PHYSICAL REVIEW C **95**, 014307 (2017)

Correlating double-difference of charge radii with quadrupole deformation and $B(E2)$ in atomic nuclei

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(Received 14 September 2016; published 6 January 2017)

A good linear correlation is found between the double-difference of charge radius $\delta R_{2p-2n}(Z,N)$ with that of quadrupole deformation data in even-even nuclei. This results in a further improved charge radius relation that holds in a precision of about 5×10^{-3} fm. The new relation can be generalized to the reduced electric quadrupole transition probability $B(E2)$ between the first 2^+ state and the 0^+ ground state, and the mean lifetime τ of the first 2^+ state. Same correlations are also seen in global nuclear models such as Hartree-Fock-Bogoliubov (HFB-24) and relativistic mean field (RMF); however, they are not consistent with the experimental data.

DOI: 10.1103/PhysRevC.95.014307

I. INTRODUCTION

Mass, charge radius, lifetime, and electric (magnetic) transition probability are among the most fundamental observables [1–4] for the many-body nuclear system. A systematic analysis of these data have been successful in bringing forth a global picture of the atomic nuclei [5–8]. For example, it has been well known that experimental binding energies or charge radii for neighboring nuclei do not differ much except at several specified regions like closed shells or onset of shape (see Refs. [9–11]).

On the other hand, comparing these observables of atomic nuclei, which differ by one or a few neutrons or protons, have yielded many empirical relations or filters for special interaction strengths between the valence nucleons [12–32]. Of them, the Garvey-Kelson relations are probably one of the best known examples [22,24–32].

The radius of the charge (proton) distribution can be assumed to be equal to that of the nuclear mass distribution, considering the nucleus as a liquid drop with the protons homogeneously distributed over the sphere of the nucleus. In this scheme, we proposed a set of nuclear charge radius relations $\delta R_{ip-jn}(Z,N)$ [11,23,33],

$$\begin{aligned}\delta R_{ip-jn}(Z,N) &= R(Z,N) - R(Z,N-j) - [R(Z-i,N) \\ &\quad - R(Z-i,N-j)] \\ &\simeq 0,\end{aligned} \tag{1}$$

where $R(Z,N)$ is the root-mean-square (rms) charge radius of the nucleus with N neutrons and Z protons. i and j are integers. The validity of such relation is a consequence of the smooth transition in the nuclear structure [34] that is often found when going from a nucleus to its neighboring nuclei. Equation (1) holds precisely over almost the whole nuclear chart except at a few regions characterized by shape transition and shape coexistence at, e.g., $N \sim 60$, $N \sim 90$, and $Z \sim 80$. These exceptions raise the possibilities that more accurate local systematics may be developed from the experimental data.

One simple case connecting only even-even nuclei is

$$\begin{aligned}\delta R_{2p-2n}(Z,N) &= R(Z,N) - R(Z,N-2) - [R(Z-2,N) \\ &\quad - R(Z-2,N-2)] \\ &\simeq 0.\end{aligned} \tag{2}$$

The term $R(Z,N) - R(Z,N-2)$, similar to the isotope shift, involves the variation of the charge density distribution when only two neutrons are added to the system. In this sense, $\delta R_{2p-2n}(Z,N)$ is nothing but the difference of isotope shifts for neighboring two isotopic chains. Hereafter we simply rewrite $\delta R_{2p-2n}(Z,N)$ as $\delta R(Z,N)$.

In this work, we aim to examine and quantify the correlation between the local charge radius relations $\delta R(Z,N)$ and those of deformation data. The correlation is made from cases that both charge radius [3] and quadrupole deformation data [4] are experimentally available. In total there are 149 even-even nuclei from Ne to Cm. This then leads us to an improved relation by correcting the contribution from quadrupole deformation effect in atomic nuclei. This new relation can be naturally extended to the reduced electric quadrupole transition probability $B(E2)$ between the first 2^+ state and the 0^+ ground state, and the mean lifetime τ of the first 2^+ state. Moreover, the same correlation has been examined in two widely used nuclear global models.

II. CORRELATING CHARGE RADIUS DATA WITH QUADRUPOLE DEFORMATION DATA

Let us start with a system of spherical nuclei, for which the rms charge radii can be empirically described by

$$R(Z,N) = \sqrt{3/5}r_0A^{1/3}, \tag{3}$$

where A is the mass number and r_0 is fixed to 1.2 fm throughout this paper. $\delta R(Z,N)$ for even-even isotopes is

$$\delta R(Z,N) \equiv \delta R(Z,N)_{\rm sph} = \sqrt{3/5}r_0\delta(A^{1/3}). \tag{4}$$

Numerically it is easy to demonstrate that $\delta R(Z,N)_{\rm sph}$ goes down to a few times 10^{-4} fm with increasing mass number [23].

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For a deformed nucleus, the charge radius can be decomposed into the spherical and deformation parts. In particular, considering the important case of an axially symmetric shape, the charge radius by neglecting the high-order corrections can be expressed as

$$R(Z,N) = R_0(Z,N)\left[1 + \frac{5}{8\pi}\beta_2^2(Z,N)\right]. \quad (5)$$

Here $\beta_2(Z,N)$ is the rms quadrupole deformation for the nuclide (Z,N) and is derived experimentally from the reduced electric quadrupole transition probability $B(E2)$. $R_0(Z,N)$ represents the spherical equivalent radius and is calculated by Eq. (3). Equation (5) may be generalized to include higher order multipoles or triaxial shape [35,36].

Accordingly, $\delta R(Z,N)$ can be rewritten as a two-parameter relation,

$$\delta R(Z,N) = \delta R(Z,N)_{\rm sph} + \delta R(Z,N)_{\rm def} = \delta R(Z,N)_{\rm sph} + \frac{5}{8\pi}\delta\left(R_0\beta_2^2\right), \quad (6)$$

where $\delta R(Z,N)_{\rm sph}$ is defined in Eq. (4) and $\delta R(Z,N)_{\rm def}$ comes from the variance of deformation in the relevant nuclei,

$$\begin{aligned}\delta R(Z,N)_{\rm def} &\equiv \frac{5}{8\pi}\delta\left(R_0\beta_2^2\right)\\ &= \sqrt{\frac{3}{5}}\frac{5}{8\pi}r_0[A^{1/3}\beta_2^2(Z,N)\\ &\quad + (A-4)^{1/3}\beta_2^2(Z-2,N-2)\\ &\quad - (A-2)^{1/3}\beta_2^2(Z-2,N)\\ &\quad - (A-2)^{1/3}\beta_2^2(Z,N-2)]\\ &\approx \sqrt{\frac{3}{5}}\frac{5}{8\pi}r_0A^{1/3}\delta\beta_2^2. \end{aligned} \quad (7)$$

The approximation of the last term is valid especially for heavier system. Because of the negligible contribution from $\delta R(Z,N)_{\rm sph}$, the resulting $\delta R(Z,N)$ is mostly determined by the terms relevant to nuclear deformation. Although dynamic deformations and higher-order multipoles are not included in this equation, they could be subsumed in principle under the deformation term in which $\delta\beta_2^2$ is replaced by $\sum_i \delta\beta_i^2$.

The correlation between the experimental $\delta R(Z,N)$ and $\delta R(Z,N)_{\rm def}$ is shown in Fig. 1. The majority of data distribute around $\delta R(Z,N)_{\rm def} = 0$ and $\delta R(Z,N) = 0$, due to the rare experimental data far from the β-stability line. Almost all the data follow a linear trend within one standard deviation (σ). A coefficient of 0.30(5) is determined with a reduced χ^2 of 0.7 by considering the uncertainties in both charge radii and deformation, indicating that experimental data of charge radii and β_2 are in well consistency. Figures 2(a) and 2(b) present the same correlation data but for two different mass regions. To see more clearly the centroid values the experimental uncertainties are not shown. The linear correlations exist for the two subgroups of data, but there seems to exist two different coefficients, 0.34(9) for the data with $A < 130$ and 0.15(5) for $A \geqslant 130$. A close look at the data with $A \geqslant 130$ shows that many data have uncertainties of more than 0.01 fm (see also the data with large error bars around (0,0) in Fig. 1), thus hardly making a reliable fit. To verify the possible impact of the experimental precision we plot in Fig. 2(c) only the correlation data which have $\delta R(Z,N) < 0.007$ fm and $\delta R(Z,N)_{\rm def} < 0.007$ fm. The coefficient is determined to be 0.36(6), which is in good agreement with the coefficient of 0.30(5) obtained from the overall experimental data.

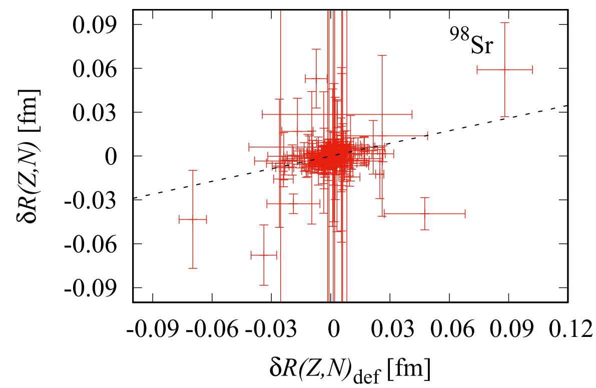

FIG. 1. $\delta R(Z,N)$ as a function of $\delta R(Z,N)(\beta_2^2)$ for all experimentally known cases. The linear fit is indicated by the dashed line. The relevant point for ^{98}Sr is labeled.

Specifically, we plot in Fig. 3 the correlation for the Sr isotopes. The sudden onset of the shape transition at $N = 60$ is reflected distinctly by both $\delta R(Z,N)_{\rm def}$ and $\delta R(Z,N)$ (see also Fig. 1). The deformation parameters for the relevant four nuclei, $^{98}_{38}$Sr, $^{96}_{38}$Sr, $^{96}_{36}$Kr, and $^{94}_{36}$Kr are 0.40(1), 0.175(6), 0.25(3), and 0.19(1), resulting in $\delta R(Z,N)_{\rm def}$ of 0.088(14) fm for ^{98}Sr. Therefore, once considering the deformation correlation, the large $\delta R(Z,N)$ value at $N = 60$, the well-known region of phase transitions, can be largely diminished. Similar correlation has been observed at $N \sim 90$ for the Nd isotopes. Unfortunately, it is not possible yet to test the region at $Z \sim 80$ due to missing deformation data.

Five cases at $^{20}_{10}$Ne, $^{46}_{20}$Ca, $^{46}_{22}$Ti, $^{76}_{34}$Se, and $^{78}_{36}$Kr show deviations from the linear trend by more than 2σ. Of them, $^{20}_{10}$Ne is the lightest nuclide with available charge radius and deformation. It is known already from the previous work [23] that the precision of the charge radius formula deteriorates with decreasing mass number (in particular for $A < 60$). This may be understood in the fact that the collective deformation property is more suitable for heavy nuclei in comparison with lighter nuclei. Further check on the mass dependence

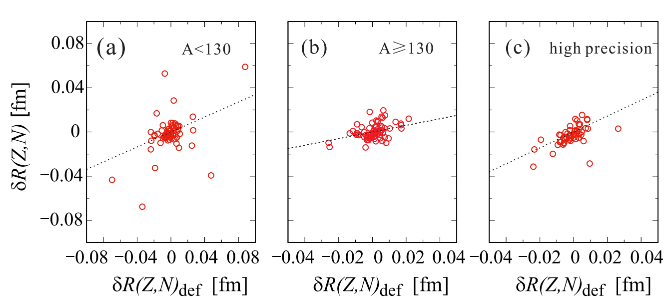

FIG. 2. Same as Fig. 1 but for the data with $A < 130$ (a), $A \geqslant 130$ (b), and $\delta R(Z,N) < 0.007$ fm and $\delta R(Z,N)_{\rm def} < 0.007$ fm (c). Experimental uncertainties are not shown. Linear fits to the relevant experimental data are indicated by dashed lines.

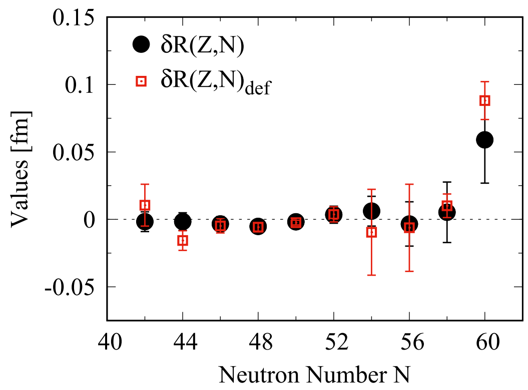

FIG. 3. Experimentally known $\delta R(Z,N)$ (filled circles) and $\delta R(Z,N)_{\rm def}$ (open squares) for the Sr isotopic chain.

confirms this argument, where 8 of 10 cases with $A < 60$ have deviations from the linear trend by more than 1 σ. It should be noted nevertheless that 1 σ discussed here is as small as 0.0040 fm.

The few cases with large deviations may be partially related to the various sources of charge radii and deformation, given the different methods of systematic errors associated with different techniques. We noted that a recent analysis [37] of $B(E2)$ measurements, from which β_2 is derived, concluded that most prevalent methods of measuring $B(E2)$ values are equivalent. Such comparison is not available yet for charge radius data across the entire chart of nuclides, and thus a consistent and equivalent set of nuclear charge radii and $B(E2)$ data are definitely crucial. A combined analysis of the cases for $^{46}_{20}$Ca and $^{46}_{22}$Ti shows that increasing the charge radius by 0.3% or decreasing the β_2 by about 40% (unlikely) for $^{44}_{20}$Ca will result in their $\delta R(Z,N)$ values in better agreements with the linear trend. Similar arguments hold also for the cases of $^{76}_{34}$Se and $^{78}_{36}$Kr. Therefore, the correlation identified here may provide us a very accurate way to investigate the consistency of charge radius and deformation surface.

In Ref. [23], it was found that Eq. (1) is remarkably successful even at nuclei with magic neutron and/or proton numbers. This can be easily understood with the correlation identified here. These nuclei are mostly in the spherical shape, i.e., their β_2 values are less than 0.1, thus leading to naturally $\delta R(Z,N) \approx \delta R(Z,N)_{\rm def} \approx 0$. This result is very different from the counterpart in nuclear masses, the valence proton-neutron interactions δV_{pn} [38–40], which depend strongly on the spatial overlap of the valence orbits and present a dramatic variation when crossing neutron shell closures.

III. DISCUSSION

A. Improved nuclear charge radius relation

The fact that $\delta R(Z,N)$ can be quantitatively reproduced with the $\delta R(Z,N)_{\rm def}$ for the existing data motivates further an improved charge radius formula:

$$\delta R(Z,N)^{\rm corr} = \delta R(Z,N) - c\delta R(Z,N)_{\rm def} \approx 0, \qquad (8)$$

where c is 0.30(5) determined from Fig. 1. $\delta R(Z,N)$ and $\delta R(Z,N)_{\rm def}$ are given in Eqs. (2) and (7), respectively. The available experimental data are used to the check the accuracy of the relation with the deformation correction. The weighted mean value of $\delta R(Z,N)^{\rm corr}$ amounts to only -8×10^{-4} fm with the weighted standard deviation of 5×10^{-3} fm. This corresponds to about 15% improvement in precision comparing with that without correction (i.e., $\delta R(Z,N)$). The significance is that Eq. (8) can now be extended to cases even when sudden variances occur in nuclear shapes [23,33].

Experimentally, the quadrupole deformation values are derived from the reduced electric quadrupole transition probability $B(E2)$, between the 0^+ ground state and the first 2^+ state in even-even nuclides, using the semiempirical approach,

$$\beta_2 = \frac{4\pi}{3ZR_0^2}\left[\frac{B(E2)}{\rm e^2}\right]^{1/2}. \qquad (9)$$

Here $R_0 = r_0 A^{1/3} = 1.2A^{1/3}$ fm and $B(E2)$ is in units of e^{2}b^2. To get Eq. (9), it is assuming a uniform charge distribution out to the distance R and zero charge beyond [36,41]. Moreover, $B(E2)$ is related to the mean lifetime τ of the first 2^+ state through

$$\tau(1+\alpha) = 40.81 \times 10^{13} E^{-5}\left[\frac{B(E2)}{\rm e^2\,b^2}\right]^{-1}, \qquad (10)$$

where E is the excitation energy of the first 2^+ state (in units of keV) and τ is in ps. The total internal conversion coefficient α for a specific E is needed for correction.

Accordingly, $\delta R(Z,N)_{\rm def}$ can be expressed in terms of $B(E2)$ and τ,

$$\begin{aligned}\delta R(Z,N)_{\rm def} &= 4.35 \times 10^3 \delta\left(\frac{B(E2)/{\rm e^2b^2}}{Z^2A}\right){\rm b}^{1/2}\\ &= 1.77 \times 10^{21}\delta\left(\frac{E^{-5}Z^{-2}A^{-1}}{\tau(1+\alpha)}\right){\rm b}^{1/2}, \qquad (11)\end{aligned}$$

where all the quantities E, $B(E2)$, Z, A, and α in the δ term are for the four neighboring even-even nuclei. In the case of no abrupt shape transition, $\delta R(Z,N)_{\rm def} \simeq 0$, and the following relation involving again four neighboring doubly even nuclei should hold well,

$$\delta(A^{1/3}\beta_2^2) \simeq \delta\left(\frac{B(E2)}{Z^2A}\right) \simeq \delta\left(\frac{E^{-5}Z^{-2}A^{-1}}{\tau(1+\alpha)}\right) \simeq 0. \qquad (12)$$

For heavy nuclear system, where the difference in Z^2A can be safely neglected, we can then get the relation

$$\begin{aligned}\delta B &= B(E2)(Z,N) - B(E2)(Z,N-2) - B(2)(Z-2,N)\\ &\quad + B(E2)[Z-2,N-2)]\\ &\simeq 0. \qquad (13)\end{aligned}$$

The same relation was proposed independently in Ref. [42] and was explored later in Ref. [43]. Using Eq. (13), $B(E2)$ transition strengths can be reproduced with an accuracy of $\pm 25\%$.

Inserting Eq. (11) into Eq. (8), we can get the correlation between four-point charge radius relation with that of $B(E2)$ or τ. This new relation, in principle, should be more accurate for predictions of unknown $B(E2)$ values than Eq. (13), because possible shape transitional effects can be at least partially compensated for by the corresponding charge radius data. When seven quantities out of eight, e.g., four charge radii

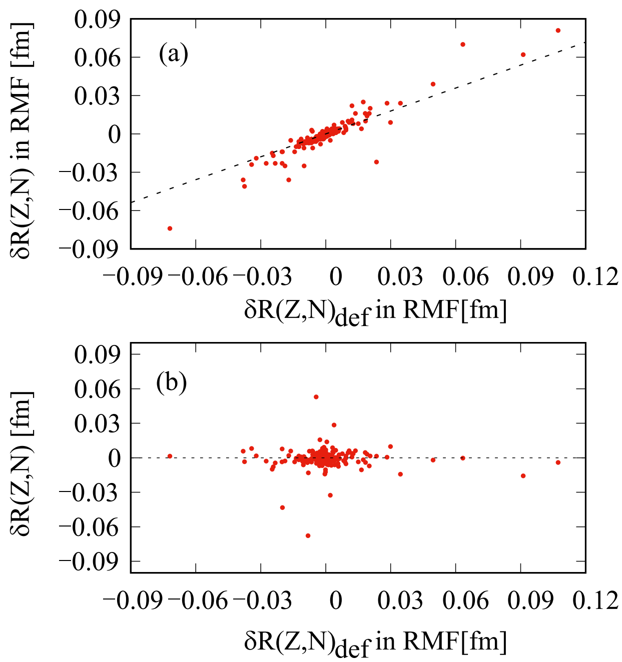

FIG. 4. Same as Fig. 1 but using the predicted $\delta R(Z,N)$ and $\delta R(Z,N)_{\rm def}$ in the RMF model (a), and using experimental $\delta R(Z,N)$ and predicted $\delta R(Z,N)_{\rm def}$ in the RMF model (b). Error bars are not shown for experimental data.

and three $B(E2)$ data, are known, then the last $B(E2)$ can be calculated. Unfortunately, this will give an uncertainty mostly too large to make meaningful predictions. The uncertainty is mainly propagated from the charge radius data and is typically about 1 order of magnitude higher than that from Eq. (13).

It should be noted that the knowledge of both experimental data on charge radii and deformation are still very limited. Therefore, it will be very useful if nuclear models can provide the relevant data either in absolute values or in differential values at a reasonable precision. For example, when one experimental deformation parameter is missing, one can resort to the theoretical predictions in nuclear models. Care should be taken that quantities like $B(E2)$ refer to the charge (proton) distribution in the nucleus and that in particular β is the charge deformation related to this charge distribution. This should be kept in mind when comparing the experimental results to nuclear models.

B. Correlation in global nuclear models

Global models such as Hartree-Fock-Bogoliubov (HFB-24) [44] and the relativistic mean-field (RMF) model [45] can provide self-consistently charge radius and deformation data. Taking RMF as an example, a linear correlation is predicted between $\delta R(Z,N)$ and $\delta R(Z,N)_{\rm def}$ as shown in Fig. 4(a). The slope is determined to be 0.60, about two times larger than that of experimental data. The same correlation has also been found in the HFB-24 but with a different coefficient parameter (0.85). Such correlations, however, vanish once we attempt to combine the model predictions with experimental data. An example is shown in Fig. 4(b), which displays experimental $\delta R(Z,N)$ vs calculated $\delta R(Z,N)_{\rm def}$ using the RMF. This indicates that current nuclear models like HFB-24 and RMF are not able yet to reproduce the fine correlation seen in the experimental data. Dedicated studies are thus required to provide a consistent description of charge radius and deformation to reproduce the experimental data.

IV. SUMMARY

To summarize, with the available experimental data we found a linear correlation between the charge radius relation $\delta R(Z,N)$ and the corresponding quadrupole deformation relation $\delta R(Z,N)_{\rm def}$. This correlation can provide a consistence check or analysis of experimental data on charge radii and deformation [and accordingly $B(E2)$]. A linear coefficient of 0.30(5) with no apparent dependence on the precision of experimental data is determined between $\delta R(Z,N)$ and $\delta R(Z,N)_{\rm def}$. It will be interesting to see whether this coefficient remains robust for more exotic nuclei, especially at shape transitional regions.

One important conclusion of the present work is that the large deviation of double-differential charge radius relation $\delta R(Z,N)$ from zero at shape transitional regions can be quantitatively reproduced with the experimental $\delta R(Z,N)_{\rm def}$. This in turn gives an improved charge radius formula and hence is very useful to make reliable short-range extrapolations of charge radii over the nuclear chart. The relation can be further generalized between charge radius and $B(E2)$ or τ. Moreover, the same correlation has been found in global nuclear models like HFB-24 and RMF, but so far the two models are not able to reproduce the experimental data.

Finally, we would like to mention that a consistent description [46–53] of radius and transition probabilities of atomic nuclei are important to understand their correlations and thus for a better interpretation of experimental results. A recent example is shown in $^{111-129}$Cd [54], in which the parabolic behavior of the charge radii is found due to the linear tendency of the quadruple deformation.

ACKNOWLEDGMENTS

This work has been supported by the NSFC under Grants No. 11475014 and the National key research and development program (2016YFA0400502). The authors thank Z. Li, Z. Niu, P. Zhao, and L. Zhu for useful comments.

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