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Axial anomaly and energy dependence of hyperon polarization in heavy-ion collisions

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We address the issue of energy and charge dependence of global polarization of Λ hyperons in peripheral Au-Au collisions recently observed by the STAR Collaboration at Relativistic Heavy Ion Collider (RHIC). We compare different contributions to the anomalous mechanism relating polarization to vorticity and hydrodynamic helicity in QCD matter. We stress that the suppression of the gravitational anomaly contribution in strongly correlated matter observed in lattice simulations confirms our earlier prediction of rapid decrease of polarization with increasing collision energy. Our mechanism leads to polarization of $\bar{\Lambda}$ of the same sign and larger magnitude than the polarization of Λ. The energy and charge dependence of polarization is suggested as a sensitive probe of fine details of QCD matter structure.

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Introduction. The local violation [1] of discrete symmetries in strongly interacting QCD matter is entering a new important phase of its investigation. It started from chiral magnetic effect (CME) [1] which uses the $(C)P$-violating (electro)magnetic field generated in heavy ion collisions in order to probe the $(C)P$-odd effects in QCD matter.

Current development is related to counterpart of this effect, chiral vortical effect (CVE) [2] due to coupling to P-odd medium vorticity leading to the induced electromagnetic and all conserved-charge currents [3], in particular the baryonic one.

What became most important is that now P-odd effects might be observable as baryon polarization. A mechanism analogous to CVE (known as axial vortical effect, see Ref. [4] and references therein) leads to an induced axial current of strange quarks which may be converted to polarization of Λ hyperons by anomalous mechanism suggested in Ref. [3] and later rediscovered in Ref. [5]. The numerical calculations exploring this mechanism were performed in Ref. [6], where hydrodynamic description of kinetic (QGSM) model results was introduced. Another mechanism of this polarization is provided by so-called thermal vorticity in the hydrodynamical approach [7,8], exploring the local thermodynamical equilibrium.

Recently pioneering preliminary experimental results on global polarization of Λ and $\bar{\Lambda}$ hyperons in peripheral Au-Au collisions in the RHIC beam energy scan were released [9], showing a decrease of polarization with increasing energy compatible to earlier negative STAR results [10] at the top RHIC energy. Such a behavior is in qualitative agreement with the prediction of Ref. [3]. Here, we address this issue and explore the relevant details of theoretical description. The decrease with energy is shown to be related to the suppression of the axial magnetic effect contribution in strongly correlated QCD matter found in lattice simulations. Consequently, accurate measurements of polarization energy dependence may serve as a sensitive probe of strongly correlated QCD matter.

Axial anomaly and hyperon polarization. We consider hyperon polarization as the observable related to vorticity and helicity [3]. We shall concentrate mostly on $\Lambda(\bar{\Lambda})$ hyperons, which are produced in large numbers, and their polarization may be easily recovered from the angular distributions of their weak decays products. These advantages are important in the current STAR measurements [9].

We explore the mechanism of generation of axial current similar to the famous axial anomaly. In the medium described by a chemical potential $\mu(x)$ there is a contribution to the interaction Lagrangian [11] proportional to the appropriate conserved charge density in the medium rest frame $\rho(x) = j_0(x)$:

$$\Delta L(x) = \mu(x)\rho(x).$$

The Lorentz covariance allows one to transform this expression using the hydrodynamical four-velocity $u_\alpha = \gamma(1,\vec{v})$, where γ is the Lorentz factor:

$$\Delta L(x) = \mu(x)u^\alpha(x)j_\alpha(x).$$

Here, the velocity $u_\alpha(x)$ and the chemical potential $\mu(x)$ play the role of the gauge field $A(x)$ and the corresponding coupling g, respectively:

$$gA^\beta(x)j_\beta(x) \to \mu(x)u^\alpha(x)j_\alpha(x). \quad (1)$$

This substitution can be applied to any diagram with the lines of external (classical) gauge fields leading to various medium effects. In the case of the famous anomalous triangle diagram (Fig. 1) it leads to the induced (classical) axial current.

This effect is quite similar to the anomalous gluon contribution to the nucleon spin (see, e.g., Refs. [12,13]). The role of the gauge gluon field is played by the velocity field while the longitudinal polarization (helicity) of gluons corresponds, as we will see below, to the hydrodynamic helicity.

Note that for massive quarks the anomalous contribution is partially compensated by the normal one (see, e.g., Ref. [12]). For a heavy quark axial current $\bar{Q}\gamma_\mu\gamma_5 Q$, the resulting matrix element between momentum eigenstates may be expanded to

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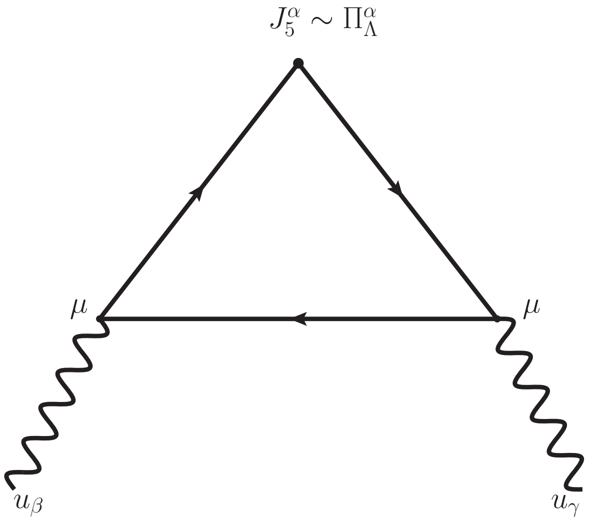

FIG. 1. Generation of Λ hyperon polarization via the axial anomaly.

inverse powers of the quark mass m_Q [14]:

$$\langle p|\bar{Q}\gamma_\mu\gamma_5 Q|p\rangle = i\frac{N_c\alpha_s}{2\pi}\varepsilon_{\mu\nu\lambda\rho}e^\nu e^{*\rho}p^\lambda\left\{1-\int_0^1 dx \times\frac{2m_Q^2(1-x)}{m_Q^2-p^2x(1-x)}\right\} = -i\frac{N_c\alpha_s}{12\pi}\varepsilon_{\mu\nu\lambda\rho}e^\nu e^{*\rho}p^\lambda\frac{p^2}{m_Q^2}+O\left(\frac{1}{m_Q^4}\right), \quad (2)$$

where e^ν, $e^{*\rho}$ are polarization vectors of these eigenstates and N_c is the number of colors. As far as the $SU(3)$ wave function of u and d quarks form the spin singlet, one might assume that in the nonrelativistic approximation Λ spin is carried predominantly by the strange quark. The strange quark may be considered as both light (with respect to the nucleon mass) and heavy (with respect to intrinsic higher twist scale) [15]. When the transition to coordinate space is performed, the Fourier transform of the corresponding matrix elements (requiring, generally speaking, the knowledge of the anomaly graph at arbitrary external momenta [16]) should contribute to the classical axial current. We do not expect that these corrections could change the scale of the effect substantially.

To quantify this expectation (see Fig. 2) let us consider the dependence of the anomaly coefficient

$$C(r) = 1-\int_0^1 dx\frac{2m^2(1-x)}{m^2-p^2x(1-x)}$$

on the ratio

$$r = \frac{2m^2}{p^2},$$

where p^2 is the virtuality corresponding to the external line of velocity field. One should expect that this virtuality is defined by the characteristic time and scale of the variations of velocity field and it is of the same order as the strange quark mass. Therefore, the effect should remain of the same order of magnitude after the account for this correction.

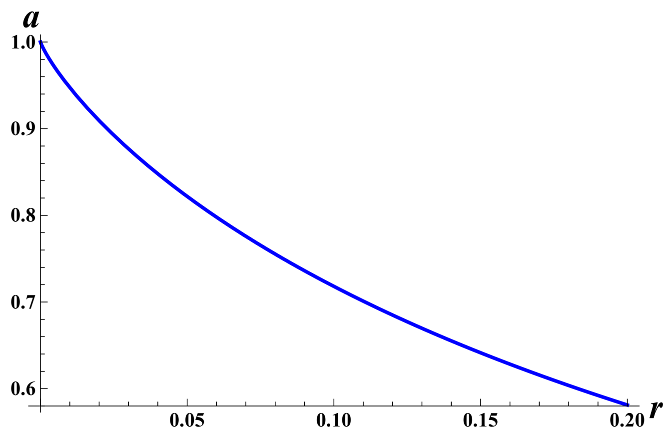

FIG. 2. Dependence of the anomaly coefficient on $r = 2m^2/k^2$ (see the text).

$C(r)$ should enter the classical strange axial charge [6]

$$Q_5^s = \frac{N_c}{2\pi^2}\int d^3x\, C(r)\,\mu_s^2(x)\gamma^2\epsilon^{ijk}v_i\partial_j v_k \quad (3)$$

induced by the anomalous triangle diagram (Fig. 1). Here, we keep actually only the first term in the chiral vorticity coefficient

$$c_V = \frac{\mu_s^2+\mu_A^2}{2\pi^2}+\frac{T^2}{6},$$
$$Q_5^s = N_c\int d^3x\, C(r)\,c_V\gamma^2\epsilon^{ijk}v_i\partial_j v_k, \quad (4)$$

since we assume that the chiral chemical potential μ_A is much smaller than the strange one μ_s. The temperature-dependent term in Eq. (4), related to the gravitational anomaly [17], can naively be considered to be quite substantial. However, lattice simulations [18] lead to a zero result in the confined phase and to suppression by one order of magnitude at high temperatures. As far as for free fermion gas the $T^2/6$ term is recovered [19] for large lattice volume at fixed temperature, the above-mentioned suppression should be attributed to the correlation effects.

In order to relate the strange axial charge Q_5^s (3) to hydrodynamical quantities one can use the mean-value theorem to evaluate it [6]:

$$Q_5^s = \frac{\langle\mu^2 C(r)\gamma^2\rangle\, N_c H}{2\pi^2}, \quad (5)$$

where hydrodynamical helicity

$$H \equiv \int d^3x(\vec{v}\cdot\vec{w})$$

is the integrated projection of the velocity $\vec{v}$ to the vorticity $\vec{w} = curl\vec{v}$.

Note that the hydrodynamic helicity is related to the zeroth component ($\mu = 0$) of the four-current

$$K^\mu(x) = \epsilon^{\mu\nu\rho\gamma}u_\nu(x)\partial_\rho u_\gamma(x), \quad H = \int d^3x\frac{K^0(x)}{\gamma^2}. \quad (6)$$

Coming back to the similarity with the spin crisis, this is the analog of the topological current, related [12] (in the axial gauge) to the gluon polarization.

The space components of (6) for sufficiently slow fields in the nonrelativistic approximation are related to the vorticity vector:

$$K^i(x)|_{\gamma\to 1} = 2\epsilon^{ijk}\partial_j v_k(x) = 2\omega^i,$$

while the exact relation involves relativistic corrections. To avoid such complications, here we discuss the complementary approach [20] relying on the $K^0(x)$, helicity, and axial charge. To pass from the classical charge (we are always dealing with) to the quantum matrix elements we will use the analogy with the conserved current case when the conserved charge Q appears in the symmetric one-particle matrix elements of the current

$$\langle p_n|j^0(0)|p_n\rangle = 2p_n^0 Q_n. \tag{7}$$

To calculate the (quantum) average charge per particle resulting from the latter equation it is sufficient to divide the total *classical* charge by particle number N:

$$\langle Q\rangle \equiv \frac{\sum_{n=1}^N Q_n}{N} = \frac{\int d^3x\, j^0_{\text{class}}(x)}{N}. \tag{8}$$

Passing now to the axial charge case, let us note that the symmetric matrix element of quark axial current of the flavor i is related to the fraction a_i of a hadron covariant polarization (strange quark axial coupling) Π^μ ($p_\mu\Pi^\mu = 0$) carried by that quark:

$$\langle p_n,\Pi_n|j^0_{5,i}(0)|p_n,\Pi_n\rangle = 2a_{i,n}m_n\Pi^0_n,$$

where m_i is a hadron i mass. By analogy with (7) and (8), the axial charge should correspond to

$$Q_{5,i,n} \to \frac{m_n a_{i,n}\Pi^0_n}{p^0_n}.$$

The average polarization is then

$$\left\langle \frac{a_i m\Pi^0}{p_0}\right\rangle \equiv \frac{\sum_{n=1}^N \frac{m_n a_{i,n}\Pi^0_n}{p^0_n}}{N} = \frac{\int d^3x\, j^0_{5,\text{class}}(x)}{N}.$$

In what follows we will consider all N particles to be $\Lambda\,(\bar\Lambda)$ and put $a_i m/p_0 = 1$ assuming (following the above-mentioned $SU(3)$ based arguments) that strange (anti)quarks carry all the polarization and considering the Λ mass is large enough with respect to its momentum. The latter assumption is a reasonable approximation at the Nuclotron-based Ion Collider fAcility (NICA) energies ($\sqrt{s_{NN}} = 4 - 11$ GeV) and will provide a lower bound estimate of the polarization. This is just an approximation, and it can be possible to go beyond this approximation in future.

This procedure bears some similarity to particlization compatible to Cooper-Frye formula and applied recently in Ref. [21]. In our case we consider the axial current, directly related to spin angular momentum tensor and polarization, while in thermodynamic approach the coupling of rotation to total angular momentum matters. Although there are indications [22] on the thermodynamical nonequivalence of spin and orbital angular momenta, the extraction of spin (polarization) part does not seem obvious. Note also, that in Cooper-Frye formula the integration over freezeout 3-surface is performed, while in our case the surface of constant time is used. In principle, the generalization for arbitrary 3-surface may be considered.

Note also some analogy[1] to the vector dominance model (VDM), where the conservation of vector current is relevant. In this sense our approach may be considered as a sort of axial dominance model. It is interesting that VDM may be related [23] to the axial anomaly considered in the framework [24,25] of the dispersive approach.

The helicity shows the phenomenon of separation [6,26] so that its sign is changed at the two sides of the reaction plane. As a result, the axial charge and the zeroth component of the hyperon polarization also manifest such a phenomenon of sign change.

As the axial charge is related to the zeroth component of the hyperon polarization in the laboratory frame Π_0^{lab}, the transformation to hyperon rest frame must be performed [20]. Taking into account that the polarization pseudovector should be directed along the y axis transverse to the reaction plane (as it has to be collinear to the angular momentum pseudovector), one gets for the components of the laboratory frame polarization

$$\Pi^{\Lambda,\text{lab}} = \left(\Pi_0^{\Lambda,\text{lab}},\Pi_x^{\Lambda,\text{lab}},\Pi_y^{\Lambda,\text{lab}},\Pi_z^{\Lambda,\text{lab}}\right) = \frac{\Pi_0^\Lambda}{m_\Lambda}(p_y,0,p_0,0). \tag{9}$$

One can use both its time $\Pi_0^{\Lambda,\text{lab}}$ and space $\Pi_y^{\Lambda,\text{lab}}$ components to recover the polarization in the hyperon rest frame. Note that for the time component the factor p_y changes sign at two sides of reaction plane, compensating for the sign change due to helicity separation. At the same time, that factor is absent for space component. As we discussed above, this component is related to the vorticity rather than helicity and should not show the sign change. Therefore, the use of either time or space component of $\Pi_0^{\Lambda,\text{lab}}$ should lead to equal polarization at both sides of the reaction plane. Leaving the exploration of space component for future work we use the time component and get

$$\langle\Pi_0^\Lambda\rangle = \frac{m_\Lambda\,\Pi_0^{\Lambda,\text{lab}}}{p_y} = \left\langle\frac{m_\Lambda}{N_\Lambda\, p_y}\right\rangle$$
$$Q_5^s \equiv \left\langle\frac{m_\Lambda}{N_\Lambda\, p_y}\right\rangle \frac{N_c}{2\pi^2}\int d^3x\, C(r)\,\mu_s^2(x)\gamma^2\epsilon^{ijk}v_i\partial_j v_k. \tag{10}$$

The appearance of p_y in the denominator is not dangerous, as the particles with zero transverse momentum do not have also the time component of polarization.

The average polarization was first roughly estimated just by dividing Q_5^s (5) by the number of Λ, leading [6] to a value of about 1%, later confirmed [20] by more detailed simulations, which are compatible with the current STAR data [9].

[1] We are indebted to an anonymous referee for pointing out to this similarity.

The appearance of μ^2 in Eqs. (3) and (10), related to the positive C parity of axial current, immediately leads to the same expressions for axial charge of strange quarks and antiquarks. As far as there is a smaller number of $\bar{\Lambda}$s than of Λs, so that the same axial charge should be distributed among smaller number of antiquarks compared with the number of quarks, the corresponding factor in the denominator in Eq. (10) is smaller for $\bar{\Lambda}$s, which results in an increase of the effect for the latter. Thus, one could expect that the polarization of $\bar{\Lambda}$ has to be of the same sign but of a larger magnitude than the polarization of Λ, which is compatible with the quite recent STAR data [9]. This effect might be partly compensated by the fact that a larger amount of axial charge in the case of strange antiquarks might be carried by more numerous K^* mesons.

It is well known that baryon chemical potential rapidly decreases with increasing energy and one might expect analogous behavior of the strange chemical potential. This is indeed confirmed by numerical simulations [27]. This property provides a natural explanation of the observed hint [9] for decrease of polarization with energy. More accurate measurements of Λ and $\bar{\Lambda}$ polarization at RHIC, and at NICA and FAIR in future, might allow one to test the suppression of T^2 term and, at best, even to check experimentally the magnitude of its theoretically predicted coefficient.

One might expect that the approach [7] based on thermal vorticity provides an extra contribution to polarization. Indeed, this approach is based entirely on the thermodynamical equilibrium in rotating medium. Our anomalous contribution is dynamical and can appear on the top of thermodynamical one. While thermodynamical contribution is universal and should be equal for all the particles, the dependence of anomalous contribution on chemical potential makes it essentially different for different particles. The experimental tests of polarization of different hyperons and vector mesons, in principle, allows one to separate thermodynamical and anomalous contributions.

Conclusions and outlook. The generation of polarization by the anomalous mechanism (axial vortical effect) naturally explains several features of the observed data:

(1) The decrease of chemical potential with energy leads to the decrease of polarization. An additional source of decrease is provided by the energy dependence of the helicity which was earlier found [6] to be maximal in the NICA energy range. The contribution related to the gravitational anomaly proportional to T^2 may be suppressed in strongly correlated matter. Moreover, the accurate measurements of the energy dependence of polarization should allow to separate the gravitational anomaly contribution and test the degree of its suppression in strongly correlated QCD matter.

(2) The proportionality of the polarization to the square of the chemical potential related to C-even parity of axial current leads to the same sign of polarization of Λ and $\bar{\Lambda}$ hyperons. The smaller number of the latter should result in a larger fraction of the axial charge, corresponding to each antihyperon and to a larger absolute value of polarization. Detailed numerical simulations may allow to quantify this prediction. Accurate measurements of Λ and $\bar{\Lambda}$ polarization should allow to check these predictions and provide an additional check of the gravitational anomaly related contribution.

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[1] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, The chiral magnetic effect, Phys. Rev. D **78**, 074033 (2008).

[2] D. Kharzeev and A. Zhitnitsky, Charge separation induced by P-odd bubbles in QCD matter, Nucl. Phys. A **797**, 67 (2007).

[3] O. V. Rogachevsky, A. S. Sorin, and O. V. Teryaev, Chiral vortaic effect and neutron asymmetries in heavy-ion collisions, Phys. Rev. C **82**, 054910 (2010).

[4] T. Kalaydzhyan, Temperature dependence of the chiral vortical effects, Phys. Rev. D **89**, 105012 (2014).

[5] J.-H. Gao, Z.-T. Liang, S. Pu, Q. Wang, and X.-N. Wang, Chiral Anomaly and Local Polarization Effect from Quantum Kinetic Approach, Phys. Rev. Lett. **109**, 232301 (2012).

[6] M. Baznat, K. Gudima, A. Sorin, and O. Teryaev, Helicity separation in heavy-ion collisions, Phys. Rev. C **88**, 061901 (2013).

[7] F. Becattini, L. P. Csernai, and D. J. Wang, Λ polarization in peripheral heavy ion collisions, Phys. Rev. C **88**, 034905 (2013).

[8] R. H. Fang, L. G. Pang, Q. Wang, and X. N. Wang, Polarization of massive fermions in a vortical fluid, Phys. Rev. C **94**, 024904 (2016).

[9] M. Lisa *et al.* (STAR Collaboration), Talks at QCD Chirality Workshop, UCLA, February 2016 and SQM conference, Berkeley, June 2016 (unpublished).

[10] B. I. Abelev *et al.* (STAR Collaboration), Global polarization measurement in Au+Au collisions, Phys. Rev. C **76**, 024915 (2007).

[11] A. V. Sadofyev, V. I. Shevchenko, and V. I. Zakharov, Notes on chiral hydrodynamics within effective theory approach, Phys. Rev. D **83**, 105025 (2011).

[12] A. V. Efremov, J. Soffer, and O. V. Teryaev, Spin structure of nucleon and the axial anomaly, Nucl. Phys. B **346**, 97 (1990).

[13] M. Anselmino, A. Efremov, and E. Leader, The theory and phenomenology of polarized deep inelastic scattering, Phys. Rep. **261**, 1 (1995); Erratum, **281**, 399(E) (1997).

[14] M. V. Polyakov, A. Schafer, and O. V. Teryaev, The intrinsic charm contribution to the proton spin, Phys. Rev. D **60**, 051502 (1999).

[15] O. V. Teryaev, Nucleon spin and orbital structure: 20 years later, Mod. Phys. Lett. A **24**, 2831 (2009).

[16] O. L. Veretin and O. V. Teryaev, Axial anomaly at the arbitrary external momenta, Yad. Fiz. **58**, 2266 (1995) [Phys. Atom. Nucl. **58**, 2150 (1995)].

[17] K. Landsteiner, E. Megias, L. Melgar, and F. Pena-Benitez, Holographic gravitational anomaly and chiral vortical effect, J. High Energy Phys. 09 (2011) 121.

[18] V. Braguta, M. N. Chernodub, K. Landsteiner, M. I. Polikarpov, and M. V. Ulybyshev, Numerical evidence of the axial magnetic effect, Phys. Rev. D **88**, 071501(R) (2013); V. Braguta, M. N. Chernodub, V. A. Goy, K. Landsteiner, A. V. Molochkov, and M. I. Polikarpov, Temperature dependence of the axial magnetic effect in two-color quenched QCD, *ibid.* **89**, 074510 (2014); V. Braguta, M. N. Chernodub, V. A. Goy, K. Landsteiner, A. V. Molochkov, and M. Ulybyshev, Study of axial magnetic effect, AIP Conf. Proc. **1701**, 030002 (2016); V. Goy, Investigation of SU(2) gluodynamics in the framework of the latice approach, Ph.D. dissertation, Far Eastern Federal University, Vladivostok, 2015 (unpublished).

[19] P. V. Buividovich, Axial magnetic effect and chiral vortical effect with free lattice chiral fermions, J. Phys. Conf. Ser. **607**, 012018 (2015).

[20] M. I. Baznat, K. K. Gudima, A. S. Sorin, and O. V. Teryaev, Femto-vortex sheets and hyperon polarization in heavy-ion collisions, Phys. Rev. C **93**, 031902 (2016).

[21] I. Karpenko and F. Becattini, Study of Λ polarization in relativistic nuclear collisions at $\sqrt{s_{NN}} = 7.7$–200 GeV, arXiv:1610.04717 (unpublished).

[22] F. Becattini and L. Tinti, Thermodynamical inequivalence of quantum stress-energy and spin tensors, Phys. Rev. D **84**, 025013 (2011); Nonequilibrium thermodynamical inequivalence of quantum stress-energy and spin tensors, **87**, 025029 (2013).

[23] Y. Klopot, A. Oganesian, and O. Teryaev, Axial anomaly and vector meson dominance model, JETP Lett. **99**, 679 (2014).

[24] A. D. Dolgov and V. I. Zakharov, On conservation of the axial current in massless electrodynamics, Nucl. Phys. B **27**, 525 (1971).

[25] J. Horejsi and O. Teryaev, Dispersive approach to the axial anomaly, the t'Hooft's principle, and QCD sum rules, Z. Phys. C **65**, 691 (1995).

[26] O. Teryaev and R. Usubov, Parity-odd effects in heavy-ion collisions in the HSD model, arXiv:1406.4451 [nucl-th]; Vorticity and hydrodynamic helicity in heavy-ion collisions in the hadron-string dynamics model, Phys. Rev. C **92**, 014906 (2015).

[27] M. Baznat, K. Gudima, A. Sorin, and O. Teryaev (unpublished).