$ of quarks and antiquark, using the charge conjugation symmetry and isospin symmetry between u and d quarks for the quark system produced at LHC, there are only eight relevant quark correlations, i.e.,

two variances $C_{uu} \equiv \sigma_u^2$ and $C_{ss} \equiv \sigma_s^2$,
two pair correlations $C_{u\bar{u}}$ and $C_{s\bar{s}}$,
four off-diagonal correlations C_{ud}, C_{us}, $C_{u\bar{d}}$ and $C_{u\bar{s}}$.

Variances of quark numbers are usually approximated to follow Poisson statistics $\sigma_u^2 \approx \langle N_u\rangle$ and $\sigma_s^2 \approx \langle N_s\rangle$ for a thermalized quark system with grand canonical ensemble. Lattice QCD calculations at vanishing chemical potential provide an important constraint on the above quark correlations [36], which show the weak off-diagonal flavor susceptibilities of quark numbers $\chi_{us}/\chi_{ss} \approx -0.05$ and $\chi_{ud}/\chi_{uu} \approx -0.05$ as temperature approaches the confinement phase boundary. Here, $\chi_{us} \equiv C_{us} + C_{\bar{u}\bar{s}} - C_{u\bar{s}} - C_{\bar{u}s} = 2(C_{us} - C_{u\bar{s}})$, and others are similarly defined. Because of the lack of further theoretical constraints on those quark number correlations at present, we have to adopt some symmetry approximations on quark correlations, i.e., $C_{u\bar{s}}/C_{us} = C_{u\bar{d}}/C_{ud} = \lambda_1$ and $C_{u\bar{u}}/\sigma_u^2 = C_{s\bar{s}}/\sigma_s^2 = \lambda_2$ where λ_1 and λ_2 are treated as parameters of this work. The value of λ_2 is smaller than 1 if we consider a slice of the quark system, e.g., the mid-rapidity region, produced in heavy ion collisions. The off-diagonal flavor correlations are usually expected to be much smaller than variances of quark numbers. Inspired by the weak off-diagonal flavor susceptibilities in lattice QCD calculations, we assume $C_{ud}/C_{uu} \sim 0.05$ (correspondingly $\lambda_1 \sim 2.0$) with some arbitrariness in this work to study effects of the weak flavor off-diagonal quark correlations on baryon and antibaryon production.

Since this work focuses on the baryon sector, we introduce the total baryon number balance coefficient $\rho_B^{(q)}$ as one physical characteristic of the quark system,

$$\rho_B^{(q)} = \frac{\sum_{f_1,f_2}\frac{1}{3}C_{f_1\bar{f}_2}}{N_B^{(q)}} = \lambda_2 - 0.1\lambda_1\frac{1-\lambda_2}{1-\lambda_1}\frac{1+2\lambda_s}{2+\lambda_s}, \quad (36)$$

where indexes f_1, f_2 run over all flavors of quarks and $N_B^{(q)} = \frac{1}{3}(\langle N_u\rangle + \langle N_d\rangle + \langle N_s\rangle)$. Note that the factor 1/3 before $C_{f_1\bar{f}_2}$ denotes the balanced baryon number if f_1 and $\bar{f}_2$ are correlated. The second equality uses the above approximated quark correlations. We also introduce the electric charge balance coefficient of the quark system, which is defined as

$$\rho_C^{(q)} = \frac{1}{N_C^{(q)}}\sum_{f_1,f_2}\min(Q_{f_1},Q_{f_2})\,C_{f_1\bar{f}_2}, \quad (37)$$

where indexes $f_1, f_2 = u,\bar{d},\bar{s}$ run over all positively charged quarks with electric charges Q_{f_1} and Q_{f_2}, respectively, and $N_C^{(q)} = \frac{1}{3}(2\langle N_u\rangle + \langle N_{\bar{d}}\rangle + \langle N_{\bar{s}}\rangle)$. The balanced charge for the $f_1\bar{f}_2$ pair is the minimum of their electric charges. The above approximated quark two-body correlations guarantee the correct boundary behavior of conserved charge for the quark system; i.e., as λ_2 goes to 1, both $\rho_B^{(q)}$ and $\rho_C^{(q)}$ go to 1. Using the measured charge balance function of thermal particles in central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [37], we can roughly constrain $\rho_C^{(q)}$ of the quark system,

$$\rho_C^{(q)}(y_w) \approx \int_0^{y_w} B(\delta\eta)\,d\delta\eta, \quad (38)$$

if we expect a small change of the charge balance property of the system during hadronization [38]. By $\rho_C^{(q)}(y_w)$ we can fix λ_2 and other off-diagonal elements of two-body correlations which are also dependent on y_w.

Three-body and four-body correlations of quarks and antiquarks have relatively less influence on the physical quantities of initial baryons in the previous subsection than the two-body correlations of quark numbers. But they will influence those of final baryons through resonance decays (as shown in next section), so we need them also. Because there are no theoretical calculations at present which we can borrow, we take the following approximation for three-body quark correlations: i.e., $C_{fff} \equiv \langle\delta N_f^3\rangle = \langle N_f\rangle$ and off-diagonal correlations $C_{f_1f_2f_3} = 0$, where f_1, f_2, and f_3 are different flavors. For four-body correlations, we approximate them using two-body correlations

$$\begin{aligned} C_{f_1f_2f_3f_4} \approx\ & C_{f_1f_2}C_{f_3f_4} + C_{f_1f_3}C_{f_2f_4} + C_{f_1f_4}C_{f_2f_3} \\ & + 3\delta_{f_1f_3}\delta_{f_2f_4}C_{f_1f_2} + 3\delta_{f_1f_4}\delta_{f_2f_3}C_{f_1f_2} \\ & + 3\delta_{f_1f_3}\delta_{f_2f_4}C_{f_1f_3}, \end{aligned} \quad (39)$$

$$\begin{aligned} C_{\bar{f}_1f_2f_3f_4} \approx\ & C_{\bar{f}_1f_2}C_{f_3f_4} + C_{\bar{f}_1f_3}C_{f_2f_4} + C_{\bar{f}_1f_4}C_{f_2f_3} \\ & + 9\delta_{f_2f_3}\delta_{f_2f_4}C_{\bar{f}_1f_2}, \end{aligned} \quad (40)$$

$$\begin{aligned} C_{\bar{f}_1\bar{f}_2f_3f_4} \approx\ & C_{\bar{f}_1\bar{f}_2}C_{f_3f_4} + C_{\bar{f}_1f_3}C_{\bar{f}_2f_4} + C_{\bar{f}_1f_4}C_{\bar{f}_2f_3} \\ & + 9\delta_{\bar{f}_1\bar{f}_2}\delta_{f_3f_4}C_{\bar{f}_1f_3}. \end{aligned} \quad (41)$$

By this approximation, the kurtosis of net baryons has the property $K_{net\,B}\sigma_{net\,B}^2 = 1$, which is suggested in ultrarelativistic heavy ion collisions [11].

D. Numerical results of multiplicity moments of identified baryons

Figure 5 shows moments of various identified baryons after taking into account effects of quark number correlations and fluctuations. The system size is taken to be the default value of unit y_w. In order to clearly present effects of quark correlations

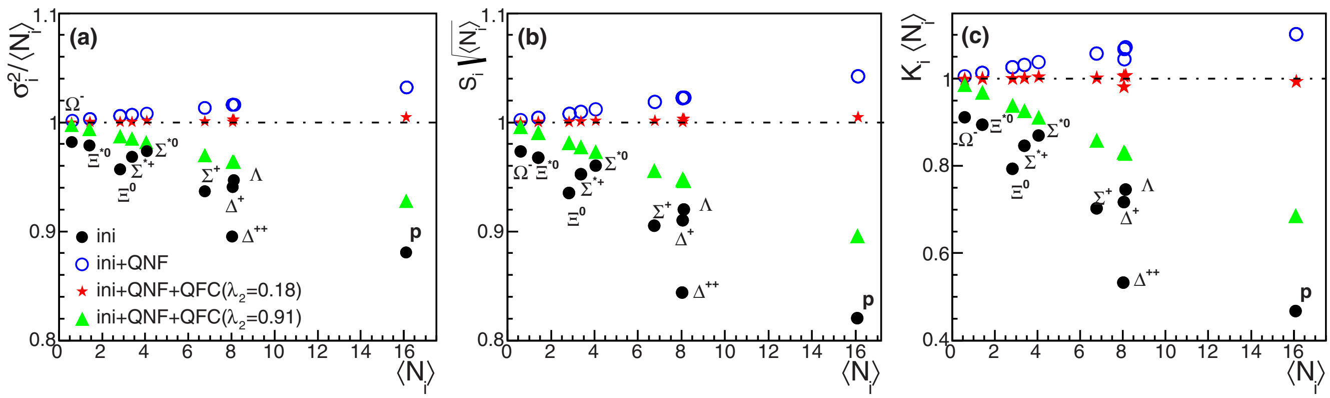

FIG. 5. Moments of identified baryons after considering quark number fluctuations (QNF) and quark number flavor conservation (QFC) with parameter λ_2. The value $\lambda_2 = 0.91$ is chosen to be consistent with the observed charge balance function of thermal particles in the unit pseudorapidity window observed in Pb+Pb collisions at 2.76 TeV [37].

and fluctuations, the variance σ_i^2, skewness S_i, and kurtosis K_i of identified baryons are multiplied by factors $1/\langle N_i\rangle$, $\sqrt{\langle N_i\rangle}$, and $\langle N_i\rangle$, respectively, to make them the order of 1. Here, the usage of $\langle N_i\rangle$ as the scaling factor is due to its insensitivity to correlations and fluctuations of quark numbers. We present results caused by the quark combination process (marked by "ini"), results including effects of quark number fluctuations (marked by "ini+QNF"), and results further including effects of quark flavor conservation (marked by "ini+QNF+QFC"). The last case is the physical result. The purpose of such a presentation is to show the contributions of different sources in the final physical results.

Solid circles in Fig. 5(a) are the variances of initial baryons directly produced by hadronization. As discussed previously, $\sigma_i^2/\langle N_i\rangle$ of identified baryons, roughly following a binomial distribution, is always smaller than 1 and usually decreases with the increase of multiplicity or production weight. For Ω^- it is only 2% smaller than 1 while for protons it is about 10% smaller. But there are several exceptions to such a decreasing trend. For example, variance of Δ^{++} is smaller than its isospin partner Δ^+ although their multiplicities are nearly the same. This is due to the effect of identical quark flavors in baryon production encoded via coefficient A_L in their variance formula in Eq. (7). Others exceptions, including those between Ξ and Σ^* and those between Σ^+ and Λ, are due to the same reasons either in the strange or light flavor sector. These properties are also observed in baryon skewness, Fig. 5(b), and kurtosis, Fig. 5(c), with larger amplitude.

Open circles show the baryon moments after considering effects of quark number fluctuations. We can see that fluctuations of quark numbers obviously increase the baryon's multiplicity fluctuations. $\sigma_i^2/\langle N_i\rangle$ of various baryons exceeds 1. For the proton it is about 3% greater than 1 while for Ω^- it also slightly exceeds 1. Skewness and kurtosis of baryons are also greater than 1, and they are more sensitive to quark number fluctuations; e.g., proton skewness increases about 5% and kurtosis about 10%, respectively. The numerical reason for such rapid increase, taking variance for example, is that quark number fluctuations contribute to baryon variance mainly via the $\sum_f(\partial\overline{N}_{B_i}/\partial N_f)^2\sigma_f^2$ term in Eq. (32) but contribute to baryon yield via the $\sum_f(\partial^2\overline{N}_{B_i}/\partial N_f^2)\sigma_f^2$ term in Eq. (31), which is much smaller than the former. We emphasize that these results are not the final physical predictions of baryon moments because we should always consider the effect of flavor (or charge) conservation in the studied rapidity window in the context of relativistic heavy ion collisions.

Solid up-triangles show baryon moments after considering further effects of quark flavor conservation with parameter $\lambda_2 = 0.91$, as well as quark number fluctuations. Here the value of parameter λ_2 is chosen so that the electric charge balance coefficient $\rho_C^{(q)}$ of the quark system, according to Eq. (38), is consistent with the measured charge balance function in the unit pseudorapidity window in central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ GeV [37]. The pair association of quarks and antiquarks will facilitate meson production and suppress baryon production. Comparing the open circles, we therefore observe a significant decrease of proton variance, skewness, and kurtosis. Such influence of flavor (or charge) conservation has been studied in Ref. [39]. For baryons with small multiplicities such as Ω^- and Ξ^*, they are weakly influenced by flavor conservation of quark numbers and their moments are always almost 1. If we choose a smaller flavor conservation parameter $\lambda_2 = 0.18$, which corresponds to the observed charge balance in small rapidity window $y_w \approx 0.15$, we can observe almost unitary baryon moments, shown as star symbols, which is similar to a Poisson distribution. However, for such small y_w, particle exchange in the window boundary due to the rapidity shift in hadronization, resonance decays, and particle rescatterings is significant, and therefore the statistic effect is dominant. The Poisson distribution is then usually expected but the microscopic dynamics of hadron production is lost at such small y_w.

E. Numerical results of two-baryon correlations

Figure 6 shows two-baryon multiplicity correlations after considering effects of quark number fluctuations and correlations. The system size is taken to be the default value of unit y_w. Solid circles show initial two-baryon correlations

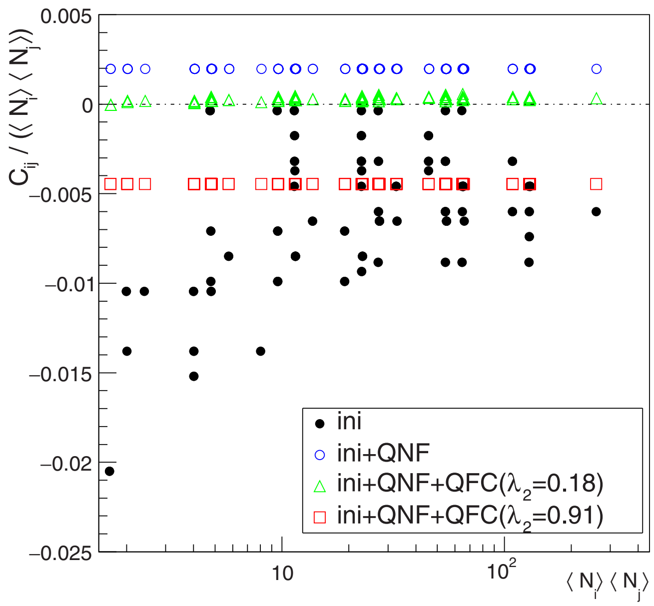

FIG. 6. Two-baryon multiplicity correlations after considering the effects of quark number fluctuations (QNF) and quark flavor conservation (QFC) just before hadronization. The labels for solid circles are the same as those in Fig. 3.

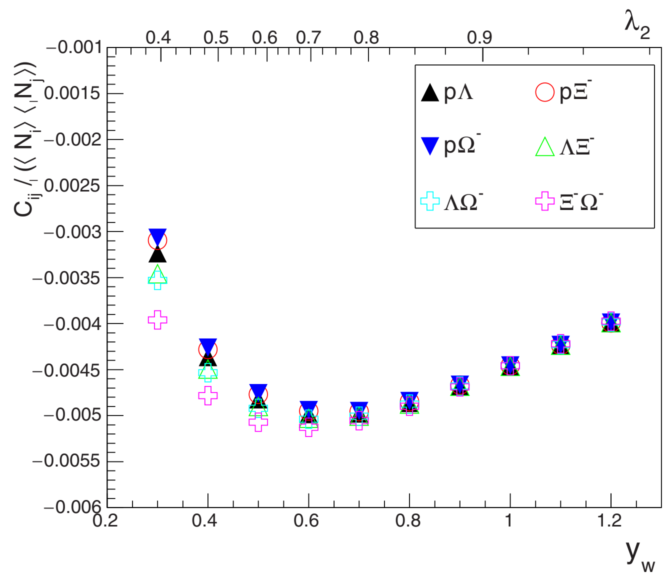

FIG. 7. Rapidity window size dependence of two-baryon correlations after taking into account effects of quark number correlations and fluctuations before hadronization. The auxiliary horizontal axis on top of the figure shows the corresponding value of flavor conservation parameter λ_2.

due to the hadronization of a quark system with given quark and antiquark numbers. They exhibit a sensitive dependence on baryon species, as discussed in detail in Sec. III B. After taking into account effects of quark number fluctuations, all two-baryon correlations, open circles, flip sign and become a positive and almost universal value. The positive value means the production of two baryons is associated, which is because both baryons jointly respond to the change of quark numbers or that of antiquark numbers. This association is suppressed and/or canceled by further taking into account the flavor conservation of quark numbers. With small flavor conservation parameter $\lambda_2 = 0.18$, all two-baryon correlations, open up-triangles, tend to be zero. With practical $\lambda_2 = 0.91$ for unit rapidity window size, we get the physical prediction of two-baryon correlations shown as open squares. We see a strong production anti-association between two baryons, and interestingly we see a universal value for all two-baryon correlations. This is a striking characteristic of two-baryon production in QCM.

Figure 7 shows two-body correlations of stable baryons p, Λ, Ξ^-, Ω^- at different rapidity window sizes y_w. In order to closely relate to the experimental measurement at specific y_w, we have introduced the electric charge balance coefficient of the quark system, $\rho_c^{(q)}$, defined in Eq. (37), and we estimate its value by Eq. (38) using the data of the charge balance function [37]. After obtaining the $\rho_c^{(q)}(y_w)$, we fix the flavor conservation parameter $\lambda_2(y_w)$. The value of λ_2 as a function of y_w is shown as an auxiliary horizontal axis on top of the figure. Note that the average quark numbers of the quark system are also linearly changed with y_w. We see a nonmonotonic behavior of two-baryon correlations with respect to y_w, which is due to the competition between the changed flavor conservation and the changed quark numbers of system. As y_w increases from 0.3 to 0.6, the flavor conservation coefficient λ_2 increases rapidly up to about 0.7, and this leads to the increased anti-association between two baryons. However, as y_w continues to enlarge, the effect of increased flavor conservation is overwhelmed by that of the increased quark numbers, and we see a decreased anti-association between two baryons. We also see that with the increased y_w the difference between different two-baryon correlations decreases, and we have an almost universal correlation magnitude for all two-baryon correlations, as shown in Fig. 6.

F. Numerical results of baryon-antibaryon correlations

Figure 8 shows various baryon-antibaryon multiplicity correlations after considering effects of quark number fluctuations and correlations. The system size is taken to be the default value of unit y_w. Solid circles show baryon-antibaryon correlations for the hadronization of a quark system with given quark and antiquark numbers. They exhibit a universal behavior; see Sec. III C. After taking into account effects of quark number fluctuations, all baryon-antibaryon correlations, open circles, flip sign and become a negative and universal value. The negative value means production of a baryon and antibaryon is anti-associated. This is because that the increase (decrease) of quark numbers will enhance (suppress) the baryon formation and suppress (enhance) antibaryon formation. It is contrary to the case of two-baryon production discussed in the above subsection.

After further taking into account the flavor conservation of quark numbers with parameter $\lambda_2 = 0.91$, we get the physical prediction of baryon-antibaryon correlations shown as open squares in Fig. 8. We find that most baryon-antibaryon

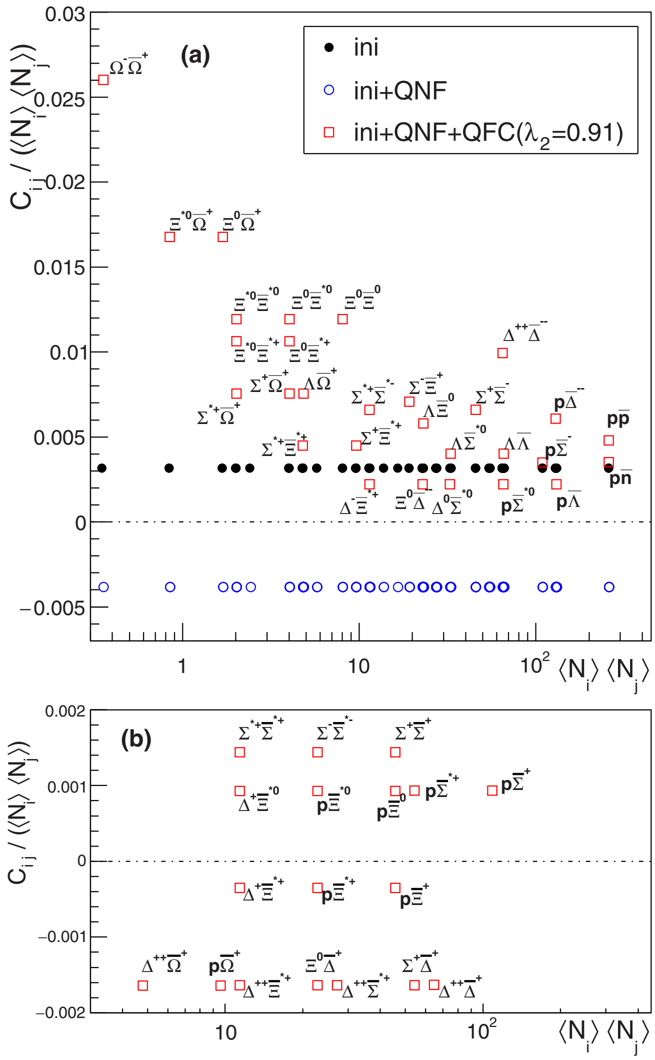

FIG. 8. Baryon-antibaryon multiplicity correlations after considering effects of quark number fluctuations (QNF) and quark flavor conservation (QFC) with parameter $\lambda_2 = 0.91$.

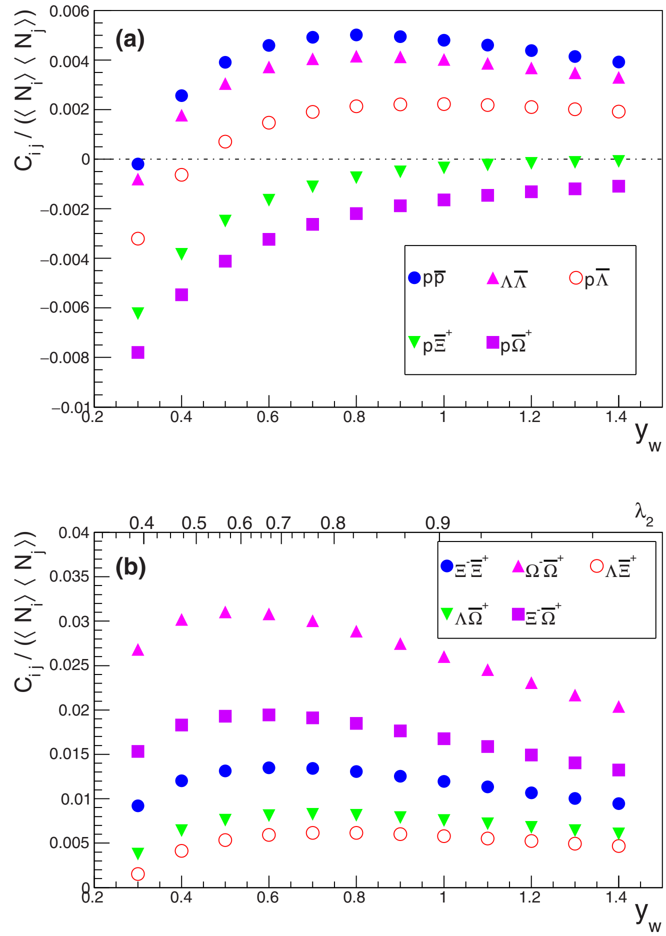

FIG. 9. The rapidity window size dependence of baryon-antibaryon correlations after taking into account quark number fluctuations and correlations just before hadronization. The auxiliary horizontal axis on top of panel (b) shows the corresponding value of flavor conservation parameter λ_2.

correlations return to the positive case, which means their production is associated. In particular, hyperon-antihyperon correlations, e.g., $\Omega^-\bar{\Omega}^+$ and $\Omega^-\bar{\Xi}^0$, are much larger than $p\bar{p}$ correlation. This suggests that strangeness conservation plays an important role in hyperon-antihyperon joint production. Surprisingly, in Fig. 8(b), some baryon-antibaryon pairs, e.g., $p\bar{\Xi}^+$, $p\bar{\Omega}^+$, have negative values. This is because these baryon-antibaryon pairs do not involve, or involve to a lesser degree, the matched $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ pairs, and thus flavor conservation less directly constrains their joint production and therefore the effect of quark number fluctuations is dominant. With small flavor conservation parameter $\lambda_2 = 0.18$, all baryon-antibaryon correlations tend to zero (with maximum deviation about 0.002) and we do not show them in Fig. 8 for clarity.

In Fig. 9, we show the rapidity window size y_w dependence of some baryon-antibaryon correlations. The relationship between λ_2 and y_w is the same as that in the above subsection. We observe from panel (a) that $C_{p\bar{\Omega}^+}$ is always negative at different y_w and $C_{p\bar{\Xi}^+}$ is negative at small y_w and tends to zero with increasing y_w due to the increasing effect of flavor conservation λ_2. For $p\bar{p}$, $\Lambda\bar{\Lambda}$ correlations in Fig. 9(a), $\Xi^-\bar{\Xi}^+$, $\Omega^-\bar{\Omega}^+$ and other hyperon-antihyperon correlations in panel (b) that largely involve the matched $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ pairs, they are all positive under the influence of quark flavor conservation. We also observe that as $y_w \gtrsim 0.6$, $p\bar{p}$, $\Lambda\bar{\Lambda}$, $\Xi^-\bar{\Xi}^+$, $\Omega^-\bar{\Omega}^+$ correlations decrease with increasing y_w, which is because of the increasing quark numbers (or system size).

V. DECAY EFFECTS

Multiplicity of final baryons observed in experiments usually contains the decay contribution of unstable resonances. In this section, we study the effect of resonance decays on the multiplicity correlations and fluctuations of final stable baryons. We first derive formulas of decay influence on stable baryons and then show numerical results of stable baryons p, Λ, Ξ^-, and Ω^-.

A. Formulas of decay effects

For baryon resonance i, its stable daughter baryons are denoted as $a,b,c,\ldots$ with decay branch ratios $\mathcal{D}_{ia},\mathcal{D}_{ib},\mathcal{D}_{ic}\ldots$, respectively. $\mathcal{D}_{ij}$ is taken from PDG [40]. The joint multiplicity distribution of daughter baryons from the parent baryon i of number N_i is taken to be the multinomial distribution $f(\{N_a^i,N_b^i,N_c^i,\ldots\},N_i,\{\mathcal{D}_{ia},\mathcal{D}_{ib},\mathcal{D}_{ic}\ldots\})$, where $N_a^i,N_b^i,N_c^i,\ldots$ denote the numbers of decayed baryons $a,b,c,\ldots$, respectively. Recalling the joint distribution of directly produced baryons in Sec. IV, we write the joint multiplicity distribution of stable baryons

$$F(N_a,N_b,N_c,\ldots)=\sum_{\{N_{h_j}\}}\mathcal{P}(\{N_{h_j}\};\{\langle N_{q_i}\rangle,\langle N_{\bar{q}_i}\rangle\})\prod_i\left(\sum_{\{N^i\}}f\left(\left\{N_a^i,N_b^i,N_c^i,\ldots\right\},N_i,\{\mathcal{D}_{ia},\mathcal{D}_{ib},\mathcal{D}_{ic},\ldots\}\right)\right)\prod_{k=a,b,c,\ldots}\delta_{N_k,\sum_i N_k^i}, \tag{42}$$

where index i runs over all kinds of directly produced baryons and k runs over all stable hadrons we study.

The inclusive yield of final-state identified baryons includes the linear superposition of resonance decays,

$$\begin{aligned}\langle N_a\rangle &= \sum_{\{N_a,N_b,\ldots\}} N_a\,F(N_a,N_b,N_c,\ldots)\\ &= \sum_{\{N_{h_j}\}}\mathcal{P}(\{N_{h_j}\})\prod_i\left(\sum_{\{N_a^i\}}f\left(N_a^i,N_i,\mathcal{D}_{ia}\right)\right)\sum_k N_a^k\\ &= \sum_k\left(\sum_{\{N_{h_j}\}}\mathcal{P}(\{N_{h_j}\};\{\langle N_{q_i}\rangle,\langle N_{\bar{q}_i}\rangle\})\,N_k\mathcal{D}_{ka}\right)\\ &= \sum_k\langle N_k\rangle\mathcal{D}_{ka}.\end{aligned} \tag{43}$$

Note that we have used the abbreviation $\mathcal{P}(\{N_{h_j}\})\equiv\mathcal{P}(\{N_{h_j}\};\{\langle N_{q_i}\rangle,\langle N_{\bar{q}_i}\rangle\})$ for the joint distribution of directly produced baryons, and we have written $\mathcal{D}_{kk}=1$ to obtain compact formulas. Similarly, we can calculate various moments of multiplicity distributions of stable baryons as

$$\left\langle N_a^m\right\rangle=\sum_{\{N_a,N_b,\ldots\}}N_a^m\,F(N_a,N_b,N_c,\ldots)=\sum_{\{N_{h_j}\}}\mathcal{P}(\{N_{h_j}\})\prod_i\left(\sum_{\{N_a^i\}}f\left(N_a^i,N_i,\mathcal{D}_{ia}\right)\right)\left(\sum_k N_a^k\right)^m,$$

and finally we have

$$\sigma_a^2=\sum_{m,n}C_{mn}\,\mathcal{D}_{ma}\mathcal{D}_{na}+\sum_m\langle N_m\rangle\mathcal{D}_{ma}(1-\mathcal{D}_{ma}), \tag{44}$$

$$S_a=\frac{1}{\sigma_a^3}\left(\sum_{k,m,n}C_{kmn}\,\mathcal{D}_{ka}\mathcal{D}_{ma}\mathcal{D}_{na}+3\sum_{m,n}C_{mn}\,\mathcal{D}_{ma}(1-\mathcal{D}_{ma})\mathcal{D}_{na}+\sum_m\langle N_m\rangle\mathcal{D}_{ma}(1-\mathcal{D}_{ma})(1-2\mathcal{D}_{ma})\right), \tag{45}$$

$$\begin{aligned}K_a+3=\frac{1}{\sigma_a^4}\Bigg(&\sum_{m,n,k,l}C_{mnkl}\,\mathcal{D}_{ma}\mathcal{D}_{na}\mathcal{D}_{ka}\mathcal{D}_{la}+6\sum_{mnk}(C_{mnk}+C_{nk}\langle N_m\rangle)\mathcal{D}_{ma}(1-\mathcal{D}_{ma})\mathcal{D}_{na}\mathcal{D}_{ka}\\ &+4\sum_{mn}C_{mn}\mathcal{D}_{ma}(1-\mathcal{D}_{ma})(1-2\mathcal{D}_{ma})\mathcal{D}_{na}+3\sum_{mn}(C_{mn}+\langle N_m\rangle\langle N_n\rangle)\mathcal{D}_{ma}(1-\mathcal{D}_{ma})\mathcal{D}_{na}(1-\mathcal{D}_{na})\\ &+\sum_m\langle N_m\rangle\mathcal{D}_{ma}(1-\mathcal{D}_{ma})[1-6\mathcal{D}_{ma}(1-\mathcal{D}_{ma})]\Bigg).\end{aligned} \tag{46}$$

The average of the multiplicity product of two stable baryons is evaluated by

$$\begin{aligned}\langle N_aN_b\rangle&=\sum_{\{N_a,N_b,\ldots\}}N_aN_b\,F(N_a,N_b,N_c,\ldots)=\sum_{\{N_{h_j}\}}\mathcal{P}(\{N_{h_j}\})\prod_i\left(\sum_{\{N_a^i,N_b^i\}}f\left(\left\{N_a^i,N_b^i\right\},N_i,\{\mathcal{D}_{ia},\mathcal{D}_{ib}\}\right)\right)\left(\sum_m N_a^m\right)\left(\sum_n N_b^n\right)\\ &=\sum_{\{N_{h_j}\}}\mathcal{P}(\{N_{h_j}\})\prod_i\left(\sum_{\{N_a^i,N_b^i\}}f\left(\left\{N_a^i,N_b^i\right\},N_i,\{\mathcal{D}_{ia},\mathcal{D}_{ib}\}\right)\right)\left(\sum_{m\neq n}N_a^mN_b^n+\sum_{m=n}N_a^mN_b^m\right)\\ &=\sum_{m,n}\langle N_mN_n\rangle\mathcal{D}_{ma}\mathcal{D}_{nb}-\sum_m\langle N_m\rangle\mathcal{D}_{ma}\mathcal{D}_{mb}.\end{aligned} \tag{47}$$

Substituting it into the definition of two-body correlation we get for $a \neq b$

$$C_{ab} = \sum_{m,n}[C_{mn} - \delta_{mn}\langle N_m\rangle]\,\mathcal{D}_{ma}\mathcal{D}_{nb}, \tag{48}$$

which includes the coherent superposition of two resonance correlations as well as the anti-association due to the possible same parent resonance.

Following the spirit of Eq. (47), we obtain the three-body correlation with different species C_{abc},

$$C_{abc} = \sum_{mnk}(C_{mnk} - (\delta_{mk} + \delta_{nk})C_{mn} - \delta_{mn}C_{mk} + 2\delta_{mn}\delta_{nk}\langle N_m\rangle)\mathcal{D}_{ma}\mathcal{D}_{nb}\mathcal{D}_{kc}, \tag{49}$$

and C_{aac} with one identical pair can be obtained by $C_{aac} = (C_{abc})_{a=b} + C_{ac}$, and the four-body correlation with different species C_{abcd} by

$$\begin{aligned} C_{abcd} = &\sum_{mnkl} C_{mnkl}\,\mathcal{D}_{ma}\mathcal{D}_{nb}\mathcal{D}_{kc}\mathcal{D}_{ld} \\ &- \sum_{mkl}(C_{mkl} + C_{kl}\langle N_m\rangle)\mathcal{D}^{(211)}_{mkl}(a,b,c,d) \\ &+ \sum_{ml}(C_{ml} + \langle N_m\rangle\langle N_l\rangle)\mathcal{D}^{(22)}_{ml}(a,b,c,d) \\ &+ 2\sum_{ml} C_{ml}\,\mathcal{D}^{(31)}_{ml}(a,b,c,d) \\ &- 6\sum_{m}\langle N_m\rangle\mathcal{D}_{ma}\mathcal{D}_{nb}\mathcal{D}_{kc}\mathcal{D}_{ld}. \end{aligned} \tag{50}$$

Here, $\mathcal{D}^{(211)}_{mkl}(a,b,c,d) = \mathcal{D}_{ma}\mathcal{D}_{mb}\mathcal{D}_{kc}\mathcal{D}_{ld} + \mathcal{D}_{ma}\mathcal{D}_{mc}\mathcal{D}_{kb}\mathcal{D}_{ld} + \mathcal{D}_{ma}\mathcal{D}_{md}\mathcal{D}_{kb}\mathcal{D}_{lc} + \mathcal{D}_{mb}\mathcal{D}_{mc}\mathcal{D}_{ka}\mathcal{D}_{ld} + \mathcal{D}_{mb}\mathcal{D}_{md}\mathcal{D}_{ka}\mathcal{D}_{lc} + \mathcal{D}_{mc}\mathcal{D}_{md}\mathcal{D}_{ka}\mathcal{D}_{lb}$ denotes the summation over all possible joint-decay probabilities for three resonances mkl into four stable baryons, where the superscript (211) denotes that one of the parent resonances has two decay channels to two different stable baryons, respectively. Similarly, we have $\mathcal{D}^{(31)}_{ml}(a,b,c,d) = \mathcal{D}_{ma}\mathcal{D}_{mb}\mathcal{D}_{mc}\mathcal{D}_{ld} + \mathcal{D}_{ma}\mathcal{D}_{mb}\mathcal{D}_{md}\mathcal{D}_{lc} + \mathcal{D}_{ma}\mathcal{D}_{mc}\mathcal{D}_{md}\mathcal{D}_{lb} + \mathcal{D}_{mb}\mathcal{D}_{mc}\mathcal{D}_{md}\mathcal{D}_{la}$ and $\mathcal{D}^{(22)}_{ml}(a,b,c,d) = \mathcal{D}_{ma}\mathcal{D}_{mb}\mathcal{D}_{lc}\mathcal{D}_{ld} + \mathcal{D}_{ma}\mathcal{D}_{mc}\mathcal{D}_{lb}\mathcal{D}_{ld} + \mathcal{D}_{ma}\mathcal{D}_{md}\mathcal{D}_{lb}\mathcal{D}_{lc}$. Other four-body correlations of stable baryons with one identical pair, two identical pairs, and three identical species can be obtained as follows:

$$C_{aabd} = (C_{abcd})_{a=c} + C_{abd} + \langle N_a\rangle C_{bd}, \tag{51}$$

$$C_{aaab} = (C_{abcd})_{a=c=d} + 3C_{aab} + (3\langle N_a\rangle - 2)C_{ab}, \tag{52}$$

$$C_{aabb} = (C_{abcd})_{a=c,b=d} + C_{aab} + C_{abb} + \langle N_a\rangle\sigma_b^2 + \langle N_b\rangle\sigma_a^2 - C_{ab} - \langle N_a\rangle\langle N_b\rangle. \tag{53}$$

B. Numerical results of stable baryons

Figure 10 shows multiplicity moments of the final proton, Λ, and Ξ^- at different rapidity window sizes. Lines show moments of baryons without including resonance decays. Open symbols show results including weak decays, strong decays, and electromagnetic decays. Solid symbols show results including only strong and electromagnetic decays. We can see that, due to the large decay contribution to the final proton and Λ, moments of the final proton and Λ, circle and square symbols, are obviously smaller than those of initial ones without including resonance decays, solid and dashed lines, respectively. The decay contribution to Ξ^- multiplicity is relatively small, and we see that both weak decays and strong and electromagnetic decays weakly influence moments of Ξ^-. In contrast to significant y_w dependence of the moments of proton and Λ, moments of Ξ^- are only weakly decreasing with increasing y_w, and the magnitudes are almost 1, which is quite close to a Poisson distribution.

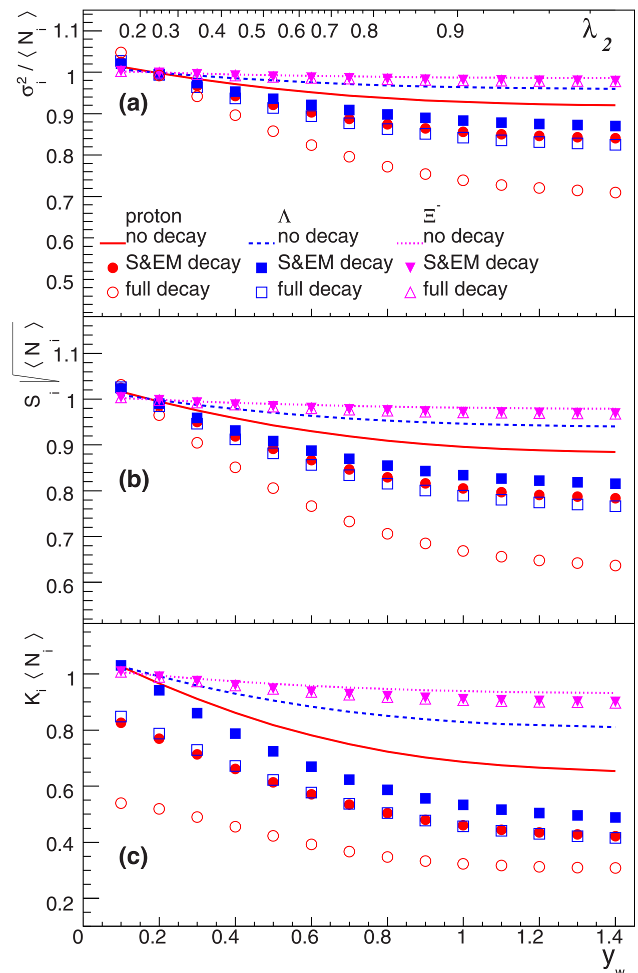

FIG. 10. Moments of final proton, Λ, and Ξ^- at different rapidity window sizes y_w. The auxiliary horizontal axis on top of panel (a) shows the corresponding value of flavor conservation parameter λ_2. Lines show moments of baryons without including resonance decays. Open symbols show results including weak decays, strong decays, and electromagnetic decays. Solid symbols show results including only strong and electromagnetic decays.

Figure 11 shows two-baryon correlations of the final proton, Λ, Ξ^-, and Ω^- at different rapidity window sizes. Surprisingly, we see that they are almost unaffected by resonance decays. However, we emphasize that the almost unchanged quantities are relative correlations $C_{ij}/(\langle N_i\rangle\langle N_j\rangle)$, and for absolute correlations C_{ij} they indeed change a lot.

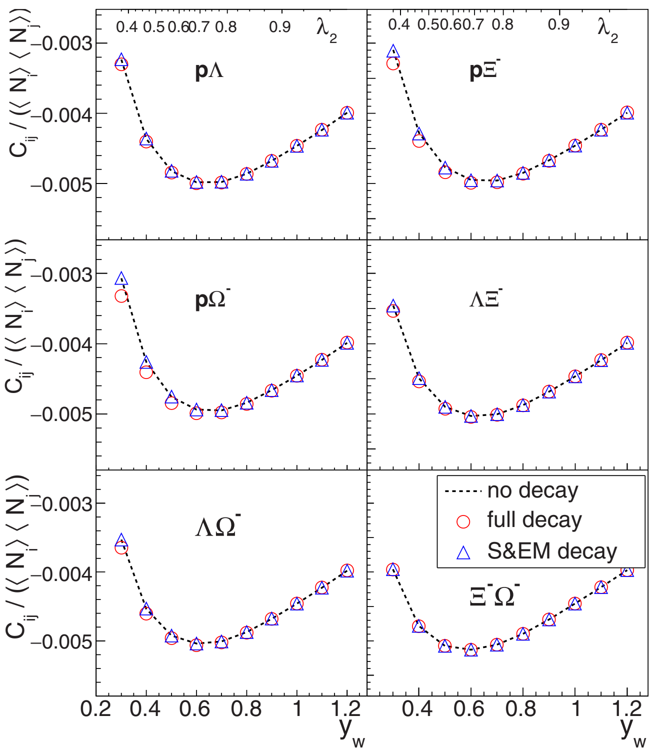

FIG. 11. Two-baryon correlations at different rapidity window sizes y_w. The auxiliary horizontal axis on top of the panels shows the corresponding value of flavor conservation parameter λ_2. Dashed lines show baryon-antibaryon correlations without including resonance decays. Open circles show results including weak decays, strong decays, and electromagnetic decays. Open up-triangles show results including only strong and electromagnetic decays.

The nonmonotonic dependence of two-baryon correlations on rapidity window size is also a striking behavior for future experimental measurement.

Figure 12 shows baryon-antibaryon correlations of the final proton, Λ, Ξ^-, and Ω^- at different rapidity window sizes. Open squares show results including only strong and electromagnetic (S&EM) decays. Comparing to initial baryon-antibaryon correlations without resonance decays (dashed lines), we can see that all correlations except $p\bar{p}$ are almost unaffected by S&EM decays. However, for baryon-antibaryon correlations except $\Xi^-\bar{\Xi}^+$ and $\Xi^-\bar{\Omega}^+$, after further including weak decays, they (open circles) are significantly changed. In addition, we observe that final $p\bar{p}$, $p\bar{\Lambda}$, $p\bar{\Xi}^+$ and $p\bar{\Omega}^+$ with full decay contributions, open circles, have almost the same correlations. This is because they all reflect such a baryon-antibaryon production association; i.e., when an antibaryon, either $\bar{p}$, $\bar{\Lambda}$, or $\bar{\Xi}^+$ is produced, a baryon of any species (via final the proton) should be produced with a certain associated probability to balance the baryon quantum number.

There are some striking properties in the above decay calculations which are suitable for future experimental measurement. First, $\Xi^-\bar{\Xi}^+$ and $\Xi^-\bar{\Omega}^+$ correlations are almost unaffected by resonance decays. Second, $p\bar{\Omega}^+$ correlation with only S&EM decays is negative, while that including weak decays is positive at moderate and large rapidity window sizes. Third, final $p\bar{\Lambda}$ correlation changes sign around moderate rapidity window size. Fourth, final $p\bar{\Xi}^+$ correlation with full decay contribution is positive, while with only S&EM decays it tends to zero at moderate and large y_w.

VI. SUMMARY AND DISCUSSION

We have studied dynamical multiplicity fluctuations and correlations of identified baryons and antibaryons produced by the hadronization of a bulk quark system in a quark combination model. We first developed a working model to discuss the most basic dynamics of the quark combination which is necessary for multiplicity study. Then, for the hadronization of a quark system with given quark and antiquark numbers, we derive moments (variance, skewness, and kurtosis) of multiplicity distributions of produced baryons, two-baryon multiplicity correlations, and baryon-antibaryon multiplicity correlations. We obtain some interesting results about baryon multiplicity as follows:

(1) Multiplicity moments of identified baryons exhibit the behavior of a binomial distribution.
(2) Anti-association of two-baryon production is mainly determined by the coinciding flavors of two baryons.
(3) All baryon-antibaryon correlations show a positive and universal magnitude, which suggests that the joint production of a baryon and antibaryon is mainly constrained by baryon quantum number conservation in combination.

These properties come from the basic dynamics of the quark combination and, therefore, can be regarded as general features of the quark combination mechanism.

We also take into account correlations and fluctuations of quark numbers and antiquark numbers before hadronization to study their effects on multiple production of baryons and antibaryons. Supposing the weak off-diagonal flavor correlations of quarks and antiquarks, we focus on effects of quark number fluctuations and flavor conservation. In order to relate the experimental measurement at specific rapidity window size y_w, we use the charge balance function of thermal particles measured in central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV to constrain the flavor conservation at different rapidity window sizes. We calculate moments of inclusive baryon multiplicity, two-baryon multiplicity correlations, and baryon-antibaryon correlations at mid-rapidity with unit window size and at different rapidity window sizes. Comparing with those results directly from the quark combination, after including quark number fluctuations and correlations we find

(1) Multiplicity moments of baryons deviate from binomial distribution, and at small flavor conservation parameter we can observe the Poisson statistics.
(2) All two-baryon correlations at unit rapidity window size tend to be a negative and universal value.
(3) Baryon-antibaryon correlations exhibit large species difference. In particular, $C_{p\bar{\Omega}^+}$ is negative, showing the anti-association between p and $\bar{\Omega}^+$ production. At moderate rapidity window size we observe the negative sign of $p\bar{\Xi}^+$ correlation but at large window size

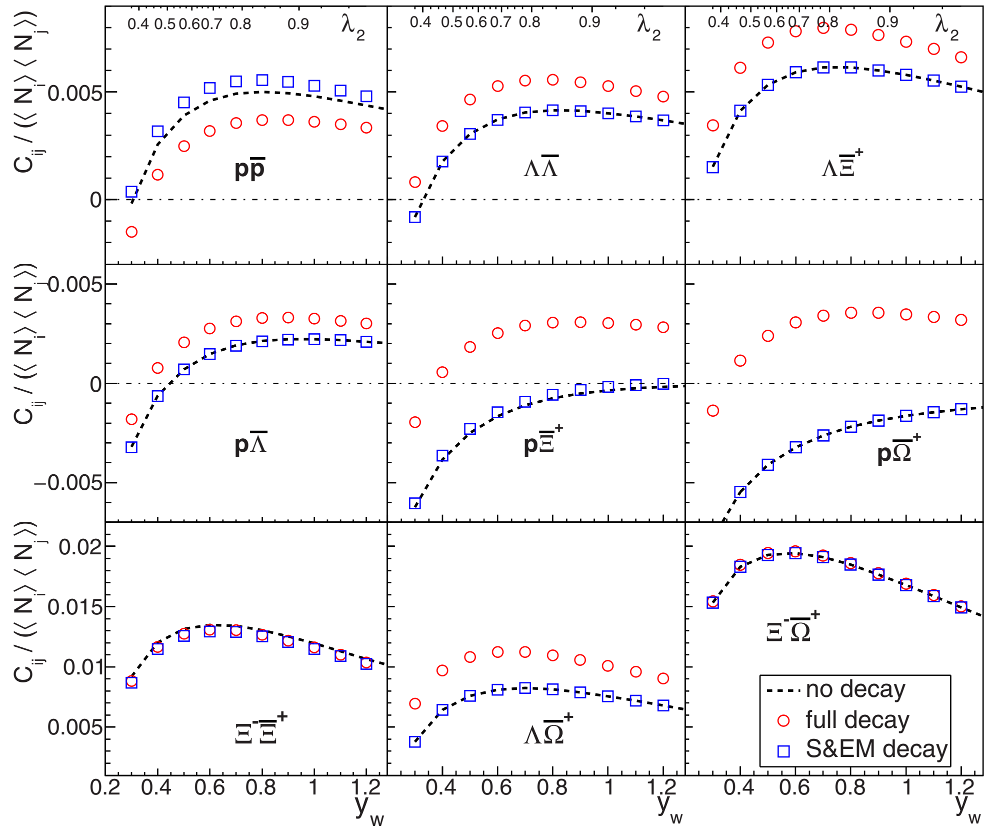

FIG. 12. Baryon-antibaryon correlations at different rapidity window sizes y_w. The auxiliary horizontal axis on top of the panels shows the corresponding value of flavor conservation parameter λ_2. Dashed lines show baryon-antibaryon correlations without including resonance decays. Open circles show results including weak decays, strong decays, and electromagnetic decays. Open squares show results including only strong and electromagnetic decays.

we observe the vanishing $p\bar{\Xi}^+$ correlation. We also observe the sign change of $p\bar{\Lambda}$ correlation at moderate window size.

We also study the influence of resonance decays. We separately calculate the above quantities including strong and electromagnetic (S&EM) decays and those further including weak decays. Our final results of stable baryons p, Λ, Ξ^-, and Ω^- show several interesting properties as follows.

(1) Moments of final proton and Λ are obviously smaller than those of directly produced baryons. However, the scaled moments of final Ξ^- are weakly influenced by resonance decays and are close to a Poisson distribution.
(2) Two-baryon correlations are hardly influenced by either S&EM decays or weak decays. In addition, they are dependent on rapidity window size in a nonmonotonic way.
(3) Effects of resonance decays on baryon-antibaryon correlations are sophisticated. $\Xi^-\bar{\Xi}^+$ and $\Xi^-\bar{\Omega}^+$ correlations are almost unaffected by S&EM and weak decays. $p\bar{\Omega}^+$ correlation with only S&EM decays is negative, while with weak decays it is positive at moderate and large rapidity window sizes. $p\bar{\Lambda}$ correlation changes sign around moderate rapidity window size.

They are striking phenomena which are suitable for the future experimental measurement.

Some discussions related to experimental observation at finite rapidity window size are in order. In Secs. III and IV, we choose a quark system of specific size which corresponds to a specific rapidity window of the quark system produced in relativistic heavy ion collisions. Here we do not consider the possible rapidity shift between (anti)quarks and the formed (anti-)baryon, which may lead the produced baryons to fly out of the studied window and baryons produced in other regions to fly into this window. However, the effect of rapidity shift in combination is quite small because of the following two reasons. First, there is small discrepancy between the total mass of three quarks and the mass of the formed baryon. Note that we usually use the constituent quark mass in QCM, i.e., $m_u \sim 330$ MeV and $m_s \sim 500$ MeV. Therefore, there is no large rapidity shift in combination due to the mass (or energy) mismatch between three neighboring quarks in phase space and the baryon they form. Second, we apply the quark combination rule as explained in Sec. II to longitudinal rapidity direction to solve the unitary issue which is necessary for multiplicity study. This approach has reproduced experimental data of rapidity distributions of identified hadrons in relativistic heavy ion collisions at different collisional energies. The rapidity interval between neighboring quarks is only of the order of 10^{-3} due to the high quark number density $dN/dy \sim 10^3$ in ultrarelativistic heavy ion collisions. Therefore, rapidity shift in baryon production is quite small and it hardly influences results in this work. In Sec. V, we also neglect the rapidity shift in resonance decays. Because the rapidity shift in baryon decays is small ($\lesssim 0.1$), its influence is also expected to be small.

ACKNOWLEDGMENTS

The authors thank Shu-qing Li for helpful discussions. The work is supported by the National Natural Science Foundation of China under Grants No. 11305076, No. 11575100, No. 11505104, and No. 11675091.

APPENDIX: DERIVATION OF EQS. (32)–(34)

Applying Eq. (30) and substituting the expansion

$$\langle N_\alpha^m N_\beta^n\rangle = \overline{N_\alpha^m N_\beta^n} + \frac{1}{2}\sum_{f_1 f_2}\partial_{12}\overline{N_\alpha^m N_\beta^n}\,C_{f_1 f_2} + \frac{1}{3!}\sum_{f_1 f_2 f_3}\partial_{123}\overline{N_\alpha^m N_\beta^n}\,C_{f_1 f_2 f_3} + \frac{1}{4!}\sum_{f_1 f_2 f_3 f_4}\partial_{1234}\overline{N_\alpha^m N_\beta^n}\,C_{f_1 f_2 f_3 f_4} \tag{A1}$$

into the definition of multiplicity moments

$$\sigma_\alpha^2 = \langle N_\alpha^2\rangle - \langle N_\alpha\rangle^2, \quad \langle \delta N_\alpha^3\rangle = \langle N_\alpha^3\rangle - 3\langle N_\alpha\rangle\sigma_\alpha^2 - \langle N_\alpha\rangle^3,$$
$$\langle \delta N_\alpha^4\rangle = \langle N_\alpha^4\rangle - 4\langle \delta N_\alpha^3\rangle\langle N_\alpha\rangle - 6\langle N_\alpha\rangle^2\sigma_\alpha^2 - \langle N_\alpha\rangle^4 \tag{A2}$$

and two-body multiplicity correlation

$$C_{\alpha\beta} = \langle N_\alpha N_\beta\rangle - \langle N_\alpha\rangle\langle N_\beta\rangle, \tag{A3}$$

we can get the expressions of Eqs. (32)–(34) up to two-body quark correlations. The complete expansions of Eqs. (A2) and (A3) up to four-body quark correlations are too long to be shown. In addition, direct calculations according to Eqs. (A2) and (A3) are numerically convenient.

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