Light-front spin-dependent spectral function and nucleon momentum distributions for a three-body system

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Poincaré covariant definitions for the spin-dependent spectral function and for the momentum distributions within the light-front Hamiltonian dynamics are proposed for a three-fermion bound system, starting from the light-front wave function of the system. The adopted approach is based on the Bakamjian–Thomas construction of the Poincaré generators, which allows one to easily import the familiar and wide knowledge on the nuclear interaction into a light-front framework. The proposed formalism can find useful applications in refined nuclear calculations, such as those needed for evaluating the European Muon Collaboration effect or the semi-inclusive deep inelastic cross sections with polarized nuclear targets, since remarkably the light-front unpolarized momentum distribution by definition fulfills both normalization and momentum sum rules. Also shown is a straightforward generalization of the definition of the light-front spectral function to an A-nucleon system.

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I. INTRODUCTION

In the analysis of the next generation of high-energy electron-nucleus scattering experiments planned at the Jefferson Laboratory (JLab) upgraded at 12 GeV [1], as well as at the future Electron-Ion Collider [2], a refined description of nuclei will play a relevant role [3], with a particular interest in the polarized ^{3}He target at JLab12. High-precision experiments, involving both protons and neutrons, are in fact necessary to clarify the flavor dependence of (i) parton distribution functions (PDFs), measured in inclusive deep inelastic scattering (DIS), and (ii) transverse-momentum-dependent parton distribution (TMDs; see, e.g., Ref. [4] for a general introduction), accessed through semi-inclusive DIS (SIDIS). In the next few years, several experiments involving a ^{3}He nuclear target will be performed at JLab12, with the aim at extracting information on the parton structure of the neutron. New DIS measurements are planned [5,6] and, in particular, the three-dimensional neutron structure in momentum space, described in terms of quark TMDs, will be probed through SIDIS off polarized ^{3}He, where a high-energy pion (kaon) is detected in coincidence with the scattered electron [7,8].

To be able to extract PDFs and TMDs in the neutron from DIS and SIDIS off ^{3}He, accurate theoretical descriptions of the structure of ^{3}He and of the scattering process are also needed. Initial studies of DIS and SIDIS off ^{3}He were performed in Ref. [9] and in Ref. [10], respectively, where the plane-wave impulse approximation (PWIA) was adopted to describe the reaction mechanism; namely, the interaction in the final state (FSI) was considered only within the two-nucleon spectator pair which recoils. The ^{3}He structure was treated nonrelativistically by using the AV18 *NN* interaction [11].

In a recent paper [12], the spectator SIDIS process off polarized ^{3}He, where a deuteron in the final state is detected, was studied by taking into account for the first time the FSI between the hadronizing quark and the detected deuteron through a distorted spin-dependent spectral function of ^{3}He. The study of the standard SIDIS process off transversely polarized ^{3}He with a fast detected pion including the FSI is presented in Ref. [13], where the FSI between the observed pion and the remnant is again taken into account through a distorted spin-dependent spectral function (preliminary results can be found in Ref. [14]). However, the description of the nuclear dynamics in Refs. [13,14], is still nonrelativistic or, more appropriately, non-Poincaré covariant, while the high energies involved in the forthcoming SIDIS experiments [7,8] should require a proper treatment of Poincaré covariance.

Our aim is to obtain a Poincaré covariant description of nuclear dynamics which considers only the nucleonic degrees of freedom and takes care of the large amount of knowledge on the nuclear interaction obtained from the nonrelativistic description of nuclei. Our approach could be used as a well-grounded relativistic starting point for further developments in the analysis of DIS or SIDIS processes, as the inclusion of other degrees of freedom, necessary for a full comprehension of these processes once the wavelength of the probe becomes more and more tiny (see, e.g., Ref. [15] and references therein). In particular, in this paper, the structure of a spin-$\frac{1}{2}$ three-nucleon system is investigated within a relativistic, Poincaré covariant framework (see Refs. [16,17] for early studies). Our approach can be straightforwardly generalized to other spin-$\frac{1}{2}$ three-body systems and even to complex nuclei. To develop a Poincaré covariant framework that allows one to embed our knowledge of the nuclear interaction, we adopt the relativistic hamiltonian dynamics (RHD) [18] with a fixed number of on-mass-shell constituents in its light-front (LF) version [19–22]. Within the LF form of RHD, the Poincaré group has a subgroup given by the LF boosts, which allows a kinematical separation of the intrinsic motion from the global one. Such a property plays a very important role for the relativistic description of DIS, SIDIS, and deeply virtual Compton scattering (DVCS) processes, where the final states can have a fast recoil. Furthermore, the LF dynamics allows a meaningful Fock expansion of the interacting system state [23] (with the caveat of zero modes). Only the valence component of the LF wave function of the system is considered here. If non-nucleonic degrees of freedom are needed for an accurate description of experimental data, as can be the case for a full evaluation of phenomena such as the European Muon Collaboration (EMC) effect, then within the LF Hamiltonian dynamics one could introduce higher Fock components of the LF wave function and the corresponding spectral functions, as suggested, e.g., in Ref. [24], or reconstruct the effects of the higher Fock states through the introduction of many-body currents by exploiting the exact LF projection technique proposed in Ref. [25]. Note that, in a field-theoretical framework, explicitly covariant, the constituent masses are off shell and the four-momenta are conserved, but the interaction must be introduced perturbatively. On the contrary, in a RHD framework (i) the explicit covariance is lost, (ii) the constituent masses are on mass shell and only three component of the momenta are conserved, but (iii) Poincaré covariance fully holds, and (iv) the interaction can be introduced nonperturbatively through the Bakamjian–Thomas construction of the Poincaré generators [26]. This last feature is essential for a realistic description of nuclei. For the sake of definiteness we consider the case of the three-nucleon systems, i.e., ^{3}He and ^{3}H.

The key quantity to be considered is the LF spectral function, depending on (i) spin and intrinsic momentum of the nucleon and (ii) the removal energy of the two-nucleon spectator system (for the definition of the nonrelativistic spin-dependent spectral function see, e.g., Ref. [27]). In general, for an A-body system the spectral function yields the probability distribution to find a constituent with a definite value of spin and momentum, while the $(A-1)$-constituent spectator system has a definite value of its mass. Such a distribution, properly convoluted with the probe-nucleon elementary cross section, leads to the description of scattering processes off nuclei in the impulse approximation. In this case, the motion of the knocked-out nucleon is free, while the spectator system is fully interacting. Therefore, one has to relativistically describe a final state where the cluster separability should be implemented. As shown in Ref. [19], this can be achieved by adopting the tensor product of a plane wave for the knocked-out constituent and a fully interacting intrinsic state for the spectator system, with given mass, all moving *in their intrinsic reference frame*. To build the spin-dependent spectral function, one needs to evaluate overlaps between the final state, previously described, and the ground state of the three-nucleon system. As a consequence, a crucial part of the paper is devoted to carefully defining interacting and noninteracting two- and three-body LF states, also providing the detailed link with the instant form counterparts. Notably, given the Bakamjian–Thomas (BT) framework we have assumed, the instant form states in turn can be safely approximated by the corresponding nonrelativistic quantities, as explained in what follows. It should be pointed out that, in order to describe the needed states, three reference frames are considered: (i) the laboratory frame of the fully interacting three-body system, (ii) the intrinsic LF frame of three free particles, and (iii) the intrinsic LF frame of a cluster of a free particle and an interacting two-particle subsystem.

With respect to previous attempts to describe DIS processes off ^{3}He in a LF framework (see, e.g., the one in Ref. [24]), in our approach special care is devoted to the definition of the intrinsic LF variables of the problem, as well as to the spin degrees of freedom in the definition of the spin-dependent spectral function. Details of the difference between our approach and the one of Ref. [24] are given in Sec. IV. Let us only anticipate here that the essential difference is the definition of the intrinsic nucleon momentum: in this paper it is the intrinsic momentum $\boldsymbol{\kappa}$ of the nucleon in a cluster of the nucleon and the fully interacting $(A-1)$-spectator system with given mass, needed to implement the cluster separability, while in Ref. [24]) it is the intrinsic nucleon momentum $\boldsymbol{k}$ for a system of A free nucleons. The difference between the two momenta $\boldsymbol{\kappa}$ and $\boldsymbol{k}$ depends on the energy of the interacting $(A-1)$ system and introduces an effect of binding which is new with respect to previous approaches within LF dynamics.

Our paper is organized as follows: in Sec. II the LF kinematics is summarized and in Sec. III the LF dynamics of two- and three-particle systems is briefly described and, whenever possible, use has been made of appendixes to collect and discuss in detail the relevant formal results. Section IV presents the definition of the LF spin-dependent spectral function in terms of the above-mentioned overlaps, as well as the LF momentum distributions and their sum rules. Conclusion and perspectives are discussed in Sec. V.

II. LIGHT-FRONT KINEMATICS

In this section, for the sake of completeness and to establish the formalism, we briefly review the LF kinematics [19].

A generic LF four-vector is $v = (v^-, \tilde{\mathbf{v}})$, with $\tilde{\mathbf{v}} = (v^+, \mathbf{v}_\perp)$ and $v^\pm = v^0 \pm v^3$; moreover, the scalar product of two vectors $\mathbf{a}$ and $\mathbf{b}$ is given by $\mathbf{a}\cdot\mathbf{b} = (a^-b^+ + a^+b^-)/2 - \mathbf{a}_\perp \cdot \mathbf{b}_\perp$.

Let us consider a system of mass M of n on-mass-shell *interacting* particles of mass m_i, momenta p_i $(i = 1,\dots,n)$, and total momentum P in the laboratory frame $(P^2 = M^2)$. The minus components of the momenta are

$$p_i^- = \frac{m_i^2 + |\mathbf{p}_{i\perp}|^2}{p_i^+}, \tag{1}$$

and the following intrinsic variables (invariant under a LF boost) can be introduced:

$$\xi_i = \frac{p_i^+}{P^+}, \quad \mathbf{k}_{i\perp} = \mathbf{p}_{i\perp} - \frac{p_i^+}{P^+}\mathbf{P}_\perp = \mathbf{p}_{i\perp} - \xi_i \mathbf{P}_\perp. \tag{2}$$

The conserved total LF momentum of the system (a three-dimensional one!) is given by

$$P^+ = \sum_{i=1,n} p_i^+, \quad \mathbf{P}_\perp = \sum_{i=1,n} \mathbf{p}_{i\perp}, \tag{3}$$

and as a consequence one has

$$\sum_i \xi_i = 1, \quad \sum_i \mathbf{k}_{i\perp} = 0. \tag{4}$$

One can complete the intrinsic variables by adding the plus and minus components of the intrinsic momenta as follows:

$$k_i^+ = \xi_i M_0,$$
$$k_i^- = \frac{P^+}{M_0}\left[p_i^- - 2\mathbf{p}_{i\perp}\cdot\frac{\mathbf{P}_\perp}{P^+} + p_i^+\left(\frac{\mathbf{P}_\perp}{P^+}\right)^2\right] = \frac{m_i^2 + |\mathbf{k}_{i\perp}|^2}{k_i^+}, \tag{5}$$

where M_0 is the invariant (for LF boosts) free mass, given by

$$M_0^2 = P^+ \sum_i \frac{m_i^2 + |\mathbf{p}_{i\perp}|^2}{p_i^+} - |\mathbf{P}_\perp|^2 = \sum_i \frac{m_i^2 + |\mathbf{k}_{i\perp}|^2}{\xi_i}. \tag{6}$$

Then, in a more compact form,

$$k_i^\mu = \left[B_{\rm LF}^{-1}(\tilde{\mathbf{P}}/M_0)\right]^\mu_\nu p_i^\nu, \tag{7}$$

with $B_{\rm LF}(\tilde{\mathbf{P}}/M_0)$ being a LF boost to the intrinsic rest frame of the system of n free particles of momenta p_i. Such a frame is defined by a total LF momentum $\tilde{\mathbf{P}}_{\rm intr} \equiv \{\sum_i k_i^+ = M_0, \mathbf{0}_\perp\}$.

Notice that $k_i^2 = p_i^2 = m_i^2$, since in the light-front Hamiltonian dynamics (LFHD) the constituents are put on the mass shell, as already mentioned. This feature, with the nice separation of the intrinsic motion from the global one, as shown in Eqs. (6) and (14) (see below), make straightforward the analogy with the nonrelativistic case.

Instead of the intrinsic variables ξ_i, one can introduce an alternative set of variables; namely,

$$k_{iz} = \frac{1}{2}[k_i^+ - k_i^-] = \frac{M_0}{2}\left[\xi_i - \frac{m_i^2 + |\mathbf{k}_{i\perp}|^2}{M_0^2 \xi_i}\right], \tag{8}$$

that fulfill the following constraint [cf. Eqs. (4) and (6)]:

$$\sum_{i=1,n} k_{iz} = 0. \tag{9}$$

Then, one can equally well use the LF intrinsic variables, $\{k_i^+, \mathbf{k}_{i\perp}\}$, or the Cartesian intrinsic variables, $\mathbf{k}_i$, that fulfill

$$\sum_{i=1,n} \mathbf{k}_i = 0. \tag{10}$$

To adopt the variables $\mathbf{k}_i$ is useful for making evident the analogy with the nonrelativistic framework, still remaining in the LFHD approach. In the case of free particles the intrinsic LF frame, defined by $\tilde{\mathbf{P}}_{\rm intr} \equiv \{M_0, \mathbf{0}_\perp\}$, can be also defined by $\mathbf{P} \equiv \mathbf{0}$. Let us recall that the bold character indicates a Cartesian vector, while the added tilde symbols indicates a LF three-vector.

Because of the positivity of ξ_i, one can invert Eq. (8), obtaining

$$\xi_i = \frac{k_{iz} + \sqrt{m_i^2 + |k_{iz}|^2 + |\mathbf{k}_{i\perp}|^2}}{M_0} = \frac{k_{iz} + E_i}{M_0}, \tag{11}$$

where $E_i = (m_i^2 + |\mathbf{k}_i|^2)^{1/2}$. Then

$$\sum_{i=1,n} E_i = M_0. \tag{12}$$

Let us stress that the minus component of the total momentum, P^-, is different from the free one [19]:

$$P^- = \frac{M^2 + \mathbf{P}_\perp^2}{P^+} \neq \sum_{i=1,n} p_i^- = \sum_{i=1,n} \frac{m_i^2 + |\mathbf{p}_{i\perp}|^2}{p_i^+}$$
$$= \frac{1}{P^+}\sum_{i=1,n}\frac{m_i^2 + |\mathbf{p}_{i\perp}|^2}{\xi_i} = P^-_{\rm free}. \tag{13}$$

In terms of the free mass, one can rewrite $P^-_{\rm free}$ as follows:

$$P^-_{\rm free} = \frac{1}{P^+}\left[M_0^2 + |\mathbf{P}_\perp|^2\right]. \tag{14}$$

For a particle of mass m, the LF spin, which has the three components $s^j_{\rm LF}$ in the particle rest frame, yields the Pauli–Lubanski four-vector in the reference where the particle has LF momentum $\tilde{\mathbf{p}}$, by applying a proper LF boost, $B_{\rm LF}(\tilde{\mathbf{p}}/m)$ (see, e.g., Ref. [20] for a detailed discussion of the LF spin). On the other hand, the canonical spin (instant form) s^i_c is obtained through a canonical boost $B_c^{-1}(\mathbf{p}/m)$ applied to the same Pauli–Lubanski four-vector. Therefore, the relation between the two spins is given by

$$s^i_c = \left[B_c^{-1}(\mathbf{p}/m)\right]^i_\nu [B_{\rm LF}(\tilde{\mathbf{p}}/m)]^\nu_j s^j_{\rm LF} = [\mathcal{R}^\dagger_M(\tilde{\mathbf{p}})]^i_j s^j_{\rm LF}, \tag{15}$$

where $\mathcal{R}_M(\tilde{\mathbf{p}})$, called Melosh rotation [28,29], is the rotation between the two rest frames reachable through LF and canonical boosts, respectively [19]. This rotation of spins implies the following relation between the plane-wave states of a particle with spin s (notice that the squared spin does not depend on the chosen RHD form) in the instant form and the LF one

$$|\mathbf{p}; s\sigma\rangle_c = \sum_{\sigma'} D^s_{\sigma'\sigma}[\mathcal{R}_M(\tilde{\mathbf{p}})]|\tilde{\mathbf{p}}; s\sigma'\rangle_{\rm LF}, \tag{16}$$

where $D^s_{\sigma'\sigma}[\mathcal{R}_M(\tilde{\mathbf{p}})]$ is the Wigner function for a spin s. Within **SL**(2C), the covering set of the four-dimensional Poincaré group, the representation of the Melosh rotation for $s = 1/2$, relevant in what follows, is a 2×2 matrix and reads

$$D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma'} = \chi^\dagger_\sigma \frac{m + k^+ - \imath\boldsymbol{\sigma}\cdot(\hat{z}\times\mathbf{k}_\perp)}{\sqrt{(m+k^+)^2 + |\mathbf{k}_\perp|^2}}\chi_{\sigma'}$$
$$= {}_{\rm LF}\langle\tilde{\mathbf{k}}; s\sigma|\mathbf{k}; s\sigma'\rangle_c, \tag{17}$$

where χ_σ is a two-dimensional spinor. The main feature of LF rotations, $R_{\rm LF}$, is given by the difference between the corresponding Wigner rotations (that occurs when the state $|\tilde{\mathbf{k}}; s\sigma'\rangle_{\rm LF}$ has to be transformed) and the rotations itself, differently from the case of instant-form rotations $R_{\rm IF}$ (where $R_{\rm IF}$ coincides with the associated Wigner rotation) [19,20]. This prevents the use of the usual Clebsch–Gordan coefficients for constructing the spin-spin and orbital-spin couplings within a LF framework, and therefore one has to exploit the relation (16) with the canonical spin.

We adopt the following normalization for the LF states $|\tilde{\mathbf{p}}; s\sigma\rangle_{\mathrm{LF}}$:

$$
{}_{\mathrm{LF}}\langle\sigma' s,\tilde{\mathbf{p}}'|\tilde{\mathbf{p}}; s\sigma\rangle_{\mathrm{LF}} = 2p^+(2\pi)^3\delta^3(\tilde{\mathbf{p}}' - \tilde{\mathbf{p}})\sum_{\mu'\mu} D^s_{\sigma'\mu'}[\mathcal{R}_M(\tilde{\mathbf{p}})]D^s_{\mu\sigma}[\mathcal{R}^\dagger_M(\tilde{\mathbf{p}})]_c\langle\mu' s|s\mu\rangle_c
$$
$$
= 2p^+(2\pi)^3\delta^3(\tilde{\mathbf{p}}' - \tilde{\mathbf{p}})\delta_{\sigma'\sigma}, \tag{18}
$$

and for the instant form states and spinors,

$$
\langle\mathbf{p}'|\mathbf{p}\rangle = 2E(2\pi)^3\delta(p'_z - p_z)\delta(\mathbf{p}'_\perp - \mathbf{p}_\perp)
$$
$$
\bar{u}u = 2m, \quad u^\dagger u = 2E, \tag{19}
$$

with $E(\mathbf{p}) = (m^2 + |\mathbf{p}|^2)^{1/2}$ and $\partial p^+/\partial p_z = 1 + p_z/p_0 = p^+/p_0$.

III. LIGHT-FRONT DYNAMICS FOR TWO- AND THREE-PARTICLE SYSTEMS

This section presents a resumé of the main features of the BT construction, which allows one to consistently include the interaction in the generators of the Poincaré group (see, e.g., Ref. [19]). In particular, since for defining the LF spectral function one needs overlaps between the three-nucleon ground state and three-nucleon states composed by the tensor product of a plane wave for one of the particles and a two-body interacting state for the spectator pair, we will focus on two- and three-body cases.

A. Dynamics of two interacting particles

In the case of a system of two identical particles, the LFHD leads to an ansatz for the two-body mass operator able to naturally embed a description based on the Schrödinger equation into a Poincaré-covariant framework (see, e.g., Refs. [30–32] for an application).

By eliminating the longitudinal LF variable ξ in favor of the third Cartesian component of the intrinsic momentum

$$
k_z = k_{1z} = M_0(1,2)\left(\xi - \tfrac{1}{2}\right), \tag{20}
$$

where $M_0^2(1,2)$ is given by

$$
M_0^2(1,2) = \frac{m^2 + |\mathbf{k}_\perp|^2}{\xi(1-\xi)} = 4[E(\mathbf{k})]^2 = 4(m^2 + |\mathbf{k}|^2), \tag{21}
$$

one can show the formal equivalence between a nonrelativistic description and a LF one. Moreover, one has

$$
k_1^+ = \xi M_0(1,2) = k^+,
$$
$$
k_2^+ = (1-\xi)M_0(1,2) = M_0(1,2) - k^+. \tag{22}
$$

The two-body Hamiltonian, with an interaction that depends upon intrinsic variables and fulfills the correct invariance properties under rotations and translations, leads to a square mass operator suitable for a Bakamjian–Thomas (BT) construction of the Poincaré generators [26]. This construction gives a simple way to introduce the interaction in the generators while satisfying the correct commutation rules. As a matter of fact, within the BT framework the two-body mass equation can be written as follows (see, e.g., Refs. [19,21,22]):

$$
\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\mathbf{k}|\left[M_0^2(1,2) + U(|\mathbf{k}|)\right]|j,j_z;\epsilon_{\mathrm{int}},\alpha;TT_z\rangle
$$
$$
= M^2\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\mathbf{k}|j,j_z;\epsilon_{\mathrm{int}},\alpha;TT_z\rangle,
$$
$$
\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\mathbf{k}|[4m^2 + 4|\mathbf{k}|^2 + U(|\mathbf{k}|)]|j,j_z;\epsilon_{\mathrm{int}},\alpha;TT_z\rangle
$$
$$
= M^2\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\mathbf{k}|j,j_z;\epsilon_{\mathrm{int}},\alpha;TT_z\rangle,
$$
$$
\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\mathbf{k}|\left[\frac{|\mathbf{k}|^2}{m} + V(|\mathbf{k}|)\right]|j,j_z;\epsilon_{\mathrm{int}},\alpha;TT_z\rangle
$$
$$
= \epsilon_{\mathrm{int}}\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\mathbf{k}|j,j_z;\epsilon_{\mathrm{int}},\alpha;TT_z\rangle, \tag{23}
$$

where $V = U/(4m)$ and

$$
\epsilon_{\mathrm{int}} = \frac{M^2 - 4m^2}{4m}. \tag{24}
$$

In the last line of Eq. (23) one formally recovers the Schrödinger equation for a two-body intrinsic eigenstate (that does not depend upon the chosen RHD) of angular momentum (j,j_z), the intrinsic energy ϵ_{int} (negative for bound states and positive for the scattering ones), and isospin (T,T_z). The symbol α represents the quantum numbers needed to completely define the state of the system. For the bound state (the deuteron in our case) one has $M = 2m - B$, and then

$$
\epsilon_{\mathrm{int}} = -B + \frac{B^2}{4m} \sim -B, \tag{25}
$$

given the small binding energy of the deuteron with respect to its mass. For the scattering states, one has $M^2 = s$, with s being one of the Mandelstam variables, and asymptotically $M^2 = 4m^2 + 4|\mathbf{t}|^2$ with $\mathbf{t}$ being the asymptotic Cartesian momentum in the intrinsic frame. Then, one can write

$$
\epsilon_{\mathrm{int}} = \frac{M^2 - 4m^2}{4m} = \frac{|\mathbf{t}|^2}{m}. \tag{26}
$$

Therefore the intrinsic eigenstates of Eq. (23) (i.e., of a Poincaré covariant mass operator) can be safely identified with the usual nonrelativistic two-body eigenstates [31,32] [only for bound states one disregards terms $O(B/(4m))$] and the overlap $\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\mathbf{k}|j,j_z;\epsilon_{\mathrm{int}},\alpha;TT_z\rangle$, which contains canonical spins, with its *nonrelativistic counterpart*.

As discussed in Appendix A, the normalized LF two-body wave function is

$$
{}_{\mathrm{LF}}\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\tilde{\mathbf{k}},\tilde{\mathbf{P}}'|\tilde{\mathbf{P}};j,j_z;\epsilon_{\mathrm{int}},\alpha;TT_z\rangle_{\mathrm{LF}}
$$
$$
= 2P^+(2\pi)^3\delta^3(\tilde{\mathbf{P}}' - \tilde{\mathbf{P}})\sqrt{(2\pi)^3E(\mathbf{k})}
$$
$$
\times\sum_{\sigma'_1,\sigma'_2} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma_1\sigma'_1}D^{\frac{1}{2}}[\mathcal{R}_M(-\tilde{\mathbf{k}})]_{\sigma_2\sigma'_2}
$$
$$
\times\langle\sigma'_1,\sigma'_2;\tau_1,\tau_2;\mathbf{k}|j,j_z;\epsilon_{\mathrm{int}},\alpha;TT_z\rangle, \tag{27}
$$

where we define [cf. Eq. (22)]

$$-\tilde{\mathbf{k}} \equiv ((M_0 - k^+), -\mathbf{k}_\perp). \quad (28)$$

It has to be emphasized that, in the intrinsic two-body wave function $\langle \sigma_1', \sigma_2'; \tau_1, \tau_2; \mathbf{k} | j, j_z; \epsilon_{\text{int}}, \alpha; T T_z \rangle$, the canonical spins can be composed with the orbital angular momenta by using the familiar Clebsch–Gordan coefficients. The state $|\mathbf{k}\rangle$ (with Cartesian variables) is normalized as follows:

$$\langle \mathbf{k}' | \mathbf{k} \rangle = \delta(\mathbf{k}' - \mathbf{k}). \quad (29)$$

Notice the difference with Eq. (19). Furthermore, for the two-body interacting case the LF completeness reads [see Eq. (A18)]

$$\int \frac{d\tilde{\mathbf{P}}}{2P^+(2\pi)^3} \sum_{j,j_z\alpha} \sum_{TT_z} \sum\!\!\!\!\!\!\int \lambda(t) dt |\tilde{\mathbf{P}}; j, j_z; \epsilon_{\text{int}}, \alpha; T T_z \rangle_{\text{LF}} \times {}_{\text{LF}}\langle T_z T; \alpha, \epsilon_{\text{int}}; j_z, j; \tilde{\mathbf{P}}| = \mathbf{I}, \quad (30)$$

where the symbol $\sum\!\!\!\!\!\!\int$ means a sum over the bound states of the pair (namely the deuteron in the present case) and an integration over the continuum. Notice the choice of the Cartesian t momentum to label the intrinsic energy. The quantity $\lambda(t)$ is the t density of the two-body states [$\lambda(t) = 1$ for the bound states and $\lambda(t) = t^2$ for the continuum]. Such a completeness follows from the one fulfilled by the eigensolutions of Eq. (23), i.e.,

$$\sum_{j,j_z\alpha} \sum_{TT_z} \sum\!\!\!\!\!\!\int \lambda(t) dt \langle \mathbf{k}' | j, j_z; \epsilon_{\text{int}}, \alpha; T T_z \rangle \langle T_z T; \alpha, \epsilon_{\text{int}}; j_z, j | \mathbf{k} \rangle = \delta^3(\mathbf{k}' - \mathbf{k}). \quad (31)$$

B. Three-interacting-particle systems

To have a Poincaré covariant description of an interacting system, such as the ^{3}He nucleus, it seems appropriate to adopt the LFHD framework combined with a Bakamjian–Thomas (BT) construction [26] of the Poincaré generators. With a suitable ansatz for the interaction (see, e.g., Refs. [19,22]), the mass operator is

$$M(1,2,3) = M_0(1,2,3) + \mathcal{V}(1,2,3) = \sum_{i=1,3} \sqrt{m_i^2 + |\mathbf{k}_i|^2} + \mathcal{V}(\mathbf{k}_i \cdot \mathbf{k}_j), \quad (32)$$

where $\mathbf{k}_i$ are the intrinsic momenta defined in Sec. II, and the interaction $\mathcal{V}$ is invariant for rotations and translations. The ground state can be written as the product of a plane wave describing the global motion with LF momentum $\tilde{\mathbf{P}}$ times eigenvectors of the three-body mass operator in Eq. (32). It reads

$$\left| \tilde{\mathbf{P}}; j, j_z; \epsilon^3_{\text{int}}, \Pi; \frac{1}{2}, T_z \right\rangle_{\text{LF}}, \quad (33)$$

where $\epsilon^3_{\text{int}} = M_3 - 3m$ is the energy, j is the total angular momentum, $1/2$ is the isospin of the system, and Π is the parity. From now on, we assume that the three particles have the same mass.

When applications like DIS or SIDIS processes are concerned, the issue of macrocausality has to be considered, i.e., if the subsystems which compose a system are brought far apart, the Poincaré generators of the system have to become the sum of the Poincaré generators corresponding to the subsystems in which the system is asymptotically separated. It is important to notice that the packing operators [19,33], which make it possible to include the macrocausality, are not considered in the present approximation for the description of the bound state. However, we implement macrocausality in the tensor product of a plane wave for the knocked-out constituent times a fully interacting intrinsic state for the spectator pair. This tensor product is needed for the definition of the LF spectral function, as shown below.

In a given frame, the LF three-body wave function can be expressed in terms of the intrinsic wave function, with canonical spins. Therefore, as in the two-body case, one can approximate such an intrinsic wave function by the corresponding nonrelativistic wave function, after checking that the nonrelativistic Schrödinger operator can be properly identified with a BT mass operator. Then the key point for actual calculations is the approximation $M(1,2,3) \sim M_{NR}(1,2,3)$, which is based on an appropriate definition of the interaction $\mathcal{V}$. This approximation is allowed since

$$M_{NR}(1,2,3) = 3m + \sum_{i=1,3} k_i^2/2m + V_{12}^{NR} + V_{23}^{NR} + V_{31}^{NR} + V_{123}^{NR} \quad (34)$$

fulfills rotational and translational invariance; namely, the general properties for making a mass operator acceptable as a BT mass operator. As a matter of fact, these properties are just those satisfied by the nonrelativistic nuclear interactions that give an accurate description of two- and three-nucleon data (see, e.g., Refs. [11,34]). An early investigation of the electromagnetic trinucleon systems, within the above-illustrated approach and using the refined nonrelativistic ground states of Ref. [35], can be found in Ref. [36].

1. Nonsymmetric intrinsic variables

To define the LF spectral function one needs the overlaps between the ground state of the three-body system and the states composed of the tensor product of a free nucleon and a fully interacting two-body system. Therefore, proper variables suited to describe these states have to be introduced. Instead of the symmetric intrinsic variables $\tilde{\mathbf{k}}_i$ ($i = 1,2,3$) that refer to the three particles moving in the three-body intrinsic frame, it is more suitable to introduce nonsymmetric variables. Let us consider the intrinsic variable $\tilde{\mathbf{k}}_j$ for particle j and the intrinsic variables for the internal motion of the spectator pair. For the sake of concreteness, let us take $j = 1$ and focus on the kinematics of the (2,3) pair, which globally moves in the three-body intrinsic frame with total LF momentum $(K_{23}^+, K_{23\perp})$. A set of intrinsic variables for the internal motion of the (2,3) pair can be defined as follows:

$$\eta = \frac{k_2^+}{k_2^+ + k_3^+} = \frac{\xi_2}{(\xi_2 + \xi_3)} = \frac{\xi_2}{1 - \xi_1} = \frac{p_2^+}{p_2^+ + p_3^+},$$

$$\mathbf{k}_{23\perp} = \mathbf{k}_{2\perp} - \eta(\mathbf{k}_{2\perp} + \mathbf{k}_{3\perp}) = \mathbf{k}_{2\perp} + \eta \mathbf{k}_{1\perp},$$

$$k_{23}^+ = \eta M_{23}, \quad k_{23z} = M_{23}\left(\eta - \frac{1}{2}\right), \quad (35)$$

where $k_i^+ = (m^2 + |\mathbf{k}_i|^2)^{1/2} + k_{iz}$ and M_{23} is the free mass for the (2,3) pair, defined as in Eq. (21),

$$M_{23}^2 = \frac{m^2 + |\mathbf{k}_{23\perp}|^2}{\eta(1-\eta)} = [2\sqrt{m^2 + |\mathbf{k}_{23}|^2}]^2. \tag{36}$$

Furthermore, the total LF momentum of the free (2,3) pair in the laboratory frame is

$$P_{23}^+ = p_2^+ + p_3^+, \quad \mathbf{P}_{23\perp} = \mathbf{p}_{2\perp} + \mathbf{p}_{3\perp}, \tag{37}$$

while in the intrinsic three-body frame the total LF momentum is

$$K_{23}^+ = k_2^+ + k_3^+, \quad \mathbf{K}_{23\perp} = \mathbf{k}_{2\perp} + \mathbf{k}_{3\perp} = -\mathbf{k}_{1\perp}. \tag{38}$$

In terms of the nonsymmetric intrinsic variables, the free mass of the three-particle system can be written as follows:

$$\begin{aligned} M_0(1,2,3) &= \sum_{i=1,3} \sqrt{m^2 + |\mathbf{k}_i|^2} \\ &= \sqrt{m^2 + |\mathbf{k}_1|^2} + \sqrt{M_{23}^2 + |\mathbf{k}_1|^2} \\ &= \frac{m^2 + |\mathbf{k}_{1\perp}|^2}{k_1^+} + \frac{M_{23}^2 + |\mathbf{k}_{1\perp}|^2}{K_{23}^+}. \end{aligned} \tag{39}$$

Then one has

$$\begin{aligned} &\frac{m^2 + |\mathbf{k}_{2\perp}|^2}{k_2^+} + \frac{m^2 + |\mathbf{k}_{3\perp}|^2}{k_3^+} \\ &= \frac{1}{k_2^+ + k_3^+}[M_{23}^2 + |\mathbf{k}_{2\perp} + \mathbf{k}_{3\perp}|^2], \end{aligned} \tag{40}$$

and therefore

$$M_{23}^2 = \frac{m^2 + |\mathbf{k}_{2\perp}|^2}{\eta} + \frac{m^2 + |\mathbf{k}_{3\perp}|^2}{(1-\eta)} - |\mathbf{k}_{1\perp}|^2. \tag{41}$$

2. *Three-body light-front wave function with nonsymmetric intrinsic variables*

For the fully interacting case, i.e., $\mathcal{V}(1,2,3) \neq 0$, the three-body LF wave function can be expressed through (i) the nonsymmetric intrinsic variables $\{\tilde{\mathbf{k}}_1, \tilde{\mathbf{k}}_{23}\}$ introduced in the previous section, instead of using the three-body standard Jacobi coordinates (defined through $\tilde{k}_1, \tilde{k}_2, \tilde{k}_3$), and (ii) canonical spins in the reference frame where $P^+ = M_0(1,2,3)$. Therefore, by repeating analogous steps as in the two-body case [cf. Eq. (27)], one has

$$\begin{aligned} &{}_{LF}\left\langle \sigma_1,\sigma_2,\sigma_3;\tau_1,\tau_2,\tau_3;\tilde{\mathbf{k}}_1,\tilde{\mathbf{k}}_{23},\tilde{\mathbf{P}}' \middle| \tilde{\mathbf{P}}; j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2},T_z \right\rangle_{LF} \\ &= 2P^+(2\pi)^3\delta^3(\tilde{\mathbf{P}}' - \tilde{\mathbf{P}}) \sum_{\sigma_1'}\sum_{\sigma_2'}\sum_{\sigma_3'} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_1)]_{\sigma_1\sigma_1'} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_2)]_{\sigma_2\sigma_2'} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_3)]_{\sigma_3\sigma_3'} \\ &\times \sqrt{\frac{(2\pi)^6 2E_1E_{23}M_{23}}{2M_0(1,2,3)}} \left\langle \sigma_1',\sigma_2',\sigma_3';\tau_1,\tau_2,\tau_3;\mathbf{k}_1,\mathbf{k}_{23} \middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z \right\rangle, \end{aligned} \tag{42}$$

where $E_{23} = (M_{23}^2 + |\mathbf{k}_1|^2)^{1/2}$ and $M_{23} = [m^2 + |\mathbf{k}_{23\perp}|^2 + (k_{23}^+)^2]/k_{23}^+$. The LF variables $\tilde{\mathbf{k}}_2$ and $\tilde{\mathbf{k}}_3$ can be easily obtained from $\tilde{\mathbf{k}}_1$ and $\tilde{\mathbf{k}}_{23}$. Indeed one has (i) $\eta = k_{23}^+/M_{23}$, (ii) $\mathbf{k}_{2\perp} = \mathbf{k}_{23\perp} - \eta\mathbf{k}_{1\perp}$, (iii) $\mathbf{k}_{3\perp} = -\mathbf{k}_{1\perp} - \mathbf{k}_{2\perp}$, (iv) $k_2^+ + k_3^+ = M_0(1,2,3) - k_1^+$ [cf. Eq. (39)], (v) $k_2^+ = \eta(k_2^+ + k_3^+)$, and (vi) $k_3^+ = M_0(1,2,3) - k_1^+ - k_2^+$.

In Eq. (42), the intrinsic wave function with canonical spins $\langle \sigma_1',\sigma_2',\sigma_3';\tau_1,\tau_2,\tau_3;\mathbf{k}_1,\mathbf{k}_{23}|j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\rangle$ is the eigensolution of the mass operator $M(1,2,3)$ of Eq. (32), which in actual calculation can be approximated by the nonrelativistic Hamiltonian operator (since, we repeat, the symmetry requirements are the same). As shown in Appendix B [see Eq. (B19)], the factors in Eq. (42) allow one to recover the normalization for the intrinsic part of the three-body bound state according to

$$\begin{aligned} &\sum_{\tau_1,\tau_2,\tau_3}\sum_{\sigma_1,\sigma_2,\sigma_3}\int d\mathbf{k}_1 \int d\mathbf{k}_{23} \\ &\times \left| \left\langle \sigma_1,\sigma_2,\sigma_3;\tau_1,\tau_2,\tau_3;\mathbf{k}_1,\mathbf{k}_{23} \middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z \right\rangle \right|^2 = 1, \end{aligned} \tag{43}$$

as in the nonrelativistic case.

3. *Free-mass and intrinsic reference frame for the (1,23) cluster*

Because of our interest in constructing the overlap between the three-nucleon ground state and a state where only the pair (2,3) is interacting, while the third nucleon is free, in what follows we investigate the corresponding mass operator, whose eigenstates are the tensor product that we have already mentioned.

By using the intrinsic variables $\{\xi_1, \mathbf{k}_{1\perp}\}$, one can introduce the squared free mass $\mathcal{M}_0^2(1,23)$ for the cluster (1,23), when the mass eigenvalue of the interacting (2,3) pair is M_S

$$\mathcal{M}_0^2(1,23) = \frac{m^2 + |\mathbf{k}_{1\perp}|^2}{\xi_1} + \frac{M_S^2 + |\mathbf{k}_{1\perp}|^2}{(1-\xi_1)}. \tag{44}$$

The intrinsic frame of the cluster (1,23) is defined by $\tilde{\mathbf{P}}_{\rm int}(1,23) \equiv \{\mathcal{M}_0, \mathbf{0}_\perp\}$. In this frame, the LF momentum of the nucleon 1 is given by

$$\begin{aligned} \kappa_1^+ &= \xi_1\mathcal{M}_0(1,23), \\ \boldsymbol{\kappa}_{1\perp} &= \mathbf{p}_{1\perp} - \xi_1\mathbf{P}_\perp = \mathbf{k}_{1\perp}, \end{aligned} \tag{45}$$

while the z Cartesian component reads [see Eq. (8)]

$$\kappa_{1z} = \frac{1}{2}[\kappa_1^+ - \kappa_1^-] = \frac{\mathcal{M}_0(1,23)}{2}\left[\xi_1 - \frac{m_1^2 + |\boldsymbol{\kappa}_{1\perp}|^2}{\mathcal{M}_0(1,23)^2\xi_1}\right]. \tag{46}$$

As a consequence one has

$$\mathcal{M}_0(1,23) = E(\kappa_1) + E_S, \tag{47}$$

with $E(\kappa_1) = (m^2 + |\boldsymbol{\kappa}_1|^2)^{1/2}$ and $E_S = (M_S^2 + |\boldsymbol{\kappa}_1|^2)^{1/2}$.

The total momentum of the (2,3) pair in the same frame is

$$\begin{aligned} K_S^+ &= (1-\xi_1)\mathcal{M}_0(1,23),\\ \mathbf{K}_{S\perp} &= -\boldsymbol{\kappa}_{1\perp} = -\mathbf{k}_{1\perp} = \mathbf{k}_{2\perp} + \mathbf{k}_{3\perp},\\ K_{Sz} &= -\kappa_{1z},\\ K_{Son}^- &= \frac{M_S^2 + |\mathbf{k}_{1\perp}|^2}{K_S^+}. \end{aligned} \tag{48}$$

Summarizing the pair (2,3), with internal variables $\{\eta,\mathbf{k}_{23\perp}\}$ and mass eigenvalue M_S [cf. Eqs. (23) and (35)], is moving with LF momentum $\tilde{\mathbf{K}}_S$ in the intrinsic frame of the *three-particle cluster* (1,23).

It should be pointed out that the intrinsic frame for the three-body system (1,2,3) and the intrinsic frame of the (1,23) cluster are related by a proper longitudinal LF boost that makes the change $P^+_{\rm int}(1,23) = \mathcal{M}_0(1,23) \to P^+_{\rm int}(1,2,3) = M_0(1,2,3)$.

4. Nonsymmetric basis for three-interacting-particle systems

In the $1 + (23)$ cluster only the interaction U_{23} between particles 2 and 3 is active; then one can introduce a three-body state given by the tensor product of an eigenstate of the total LF momentum $\tilde{\mathbf{P}}$ times *the intrinsic state of the cluster with a given mass for the interacting pair*. In turn, such an intrinsic state, which fulfills macrocausality [19], is given by the tensor product of a plane wave for particle 1 with LF momentum $\tilde{\boldsymbol{\kappa}}_1$, times the fully interacting state of the pair corresponding to the given energy eigenvalue. Therefore, one can write

$$|\tilde{\mathbf{P}};\tilde{\boldsymbol{\kappa}}_1\sigma_1\tau_1;\,j_{23}j_{23z}\epsilon_{23},\alpha;\,T_{23},\tau_{23}\rangle_{\rm LF}, \tag{49}$$

which is an eigenstate of the mass operator

$$\begin{aligned} M'(1,23) &= E(\kappa_1) + \sqrt{M_{23}^2(|\mathbf{k}_{23}|) + U_{23} + |\tilde{\boldsymbol{\kappa}}_1|^2}\\ &= E(\kappa_1) + \sqrt{M_{23}^{*2}(|\mathbf{k}_{23}|) + |\tilde{\boldsymbol{\kappa}}_1|^2}, \end{aligned} \tag{50}$$

with eigenvalue $\mathcal{M}_0(1,23) = E(\kappa_1) + E_S$ [$E_S = (M_S^2 + |\boldsymbol{\kappa}_1|^2)^{1/2}$]. The operator $M_{23}^{*2}(|\mathbf{k}_{23}|) = M_{23}^2(|\mathbf{k}_{23}|) + U_{23}(|\mathbf{k}_{23}|)$ is the square of the intrinsic mass operator of the interacting (2,3) pair, with eigenvalue $M_S^2 = 4(m^2 + m\epsilon_{23})$ [see Eq. (23)].

The set of eigenstates (49) is complete with the following completeness relation:

$$\begin{aligned} \mathbf{I} = \int \frac{d\tilde{\mathbf{P}}}{2P^+(2\pi)^3}\sum_{T_{23}\tau_{23}}\sum\!\!\!\!\!\!\int \lambda(t)dt \sum_{j_{23}j_{23z}\alpha}\sum_{\sigma_1\tau_1}\int\frac{d\tilde{\boldsymbol{\kappa}}_1}{2\kappa_1^+(2\pi)^3}\\ \times|\tilde{\mathbf{P}};\tilde{\boldsymbol{\kappa}}_1\sigma_1\tau_1;\,j_{23},j_{23z};\epsilon_{23},\alpha;\,T_{23},\tau_{23}\rangle_{\rm LF}\\ \times\,{}_{\rm LF}\langle T_{23},\tau_{23};\alpha,\epsilon_{23};\,j_{23z},j_{23};\,\tau_1\sigma_1\tilde{\boldsymbol{\kappa}}_1;\tilde{\mathbf{P}}|. \end{aligned} \tag{51}$$

Since it will play a relevant role for a proper definition of the LF spectral function, let us consider the overlap between the eigenstates (49) and the product of plane waves for (i) the total LF momentum P' for a system of three free particles, (ii) the LF momentum of particle 1, $\tilde{\mathbf{k}}_1'$, in the intrinsic frame of *the three free particles*, and (iii) the LF momentum $\tilde{\mathbf{k}}_{23}'$ for the intrinsic motion of the free subsystem (2,3). One has

$$\begin{aligned} &{}_{\rm LF}\langle\sigma_1',\sigma_2',\sigma_3';\tau_1',\tau_2',\tau_3';\tilde{\mathbf{P}}',\tilde{\mathbf{k}}_1',\tilde{\mathbf{k}}_{23}'|\tilde{\mathbf{P}};\tilde{\boldsymbol{\kappa}}_1\sigma_1\tau_1;\,j_{23},j_{23z};\epsilon_{23},\alpha;\,T_{23},\tau_{23}\rangle_{\rm LF}\\ &\quad= 2P^+(2\pi)^3\delta^3(\tilde{\mathbf{P}}'-\tilde{\mathbf{P}})\delta_{\tau_1\tau_1'}\,{}_{\rm LF}\langle\sigma_1'\tilde{\mathbf{k}}_1'|\tilde{\boldsymbol{\kappa}}_1\sigma_1\rangle_{\rm LF}\,{}_{\rm LF}\langle\sigma_2',\sigma_3';\tau_2',\tau_3';\tilde{\mathbf{k}}_{23}'|j_{23},j_{23z};\epsilon_{23},\alpha;T_{23},\tau_{23}\rangle_{\rm LF}\\ &\quad= 2P^+(2\pi)^3\delta^3(\tilde{\mathbf{P}}'-\tilde{\mathbf{P}})\delta_{\tau_1\tau_1'}\delta_{\sigma_1\sigma_1'}(2\pi)^3 2k_1'^+\delta^3\big(\tilde{\mathbf{k}}_1'-\tilde{\mathbf{k}}_1^{(a)}\big)\sqrt{\frac{\kappa_1^+E_{23}'}{k_1'^+E_S}}\sqrt{(2\pi)^3\frac{E_{23}'M_{23}'}{2M_0'(1,2,3)}}\\ &\qquad\times\sum_{\sigma_2}\sum_{\sigma_3}D^{\frac12}[\mathcal{R}_M(\tilde{\mathbf{k}}_{23}')]_{\sigma_2'\sigma_2}D^{\frac12}[\mathcal{R}_M(-\tilde{\mathbf{k}}_{23}')]_{\sigma_3'\sigma_3}\langle\sigma_2,\sigma_3;\tau_2',\tau_3';\mathbf{k}_{23}'|j_{23},j_{23z};\epsilon_{23},\alpha;T_{23},\tau_{23}\rangle, \end{aligned} \tag{52}$$

where $E_{23}' = (M_{23}'^2 + \mathbf{k}_1'^2)^{1/2}$, $M_{23}' = [m^2 + |\mathbf{k}_{23\perp}'|^2 + (k_{23}'^+)^2]/k_{23}'^+$,

$$M_0'(1,2,3) = \sqrt{m^2 + |\mathbf{k}_1'|^2} + \sqrt{M_{23}'^2 + |\mathbf{k}_1'|^2}, \tag{53}$$

and $-\tilde{\mathbf{k}}_{23}' \equiv ((M_{23}' - k_{23}'^+), -\mathbf{k}_{23\perp}')$.

The right-hand side of Eq. (52) reflects (i) the normalization properties of $|\tilde{\mathbf{k}}_1'\rangle_{\rm LF}$ and $|\tilde{\boldsymbol{\kappa}}_1\rangle_{\rm LF}$, (ii) the expression for the intrinsic wave function of the interacting pair (2,3), (iii) the proper overall normalization factors.

In Appendix C the correctness of the normalization factors in Eq. (52) is checked.

To obtain the last step in Eq. (52), one has to notice that the states $|\tilde{\mathbf{k}}_1\sigma_1\rangle_{\rm LF}$ and $|\tilde{\boldsymbol{\kappa}}_1\sigma_1\rangle_{\rm LF}$ are immediately related to the same LF state $|\xi_1,\boldsymbol{\kappa}_{1\perp} = \mathbf{k}_{1\perp},\sigma_1\rangle$, since $\xi_1 = \kappa_1^+/\mathcal{M}_0(1,23) = k_1^+/M_0(1,2,3)$. The two states differ for their normalization, i.e.,

$${}_{\rm LF}\langle\tilde{\mathbf{k}}_1'|\tilde{\mathbf{k}}_1\rangle_{\rm LF} = (2\pi)^3 2k_1^+\delta^3(\tilde{\mathbf{k}}_1' - \tilde{\mathbf{k}}_1), \tag{54}$$

and

$$_{\rm LF}\langle\tilde{\boldsymbol{\kappa}}'_1|\tilde{\boldsymbol{\kappa}}_1\rangle_{\rm LF} = (2\pi)^3 2\kappa_1^+ \delta^3(\tilde{\boldsymbol{\kappa}}'_1 - \tilde{\boldsymbol{\kappa}}_1). \tag{55}$$

In Eq. (52), $k_1^{+(a)}$ is obtained by transforming κ_1^+ from the frame where $P^+ = \mathcal{M}_0(1,23)$ to the frame where $P^+ = M_0(1,2,3)$ through a longitudinal LF boost, while $\mathbf{k}_{1\perp}^{(a)}$ remains unchanged, i.e., one has $\mathbf{k}_{1\perp}^{(a)} = \tilde{\boldsymbol{\kappa}}_{1\perp}$ [see Eq. (45)].

To determine $k_1^{+(a)}$ one can first evaluate $\mathcal{M}_0(1,23)$ from Eq. (47):

$$\mathcal{M}_0(1,23) = \frac{(\kappa_1^+)^2 + \left(m^2 + k_{1\perp}^2\right)}{2\kappa_1^+} + \left\{\left[\frac{(\kappa_1^+)^2 + \left(m^2 + k_{1\perp}^2\right)}{2\kappa_1^+}\right]^2 + M_S^2 - m^2\right\}^{1/2}. \tag{56}$$

Then one can obtain ξ_1,

$$\xi_1 = \frac{\kappa_1^+}{\mathcal{M}_0(1,23)}, \tag{57}$$

the three-body system free mass $M_0(1,2,3)$,

$$M_0^2(1,2,3) = \frac{m^2 + k_{1\perp}^2}{\xi_1} + \frac{M_{23}'^2 + k_{1\perp}^2}{1 - \xi_1}, \tag{58}$$

and

$$k_1^{+(a)} = \xi_1 M_0(1,2,3). \tag{59}$$

5. Overlaps between cluster states and bound state of three-particle system

The overlap between a state of the cluster $1 + (23)$ and the bound state of the three-particle system is the quantity needed for defining the LF spin-dependent spectral function. As a matter of fact, from Eqs. (33) and (49), one has

$$\begin{aligned}&_{\rm LF}\langle T_{23},\tau_{23};\alpha,\epsilon_{23};\, j_{23}j_{23z};\,\tau_1\sigma_1\tilde{\boldsymbol{\kappa}}_1;\tilde{\mathbf{P}}'\big|\tilde{\mathbf{P}};\, j,j_z;\epsilon^3_{\rm int},\Pi;\,\tfrac{1}{2}T_z\rangle\\ &\quad = 2P^+(2\pi)^3\delta^3(\tilde{\mathbf{P}}' - \tilde{\mathbf{P}})_{\rm LF}\langle T_{23},\tau_{23};\alpha,\epsilon_{23};\, j_{23}j_{23z};\,\tau_1\sigma_1\tilde{\boldsymbol{\kappa}}_1\big|\, j,j_z;\epsilon^3_{\rm int},\Pi;\,\tfrac{1}{2}T_z\rangle.\end{aligned} \tag{60}$$

As shown in Appendix C 2, after inserting in the intrinsic part of the overlap (60) (i) the completeness operator expressed through plane waves, i.e. [cf. Eq. (B10)],

$$\int \frac{d\tilde{\mathbf{k}}'_{23}}{k_{23}'^{+}(2\pi)^3}|\tilde{\mathbf{k}}'_{23}\rangle\langle\tilde{\mathbf{k}}'_{23}| \int \frac{M'_0(1,2,3)d\tilde{\mathbf{k}}'_1}{2k_1'^{+}E'_{23}(2\pi)^3}|\tilde{\mathbf{k}}'_1\rangle\langle\tilde{\mathbf{k}}'_1| = \mathbf{I}, \tag{61}$$

and (ii) Eqs. (42) and (52), one gets

$$\begin{aligned}&_{\rm LF}\left\langle T_{23},\tau_{23};\alpha,\epsilon_{23};\, j_{23}j_{23z};\,\tau_1\sigma_1\tilde{\boldsymbol{\kappa}}_1\middle|\, j,j_z;\epsilon^3_{\rm int},\Pi;\,\frac{1}{2}T_z\right\rangle\\ &= \sum_{\tau_2\tau_3}\int d\mathbf{k}_{23}\sum_{\sigma'_1} D^{\frac{1}{2}}\left[\mathcal{R}_M\left(\tilde{\mathbf{k}}_1^{(a)}\right)\right]_{\sigma_1\sigma'_1}\sqrt{(2\pi)^3 2E\left(\mathbf{k}_1^{(a)}\right)}\sqrt{\frac{\kappa_1^+E_{23}}{k_1^{+(a)}E_S}}\sum_{\sigma''_2,\sigma''_3}\sum_{\sigma'_2,\sigma'_3}\mathcal{D}_{\sigma''_2,\sigma'_2}(\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_2)\mathcal{D}_{\sigma''_3,\sigma'_3}(-\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_3)\\ &\quad\times\langle T_{23},\tau_{23};\alpha,\epsilon_{23};\, j_{23}j_{23z}|\mathbf{k}_{23},\sigma''_2,\sigma''_3;\tau_2,\tau_3\rangle\left\langle\sigma'_3,\sigma'_2,\sigma'_1;\tau_3,\tau_2,\tau_1;\mathbf{k}_{23},\mathbf{k}_1^{(a)}\middle|\, j,j_z;\epsilon^3_{\rm int},\Pi;\,\frac{1}{2}T_z\right\rangle,\end{aligned} \tag{62}$$

where the unitary matrices $\mathcal{D}_{\sigma''_i,\sigma'_i}$ are defined by the equation

$$\mathcal{D}_{\sigma''_i,\sigma'_i}(\pm\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_i) = \sum_{\sigma_i} D^{\frac{1}{2}}[\mathcal{R}_M^\dagger(\pm\tilde{\mathbf{k}}_{23})]_{\sigma''_i\sigma_i} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_i)]_{\sigma_i\sigma'_i}, \tag{63}$$

with the $+$ sign corresponding to $i = 2$ and the $-$ sign corresponding to $i = 3$.

Then the overlap of Eq. (60) can be evaluated by approximating $\langle T_{23},\tau_{23};\alpha,\epsilon_{23};\, j_{23}j_{23z}|\mathbf{k}_{23},\sigma''_2,\sigma''_3;\tau_2,\tau_3\rangle$ and $\langle\sigma'_3,\sigma'_2,\sigma'_1;\tau_3,\tau_2,\tau_1;\mathbf{k}_{23},\mathbf{k}_1|\, j,j_z;\epsilon^3_{\rm int},\Pi;\,\frac{1}{2}T_z\rangle$ with the corresponding nonrelativistic quantities. It should be recalled that the spins involved are canonical spins.

The normalization for the intrinsic LF overlap in Eq. (60) follows immediately from the completeness relation (51)

$$\sum_{T_{23}\tau_{23}\tau_1}\int\frac{d\tilde{\kappa}_1}{2\kappa_1^+(2\pi)^3}\sum\!\!\!\!\!\!\!\int\lambda(t)dt\sum_{\sigma_1}\sum_{j_{23}j_{23z}\alpha}\left|{}_{\rm LF}\left\langle T_{23},\tau_{23};\alpha,\epsilon_{23};\, j_{23}j_{23z};\tau_1\sigma_1,\tilde{\kappa}_1\right|j,j_z;\epsilon^3_{\rm int},\Pi;\frac{1}{2}T_z\right\rangle\right|^2=\left|j,j_z;\epsilon^3_{\rm int},\Pi;\frac{1}{2}T_z\right\rangle\right|^2=1. \tag{64}$$

As shown in Appendix C 3, this normalization can be recovered by using the explicit expression for the overlaps given in Eq. (62).

IV. LIGHT-FRONT SPIN-DEPENDENT SPECTRAL FUNCTION

The nonrelativistic spin-dependent spectral function $\hat{\mathbf{P}}^\tau_{\mathcal{M}}(\vec{p},E)$ for a nucleus of mass number A is a 2×2 matrix whose elements are

$$P^\tau_{\sigma,\sigma',\mathcal{M}}(\vec{p},E)=\sum_{f_{(A-1)}}\langle\vec{p},\sigma\tau;\psi_{f_{(A-1)}}|\psi_{\mathcal{J}\mathcal{M}}\rangle \times\langle\psi_{\mathcal{J}\mathcal{M}}|\psi_{f_{(A-1)}};\vec{p},\sigma'\tau\rangle\delta(E-E_{f_{(A-1)}}+E_A), \tag{65}$$

where $|\psi_{\mathcal{J}\mathcal{M}}\rangle$ is the ground state of the nucleus with energy E_A and polarized along $\vec{S}$, $|\psi_{f_{(A-1)}}\rangle$ is an eigenstate of the $(A-1)$-nucleon system with energy $E_{f_{(A-1)}}$ interacting with the same interaction of the nucleus, and $|\vec{p},\sigma\tau\rangle$ is the plane wave for the nucleon $\tau=\pm 1/2$, with momentum $\vec{p}$ in the nucleus rest frame and spin along the z axis equal to σ [37–39]. The state $|\psi_{\mathcal{J}\mathcal{M}}\rangle$ polarized along $\vec{S}$ can be expressed through the states $|\psi_{\mathcal{J}m}\rangle_z$ polarized along the z axis [38,40] as follows:

$$|\psi_{\mathcal{J}\mathcal{M}}\rangle_{\vec{S}}=\sum_m|\psi_{\mathcal{J}m}\rangle_z D^{\mathcal{J}}_{m,\mathcal{M}}(\alpha,\beta,\gamma), \tag{66}$$

where α, β, and γ are the Euler angles describing the proper rotation from the z axis to the polarization vector $\vec{S}$. Let us recall that the rotations involved act on the three-nucleon bound system as a whole and therefore are interaction free.

In a more compact form, for $\mathcal{J}=1/2$, the 2×2 matrix $\hat{\mathbf{P}}^\tau_{\mathcal{M}}(\vec{p},E)$ is given by

$$\hat{\mathbf{P}}^\tau_{\mathcal{M}}(\vec{p},E)=\tfrac{1}{2}\left[B^\tau_{0,\mathcal{M}}(|\vec{p}|,E)+\vec{\sigma}\cdot\vec{f}^\tau_{\mathcal{M}}(\vec{p},E)\right], \tag{67}$$

where the function $B^\tau_{0,\mathcal{M}}(|\vec{p}|,E)$ is the trace of $\hat{\mathbf{P}}^\tau_{\mathcal{M}}(\vec{p},E)$ and yields the usual unpolarized spectral function $P^\tau(|\vec{p}|,E)$. It should be noticed that the matrix $\hat{\mathbf{P}}^\tau_{\mathcal{M}}(\vec{p},E)$ and the pseudovector $\vec{f}^\tau_{\mathcal{M}}(\vec{p},E)$ depend on the direction of the polarization vector $\vec{S}$. Since $\vec{f}^\tau_{\mathcal{M}}(\vec{p},E)$ is a pseudovector, it is a linear combination of the pseudovectors at our disposal, viz. $\vec{S}$ and $\hat{p}(\hat{p}\cdot\vec{S})$, and therefore it can be put in the following form, where any angular dependence is explicitly given:

$$\vec{f}^\tau_{\mathcal{M}}(\vec{p},E)=\vec{S}B^\tau_{1,\mathcal{M}}(|\vec{p}|,E)+\hat{p}(\hat{p}\cdot\vec{S})B^\tau_{2,\mathcal{M}}(|\vec{p}|,E). \tag{68}$$

Let us focus on the $A=3$ case. To obtain a Poincaré covariant definition of the spin-dependent spectral function for a three-particle system within the LF dynamics, one replaces the nonrelativistic overlaps $\langle\vec{p},\sigma\tau;\psi_{f_{(A-1)}}|\psi_{\mathcal{J}\mathcal{M}}\rangle$, which define the nonrelativistic spectral function with their LF counterparts ${}_{\rm LF}\langle\tau_S,T_S;\alpha,\epsilon;J_zJ;\tau\sigma,\tilde{\kappa}|\Psi_0;S,T_z\rangle$, dependent upon the energy ϵ of the two-body system and upon the intrinsic momentum $\tilde{\kappa}$ of the third particle in the intrinsic reference frame of the cluster $1+(23)$ (cf. Sec. III B 5). The LF overlaps ${}_{\rm LF}\langle\tau_S,T_S;\alpha,\epsilon;J_zJ;\tau\sigma,\tilde{\kappa}|\Psi_0;S,T_z\rangle$ can be easily obtained from the overlaps of Eq. (62), writing through Eq. (66) the ground state $|\Psi_0;S,T_z\rangle$ of the three-body system, polarized along $\vec{S}$, in terms of the states $|j,j_z;\epsilon^3_{\rm int},\Pi;\frac{1}{2}T_z\rangle$, polarized along the z axis.

Then, within the LFHD one can define the spin-dependent nucleon spectral function for the three-nucleon system (^{3}He or ^{3}H) in the bound state $|\Psi_0;S,T_z\rangle$, as follows:

$$\begin{aligned}\mathcal{P}^\tau_{\sigma'\sigma}(\kappa^+,\boldsymbol{\kappa}_\perp,\kappa^-,S)&=\sum\!\!\!\!\!\!\!\int d\epsilon\rho(\epsilon)\delta\left(\kappa^--M_3+\frac{M_S^2+|\boldsymbol{\kappa}_\perp|^2}{(1-\xi)M_3}\right)\sum_{JJ_z\alpha}\sum_{T_S\tau_S}{}_{\rm LF}\langle\tau_S,T_S,\alpha,\epsilon;JJ_z;\tau\sigma',\tilde{\kappa}|\Psi_0;S,T_z\rangle\langle S,T_z;\Psi_0|\tilde{\kappa},\sigma\tau;JJ_z;\epsilon,\alpha,T_S,\tau_S\rangle_{\rm LF}\\&=\frac{1}{\left|\frac{\partial\kappa^-}{\partial\epsilon}\right|}\rho(\epsilon)\sum_{JJ_z\alpha}\sum_{T_S\tau_S}{}_{\rm LF}\langle\tau_S,T_S;\alpha,\epsilon;JJ_z;\tau\sigma',\tilde{\kappa}|\Psi_0;S,T_z\rangle\langle S,T_z;\Psi_0|\tilde{\kappa},\sigma\tau;JJ_z;\epsilon,\alpha;T_S,\tau_S\rangle_{\rm LF}=\left|\frac{\partial\epsilon}{\partial\kappa^-}\right|\mathcal{P}^\tau_{\sigma'\sigma}(\tilde{\kappa},\epsilon,S),\end{aligned} \tag{69}$$

where

$$\epsilon=\frac{(M_3-\kappa^-)(1-\xi)M_3-|\boldsymbol{\kappa}_\perp|^2}{4m}-m \tag{70}$$

is the intrinsic energy of the fully interacting two-nucleon eigenstate, $\rho(\epsilon)$ is the density of the two-body states [$\rho(\epsilon)=tm/2$ for the two-body continuum states and $\rho(\epsilon)=1$ for the deuteron bound state], M_3 is the nucleus mass, $\xi=\kappa^+/\mathcal{M}_0(1,23)$ [cf. Eqs. (45) and (56)], and

$$\left|\frac{\partial\epsilon}{\partial\kappa^-}\right|=\frac{(1-\xi)M_3}{4m}. \tag{71}$$

Let us notice that the variable κ^- is the *minus* component of the momentum of an off-mass-shell nucleon, as is clear from the δ function in Eq. (69). In Eq. (69), $\tau = \pm 1/2$, J, J_z is the spin, T_S, τ_S is the isospin, α is the set of quantum numbers needed to completely specify the two-body eigenstate, and $M_S^2 = 4(m^2 + m\epsilon)$.

The overlap ${}_{\rm LF}\langle \tau_S, T_S; \alpha, \epsilon; J_z J; \tau\sigma, \tilde{\boldsymbol{\kappa}} | \Psi_0; S, T_z\rangle$ is the one defined by Eqs. (66) and (62). In the special case where $\vec{S}$ is along the z axis, one obtains

$$\begin{aligned}
&\mathcal{P}^\tau_{\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S) \\
&= \rho(\epsilon) \sum_{J J_z \alpha} \sum_{T_S \tau_S} {}_{\rm LF}\left\langle \tau_S, T_S; \alpha, \epsilon; J J_z; \tau\sigma', \tilde{\boldsymbol{\kappa}} \middle| j, j_z; \epsilon^3_{\rm int}, \Pi; \frac{1}{2} T_z \right\rangle \\
&\quad \times \left\langle \frac{1}{2} T_z; \Pi, \epsilon^3_{\rm int}; j, j_z; \middle| \tilde{\boldsymbol{\kappa}}, \sigma\tau; J J_z; \epsilon, \alpha; T_S, \tau_S \right\rangle_{\rm LF},
\end{aligned} \tag{72}$$

and the LF spectral function can be evaluated through the explicit expression (62) for the overlap ${}_{\rm LF}\langle \tau_S, T_S; \alpha, \epsilon; J J_z; \tau\sigma, \tilde{\boldsymbol{\kappa}} | j, j_z; \epsilon^3_{\rm int}, \Pi; \frac{1}{2} T_z\rangle$ in terms of canonical two- and three-body wave functions. In turn, these wave functions can be replaced by the nonrelativistic wave functions. We emphasize once more that the two- and three-body nonrelativistic wave functions have all the needed properties with respect to rotations and translations of the corresponding canonical wave functions.

Let us now illustrate the differences between our LF spectral function and the one proposed in Ref. [24]. There are two main differences: The first difference is in the definition of the intrinsic momentum of the nucleon to be used in the overlaps ${}_{\rm LF}\langle \tau_S, T_S; \alpha, \epsilon; J J_z; \tau\sigma, \tilde{\boldsymbol{\kappa}} | j, j_z; \epsilon^3_{\rm int}, \Pi; \frac{1}{2} T_z\rangle$ needed in the definition of the spectral function. As explained in the previous sections, in our case the momentum $\boldsymbol{\kappa}$ is the intrinsic momentum of particle 1 in the intrinsic reference frame of the cluster $1 + (23)$. At variance, in Ref. [24], the spectral function is defined in terms of the intrinsic nucleon momentum $\mathbf{k}_1$ in the intrinsic reference frame of three free nucleons. As a consequence, the states (49) used for the definition of the spectral function fulfill the macrocausality, whereas this is not the case for the states $\langle T_{23}, \tau_{23}; \alpha, \epsilon_{23}; j_{23} j_{23z}; \sigma_1, \tau_1, \mathbf{k}_1|$. The use of the variable $\boldsymbol{\kappa}$ in the spectral function is new with respect to previous LF approaches for DIS and introduces a new dependence upon the energy of the $(2-3)$ fully interacting state and therefore opens the possibility to obtain different, better results in the description of experimental data for DIS (see Ref. [41]).

The second difference with Ref. [24] is in the Melosh rotations to be used in the definition of the overlaps (62). It is again a consequence of the use of the momentum $\boldsymbol{\kappa}$, which implies a more elaborate treatment of the Melosh rotations with respect to Ref. [24]: in our case when the spectral function is evaluated the Melosh rotations for particles 2 and 3 cannot be eliminated by the sum on the angular momentum J, J_z of the pair (23).

According to the completeness relation (51), the normalization of the spectral function reads [see also Eq. (64) and Appendix C]

$$\sum\!\!\!\!\!\!\int d\epsilon \int \frac{d\boldsymbol{\kappa}}{2E(\kappa)(2\pi)^3} \sum_\tau Tr\mathcal{P}^\tau(\tilde{\boldsymbol{\kappa}},\epsilon,S) = 1. \tag{73}$$

However, in applications one can normalize the spectral function $\mathcal{P}^\tau(\tilde{\boldsymbol{\kappa}},\epsilon,S)$ for each isospin channel, i.e.,

$$\sum\!\!\!\!\!\!\int d\epsilon \int \frac{d\boldsymbol{\kappa}}{2E(\kappa)(2\pi)^3} Tr\mathcal{P}^\tau(\tilde{\boldsymbol{\kappa}},\epsilon,S) = 1. \tag{74}$$

As it occurs for the nonrelativistic spectral function [see Eqs. (67) and (68)], the LF nucleon spin-dependent spectral function can be expressed by means of three scalar functions, $\mathcal{B}^\tau_{0,S}(|\boldsymbol{\kappa}|,\epsilon)$, $\mathcal{B}^\tau_{1,S}(|\boldsymbol{\kappa}|,\epsilon)$, and $\mathcal{B}^\tau_{2,S}(|\boldsymbol{\kappa}|,\epsilon)$:

$$\mathcal{P}^\tau_{\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S) = \tfrac{1}{2}\left[\mathcal{B}^\tau_{0,S}(|\boldsymbol{\kappa}|,\epsilon) + \boldsymbol{\sigma}\cdot\mathbf{f}^\tau_S(\boldsymbol{\kappa},\epsilon)\right]_{\sigma'\sigma}, \tag{75}$$

where

$$\mathbf{f}^\tau_S(\boldsymbol{\kappa},\epsilon) = \mathbf{S}\mathcal{B}^\tau_{1,S}(|\boldsymbol{\kappa}|,\epsilon) + \hat{\kappa}(\hat{\kappa}\cdot\mathbf{S})\mathcal{B}^\tau_{2,S}(|\boldsymbol{\kappa}|,\epsilon). \tag{76}$$

The function $\mathcal{B}^\tau_{0,S}(|\boldsymbol{\kappa}|,\epsilon)$ is the trace of $\mathcal{P}^\tau_{\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S)$ and yields the unpolarized spectral function.

A. The light-front nucleon momentum distributions and momentum sum rule

Within the LFHD, one can define the LF spin-independent nucleon momentum distribution, averaged on the spin directions, through the spectral function $\mathcal{P}^\tau_{\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S)$ as follows:

$$\begin{aligned}
n^\tau(\xi,\mathbf{k}_\perp) &= \sum\!\!\!\!\!\!\int d\epsilon \frac{1}{2\kappa^+(2\pi)^3}\frac{\partial\kappa^+}{\partial\xi} Tr\mathcal{P}^\tau(\tilde{\boldsymbol{\kappa}},\epsilon,S) \\
&= \sum\!\!\!\!\!\!\int d\epsilon \frac{1}{2(2\pi)^3}\frac{E_S}{(1-\xi)\kappa^+}\rho(\epsilon)\sum_\sigma\sum_{J J_z\alpha}\sum_{T_S\tau_S} {}_{\rm LF}\langle \tau_S, T_S; \alpha, \epsilon; J_z J; \tau\sigma, \tilde{\boldsymbol{\kappa}} | \Psi_0; S T_z\rangle \langle S, T_z; \Psi_0 | \tilde{\boldsymbol{\kappa}}, \sigma\tau; J J_z; \epsilon, \alpha; T_S, \tau_S\rangle_{\rm LF},
\end{aligned} \tag{77}$$

where Eq. (B17) has been used. From the completeness relation (51), one gets immediately the normalization of the nucleon momentum distribution:

$$\int d\xi \int d\mathbf{k}_\perp n^\tau(\xi,\mathbf{k}_\perp) = 1. \tag{78}$$

An explicit expression for the spin-averaged momentum distribution can be obtained by inserting in Eq. (77) the LF spectral function as written in Eq. (72) and in turn the expression for the overlaps given in Eq. (62).

Then, by using again the two-body completeness of Eq. (31) and the unitarity of the $\mathcal{D}$ and $D^{1/2}$ matrices, one obtains

$$n^\tau(\xi,\mathbf{k}_\perp) = \frac{1}{1-\xi}\sum_\sigma\sum_{\tau_2'\tau_3'}\sum_{\sigma_2',\sigma_3'}\int d\mathbf{k}_{23}\frac{E(\mathbf{k}_1)E_{23}}{k_1^+}\left|\left\langle\sigma_3',\sigma_2',\sigma;\tau_3',\tau_2',\tau;\mathbf{k}_{23},\mathbf{k}_1\middle|j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle\right|^2, \tag{79}$$

where $\mathbf{k}_{1\perp} = \mathbf{k}_\perp$ and $k_1^+ = \xi M_0(1,2,3)$ [see Eq. (59)]. Combining Eqs. (B11) and (B14), the normalization of the LF nucleon momentum distribution (78) can be rewritten as follows:

$$\begin{aligned}\int d\xi\int d\mathbf{k}_\perp n^\tau(\xi,\mathbf{k}_\perp) &= \int d\mathbf{k}_\perp\sum_\sigma\sum_{\tau_2\tau_3}\sum_{\sigma_2,\sigma_3}\int d\mathbf{k}_{23}\int\frac{\partial\xi}{\partial k_z}dk_z\frac{\partial k_z}{\partial k^+}\frac{E_{23}}{(1-\xi)}\left|\left\langle\sigma_3,\sigma_2,\sigma;\tau_3,\tau_2,\tau;\mathbf{k}_{23},\mathbf{k}\middle|j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle\right|^2\\
&= \int d\mathbf{k}_\perp\sum_\sigma\sum_{\tau_2\tau_3}\sum_{\sigma_2,\sigma_3}\int d\mathbf{k}_{23}\int dk_z\left|\left\langle\sigma_3,\sigma_2,\sigma;\tau_3,\tau_2,\tau;\mathbf{k}_{23},\mathbf{k}\middle|j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle\right|^2\\
&= \int d\mathbf{k}_\perp\int dk_z f^\tau(k_z,\mathbf{k}_\perp) = 1,\end{aligned} \tag{80}$$

where $f^\tau(k_z,\mathbf{k}_\perp)$ is the instant form momentum distribution in terms of the intrinsic nucleon momentum $\mathbf{k} = \mathbf{k}_1$, defined by Eqs. (2) and (8) of Sec. II,

$$f^\tau(k_z,\mathbf{k}_\perp) = \sum_\sigma\sum_{\tau_2\tau_3}\sum_{\sigma_2,\sigma_3}\int d\mathbf{k}_{23}\left|\left\langle\sigma_3,\sigma_2,\sigma;\tau_3,\tau_2,\tau;\mathbf{k}_{23},\mathbf{k}\middle|j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle\right|^2. \tag{81}$$

Let us show that the momentum sum rule

$$\int\xi d\xi\int d\mathbf{k}_\perp n^\tau(\xi,\mathbf{k}_\perp) = \frac{1}{3} \tag{82}$$

is satisfied by the LF momentum distribution $n^\tau(\xi,\mathbf{k}_\perp)$. Indeed, because of the symmetry of the three-body bound state, one has

$$\begin{aligned}\int\xi d\xi\int d\mathbf{k}_\perp n^\tau(\xi,\mathbf{k}_\perp) &= \sum_{\tau_2\tau_3}\sum_{\sigma_1\sigma_2,\sigma_3}\int d\mathbf{k}_1\int d\mathbf{k}_{23}\frac{k_1^+}{M_0(1,2,3)}\left|\left\langle\sigma_3,\sigma_2,\sigma_1;\tau_3,\tau_2,\tau;\mathbf{k}_{23},\mathbf{k}_1\middle|j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle\right|^2\\
&= \sum_{\tau_2\tau_3}\sum_{\sigma_1\sigma_2,\sigma_3}\int d\mathbf{k}_2\int d\mathbf{k}_{31}\frac{k_2^+}{M_0(1,2,3)}\left|\left\langle\sigma_3,\sigma_2,\sigma_1;\tau_3,\tau_2,\tau;\mathbf{k}_{31},\mathbf{k}_2\middle|j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle\right|^2\\
&= \sum_{\tau_2\tau_3}\sum_{\sigma_1\sigma_2,\sigma_3}\int d\mathbf{k}_3\int d\mathbf{k}_{12}\frac{k_3^+}{M_0(1,2,3)}\left|\left\langle\sigma_3,\sigma_2,\sigma_1;\tau_3,\tau_2,\tau;\mathbf{k}_{12},\mathbf{k}_3\middle|j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle\right|^2\\
&= \frac{1}{3}\sum_{\tau_2\tau_3}\sum_{\sigma_1\sigma_2,\sigma_3}\int d\mathbf{k}_1\int d\mathbf{k}_{23}\frac{(k_1^+ + k_2^+ + k_3^+)}{M_0(1,2,3)}\left|\left\langle\sigma_3,\sigma_2,\sigma_1;\tau_3,\tau_2,\tau;\mathbf{k}_{23},\mathbf{k}_1\middle|j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle\right|^2\\
&= \frac{1}{3},\end{aligned} \tag{83}$$

since [see Eqs. (B13) and (42)]

$$\left[\frac{\partial(\mathbf{k}_1,\mathbf{k}_{23})}{\partial(\mathbf{k}_2,\mathbf{k}_{31})}\right] = \frac{M_{23}E_1E_{23}}{M_{31}E_2E_{31}},\quad \left[\frac{\partial(\mathbf{k}_1,\mathbf{k}_{23})}{\partial(\mathbf{k}_3,\mathbf{k}_{12})}\right] = \frac{M_{23}E_1E_{23}}{M_{12}E_3E_{12}}, \tag{84}$$

$$\sqrt{E_1E_{23}M_{23}}|\mathbf{k}_1,\mathbf{k}_{23}\rangle = \sqrt{E_2E_{31}M_{31}}|\mathbf{k}_2,\mathbf{k}_{31}\rangle = \sqrt{E_3E_{12}M_{12}}|\mathbf{k}_3,\mathbf{k}_{12}\rangle, \tag{85}$$

and $k_1^+ + k_2^+ + k_3^+ = M_0(1,2,3)$. The momentum sum rule (82) has also been successfully checked by calculating numerically Eq. (83) in an actual case using the three-body wave function of Ref. [35] with the nuclear interaction of Ref. [11]. In the case of the proton (with accuracy produced by the normalization of the nonrelativistic wave function) we obtain 0.9989 for the normalization and 0.3324 for the sum rule, while for the neutron we have 0.9981 and 0.3336, respectively (see also Ref. [41]).

Within the BT framework one can obtain LF momentum distributions dependent upon the spin directions, $n^\tau_{\sigma'\sigma}(\xi,\mathbf{k}_\perp;\vec{S})$, for any direction of the polarization vector $\vec{S}$ of the three-body system, by using Eq. (66) and the expression for the LF spin-dependent spectral function given by Eq. (72),

$$n^{\tau}_{\sigma'\sigma}(\xi,\mathbf{k}_\perp;\vec{S}) = \frac{1}{(1-\xi)}\sum_{\tau_2\tau_3}\int d\mathbf{k}_{23}\sum_{\sigma_1'} D^{\frac{1}{2}}\big[\mathcal{R}_M\big(\tilde{\mathbf{k}}_1^{(a)}\big)\big]_{\sigma'\sigma_1'} E\big(\mathbf{k}_1^{(a)}\big)\frac{E_{23}}{k_1^{+(a)}} \times \sum_{\sigma_2',\sigma_3'}\sum_m D^j_{m,\mathcal{M}}(\alpha,\beta,\gamma)\Big\langle\sigma_3',\sigma_2',\sigma_1';\tau_3,\tau_2,\tau;\mathbf{k}_{23},\mathbf{k}_1^{(a)}\Big| j,j_z=m;\epsilon^3_{\rm int},\Pi;\frac{1}{2}T_z\Big\rangle \times \sum_{\bar{\sigma}_1'} D^{\frac{1}{2}*}\big[\mathcal{R}_M\big(\tilde{\mathbf{k}}_1^{(a)}\big)\big]_{\sigma\bar{\sigma}_1'}\sum_{m'}\big[D^j_{m',\mathcal{M}}(\alpha,\beta,\gamma)\big]^*\Big\langle\sigma_3',\sigma_2',\bar{\sigma}_1';\tau_3,\tau_2,\tau;\mathbf{k}_{23},\mathbf{k}_1^{(a)}\Big| j,j_z=m';\epsilon^3_{\rm int},\Pi;\frac{1}{2}T_z\Big\rangle^*. \quad (86)$$

Recall that α, β, and γ are the Euler angles describing the rotation from the z axis to the polarization vector $\vec{S}$. In Eq. (86) the explicit expression (62) for the overlaps is used, as well as the two-body completeness and once again the unitarity of the $\mathcal{D}$ and $D^{1/2}$ matrices.

V. CONCLUSIONS AND PERSPECTIVES

In this paper, within the BT approach for the Poincaré generators, a LF spin-dependent spectral function and LF spin-dependent momentum distributions have been defined by starting from the LF wave function for a three-body system, having in mind the ^{3}He and the ^{3}H nuclei. The spectral function is defined through the overlaps between the ground-state wave function of the three-body system and the tensor product of a plane wave for one of the nucleons in the intrinsic reference frame of the cluster (1,23) and the state which describes the intrinsic motion of the fully interacting two-nucleon spectator subsystem. In the present approach the packing operators, which are needed to implement the macrocausality, are not considered in the description of the ground state of the three-body system, but the macrocausality is fully considered in the mentioned tensor product.

A generalization to A-nucleon nuclei is straightforward: one has only to generalize the definition of the intrinsic momentum κ as the momentum of one of the nucleons in the intrinsic reference frame of the cluster composed by this free nucleon and by the fully interacting system of the remaining $A-1$ nucleons. Then the LF spin-dependent spectral function for the A-nucleon nucleus is

$$\mathcal{P}^{\tau}_{\sigma'\sigma}(\kappa^+,\boldsymbol{\kappa}_\perp,\kappa^-,S,A) = \sum\!\!\!\!\!\!\int d\epsilon_{A-1}\rho(\epsilon_{A-1})_{A-1}\delta\left(\kappa^- - M_A + \frac{M^2_{A-1}+|\boldsymbol{\kappa}_\perp|^2}{(1-\xi)M_A}\right) \times\sum_{JJ_z\alpha}\sum_{T_{A-1}\tau_{A-1}} {}_{LF}\langle\tau_{A-1},T_{A-1},\alpha,\epsilon_{A-1};JJ_z;\tau\sigma',\tilde{\boldsymbol{\kappa}}|A,\Psi_0;S,T_z\rangle\langle S,T_z;\Psi_0,A|\tilde{\boldsymbol{\kappa}},\sigma\tau;JJ_z;\epsilon_{A-1},\alpha,T_{A-1},\tau_{A-1}\rangle_{\rm LF}, \quad (87)$$

where $|A,\Psi_0;S,T_z\rangle$ is the ground-state of the A-nucleon nucleus, while M_{A-1} and ϵ_{A-1} are the mass and the intrinsic energy, $\rho(\epsilon_{A-1})_{A-1}$ is the density, J, J_z is the spin, T_{A-1}, τ_{A-1} is the isospin of the $(A-1)$-nucleon system, and α is the set of quantum numbers needed to fully specify this system.

Notably within the LF Hamiltonian dynamics, both normalization and the momentum sum rule can be exactly satisfied at the same time. With respect to previous attempts to describe DIS processes off ^{3}He in a LF framework (see, e.g., the one in Ref. [24]), in our approach for the spin-dependent spectral function special care is devoted to the definition of the intrinsic LF variables of the problem, as well as to the spin degrees of freedom through the Melosh rotations. Let us stress once again that the definition of the nucleon momentum $\boldsymbol{\kappa}$ in the intrinsic reference frame of the cluster (1,23) and the use for the calculation of the LF spectral function of the tensor product of a plane wave of momentum $\boldsymbol{\kappa}$ times the state which describes the intrinsic motion of the fully interacting spectator subsystem allows one on one hand to take care of macrocausality and on the other one to introduce a new effect of binding in the spectral function.

Our approach allows one to embed in a Poincaré covariant framework the large amount of knowledge on the nuclear interaction obtained from the nonrelativistic description of nuclei, since we adopt the LF version of the relativistic Hamiltonian dynamics with a fixed number of on-mass-shell constituents. The LF form of RHD has a subgroup composed by the LF boosts, which allows a separation of the intrinsic motion from the global one, very important for the description of DIS, SIDIS and deeply virtual Compton scattering processes, since it is possible to unambiguously identify the effects due to the inner dynamics.

Therefore, our LF spectral functions can be useful in many problems that require both a proper relativistic treatment and at the same time a good description of the internal structure of the system. A calculation of DIS processes based on our spectral function will indicate which is the gap with respect to the experimental data to be filled by effects of non-nucleonic degrees of freedom or by modifications of nucleon structure in nuclei.

As a first example of forthcoming applications, we can mention the study of the role played by relativity in the EMC effect on ^{3}He, for which JLab data have been taken at 6 GeV [42] in the standard inclusive DIS sector. Encouraging results of the new effects of binding introduced by the definition of the momentum κ, obtained including an exact treatment of the deuteron channel and an approximated treatment for the continuum of the LF spectral function can be found in Ref. [41]. We plan to complete this study by using the full LF spectral function, as defined in Eq. (69).

A second example for an application of the LF approach proposed in this paper is the study of the effect of relativity in the evaluation of SIDIS cross section off ^{3}He, taking into account both the relativity and the interaction in the final state between the observed pion and the remnant. In Refs. [13,14], by adopting a nonrelativistic spectral function evaluated from the ^{3}He wave function of Ref. [35], a distorted spin-dependent spectral function was obtained by using a generalized eikonal approximation to deal with the final-state interaction, and it was shown that, within this framework, it is actually possible to get reliable information on the quark TMDs in the neutron from SIDIS experiments off ^{3}He. By considering the new LF spin-dependent spectral function, we plan to evaluate SIDIS cross sections off ^{3}He through a LF distorted spin-dependent spectral function obtained by applying again the generalized eikonal approximation for the description of the final-state interaction. Preliminary results can be found in Ref. [17].

In view of the large efforts in the determination of the TMDs to study the three-dimensional structure of the nucleon, the same concepts and definitions that are used in this paper to build up the LF spin-dependent spectral function for a three-nucleon system could tentatively be applied to a system of three valence quarks to define a nucleon spectral function in the valence approximation and then to describe the nucleon TMDs in terms of a valence wave function for the nucleon.

It will be also interesting to study in detail the relation between the LF spin-dependent spectral function and the correlator, $\Phi(k,P,S)$, of a nucleon of momentum k in a nucleus of momentum P and spin polarization S, defined in terms of the nucleon fields, in analogy with the quark correlator in a nucleon, defined in terms of the quark fields [4]. In Refs. [16,17], preliminary results were presented and it was shown that, in the valence approximation, a simple relation between the correlator and the LF spin-dependent spectral function naturally emerges and that only three of the six time-reversal even TMDs at the leading twist [4] are independent. The relations among these TMDs could be experimentally checked to test our LF description of the spin-dependent spectral function.

APPENDIX A: TWO-BODY LIGHT-FRONT WAVE FUNCTION

In this Appendix, some details are given on the two-body light-front wave function that are useful for the general discussion presented in Sec. III.

1. Completeness of two-body free states

Let $\tilde{\mathbf{P}}$ be the total LF momentum for a two-particle system,

$$\tilde{\mathbf{P}} = \tilde{\mathbf{p}}_1 + \tilde{\mathbf{p}}_2. \tag{A1}$$

The Jacobian from $\{\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2\}$ to $\{\tilde{\mathbf{P}},\xi,\mathbf{k}_\perp\}$ is

$$\left[\frac{\partial(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2)}{\partial(\tilde{\mathbf{P}},\xi,\mathbf{k}_\perp)}\right] = P^+, \tag{A2}$$

and the Jacobian from $\{\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2\}$ to $\{\tilde{\mathbf{P}},k^+,\mathbf{k}_\perp\}$ is given by

$$\left[\frac{\partial(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2)}{\partial(\tilde{\mathbf{P}},k^+,\mathbf{k}_\perp)}\right] = \frac{2(1-\xi)}{M_0(1,2)}P^+ = \frac{2\xi(1-\xi)}{k^+}P^+, \tag{A3}$$

with $M_0(1,2)$ defined by Eq. (21), since

$$\frac{\partial k^+}{\partial \xi} = M_0(1,2) - \xi\frac{1}{2M_0(1,2)}\frac{m^2+|\mathbf{k}_\perp|^2}{\xi^2(1-\xi)^2}(1-2\xi) = \frac{M_0(1,2)}{2(1-\xi)} = \frac{k^+}{2\xi(1-\xi)}. \tag{A4}$$

Furthermore, the Jacobian from $\{\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2\}$ to $\{\tilde{\mathbf{P}},k_z,\mathbf{k}_\perp\}$ is given by

$$\left[\frac{\partial(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2)}{\partial(\tilde{\mathbf{P}},k_z,\mathbf{k}_\perp)}\right] = \frac{2\xi(1-\xi)}{E(\mathbf{k})}P^+, \tag{A5}$$

since [cf. Eq. (8)]

$$\frac{\partial k_z}{\partial \xi} = M_0(1,2) - \left(\xi-\frac{1}{2}\right)\frac{1}{2M_0(1,2)}\frac{m^2+|\mathbf{k}_\perp|^2}{\xi^2(1-\xi)^2}(1-2\xi) = \frac{E(\mathbf{k})}{2\xi(1-\xi)}. \tag{A6}$$

From Eqs. (A4) and (A6) one has

$$\frac{\partial k^+}{\partial k_z} = \frac{\partial k^+}{\partial \xi}\frac{\partial \xi}{\partial k_z} = \frac{k^+}{E(\mathbf{k})}. \tag{A7}$$

Keeping separate the global motion from the intrinsic one, the completeness reads

$$\begin{aligned}\mathbf{I} &= \int \frac{d\tilde{\mathbf{p}}_1}{2p_1^+(2\pi)^3}\frac{d\tilde{\mathbf{p}}_2}{2p_2^+(2\pi)^3}|\tilde{\mathbf{p}}_1\rangle|\tilde{\mathbf{p}}_2\rangle\langle\tilde{\mathbf{p}}_1|\langle\tilde{\mathbf{p}}_2| = 2\int\frac{d\tilde{\mathbf{P}}}{2P^+(2\pi)^3}|\tilde{\mathbf{P}}\rangle\langle\tilde{\mathbf{P}}|\int\frac{d\xi}{(2\pi)^3 4\xi(1-\xi)}\int d\mathbf{k}_\perp|\tilde{\mathbf{k}}\rangle\langle\tilde{\mathbf{k}}| \\ &= 2\int\frac{d\tilde{\mathbf{P}}}{2P^+(2\pi)^3}|\tilde{\mathbf{P}}\rangle\langle\tilde{\mathbf{P}}|\int\frac{d\tilde{\mathbf{k}}}{2k^+(2\pi)^3}|\tilde{\mathbf{k}}\rangle\langle\tilde{\mathbf{k}}| = 2\int\frac{d\tilde{\mathbf{P}}}{2P^+(2\pi)^3}|\tilde{\mathbf{P}}\rangle\langle\tilde{\mathbf{P}}|\int\frac{d\mathbf{k}}{(2\pi)^3 2E(\mathbf{k})}|\tilde{\mathbf{k}}\rangle\langle\tilde{\mathbf{k}}|.\end{aligned} \tag{A8}$$

Notice in the last step the hybrid notation in the intrinsic part. It will be used in what follows.

The normalization of the free state $|\tilde{\mathbf{P}}\rangle|\tilde{\mathbf{k}}\rangle = |\tilde{\mathbf{p}}_1\rangle|\tilde{\mathbf{p}}_2\rangle$ is

$$\begin{aligned}\langle\tilde{\mathbf{p}}_2'|\tilde{\mathbf{p}}_2\rangle\langle\tilde{\mathbf{p}}_1'|\tilde{\mathbf{p}}_1\rangle &= 2p_1^+(2\pi)^3\delta^3(\tilde{\mathbf{p}}_1' - \tilde{\mathbf{p}}_1)2p_2^+(2\pi)^3\delta^3(\tilde{\mathbf{p}}_2' - \tilde{\mathbf{p}}_2)\\ &= \left[\frac{\partial(\tilde{\mathbf{P}},k^+,\mathbf{k}_\perp)}{\partial(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2)}\right]2p_1^+(2\pi)^3 2p_2^+(2\pi)^3\delta^3(\tilde{\mathbf{P}}' - \tilde{\mathbf{P}})\delta^3(\tilde{\mathbf{k}}' - \tilde{\mathbf{k}})\\ &= 2P^+(2\pi)^3\delta^3(\tilde{\mathbf{P}}' - \tilde{\mathbf{P}})k^+(2\pi)^3\delta^3(\tilde{\mathbf{k}}' - \tilde{\mathbf{k}}) = \langle\tilde{\mathbf{P}}'|\tilde{\mathbf{P}}\rangle\langle\tilde{\mathbf{k}}'|\tilde{\mathbf{k}}\rangle. \end{aligned} \tag{A9}$$

It should be pointed out that $\langle\tilde{\mathbf{k}}'|\tilde{\mathbf{k}}\rangle = k^+(2\pi)^3\delta^3(\tilde{\mathbf{k}}' - \tilde{\mathbf{k}})$, i.e., without a factor of two, since it refers to a two-body intrinsic state.

The overlap between the free two-body intrinsic states $|\tilde{\mathbf{k}};\sigma_2,\sigma_1\rangle_{\rm LF}$ and the corresponding states with canonical spin and Cartesian coordinates is relevant for the following discussion. Let us recall that $\delta(k'^+ - k^+) = \delta(k_z' - k_z)/(\partial k^+/\partial k_z)$. Then by using Eq. (16), one has

$$_c\langle\sigma_1',\sigma_2';\mathbf{k}'|\tilde{\mathbf{k}};\sigma_2,\sigma_1\rangle_{\rm LF} = \sqrt{(2\pi)^3 k^+\frac{\partial k_z}{\partial k^+}}\delta(\mathbf{k}' - \mathbf{k})D^{\frac{1}{2}*}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma_1\sigma_1'}D^{\frac{1}{2}*}[\mathcal{R}_M(-\tilde{\mathbf{k}})]_{\sigma_2\sigma_2'}, \tag{A10}$$

where the normalization and the completeness of the plane waves with Cartesian variables, $|\mathbf{k}\rangle$, are

$$\langle\mathbf{k}'|\mathbf{k}\rangle = \delta(\mathbf{k}' - \mathbf{k}), \quad \int d\mathbf{k}|\mathbf{k}\rangle\langle\mathbf{k}| = \mathbf{I}, \tag{A11}$$

and

$$-\tilde{\mathbf{k}} \equiv ((M_0 - k^+), -\mathbf{k}_\perp). \tag{A12}$$

2. Light-front wave function for a system of two interacting particles

By using the subgroup properties of the LF boosts, the LF wave function for an interacting two-body system, in a given frame, can be expressed through the intrinsic variables as follows [see Eq. (A10)]:

$$\begin{aligned}&{}_{\rm LF}\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\tilde{\mathbf{k}},\tilde{\mathbf{P}}'|\tilde{\mathbf{P}};j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle_{\rm LF}\\ &\quad = 2P^+(2\pi)^3\delta^3(\tilde{\mathbf{P}}' - \tilde{\mathbf{P}})\sqrt{(2\pi)^3 k^+\partial k_z/\partial k^+}\sum_{\sigma_1',\sigma_2'}D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma_1\sigma_1'}D^{\frac{1}{2}}[\mathcal{R}_M(-\tilde{\mathbf{k}})]_{\sigma_2\sigma_2'}\langle\sigma_1',\sigma_2';\tau_1,\tau_2;\mathbf{k}|j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle,\end{aligned} \tag{A13}$$

where a canonical completeness has been inserted for obtaining the final step.

Notice that the intrinsic two-body wave function $\langle\sigma_1',\sigma_2';\tau_1,\tau_2;\mathbf{k}|j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle$ contains *canonical spins* and therefore it can be composed by using the Clebsch–Gordan coefficients. Moreover, j is the total angular momentum of the pair, T is the isospin, α is the set of the parity and quantum numbers that label the coupled waves, and $\epsilon_{\rm int}$ is the eigenvalue of the mass operator [see Eqs. (23)–(25)].

The normalization of the *intrinsic part* of a LF *bound state* follows from the normalization fulfilled by $\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\mathbf{k}|j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle$. Indeed, if we adopt the following normalization, suitable for bound states,

$$\sum_{\tau_1,\tau_2}\sum_{\sigma_1,\sigma_2}\int d\mathbf{k}|\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\mathbf{k}|j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle|^2 = 1, \tag{A14}$$

from Eq. (A13), one has for the intrinsic part of the two-body LF wave function,

$$\begin{aligned}&\sum_{\tau_1,\tau_2}\sum_{\sigma_1,\sigma_2}\int\frac{dk^+d\mathbf{k}_\perp}{k^+(2\pi)^3}|_{\rm LF}\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\tilde{\mathbf{k}}|j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle|^2\\ &\quad = \sum_{\tau_1,\tau_2}\sum_{\sigma_1,\sigma_2}\int\frac{d\mathbf{k}}{E(\mathbf{k})(2\pi)^3}|_{\rm LF}\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\tilde{\mathbf{k}}|j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle|^2\\ &\quad = \sum_{\tau_1,\tau_2}\sum_{\sigma_1,\sigma_2}\int\frac{d\mathbf{k}}{E(\mathbf{k})}E(\mathbf{k})\left|\sum_{\sigma_1',\sigma_2'}D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma_1\sigma_1'}D^{\frac{1}{2}}[\mathcal{R}_M(-\tilde{\mathbf{k}})]_{\sigma_2\sigma_2'}\langle\sigma_1',\sigma_2';\tau_1,\tau_2;\mathbf{k}|j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle\right|^2\\ &\quad = \sum_{\tau_1,\tau_2}\sum_{\sigma_1,\sigma_2}\int d\mathbf{k}|\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\mathbf{k}|j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle|^2 = 1.\end{aligned} \tag{A15}$$

In the last step of (A15) the unitarity of the $D^{1/2}$ matrices has been used.

The normalization for the LF *scattering states* follows from (i) the orthogonality condition adopted for the canonical scattering wave function $\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\mathbf{k}|j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle$ given by [see also Eq. (A19) below for the completeness of the canonical states]

$$\sum_{\sigma_1'',\sigma_2''}\sum_{\tau_1'',\tau_2''}\int d\mathbf{k}\langle T_z'T';\alpha'\epsilon_{\rm int}';j_z'j'|\mathbf{k};\tau_2'',\tau_1'';\sigma_2'',\sigma_1''\rangle\langle\sigma_1'',\sigma_2'';\tau_1'',\tau_2'';\mathbf{k}|j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle=\delta_{T',T}\delta_{T_z',T_z}\delta_{\alpha',\alpha}\delta_{j',j}\delta_{j_z',j_z}\,\frac{\delta(t'-t)}{t^2},\tag{A16}$$

where $t=\sqrt{m\epsilon_{\rm int}}$, and (ii) the orthogonality adopted for the LF scattering states, which reads [see also the completeness of the free states for a two-body system $|\tilde{\mathbf{P}}\rangle|\tilde{\mathbf{k}}\rangle$ in Eq. (A8)]

$$\begin{aligned}
&{}_{\rm LF}\langle T_z'T';\alpha'\epsilon_{\rm int}'j_z'j';\tilde{\mathbf{P}}'|\tilde{\mathbf{P}};j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle_{\rm LF}\\
&=\sum_{\sigma_1'',\sigma_2''}\sum_{\tau_1'',\tau_2''}\int\frac{d\tilde{\mathbf{P}}''}{2P''^{+}(2\pi)^3}\int\frac{d\mathbf{k}}{E(\mathbf{k})(2\pi)^3}{}_{\rm LF}\langle T_z'T';\alpha'\epsilon_{\rm int}';j_z'j';\tilde{\mathbf{P}}'|\tilde{\mathbf{P}}'',\tilde{\mathbf{k}};\tau_2'',\tau_1'';\sigma_2'',\sigma_1''\rangle_{\rm LF}\\
&\quad\times{}_{\rm LF}\langle\sigma_1'',\sigma_2'';\tau_1'',\tau_2'';\tilde{\mathbf{k}},\tilde{\mathbf{P}}''|\tilde{\mathbf{P}};j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle_{\rm LF}\\
&=2P^+(2\pi)^3\delta^3(\tilde{\mathbf{P}}'-\tilde{\mathbf{P}})\sum_{\sigma_1'',\sigma_2''}\sum_{\tau_1'',\tau_2''}\int d\mathbf{k}\langle T_z'T';\alpha'\epsilon_{\rm int}';j_z'j'|\mathbf{k};\tau_2'',\tau_1'';\sigma_2'',\sigma_1''\rangle\langle\sigma_1'',\sigma_2'';\tau_1'',\tau_2'';\mathbf{k}|j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle\\
&=2P^+(2\pi)^3\delta^3(\tilde{\mathbf{P}}'-\tilde{\mathbf{P}})\delta_{T',T}\delta_{T_z',T_z}\delta_{\alpha',\alpha}\delta_{j',j}\delta_{j_z',j_z}\frac{\delta(t'-t)}{t^2}.
\end{aligned}\tag{A17}$$

Then for the two-body interacting case the LF completeness reads

$$\begin{aligned}
&\int\frac{d\tilde{\mathbf{P}}}{2P^+(2\pi)^3}\sum_{j,j_z\alpha}\sum_{TT_z}\sum\!\!\!\!\!\!\int\lambda(t)dt\,{}_{\rm LF}\langle\sigma_1,\sigma_2;\tau_1,\tau_2;\tilde{\mathbf{k}},\tilde{\mathbf{P}}'|\tilde{\mathbf{P}};j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle_{\rm LF}\,{}_{\rm LF}\langle T_zT;\alpha,\epsilon_{\rm int};j_z,j;\tilde{\mathbf{P}}|\tilde{\mathbf{P}}'',\tilde{\mathbf{k}}';\tau_2',\tau_1';\sigma_2',\sigma_1'\rangle_{\rm LF}\\
&=2P'^{+}(2\pi)^3\delta^3(\tilde{\mathbf{P}}'-\tilde{\mathbf{P}}'')\sum_{j,j_z\alpha}\sum_{TT_z}\sum\!\!\!\!\!\!\int\lambda(t)dt\sqrt{(2\pi)^3E(\mathbf{k})}\sum_{\bar\sigma_1,\bar\sigma_2}D^{\frac12}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma_1\bar\sigma_1}D^{\frac12}[\mathcal{R}_M(-\tilde{\mathbf{k}})]_{\sigma_2\bar\sigma_2}\langle\bar\sigma_1,\bar\sigma_2;\tau_1,\tau_2;\mathbf{k}|j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle\\
&\quad\times\sqrt{(2\pi)^3E(\mathbf{k}')}\sum_{\bar\sigma_1',\bar\sigma_2'}D^{\frac12\dagger}[\mathcal{R}_M(\tilde{\mathbf{k}}')]_{\bar\sigma_1'\sigma_1'}D^{\frac12\dagger}[\mathcal{R}_M(-\tilde{\mathbf{k}}')]_{\bar\sigma_2'\sigma_2'}\langle j,j_z;\epsilon_{\rm int},\alpha;TT_z|\bar\sigma_1',\bar\sigma_2';\tau_1',\tau_2';\mathbf{k}'\rangle\\
&=2P'^{+}(2\pi)^3\delta^3(\tilde{\mathbf{P}}'-\tilde{\mathbf{P}}'')\delta_{\tau_1',\tau_1}\delta_{\tau_2',\tau_2}\delta_{\sigma_1',\sigma_1}\delta_{\sigma_2',\sigma_2}\delta^3(\tilde{\mathbf{k}}'-\tilde{\mathbf{k}})(2\pi)^3k^+,
\end{aligned}\tag{A18}$$

where the symbol $\sum\!\!\!\!\!\!\int$ means a sum over the bound states of the pair (namely the deuteron in the present case) and the integration over the continuum. The quantity $\lambda(t)$ is the t density of the two-body states [$\lambda(t)=1$ for the bound states and $\lambda(t)=t^2$ for the continuum]. To obtain Eq. (A18), one has to use (i) the expression (A13) for the LF wave function, (ii) the unitarity of the $D^{1/2}$ matrices, (iii) the completeness for the eigensolutions of Eq. (23), i.e.,

$$\sum_{j,j_z\alpha}\sum_{TT_z}\sum\!\!\!\!\!\!\int\lambda(t)dt\langle\mathbf{k}'|j,j_z;\epsilon_{\rm int},\alpha;TT_z\rangle\langle T_zT;\alpha,\epsilon_{\rm int};j_z,j|\mathbf{k}\rangle=\delta^3(\mathbf{k}'-\mathbf{k}),\tag{A19}$$

and (iv) Eq. (A7).

APPENDIX B: THREE-BODY STATES

In this Appendix, the three-body free and interacting states are analyzed in analogy to the two-body case.

1. Completeness of three-body free states with symmetric intrinsic variables

Let $\tilde{\mathbf{P}}$ be the total LF momentum for a three-particle system

$$\tilde{\mathbf{P}}=\tilde{\mathbf{p}}_1+\tilde{\mathbf{p}}_2+\tilde{\mathbf{p}}_3\tag{B1}$$

of free mass $M_0(1,2,3)$:

$$M_0^2(1,2,3)=\frac{m^2+|\mathbf{k}_{1\perp}|^2}{\xi_1}+\frac{m^2+|\mathbf{k}_{2\perp}|^2}{\xi_2}+\frac{m^2+|\mathbf{k}_{3\perp}|^2}{\xi_3}=(E_1+E_2+E_3)^2,\tag{B2}$$

where $E_i=(m^2+|\mathbf{k}_i|^2)^{1/2}$ and $\sum_i\mathbf{k}_i=0$.

The completeness for the different set of variables, $\{\tilde{\mathbf{p}}_i\} \to \{\xi_i, \mathbf{k}_{i\perp}\} \to \mathbf{k}_i$, is given by

$$\begin{aligned}
\mathbf{I} &= \int \frac{d\tilde{\mathbf{p}}_1}{2p_1^+(2\pi)^3}\frac{d\tilde{\mathbf{p}}_2}{2p_2^+(2\pi)^3}\frac{d\tilde{\mathbf{p}}_3}{2p_3^+(2\pi)^3}|\tilde{\mathbf{p}}_3\rangle|\tilde{\mathbf{p}}_2\rangle|\tilde{\mathbf{p}}_1\rangle\langle\tilde{\mathbf{p}}_1|\langle\tilde{\mathbf{p}}_2|\langle\tilde{\mathbf{p}}_3| \\
&= \int \frac{d\tilde{\mathbf{P}}}{2P^+(2\pi)^3}|\tilde{\mathbf{P}}\rangle\langle\tilde{\mathbf{P}}|\int \frac{d\xi_1}{2\xi_1(2\pi)^3}d\mathbf{k}_{1\perp}|\xi_1\mathbf{k}_{1\perp}\rangle\langle\mathbf{k}_{1\perp}\xi_1|\int \frac{d\xi_2}{2\xi_2(2\pi)^3}d\mathbf{k}_{2\perp}\frac{1}{\xi_3}|\xi_2\mathbf{k}_{2\perp}\rangle\langle\mathbf{k}_{2\perp}\xi_2| \\
&= \int \frac{d\tilde{\mathbf{P}}}{2P^+(2\pi)^3}|\tilde{\mathbf{P}}\rangle\langle\tilde{\mathbf{P}}|\int \frac{d\mathbf{k}_1}{2E_1(2\pi)^3}\int \frac{d\mathbf{k}_2}{2E_2(2\pi)^3}\frac{M_0(1,2,3)}{E_3}|\tilde{\mathbf{k}}_1\rangle|\tilde{\mathbf{k}}_2\rangle\langle\tilde{\mathbf{k}}_2|\langle\tilde{\mathbf{k}}_1|, \qquad \text{(B3)}
\end{aligned}$$

where $|\tilde{\mathbf{p}}_3\rangle|\tilde{\mathbf{p}}_2\rangle|\tilde{\mathbf{p}}_1\rangle = |\tilde{\mathbf{P}}\rangle|\tilde{\mathbf{k}}_1\rangle|\tilde{\mathbf{k}}_2\rangle = |\tilde{\mathbf{P}}\rangle|\xi_1,\mathbf{k}_{1\perp}\rangle|\xi_2,\mathbf{k}_{2\perp}\rangle$ and the Jacobians

$$\left[\frac{\partial(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{p}}_3)}{\partial(\tilde{\mathbf{P}},\xi_1,\mathbf{k}_{1\perp},\xi_2,\mathbf{k}_{2\perp})}\right] = (P^+)^2, \qquad \text{(B4)}$$

$$\left[\frac{\partial(\tilde{\mathbf{p}}_1,\tilde{\mathbf{p}}_2,\tilde{\mathbf{p}}_3)}{\partial(\tilde{\mathbf{P}},\mathbf{k}_1,\mathbf{k}_2)}\right] = \frac{p_1^+p_2^+p_3^+M_0(1,2,3)}{P^+E_1E_2E_3} \qquad \text{(B5)}$$

have been used.

2. Completeness of three-body free states with nonsymmetric intrinsic variables

Instead of the symmetric intrinsic variables in the three-body frame, one can introduce nonsymmetric intrinsic variables, corresponding to the intrinsic frame of the (2,3) pair, i.e., $\{\tilde{\mathbf{p}}_2,\tilde{\mathbf{p}}_3\} \to \{\tilde{\mathbf{P}}_{23},\eta,\mathbf{k}_{23\perp}\}$ [see Eqs. (35) and (37)].

The completeness

$$\int \frac{d\tilde{\mathbf{p}}_1}{2p_1^+(2\pi)^3}\frac{d\tilde{\mathbf{p}}_2}{2p_2^+(2\pi)^3}\frac{d\tilde{\mathbf{p}}_3}{2p_3^+(2\pi)^3}|\tilde{\mathbf{p}}_1\rangle|\tilde{\mathbf{p}}_2\rangle|\tilde{\mathbf{p}}_3\rangle\langle\tilde{\mathbf{p}}_3|\langle\tilde{\mathbf{p}}_2|\langle\tilde{\mathbf{p}}_1| = \mathrm{I} \qquad \text{(B6)}$$

can be arranged in different ways, depending upon the the choice of variables one needs. In particular:

(1) For the variables $\tilde{\mathbf{p}}_1$, $\tilde{\mathbf{P}}_{23}$, and $\tilde{\mathbf{k}}_{23}$ one can exploit Eq. (A8), obtaining

$$\mathrm{I} = \int \frac{d\tilde{\mathbf{p}}_1}{2p_1^+(2\pi)^3}|\tilde{\mathbf{p}}_1\rangle\langle\tilde{\mathbf{p}}_1|\int \frac{d\tilde{\mathbf{P}}_{23}}{2P_{23}^+(2\pi)^3}|\tilde{\mathbf{P}}_{23}\rangle\langle\tilde{\mathbf{P}}_{23}|\int \frac{d\tilde{\mathbf{k}}_{23}}{k_{23}^+(2\pi)^3}|\tilde{\mathbf{k}}_{23}\rangle\langle\tilde{\mathbf{k}}_{23}|. \qquad \text{(B7)}$$

(2) For the variables $\tilde{\mathbf{P}}$, $\{\xi_1,\mathbf{k}_{1\perp}\}$, and $\{\eta,\mathbf{k}_{23\perp}\}$ one has from Eq. (B3)

$$\mathrm{I} = \int \frac{d\tilde{\mathbf{P}}}{2P^+(2\pi)^3}|\tilde{\mathbf{P}}\rangle\langle\tilde{\mathbf{P}}|\int \frac{d\xi_1 d\mathbf{k}_{1\perp}}{2\xi_1(1-\xi_1)(2\pi)^3}|\xi_1\mathbf{k}_{1\perp}\rangle\langle\mathbf{k}_{1\perp}\xi_1|\int \frac{d\eta d\mathbf{k}_{23\perp}}{2\eta(1-\eta)(2\pi)^3}|\eta\mathbf{k}_{23\perp}\rangle\langle\mathbf{k}_{23\perp}\eta|, \qquad \text{(B8)}$$

after recalling Eq. (35) that yields

$$\frac{d\xi_2}{\xi_2\xi_3} = \frac{d\eta}{\eta(1-\eta)(1-\xi_1)} \quad \text{and} \quad d\mathbf{k}_{2\perp} = d\mathbf{k}_{23\perp}. \qquad \text{(B9)}$$

(3) For the variables $\tilde{\mathbf{P}}$, $\tilde{\mathbf{k}}_1$, and $\tilde{\mathbf{k}}_{23}$ one has

$$\mathrm{I} = \int \frac{d\tilde{\mathbf{P}}}{2P^+(2\pi)^3}|\tilde{\mathbf{P}}\rangle\langle\tilde{\mathbf{P}}|\int \frac{d\tilde{\mathbf{k}}_{23}}{k_{23}^+(2\pi)^3}|\tilde{\mathbf{k}}_{23}\rangle\langle\tilde{\mathbf{k}}_{23}|\int \frac{M_0(1,2,3)d\tilde{\mathbf{k}}_1}{2k_1^+E_{23}(2\pi)^3}|\tilde{\mathbf{k}}_1\rangle\langle\tilde{\mathbf{k}}_1|, \qquad \text{(B10)}$$

where the following relations have been used (recall that $k_1^+ = \xi_1 M_0(1,2,3)$ and $k_{23}^+ = \eta M_{23}$):

$$\begin{aligned}
\frac{\partial k_1^+}{\partial \xi_1} &= M_0(1,2,3) + \xi_1\frac{\partial M_0(1,2,3)}{\partial \xi_1} = M_0(1,2,3) + \frac{\xi_1}{2M_0(1,2,3)}\frac{\partial M_0^2(1,2,3)}{\partial \xi_1} \\
&= \frac{1}{2M_0(1,2,3)}\left[M_0^2(1,2,3) + \frac{M_{23}^2 + |\mathbf{k}_{1\perp}|^2}{(1-\xi_1)^2}\right] \\
&= \frac{1}{2(1-\xi_1)}\left[M_0(1,2,3)(1-\xi_1) + \frac{M_{23}^2 + |\mathbf{k}_{1\perp}|^2}{K_{23}^+}\right] \\
&= \frac{1}{2(1-\xi_1)}[K_{23}^+ + K_{23on}^-] = \frac{E_{23}}{(1-\xi_1)} \qquad \text{(B11)}
\end{aligned}$$

$$\frac{\partial k_{23}^{+}}{\partial \eta} = M_{23} - \eta \frac{1}{2M_{23}} \frac{m^2 + |\mathbf{k}_{1\perp}|^2}{\eta^2(1-\eta)^2}(1-2\eta)$$
$$= \frac{M_{23}}{2(1-\eta)}[2(1-\eta) - 1 + 2\eta)] = \frac{M_{23}}{2(1-\eta)} = \frac{k_{23}^{+}}{2\eta(1-\eta)}, \tag{B12}$$

with K_{23} being the total momentum of the free (2,3) pair in the intrinsic frame of the three particles, i.e., $K_{23}^{+} = M_0(1,2,3)(1-\xi_1)$, $K_{23\perp} = \mathbf{k}_{2\perp} + \mathbf{k}_{3\perp} = -\mathbf{k}_{1\perp}$, $K_{23on}^{-} = (M_{23}^2 + |\mathbf{k}_{1\perp}|^2)/K_{23}^{+}$, and $E_{23} = (M_{23}^2 + |\mathbf{k}_1|^2)^{1/2}$.

(4) For the variables $\tilde{\mathbf{P}}$, $\mathbf{k}_1$, and $\mathbf{k_{23}}$ one has

$$\mathrm{I} = \int \frac{d\tilde{\mathbf{P}}}{2P^{+}(2\pi)^3}|\tilde{\mathbf{P}}\rangle\langle\tilde{\mathbf{P}}| \int \frac{2d\mathbf{k}_{23}}{M_{23}(2\pi)^3}|\tilde{\mathbf{k}}_{23}\rangle\langle\tilde{\mathbf{k}}_{23}| \int \frac{M_0(1,2,3)d\mathbf{k}_1}{2E_1E_{23}(2\pi)^3}|\tilde{\mathbf{k}}_1\rangle\langle\tilde{\mathbf{k}}_1|. \tag{B13}$$

To obtain the above results, the following properties have been used:

$$\frac{\partial k_{1z}}{\partial k_1^{+}} = \frac{1}{2}\left[1 + \frac{m^2 + |\mathbf{k}_{1\perp}|^2}{k_1^{+2}}\right] = \frac{E(\mathbf{k_1})}{k_1^{+}} = \frac{E(\mathbf{k_1})}{M_0(1,2,3)\xi_1}, \tag{B14}$$

$$\frac{\partial k_{23z}}{\partial \eta} = M_{23} - \left(\eta - \frac{1}{2}\right)\frac{1}{2M_{23}}\frac{m^2 + |\mathbf{k}_{1\perp}|^2}{\eta^2(1-\eta)^2}(1-2\eta)$$
$$= \frac{M_{23}}{4\eta(1-\eta)}[4\eta(1-\eta) + (2\eta-1))^2] = \frac{M_{23}}{4\eta(1-\eta)}. \tag{B15}$$

(5) For the variables $\tilde{\mathbf{P}}$, $\tilde{\mathbf{k}}_1$, and $\mathbf{k_{23}}$ one has

$$\mathrm{I} = \int \frac{d\tilde{\mathbf{P}}}{2P^{+}(2\pi)^3}|\tilde{\mathbf{P}}\rangle\langle\tilde{\mathbf{P}}| \int \frac{2d\mathbf{k_{23}}}{M_{23}(2\pi)^3}|\tilde{\mathbf{k}}_{23}\rangle\langle\tilde{\mathbf{k}}_{23}| \int \frac{M_0(1,2,3)d\tilde{\mathbf{k}}_1}{2k_1^{+}E_{23}(2\pi)^3}|\tilde{\mathbf{k}}_1\rangle\langle\tilde{\mathbf{k}}_1|. \tag{B16}$$

3. Useful derivatives involving nonsymmetric intrinsic variables

Let us evaluate the derivatives $\partial\kappa_1^{+}/\partial\xi_1$ and $\partial\kappa_{1z}/\partial\kappa_1^{+}$:

$$\frac{\partial\kappa_1^{+}}{\partial\xi_1} = \mathcal{M}_0(1,23) + \xi_1\frac{\partial\mathcal{M}_0(1,23)}{\partial\xi_1} = \mathcal{M}_0(1,23) + \frac{\xi_1}{2\mathcal{M}_0(1,23)}\frac{\partial\mathcal{M}_0(1,23)^2}{\partial\xi_1}$$
$$= \frac{1}{2\mathcal{M}_0(1,23)}\left[\mathcal{M}_0(1,23)^2 + \frac{M_S^2 + |\mathbf{k}_{1\perp}|^2}{(1-\xi_1)^2}\right] = \frac{1}{2(1-\xi_1)}\left[\mathcal{M}_0(1,23)(1-\xi_1) + \frac{M_S^2 + |\mathbf{k}_{1\perp}|^2}{P_S^{+}}\right]$$
$$= \frac{1}{2(1-\xi_1)}[P_S^{+} + P_{Son}^{-}] = \frac{E_S}{(1-\xi_1)}, \tag{B17}$$

$$\frac{\partial\kappa_{1z}}{\partial\kappa_1^{+}} = \frac{1}{2}\left[1 + \frac{m^2 + |\mathbf{k}_{1\perp}|^2}{\kappa_1^{+2}}\right] = \frac{E(\kappa_1)}{\kappa_1^{+}} = \frac{E(\kappa_1)}{\mathcal{M}_0(1,23)\xi_1}. \tag{B18}$$

4. Normalization of the light-front wave function

Let us check that the factors in the expression of the intrinsic part of the LF wave function given by the second and the third lines of Eq. (42), which allow one to obtain the normalization of the bound state $|j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2},T_z\rangle$. Indeed, by using Eqs. (B10) and (B13), one has

$$\left\langle T_z\frac{1}{2};\Pi,\epsilon_{\rm int}^3;j_z,j\middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle$$
$$= \sum_{\tau_1,\tau_2,\tau_3}\sum_{\sigma_1,\sigma_2,\sigma_3}\int\frac{d\tilde{\mathbf{k}}_1}{2k_1^{+}(2\pi)^3}\int\frac{M_0(1,2,3)d\tilde{\mathbf{k}}_{23}}{k_{23}^{+}E_{23}(2\pi)^3}\left|{}_{\rm LF}\left\langle\sigma_1,\sigma_2,\sigma_3;\tau_1,\tau_2,\tau_3;\tilde{\mathbf{k}}_1,\tilde{\mathbf{k}}_{23}\middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2},T_z\right\rangle\right|^2$$
$$= \sum_{\tau_1,\tau_2,\tau_3}\sum_{\sigma_1\sigma_2\sigma_3}\int\frac{d\mathbf{k}_1}{E_1(2\pi)^3}\int\frac{d\mathbf{k}_{23}}{E_{23}}\frac{M_0(1,2,3)}{(2\pi)^3M_{23}}\left|{}_{\rm LF}\left\langle\sigma_1,\sigma_2,\sigma_3;\tau_1,\tau_2,\tau_3;\tilde{\mathbf{k}}_1,\tilde{\mathbf{k}}_{23}\middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle\right|^2$$
$$= \sum_{\tau_1,\tau_2,\tau_3}\sum_{\sigma_1,\sigma_2,\sigma_3}\int\frac{d\mathbf{k}_1}{E_1(2\pi)^3}\int\frac{M_0(1,2,3)d\mathbf{k}_{23}}{M_{23}E_{23}(2\pi)^3}\frac{2E_1M_{23}E_{23}(2\pi)^6}{2M_0(1,2,3)}$$

$$\times \left| \sum_{\sigma_1'} \sum_{\sigma_2'} \sum_{\sigma_3'} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_1)]_{\sigma_1\sigma_1'} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_2)]_{\sigma_2\sigma_2'} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_3)]_{\sigma_3\sigma_3'} \left\langle \sigma_1',\sigma_2',\sigma_3';\tau_1,\tau_2,\tau_3;\mathbf{k}_1,\mathbf{k}_{23} \middle| j,j_z;\epsilon^3_{\rm int},\Pi;\frac{1}{2},T_z \right\rangle \right|^2$$
$$= \sum_{\tau_1,\tau_2,\tau_3} \sum_{\sigma_1,\sigma_2,\sigma_3} \int d\mathbf{k}_1 \int d\mathbf{k}_{23} \left| \left\langle \sigma_1,\sigma_2,\sigma_3;\tau_1,\tau_2,\tau_3;\mathbf{k}_1,\mathbf{k}_{23} \middle| j,j_z;\epsilon^3_{\rm int},\Pi;\frac{1}{2},T_z \right\rangle \right|^2$$
$$= 1, \tag{B19}$$

given the unitarity of the Melosh rotations and the normalization of the canonical wave function (43).

APPENDIX C: PROPERTIES OF THE BASIS STATES OF THE CLUSTER {1,(23)}

In this Appendix, the general formalism, suitable for describing the cluster {1,(23)} is presented. Recall that the final goal is to construct states where the interaction is acting only between the particles 2 and 3; namely, the three-body states we are interested in are the tensor product of free one-body states and interacting two-body states.

1. Completeness relation for the nonsymmetric basis states and orthogonality properties of three-body free states

The correctness of the normalization factors in Eq. (52) can be checked as follows:
Indeed, let us consider the product of two three-body free states

$$A = {}_{\rm LF}\langle\sigma_1',\sigma_2',\sigma_3';\tau_1',\tau_2',\tau_3';\tilde{\mathbf{P}}',\tilde{\mathbf{k}}_1',\tilde{\mathbf{k}}_{23}'|\tilde{\mathbf{k}}_{23}'',\tilde{\mathbf{k}}_1'',\tilde{\mathbf{P}}'';\tau_1'',\tau_2'',\tau_3'';\sigma_1'',\sigma_2'',\sigma_3''\rangle_{\rm LF}. \tag{C1}$$

Then, let us insert in Eq. (C1) the completeness relation (51) for the nonsymmetric basis states (49):

$$A = \int \frac{d\tilde{\mathbf{P}}}{2P^+(2\pi)^3} \sum_{\sigma_1\tau_1} \int \frac{d\tilde{\boldsymbol{\kappa}}_1}{2\kappa_1^+(2\pi)^3} \sum_{T_{23}\tau_{23}} \int\!\!\!\!\!\!\sum \lambda(t)dt$$
$$\times \sum_{j_{23}j_{23z}\alpha} {}_{\rm LF}\langle\sigma_1',\sigma_2',\sigma_3';\tau_1',\tau_2',\tau_3';\tilde{\mathbf{P}}',\tilde{\mathbf{k}}_1',\tilde{\mathbf{k}}_{23}'|\tilde{\mathbf{P}};\tilde{\boldsymbol{\kappa}}_1\sigma_1\tau_1;j_{23},j_{23z};\epsilon_{23},\alpha;T_{23},\tau_{23}\rangle_{\rm LF}$$
$$\times {}_{\rm LF}\langle T_{23},\tau_{23};\alpha,\epsilon_{23};j_{23z},j_{23};\tau_1\sigma_1\tilde{\boldsymbol{\kappa}}_1;\tilde{\mathbf{P}}|\tilde{\mathbf{k}}_{23}'',\tilde{\mathbf{k}}_1'',\tilde{\mathbf{P}}'';\tau_1'',\tau_2'',\tau_3'';\sigma_1'',\sigma_2'',\sigma_3''\rangle_{\rm LF}. \tag{C2}$$

With the help of the overlap in Eq. (52), the above equation reads

$$A = 2P'^+(2\pi)^3\delta^3(\tilde{\mathbf{P}}''-\tilde{\mathbf{P}}')\delta_{\tau_1'\tau_1''}\sum_{\sigma_1}\int \frac{d\tilde{\boldsymbol{\kappa}}_1}{2\kappa_1^+(2\pi)^3}\sum_{T_{23}\tau_{23}}\int\!\!\!\!\!\!\sum \lambda(t)dt \sum_{j_{23}j_{23z}\alpha}\delta_{\sigma_1'\sigma_1}(2\pi)^3 2k_1'^+\delta^3\left(\tilde{\mathbf{k}}_1'-\tilde{\mathbf{k}}_1^{(a)}\right)\sqrt{\frac{\kappa_1^+E_{23}'}{k_1'^+E_S}}\sqrt{(2\pi)^3\frac{E_{23}'M_{23}'}{2M_0'(1,2,3)}}$$
$$\times \sum_{\sigma_2}\sum_{\sigma_3} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_{23}')]_{\sigma_2'\sigma_2} D^{\frac{1}{2}}[\mathcal{R}_M(-\tilde{\mathbf{k}}_{23}')]_{\sigma_3'\sigma_3}\langle\sigma_2,\sigma_3;\tau_2',\tau_3';\mathbf{k}_{23}'|j_{23},j_{23z};\epsilon_{23},\alpha;T_{23},\tau_{23}\rangle$$
$$\times \delta_{\sigma_1\sigma_1''}(2\pi)^3 2k_1''^+\delta^3\left(\tilde{\mathbf{k}}_1''-\tilde{\mathbf{k}}_1''^{(a)}\right)\sqrt{\frac{\kappa_1^+E_{23}''}{k_1''^+E_S}}\sqrt{(2\pi)^3\frac{E_{23}''M_{23}''}{2M_0''(1,2,3)}}$$
$$\times \sum_{\bar\sigma_2}\sum_{\bar\sigma_3} D^{\frac{1}{2}*}[\mathcal{R}_M(\tilde{\mathbf{k}}_{23}'')]_{\sigma_2''\bar\sigma_2} D^{\frac{1}{2}*}[\mathcal{R}_M(-\tilde{\mathbf{k}}_{23}'')]_{\sigma_3''\bar\sigma_3}\langle\bar\sigma_2,\bar\sigma_3;\tau_2'',\tau_3'';\mathbf{k}_{23}''|j_{23},j_{23z};\epsilon_{23},\alpha;T_{23},\tau_{23}\rangle^*$$
$$= 2P'^+(2\pi)^3\delta^3(\tilde{\mathbf{P}}''-\tilde{\mathbf{P}}')\delta_{\tau_1'\tau_1''}\delta_{\sigma_1'\sigma_1''}\int d\mathbf{k}_{1\perp}\int \frac{d\xi_1}{(1-\xi_1)}$$
$$\times (2\pi)^3 k_1'^+\delta^3\left(\tilde{\mathbf{k}}_1'-\tilde{\mathbf{k}}_1^{(a)}\right)\sqrt{\frac{E_{23}'}{k_1'^+}}\sqrt{\frac{E_{23}'M_{23}'}{M_0'(1,2,3)}}\sum_{\sigma_2}\sum_{\sigma_3} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_{23}')]_{\sigma_2'\sigma_2} D^{\frac{1}{2}}[\mathcal{R}_M(-\tilde{\mathbf{k}}_{23}')]_{\sigma_3'\sigma_3}$$
$$\times (2\pi)^3 k_1''^+\delta^3(\tilde{\mathbf{k}}_1''-\tilde{\mathbf{k}}_1''^{(a)})\sqrt{\frac{E_{23}''}{k_1''^+}}\sqrt{\frac{E_{23}''M_{23}''}{M_0''(1,2,3)}}\sum_{\bar\sigma_2}\sum_{\bar\sigma_3} D^{\frac{1}{2}*}[\mathcal{R}_M(\tilde{\mathbf{k}}_{23}'')]_{\sigma_2''\bar\sigma_2} D^{\frac{1}{2}*}[\mathcal{R}_M(-\tilde{\mathbf{k}}_{23}'')]_{\sigma_3''\bar\sigma_3}\sum_{j_{23}j_{23z}\alpha}\sum_{T_{23}\tau_{23}}\int\!\!\!\!\!\!\sum \lambda(t)dt$$
$$\times \langle\sigma_2,\sigma_3;\tau_2',\tau_3';\mathbf{k}_{23}'|j_{23},j_{23z};\epsilon_{23},\alpha;T_{23},\tau_{23}\rangle\langle\bar\sigma_2,\bar\sigma_3;\tau_2'',\tau_3'';\mathbf{k}_{23}''|j_{23},j_{23z};\epsilon_{23},\alpha;T_{23},\tau_{23}\rangle^*, \tag{C3}$$

where the integration-variable change $d\kappa_1^+ = d\xi_1 E_S/(1-\xi_1)$ was performed [see Eq. (B17)]. In Eq. (C3) $\mathbf{k}''^{(a)}_{1\perp} = \tilde{\boldsymbol{\kappa}}_{1\perp}$ and $k''^{+(a)} = \xi_1 \bar{M}_0(1,2,3)$ with

$$\bar{M}_0^2(1,2,3) = \frac{m^2 + k_{1\perp}^2}{\xi_1} + \frac{M''^2_{23} + k_{1\perp}^2}{1-\xi_1}. \tag{C4}$$

Then, taking into account the completeness for the two-body intrinsic states $\langle \sigma_2,\sigma_3;\tau'_2,\tau'_3;\mathbf{k}'_{23}|j_{23},j_{23z};\epsilon_{23},\alpha;T_{23},\tau_{23}\rangle$ for the (2,3) pair [see Eqs. (31) and (A19)], one obtains

$$A = 2P'^+(2\pi)^3\delta^3(\tilde{\mathbf{P}}''-\tilde{\mathbf{P}}')\delta_{\tau'_1\tau''_1}\delta_{\sigma'_1\sigma''_1}\int d\mathbf{k}_{1\perp}\int \frac{d\xi_1}{(1-\xi_1)}$$
$$\times (2\pi)^3 k_1'^+\delta^3\big(\tilde{\mathbf{k}}'_1 - \tilde{\mathbf{k}}_1^{(a)}\big)\sqrt{\frac{E'_{23}}{k_1'^+}}\sqrt{\frac{E'_{23}M'_{23}}{M'_0(1,2,3)}}\sum_{\sigma_2}\sum_{\sigma_3} D^{\frac12}\big[\mathcal{R}_M\big(\tilde{\mathbf{k}}'_{23}\big)\big]_{\sigma'_2\sigma_2} D^{\frac12}\big[\mathcal{R}_M\big(-\tilde{\mathbf{k}}'_{23}\big)\big]_{\sigma'_3\sigma_3}(2\pi)^3 k_1''^+\delta^3\big(\tilde{\mathbf{k}}''_1 - \tilde{\mathbf{k}}''^{(a)}_1\big)$$
$$\times \sqrt{\frac{E''_{23}}{k_1''^+}}\sqrt{\frac{E''_{23}M''_{23}}{M''_0(1,2,3)}}\sum_{\bar{\sigma}_2}\sum_{\bar{\sigma}_3} D^{\frac12 *}[\mathcal{R}_M(\tilde{\mathbf{k}}''_{23})]_{\sigma''_2\bar{\sigma}_2} D^{\frac12 *}[\mathcal{R}_M(-\tilde{\mathbf{k}}''_{23})]_{\sigma''_3\bar{\sigma}_3}\delta_{\tau'_2,\tau''_2}\delta_{\tau'_3,\tau''_3}\delta_{\sigma_2,\bar{\sigma}_2}\delta_{\sigma_3,\bar{\sigma}_3}\delta^3(\mathbf{k}'_{23}-\mathbf{k}''_{23}). \tag{C5}$$

Therefore, using the unitarity of the $D^{\frac12}$ matrices and changing the integration variable from $d\xi_1 1/(1-\xi_1)$ to $1/E'_{23}dk_1^{+(a)}$ [see Eq. (B11)], one obtains

$$A = \delta_{\sigma'_1,\sigma''_1}\delta_{\sigma'_2,\sigma''_2}\delta_{\sigma'_3,\sigma''_3}\delta_{\tau'_1,\tau''_1}\delta_{\tau'_2,\tau''_2}\delta_{\tau'_3,\tau''_3}2P'^+(2\pi)^9\delta^3(\tilde{\mathbf{P}}''-\tilde{\mathbf{P}}')k_1'^+\delta^3(\tilde{\mathbf{k}}''_1-\tilde{\mathbf{k}}'_1)\frac{E'_{23}M'_{23}}{M'_0(1,2,3)}\delta^3(\mathbf{k}''_{23}-\mathbf{k}'_{23})$$
$$= \delta_{\sigma'_1,\sigma''_1}\delta_{\sigma'_2,\sigma''_2}\delta_{\sigma'_3,\sigma''_3}\delta_{\tau'_1,\tau''_1}\delta_{\tau'_2,\tau''_2}\delta_{\tau'_3,\tau''_3}2P'^+(2\pi)^9\delta^3(\tilde{\mathbf{P}}''-\tilde{\mathbf{P}}')2k_1'^+\delta^3(\tilde{\mathbf{k}}''_1-\tilde{\mathbf{k}}'_1)\frac{E'_{23}k'^+_{23}}{M'_0(1,2,3)}\delta^3(\tilde{\mathbf{k}}''_{23}-\tilde{\mathbf{k}}'_{23})$$
$$= \delta_{\sigma'_1,\sigma''_1}\delta_{\sigma'_2,\sigma''_2}\delta_{\sigma'_3,\sigma''_3}\delta_{\tau'_1,\tau''_1}\delta_{\tau'_2,\tau''_2}\delta_{\tau'_3,\tau''_3}2P'^+(2\pi)^9\delta^3(\tilde{\mathbf{P}}''-\tilde{\mathbf{P}}')E(k'_1)\delta^3(\mathbf{k}''_1-\mathbf{k}'_1)\frac{E'_{23}M'_{23}}{M'_0(1,2,3)}\delta^3(\mathbf{k}''_{23}-\mathbf{k}'_{23}). \tag{C6}$$

The above expressions are the proper orthogonality relations for the free case, to be related to the completeness relations of Eqs. (B16), (B10), and (B13), respectively.

2. Product of the nonsymmetric basis states and the bound state of the three-particle system

Let us express the overlaps between the states of the nonsymmetric basis (49) and the bound state of the three-particle system in terms of the canonical wave functions for the two-body and the three-body systems. To this end, the plane-wave completeness operator (61) is inserted in the intrinsic part of the overlap (60), viz.

$$_{\rm LF}\left\langle T_{23},\tau_{23};\alpha,\epsilon_{23};\,j_{23}j_{23z};\,\tau_1\sigma_1\tilde{\boldsymbol{\kappa}}_1\middle| j,j_z;\epsilon^3_{\rm int},\Pi;\frac12 T_z\right\rangle$$
$$= \sum_{\tau_2\tau_3}\sum_{\sigma_2\sigma_3}\int\frac{d\tilde{\mathbf{k}}'_{23}}{k'^+_{23}(2\pi)^3}\sum_{\sigma'_1}\int\frac{M'_0(1,2,3)d\tilde{\mathbf{k}}'_1}{2k_1'^+E'_{23}(2\pi)^3}{}_{\rm LF}\langle T_{23},\tau_{23};\alpha,\epsilon_{23};\,j_{23}j_{23z};\,\tau_1\sigma_1\tilde{\boldsymbol{\kappa}}_1|\tilde{\mathbf{k}}'_{23},\tau_2\tau_3,\sigma_2\sigma_3;\tilde{\mathbf{k}}'_1\sigma'_1\tau_1\rangle_{\rm LF}$$
$$\times\, {}_{\rm LF}\left\langle \sigma_2\sigma_3,\tau_2\tau_3,\tilde{\mathbf{k}}'_{23};\,\tau_1\sigma'_1\tilde{\mathbf{k}}'_1\middle| j,j_z;\epsilon^3_{\rm int},\Pi;\frac12 T_z\right\rangle. \tag{C7}$$

We can notice that the LF spin states do not change for LF boosts. Therefore the spin states $|\sigma_2\sigma_3\rangle_{\rm LF}$ in the intrinsic reference frame of the pair (23) or in the intrinsic reference frame of the three-particle system, with momenta related by the LF boost $B^{-1}_{\rm LF}(\tilde{\mathbf{K}}_{23}/M_{23})$, are equal. Then we can take ${}_{\rm LF}\langle T_{23},\tau_{23};\alpha,\epsilon_{23};\,j_{23}j_{23z};\,\tau_1\sigma_1\tilde{\boldsymbol{\kappa}}_1|\tilde{\mathbf{k}}'_{23},\tau_2\tau_3,\sigma_2\sigma_3;\tilde{\mathbf{k}}'_1\sigma'_1\tau_1\rangle_{\rm LF}$ as the intrinsic part of the overlap (52) and ${}_{\rm LF}\langle\sigma_2\sigma_3,\tau_2\tau_3,\tilde{\mathbf{k}}'_{23};\,\tau_1\sigma'_1\tilde{\mathbf{k}}'_1|j,j_z;\epsilon^3_{\rm int},\Pi;\frac12 T_z\rangle_{\rm LF}$ as the intrinsic three-body wave function of Eq. (42) and we obtain

$$_{\rm LF}\left\langle T_{23},\tau_{23};\alpha,\epsilon_{23};\,j_{23}j_{23z};\,\tau_1\sigma_1\tilde{\boldsymbol{\kappa}}_1\middle| j,j_z;\epsilon^3_{\rm int},\Pi;\frac12 T_z\right\rangle$$
$$= \sum_{\tau_2\tau_3}\sum_{\sigma_2\sigma_3}\int\frac{d\tilde{\mathbf{k}}'_1}{2k_1'^+(2\pi)^3}\sum_{\sigma'_1}\int\frac{2M'_0(1,2,3)d\mathbf{k}'_{23}}{E'_{23}M'_{23}(2\pi)^3}\delta_{\sigma_1\sigma'_1}(2\pi)^3 2k_1'^+\delta^3\big(\tilde{\mathbf{k}}'_1-\tilde{\mathbf{k}}^{(a)}_1\big)\sqrt{\frac{\kappa_1^+E'_{23}}{k_1'^+E_S}}\sqrt{\frac{(2\pi)^3E'_{23}M'_{23}}{2M'_0(1,2,3)}}$$
$$\times\sum_{\sigma''_2,\sigma''_3}\langle T_{23},\tau_{23};\alpha,\epsilon_{23};\,j_{23}j_{23z}|\mathbf{k}'_{23},\sigma''_2\sigma''_3,\tau_2\tau_3\rangle D^{\frac12}[\mathcal{R}^\dagger_M(\tilde{\mathbf{k}}'_{23})]_{\sigma''_2\sigma_2}D^{\frac12}[\mathcal{R}^\dagger_M(-\tilde{\mathbf{k}}'_{23})]_{\sigma''_3\sigma_3}$$

$$\times \sum_{\sigma_1'',\sigma_2',\sigma_3'} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_1')]_{\sigma_1'\sigma_1''} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_2')]_{\sigma_2\sigma_2'} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_3')]_{\sigma_3\sigma_3'}$$

$$\times \sqrt{\frac{(2\pi)^6 2E(\mathbf{k}_1')E_{23}' M_{23}'}{2M_0'(1,2,3)}} \left\langle \sigma_1'',\sigma_2',\sigma_3';\tau_1,\tau_2,\tau_3;\mathbf{k}_{23}',\mathbf{k}_1' \middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle. \quad \text{(C8)}$$

In the previous equation the integration variable $k_{23}'^{+}$ has been changed in k_{23z}', using the equality $\partial k_{23}^{+}/\partial k_{23z} = 2k_{23}^{+}/M_{23}$ [see Eqs. (B12) and (B15)]. Then one obtains

$$_{\rm LF}\left\langle T_{23},\tau_{23};\alpha,\epsilon_{23};j_{23}j_{23z};\tau_1\sigma_1\tilde{\kappa}_1 \middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle$$

$$= \sum_{\tau_2\tau_3}\int d\mathbf{k}_{23}' \sum_{\sigma_1'} D^{\frac{1}{2}}\left[\mathcal{R}_M\left(\tilde{\mathbf{k}}_1'^{(a)}\right)\right]_{\sigma_1\sigma_1'} \sqrt{(2\pi)^3 2E\left(\mathbf{k}_1'^{(a)}\right)} \sqrt{\frac{\kappa_1^+ E_{23}'}{k_1'^{+(a)} E_S}}$$

$$\times \sum_{\sigma_2'',\sigma_3''}\sum_{\sigma_2',\sigma_3'}\sum_{\sigma_2\sigma_3} D^{\frac{1}{2}}[\mathcal{R}_M^\dagger(\tilde{\mathbf{k}}_{23}')]_{\sigma_2''\sigma_2} D^{\frac{1}{2}}[\mathcal{R}_M^\dagger(-\tilde{\mathbf{k}}_{23}')]_{\sigma_3''\sigma_3} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_2')]_{\sigma_2\sigma_2'} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_3')]_{\sigma_3\sigma_3'}$$

$$\times \langle T_{23},\tau_{23};\alpha,\epsilon_{23};j_{23}j_{23z}|\mathbf{k}_{23}',\sigma_2''\sigma_3'';\tau_2,\tau_3\rangle \left\langle \sigma_3',\sigma_2',\sigma_1';\tau_3,\tau_2,\tau_1;\mathbf{k}_{23}',\mathbf{k}_1'^{(a)} \middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle$$

$$= \sum_{\tau_2\tau_3}\int d\mathbf{k}_{23} \sum_{\sigma_1'} D^{\frac{1}{2}}\left[\mathcal{R}_M\left(\tilde{\mathbf{k}}_1^{(a)}\right)\right]_{\sigma_1\sigma_1'} \sqrt{(2\pi)^3 2E\left(\mathbf{k}_1^{(a)}\right)} \sqrt{\frac{\kappa_1^+ E_{23}}{k_1^{+(a)} E_S}} \sum_{\sigma_2'',\sigma_3''}\sum_{\sigma_2',\sigma_3'} \mathcal{D}_{\sigma_2'',\sigma_2'}(\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_2)\mathcal{D}_{\sigma_3'',\sigma_3'}(-\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_3)$$

$$\times \langle T_{23},\tau_{23};\alpha,\epsilon_{23};j_{23}j_{23z}|\mathbf{k}_{23},\sigma_2''\sigma_3'';\tau_2,\tau_3\rangle \left\langle \sigma_3',\sigma_2',\sigma_1';\tau_3,\tau_2,\tau_1;\mathbf{k}_{23},\mathbf{k}_1^{(a)} \middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle, \quad \text{(C9)}$$

where

$$\mathcal{D}_{\sigma_i'',\sigma_i'}(\pm\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_i) = \sum_{\sigma_i} D^{\frac{1}{2}}[\mathcal{R}_M^\dagger(\pm\tilde{\mathbf{k}}_{23})]_{\sigma_i''\sigma_i} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_i)]_{\sigma_i\sigma_i'}, \quad \text{(C10)}$$

with the $+$ corresponding to $i=2$ and the $-$ corresponding to $i=3$.

Let us notice that the matrices $\mathcal{D}_{\sigma_i'',\sigma_i'}(\pm\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_i)$ are unitary, i.e.,

$$\sum_{\sigma_i} \mathcal{D}^\dagger_{\sigma_i'',\sigma_i}(\pm\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_i)\mathcal{D}_{\sigma_i,\sigma_i'}(\pm\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_i) = \delta_{\sigma_i'',\sigma_i'}, \quad \text{(C11)}$$

because of the unitarity of the $D^{1/2}$ matrices.

3. Normalization of overlaps between a state of cluster {1,(23)} and the bound state of three-particle system

The normalization of the intrinsic LF overlaps $_{\rm LF}\langle T_{23},\tau_{23};\alpha,\epsilon_{23};j_{23}j_{23z};\tau_1\sigma_1,\tilde{\kappa}_1|j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\rangle$ can be easily recovered by using Eq. (C9); viz.

$$N = \sum_{T_{23}\tau_{23}}\sum\!\!\!\!\!\!\int \lambda(t)dt \sum_{j_{23}j_{23z}\alpha}\sum_{\sigma_1\tau_1}\int \frac{d\tilde{\kappa}_1}{2\kappa_1^+(2\pi)^3}\left|_{\rm LF}\left\langle T_{23},\tau_{23};\alpha,\epsilon_{23};j_{23}j_{23z};\tau_1\sigma_1,\tilde{\kappa}_1 \middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle\right|^2$$

$$= \sum_{T_{23}\tau_{23}}\sum\!\!\!\!\!\!\int \lambda(t)dt \sum_{j_{23}j_{23z}\alpha}\sum_{\sigma_1\tau_1}\int \frac{d\tilde{\kappa}_1}{2\kappa_1^+(2\pi)^3}\left|\sum_{\tau_2\tau_3}\int d\mathbf{k}_{23}\sum_{\sigma_1'} D^{\frac{1}{2}}\left[\mathcal{R}_M\left(\tilde{\mathbf{k}}_1^{(a)}\right)\right]_{\sigma_1\sigma_1'}\sqrt{(2\pi)^3 2E\left(\mathbf{k}_1^{(a)}\right)}\right.$$

$$\times \sqrt{\frac{\kappa_1^+ E_{23}}{k_1^{+(a)} E_S}} \sum_{\sigma_2'',\sigma_3''}\sum_{\sigma_2',\sigma_3'} \mathcal{D}_{\sigma_2'',\sigma_2'}(\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_2)\mathcal{D}_{\sigma_3'',\sigma_3'}(-\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_3)$$

$$\left.\times \langle T_{23},\tau_{23};\alpha,\epsilon_{23};j_{23}j_{23z}|\mathbf{k}_{23},\sigma_2''\sigma_3'';\tau_2,\tau_3\rangle \left\langle \sigma_3',\sigma_2',\sigma_1';\tau_3,\tau_2,\tau_1;\mathbf{k}_{23},\mathbf{k}_1^{(a)} \middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle\right|^2$$

$$= \sum_{T_{23}\tau_{23}} \sum\!\!\!\!\!\!\int \lambda(t)dt \sum_{j_{23}j_{23z}\alpha} \sum_{\sigma_1\tau_1} \sum_{\tau_2\tau_3} \int d\mathbf{k}_{23} \int d\tilde{\mathbf{k}}_1^{(a)} \sum_{\sigma_1'} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_1^{(a)})]_{\sigma_1\sigma_1'} \sqrt{\frac{E(\mathbf{k}_1^{(a)})}{E_{23}}} \sqrt{\frac{1}{k_1^{+(a)}}} \sum_{\sigma_2'',\sigma_3''} \sum_{\sigma_2',\sigma_3'} \mathcal{D}_{\sigma_2'',\sigma_2'}(\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_2)$$
$$\times \mathcal{D}_{\sigma_3'',\sigma_3'}(-\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_3)\langle T_{23},\tau_{23};\alpha,\epsilon_{23};j_{23}j_{23z}|\mathbf{k}_{23},\sigma_2''\sigma_3'';\tau_2,\tau_3\rangle\left\langle \sigma_3',\sigma_2',\sigma_1';\tau_3,\tau_2,\tau_1;\mathbf{k}_{23},\mathbf{k}_1^{(a)}\middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle$$
$$\times \sum_{\bar{\tau}_2\bar{\tau}_3} \int d\mathbf{k}_{23}'' \sum_{\bar{\sigma}_1'} D^{\frac{1}{2}*}[\mathcal{R}_M(\tilde{\mathbf{k}}_1''^{(a)})]_{\sigma_1\bar{\sigma}_1'} \sqrt{E(\mathbf{k}_1''^{(a)})} \sqrt{\frac{E_{23}''}{k_1''^{+(a)}}} \sum_{\bar{\sigma}_2'',\bar{\sigma}_3''} \sum_{\bar{\sigma}_2',\bar{\sigma}_3'} \mathcal{D}^*_{\bar{\sigma}_2'',\bar{\sigma}_2'}(\tilde{\mathbf{k}}_{23}'',\tilde{\mathbf{k}}_2'')\mathcal{D}^*_{\bar{\sigma}_3'',\bar{\sigma}_3'}(-\tilde{\mathbf{k}}_{23}'',\tilde{\mathbf{k}}_3'')$$
$$\times \langle T_{23},\tau_{23};\alpha,\epsilon_{23};j_{23}j_{23z}|\mathbf{k}_{23}'',\bar{\sigma}_2''\bar{\sigma}_3'';\bar{\tau}_2,\bar{\tau}_3\rangle^*\left\langle \bar{\sigma}_3',\bar{\sigma}_2',\bar{\sigma}_1';\bar{\tau}_3,\bar{\tau}_2,\tau_1;\mathbf{k}_{23}'',\mathbf{k}_1''^{(a)}\middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle^*. \quad \text{(C12)}$$

In the last step of Eq. (C12) the change of integration variable $d\kappa_1^+ = dk_1'^{+(a)}E_S/E_{23}'$ [see Eqs. (B11) and (B17)] was performed. Then, using the completeness for the two-body system (2,3) [see Eq. (A19)] one obtains

$$N = \sum_{\sigma_1\sigma_2,\sigma_3} \sum_{\tau_1\tau_2\tau_3} \int d\mathbf{k}_{23} \int d\tilde{\mathbf{k}}_1^{(a)} \frac{E(\mathbf{k}_1^{(a)})}{k_1^{+(a)}} \left| \sum_{\sigma_1'} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_1^{(a)})]_{\sigma_1\sigma_1'} \right.$$
$$\left. \times \sum_{\sigma_2',\sigma_3'} \mathcal{D}_{\sigma_2,\sigma_2'}(\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_2)\mathcal{D}_{\sigma_3,\sigma_3'}(-\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_3)\left\langle \sigma_3',\sigma_2',\sigma_1';\tau_3,\tau_2,\tau_1;\mathbf{k}_{23},\mathbf{k}_1^{(a)}\middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle \right|^2. \quad \text{(C13)}$$

Finally, exploiting the unitarity of $D^{1/2}$ and $\mathcal{D}$ matrices [see Eq. (C11)], one has

$$N = \sum_{\tau_1} \sum_{\tau_2\tau_3} \int d\mathbf{k}_{23} \int d\tilde{\mathbf{k}}_1^{(a)} \frac{E(\mathbf{k}_1^{(a)})}{k_1^{+(a)}} \sum_{\sigma_1'} \sum_{\sigma_2',\sigma_3'} \left\langle \sigma_3',\sigma_2',\sigma_1';\tau_3,\tau_2,\tau_1;\mathbf{k}_{23},\mathbf{k}_1^{(a)}\middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle$$
$$\times \left\langle \sigma_3',\sigma_2',\sigma_1';\tau_3,\tau_2,\tau_1;\mathbf{k}_{23},\mathbf{k}_1^{(a)}\middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle^*$$
$$= \sum_{\tau_1} \sum_{\sigma_1'} \int d\mathbf{k}_{23} \int d\mathbf{k}_1^{(a)} \sum_{\tau_2\tau_3} \sum_{\sigma_2',\sigma_3'} \left| \left\langle \sigma_3',\sigma_2',\sigma_1';\tau_3,\tau_2,\tau_1;\mathbf{k}_{23},\mathbf{k}_1^{(a)}\middle| j,j_z;\epsilon_{\rm int}^3,\Pi;\frac{1}{2}T_z\right\rangle \right|^2$$
$$= 1. \quad \text{(C14)}$$

where Eqs. (B14) and (43) were used.

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