Extraction of structure functions for lepton-nucleus scattering in the quasi-elastic region

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Within the framework of a relativistic single-particle model, we calculate inclusive electron-nucleus scattering by electromagnetic current, and neutrino-nucleus scattering by neutral and charged current in the quasi-elastic region. The longitudinal, the transverse, and the transverse-interference structure functions are extracted from the theoretical cross section by using the Rosenbluth separation method at fixed momentum transfer and scattering angle and then compared with each other from the viewpoint of these current interactions. The position of peak for the electron scattering shifts to higher energy transfer than that for the neutrino scattering. The axial and pseudoscalar terms turn out to play an important role in the neutrino-nucleus scattering.

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I. INTRODUCTION

Neutrino scattering with a complex nucleus provides varied and important physical information for various fields of physics, such as astrophysics, particle physics, and nuclear physics. Furthermore, neutrino-nucleus scattering is a useful tools for extracting information not only on nucleon properties such as axial or strange form factors but also the nuclear structure. In particular, neutral-current (NC) neutrino scattering is a more effective tool for studying the strangeness contribution as well as the nuclear structure because of the lack of Coulomb distortion of incoming and outgoing leptons from the target nucleus.

Since the measurement of NC muon neutrino scattering was first performed at Brookhaven National Laboratory [1], two experiments on the charged-current reaction (CC) [2] and NC [3] reactions from a CH_2 target were performed by the MiniBooNE Collaboration. From these MiniBooNE data, nonstandard values of the axial mass and the strange axial form factor at $Q^2 = 0$, $M_A = 1.39$ GeV$/c^2$, and $g_A^s = 0.08$ were extracted by measuring the flux averaged NC elastic cross sections in the quasi-elastic region. Other MiniBooNE data for the muon antineutrino CC reaction [4] show that the observed cross section is higher than the theoretical cross section predicted with a relativistic Fermi gas (RFG) model [5].

The antineutrino NC reaction cross sections were also measured, and the ratio of the antineutrino to neutrino NC cross section and the ratio of the antineutrino NC to antineutrino CC cross section were shown [6]. From this experiment, the antineutrino NC cross section shows good agreement with the RFG model, with $M_A = 1.35$ MeV$/c^2$. Moreover, other muon antineutrino and neutrino quasi-elastic scattering experiments were performed by the MINERνA Collaboration [7], but they obtained good agreement with a model in which M_A is 0.99 GeV$/c^2$.

Since these neutrino and antineutrino data were published, there have been many theoretical works regarding the nonstandard axial mass and the axial strangeness coupling constant [8–11]. In Ref. [8], the CC quasi-elastic cross section was calculated by using the relativistic Green function method, and the results describe the experimental data without additional adjustment of the nucleon axial mass. Butkevich and Perevalov [9] calculated the NC reaction with the relativistic distorted-wave impulse approximation (RDWIA) and then extracted the axial strange form factor $g_A^s = -0.11$ by using the MiniBooNE data in the high energy region. Nieves *et al.* [10] deduced that the increases of the axial mass may be associated with the concept of underestimated neutrino flux, to calculate the CC double cross section within random phase approximation (RPA) correlations and multinucleon contributions based on the Fermi gas model. Ankowski's results [11] underestimated the MiniBooNE data by 20% with the axial mass 1.23 GeV$/c^2$, but overestimated the data by 15% with the axial mass 1.39 GeV$/c^2$ for the $^{12}C(\nu_\mu,\mu^-)$ reaction. Another value of the axial mass [12] was extracted, about $M_A = 0.84$ GeV$/c^2$ at $Q^2 = 1.0$ (GeV$/c)^2$, using the antineutrino CC scattering data from MinBooNE [4]. The Granada group [13] study the role of the axial form factor by using an axial-vector-meson-dominance model, and the most sensitive region in the MiniBooNE data is in the range $0.2 \lesssim Q^2 \lesssim 0.6$ (GeV$/c)^2$.

In our previous papers [14–19], we studied various uncertainties for the NC and CC neutrino scattering in the quasi-elastic region. In Ref. [14], we investigated the effect of g_A^s on the NC cross section, the asymmetries, and the ratios

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of the NC to the CC reactions, and then we studied the effect of the Coulomb distortion of the outgoing leptons in (ν_μ,μ^-) and $(\bar{\nu}_\mu,\mu^+)$ reactions [15]. As an another uncertainty, we examine the final state interaction (FSI) on the NC reaction not only from the cross section but from the asymmetry and each response cross section; as a result, the FSI turns out to play an important role [16]. To include the nuclear medium effect, we investigated the contribution of the density-dependent weak form factors in the CC [17] and the NC [18] reactions, in which the form factors of the density dependence are obtained from the quark-meson-coupling (QMC) model generated by Thomas and collaborators [20]. Recently, we also studied the influence of M_A and g_A^s on the cross section, the separated cross sections associated with the longitudinal and transverse response functions, the asymmetry of the NC reaction, and the various ratios of the NC to CC reactions [19].

On the other hand, in the nuclear structure, it is considerably interesting to look for a way to extract longitudinal and transverse structure functions as a function of energy transfer at fixed three-momentum transfer for the inclusive (e,e') reaction. The Fermi gas model in the impulse approximation describes roughly the inclusive (e,e') cross sections but fails to reproduce the structure functions. In particular, there appears to be a large suppression (about 50%) of the longitudinal structure function, referring to the missing strength in the Coulomb sum rule (CSR) [21]. While Traini [22] supports the suppression of the CSR, Jourdan [23] examined the world (e,e') data set on ^{12}C, ^{40}Ca, and ^{56}Fe at momentum transfers of 300, 380, and 570 MeV$/c$, and did not find any evidence of the suppression. The CSR was also examined in Ref. [24] by using the effective momentum approximation (EMA) in medium and heavy nuclei, and as a result the longitudinal structure function was quenched about 20%. The authors in Ref. [25] found that there is no drastic suppression of the longitudinal structure function in a relativistic mean field (RMF) model. Recently, relativistic and nuclear medium effects on the CSR were investigated by using the QCD model [26], and it was found that there is a dramatic quenching of the CSR for three-momentum transfer $q \geqslant 0.5$ GeV$/c$.

In this work, we calculate the inclusive (e,e'), (ν,ν'), $(\bar{\nu},\bar{\nu}')$, (ν_e,e^-), and $(\bar{\nu}_e,e^+)$ reactions from ^{40}Ca within a relativistic single-particle model in the quasi-elastic region where the inelastic processes are excluded. Of course, the structure functions for the electron scattering can be theoretically calculated and can be experimentally extracted by using the Rosenbluth separation. But they cannot be directly extracted from experimental neutrino scattering, so we modify the Rosenbluth separation with the incident neutrino plus and/or minus antineutrino scattering cross sections. For the NC reaction the structure functions can be extracted by combining the (ν,ν') and the $(\bar{\nu},\bar{\nu}')$ reactions by using Rosenbluth separation, but for the (ν_e,e^-) and the $(\bar{\nu}_e,e^+)$ reactions only the transverse-interference structure function can be extracted because the longitudinal structure function consists of three terms. The bound nucleon wave functions are solutions to the Dirac equation in the presence of the strong scalar and vector potentials based on the $\sigma-\omega$ model [27]. The wave functions of the knocked-out nucleons are generated by the same potential as the bound nucleons; this is the RMF theory. This model describes the experimental data very well. Finally, to include the Coulomb distortion of incoming and outgoing leptons, we use an approximate treatment of the Coulomb distortion developed by the Ohio University group [28].

The outline of this paper is as follows: In Sec. II, we briefly present the formalism for the inclusive (e,e'), NC, and CC reactions. In Sec. III, our numerical results are presented for the structure functions extracted from each reaction, and in Sec. IV the summary and conclusion are given.

II. FORMALISM

A. Inclusive (e,e') reaction

In the plane-wave Born approximation (PWBA) in which the electrons are described as Dirac plane waves, the cross section for the inclusive (e,e') scattering can be written as

$$\frac{d^2\sigma}{d\omega\, d\Omega_e} = \sigma_M\left[\frac{Q^4}{q^4}S_L(q,\omega) + \left(\tan^2\frac{\theta_e}{2} + \frac{Q^2}{2q^2}\right)S_T(q,\omega)\right], \tag{1}$$

where $Q^2 = \mathbf{q}^2 - \omega^2 = -q_\mu^2$ is the four-momentum transfer, σ_M is the Mott cross section given by $\sigma_M = (\frac{\alpha}{2E_i})^2\cos^2(\frac{\theta_e}{2})/\sin^4(\frac{\theta_e}{2})$, and S_L and S_T are the longitudinal and transverse structure functions which depend only on the three-momentum transfer q and the energy transfer ω. By keeping the momentum and energy transfers fixed while varying the electron incident energy E_i and scattering angle θ_e, it is possible to extract the two structure functions with two measurements. The longitudinal and transverse structure functions in Eq. (1) are squares of the Fourier transform of the components of the nuclear transition current density integrated over outgoing nucleon angles, Ω_p. Explicitly, the structure functions for a given bound state with angular momentum j_b are given by

$$S_L(q,\omega) = \sum_{\mu_b s_p}\frac{\rho_p}{2(2j_b+1)}\int |N_0|^2 d\Omega_p, \tag{2}$$

$$S_T(q,\omega) = \sum_{\mu_b s_p}\frac{\rho_p}{2(2j_b+1)}\int (|N_x|^2 + |N_y|^2) d\Omega_p \tag{3}$$

with the outgoing nucleon density of states $\rho_p = \frac{pE_p}{(2\pi)^3}$. The $\hat{\mathbf{z}}$ axis is taken to be along the momentum transfer $\mathbf{q}$ and the z components of the angular momentum of the bound and continuum state nucleons are μ_b and s_p, respectively. The Fourier transform of the nuclear current $J^\mu(\mathbf{r})$ is simply given by

$$N^\mu = \int J^\mu(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}d^3r, \tag{4}$$

where $J^\mu(\mathbf{r})$ denotes the nucleon transition current. The continuity equation could be used to eliminate the z component (N_z) via the equation $N_z = -\frac{\omega}{q}N_0$ if the current is conserved. The nucleon transition current in the relativistic single particle model is given by

$$J^\mu(\mathbf{r}) = e\bar{\psi}_p(\mathbf{r})\hat{\mathbf{J}}^\mu\psi_b(\mathbf{r}), \tag{5}$$

where $\hat{\mathbf{J}}^\mu$ is a free nucleon current operator, and ψ_p and ψ_b are the wave functions of the knocked-out nucleon and the bound state, respectively. For a free nucleon, the operator comprises the Dirac contribution and the contribution of an anomalous magnetic moment μ_T given by $\hat{\mathbf{J}}^\mu = F_1(q_\mu^2)\gamma^\mu + F_2(q_\mu^2)\frac{i\mu_T}{2M_N}\sigma^{\mu\nu}q_\nu$, where M_N is the mass of a nucleon. The form factors F_1 and F_2 are related to the electric and magnetic Sachs form factors given by $G_E = F_1 + \frac{\mu_T Q^2}{4M_N^2}F_2$ and $G_M = F_1 + \mu_T F_2$ which are assumed to take the following standard form:

$$G_E = \frac{1}{\left(1+\frac{Q^2}{\Lambda^2}\right)^2} = \frac{G_M}{(\mu_T+1)}, \tag{6}$$

where the standard value for Λ^2 is 0.71 $(\mathrm{GeV}/c)^2$.

From the measured cross section in Eq. (1), the total structure function is defined as

$$S_{\mathrm{tot}}(q,\omega,\theta_e) = \left(\frac{\epsilon(\theta_e)}{\sigma_M}\right)\left(\frac{q^4}{Q^4}\right)\frac{d^2\sigma}{d\omega\, d\Omega_e}, \tag{7}$$

where the $\epsilon(\theta_e)$ is the virtual photon polarization given by $\epsilon(\theta_e) = [1 + (\frac{2q^2}{Q^2})\tan^2(\theta_e/2)]^{-1}$. Therefore, the total structure function in Eq. (7) becomes

$$S_{\mathrm{tot}}(q,\omega,\theta_e) = \epsilon(\theta_e)S_L(q,\omega) + \left(\frac{q^2}{2Q^2}\right)S_T(q,\omega). \tag{8}$$

S_{tot} is described as a straight line in terms of the independent variable $\epsilon(\theta_e)$ with slope $S_L(q,\omega)$ and intercept proportional to $S_T(q,\omega)$ by keeping the momentum transfer q and the energy transfer ω fixed. This is called Rosenbluth separation.

B. Inclusive neutrino-nucleus scattering

In order to model the neutrino-nucleus scattering mathematically, we choose the nucleus rest frame where the target nucleus is positioned at the origin of the coordinate system. The four-momenta of the incoming and outgoing neutrinos (antineutrinos) are labeled $p_i^\mu = (E_i,\mathbf{p}_i)$ and $p_f^\mu = (E_f,\mathbf{p}_f)$. $p_A^\mu = (E_A,\mathbf{p}_A)$, $p_{A-1}^\mu = (E_{A-1},\mathbf{p}_{A-1})$, and $p^\mu = (E_N,\mathbf{p})$ represent the four-momenta of the target nucleus, the residual nucleus, and the knocked-out nucleon, respectively. In the laboratory frame, the inclusive cross section for the NC and CC reactions is given by the contraction between the lepton and hadron tensor:

$$\frac{d\sigma}{d\omega\, d\Omega_l} = 4\pi^2\frac{M_N M_{A-1}}{(2\pi)^3 M_A}\int \sin\theta_N\, d\theta_N\, p\, f_{\mathrm{rec}}^{-1}\sigma_M^{Z,\,W^\pm} \times [v_L S_L + v_T S_T + h v_T' S_T'], \tag{9}$$

where Ω_l denotes the solid angle of the incident lepton, θ_N is the polar angle of the knocked-out nucleons, and $h = -1$ $(h = +1)$ corresponds to the helicity of the incident neutrino (antineutrino). For the NC reaction, the kinematic factor (σ_M^Z) is defined by

$$\sigma_M^Z = \left(\frac{G_F\cos(\theta_l/2)E_f M_Z^2}{\sqrt{2}\pi\left(Q^2+M_Z^2\right)}\right)^2, \tag{10}$$

and for the CC reaction

$$\sigma_M^{W^\pm} = \sqrt{1-\frac{M_l^2}{E_f^2}}\left(\frac{G_F\cos(\theta_C)E_f M_W^2}{2\pi\left(Q^2+M_W^2\right)}\right)^2, \tag{11}$$

where M_Z and M_W are the rest masses of the Z and W bosons, respectively. θ_l denotes the scattering angle of the lepton. G_F is the Fermi constant given by $G_F \simeq 1.166\,39\times10^{-11}$ MeV^{-1}. θ_C represents the Cabibbo angle given by $\cos^2\theta_C \simeq 0.9749$. The recoil factor (f_{rec}) is expressed as

$$f_{\mathrm{rec}} = \frac{E_{A-1}}{M_A}\left|1 + \frac{E_p}{E_{A-1}}\left[1-\frac{\mathbf{q}\cdot\mathbf{p}}{p^2}\right]\right|. \tag{12}$$

The recoil factor depends on the polar angle of the knocked-out nucleon but the magnitude is almost 1. For simplicity, the recoil factor is just assumed to be 1.

For the NC reaction, the kinematical coefficients v are given by

$$v_L = 1, \quad v_T = \tan^2\frac{\theta_l}{2} + \frac{Q^2}{2q^2},$$
$$v_T' = \tan\frac{\theta_l}{2}\left[\tan^2\frac{\theta_l}{2} + \frac{Q^2}{q^2}\right]^{1/2}, \tag{13}$$

and corresponding response functions are expressed as

$$S_L = \left|N^0 - \frac{\omega}{q}N^z\right|^2, \quad S_T = |N^x|^2 + |N^y|^2,$$
$$S_T' = 2\,\mathrm{Im}(N^{x*}N^y). \tag{14}$$

For the CC reaction, the coefficients v are given by

$$v_L^0 = 1 + \sqrt{1-\frac{M_l^2}{E_f^2}}\cos\theta_l,$$
$$v_L^z = 1 + \sqrt{1-\frac{M_l^2}{E_f^2}}\cos\theta_l - \frac{2E_iE_f}{q^2}\left(1-\frac{M_l^2}{E_f^2}\right)\sin^2\theta_l,$$
$$v_L^{0z} = \frac{\omega}{q}\left(1+\sqrt{1-\frac{M_l^2}{E_f^2}}\cos\theta_l\right) + \frac{M_l^2}{E_f q},$$
$$v_T = 1 - \sqrt{1-\frac{M_l^2}{E_f^2}}\cos\theta_l + \frac{E_iE_f}{q^2}\left(1-\frac{M_l^2}{E_f^2}\right)\sin^2\theta_l,$$
$$v_T' = \frac{E_i+E_f}{q}\left(1-\sqrt{1-\frac{M_l^2}{E_f^2}}\cos\theta_l\right) - \frac{M_l^2}{E_f q}. \tag{15}$$

The corresponding response functions are given by

$$S_L^0 = |N^0|^2, \quad S_L^z = |N^z|^2, \quad S_L^{0z} = -2\mathrm{Re}(N^0N^{z*}),$$
$$S_T = |N^x|^2 + |N^y|^2, \quad S_T' = 2\,\mathrm{Im}(N^{x*}N^y), \tag{16}$$

and

$$v_L S_L = v_L^0 S_L^0 + v_L^z S_L^z + v_L^{0z}S_L^{0z}. \tag{17}$$

Here, N^μ is the same as that in Eq. (4).

For a free nucleon, the current operator comprises the weak vector and the axial vector form factors,

$$\hat{\mathbf{J}}^\mu = F_1^V(Q^2)\gamma^\mu + F_2^V(Q^2)\frac{i\mu_T}{2M_N}\sigma^{\mu\nu}q_\nu + G_A(Q^2)\gamma^\mu\gamma^5 + \frac{1}{2M_N}G_P(Q^2)q^\mu\gamma^5, \tag{18}$$

where M_N denotes the nucleon mass. By the conservation of the vector current (CVC) hypothesis, the vector form factors for the proton (neutron) $[F_i^{V,p(n)}(Q^2)]$ are expressed as

$$F_i^{V,p(n)}(Q^2) = \left(\frac{1}{2} - 2\sin^2\theta_W\right)F_i^{p(n)}(Q^2) - \frac{1}{2}F_i^{n(p)}(Q^2) - \frac{1}{2}F_i^s(Q^2), \quad \text{for the NC},$$
$$F_i^V(Q^2) = F_i^p(Q^2) - F_i^n(Q^2), \quad \text{for the CC}, \tag{19}$$

where θ_W is the Weinberg angle given by $\sin^2\theta_W = 0.2224$.

The strange vector form factors $(F_i^s(Q^2))$ in Eq. (19) are usually taken to be of dipole form as

$$F_1^s(Q^2) = \frac{F_1^s(0)Q^2}{(1+\tau)\left(1+Q^2/M_V^2\right)^2},$$
$$F_2^s(Q^2) = \frac{F_2^s(0)}{(1+\tau)\left(1+Q^2/M_V^2\right)^2}, \tag{20}$$

where $\tau = Q^2/(4M_N^2)$ and $M_V = 0.843$ GeV is the cut-off mass parameter usually adopted for nucleon electromagnetic form factors. $F_1^s(0)$ is defined as $F_1^s(0) = dG_E^s(Q^2)/dQ^2|_{Q^2=0} = 0.53\text{GeV}^{-2}$ and $F_2^s(0) = \mu_s = -0.4$ is an anomalous strange magnetic moment.

The axial form factors are given by

$$G_A(Q^2) = \tfrac{1}{2}\left(\mp g_A + g_A^s\right)/\left(1 + Q^2/M_A^2\right)^2, \tag{21}$$

where $g_A = 1.262$. The axial mass $M_A = 1.032$ GeV is used in this work although there have been some disputes about this value recently. The $-$ $(+)$ signs are from the isospin dependence, and correspond to the knocked-out proton (neutron). All parameters exploited here are summarized with relevant experimental data in Ref. [29].

The induced pseudoscalar form factor is parametrized by the Goldberger–Treimann relation

$$G_P(Q^2) = \frac{2M_N}{Q^2 + m_\pi^2}G_A(Q^2), \tag{22}$$

where m_π is the pion mass. However, note that the contribution of the pseudoscalar form factor vanishes for the NC reaction, because of the negligible final lepton mass participating in this reaction.

To extract the structure functions for the NC reaction, the same method as the electron scattering is used as follows:

$$\frac{\sigma(+h) + \sigma(-h)}{2K} = v_L S_L + v_T S_T, \tag{23}$$

where σ denotes the differential cross section and $K = 4\pi^2 \frac{M_N M_{A-1}}{(2\pi)^3 M_A} p f_{\text{rec}}^{-1} \sigma_M^{Z,W^\pm}$ denotes the kinematics factor in front of integration in Eq. (9) with the recoil factor $f_{\text{rec}}^{-1} \sim 1$. The transverse structure function S_T becomes the slope in term of variable v_T and the intercept point is S_L by keeping q and ω fixed because of $v_L = 1$. However, for the CC reaction, these two structure functions cannot be extracted with this way because the longitudinal structure function comprises three terms. It is easy to obtain the transverse-interference structure function for the NC and the CC reactions as follows:

$$\frac{\sigma(+h) - \sigma(-h)}{2K} = v_T' S_T', \tag{24}$$

where S_T' is the coefficient in terms of the variable v_T'.

III. RESULTS

To extract the structure functions, we calculate the cross sections from the inclusive (e,e'), (ν,ν'), $(\bar{\nu},\bar{\nu}')$, (ν_e,e^-), and $(\bar{\nu}_e,e^+)$ reactions for three different scattering angles, 45.5°, 90°, and 140°, which were used at the Bates accelerator. The energies of the incident lepton are between 200 and 800 MeV and the target is ^{40}Ca. Some of the theoretical cross sections were already presented in our previous papers [18,19,25]. The structure functions are extracted at three-momentum transfers $q = 300, 400, 500$ MeV/c. We compare the structure functions extracted from the electron scattering to those from the neutrino scattering; that is, we compare them by the electromagnetic current and the electroweak current operators. Note that the Rosenbluth separation cannot be used in the CC reaction because of the longitudinal term in Eq. (17).

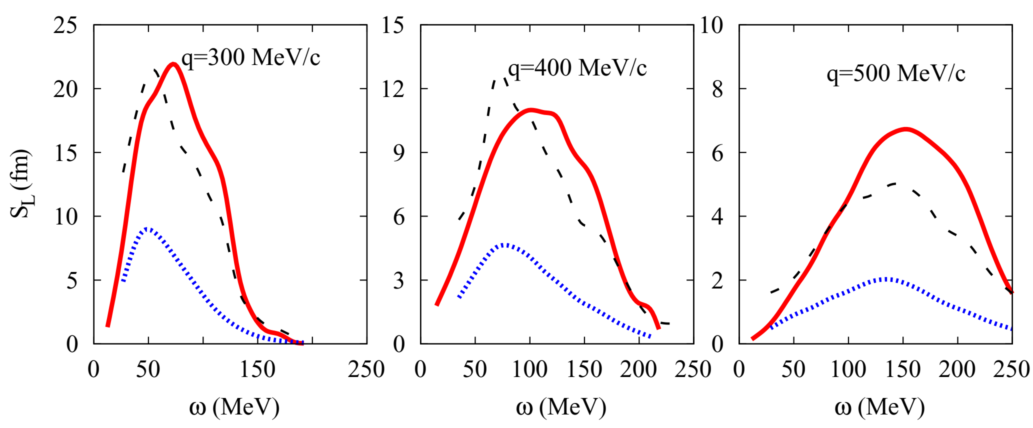

FIG. 1. The longitudinal structure functions in terms of the energy transfer by keeping the momentum transfer. The solid curves are the results for the electron scattering, the dashed lines are for the NC neutrino scattering, and the dotted lines are without an axial form factor of the NC neutrino scattering.

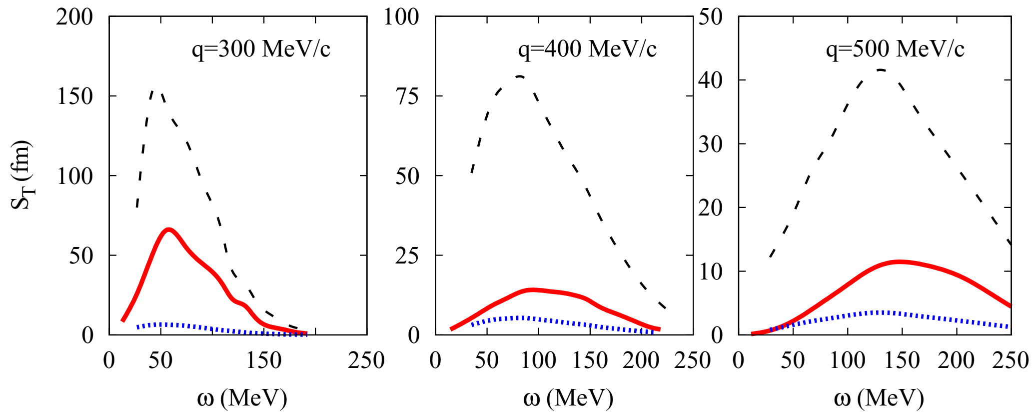

FIG. 2. The transverse structure functions in terms of the energy transfer by keeping the momentum transfer. The solid curves are the results for the electron scattering, the dashed lines are for the NC neutrino scattering, and the dotted lines are without an axial form factor of the NC neutrino scattering.

In Fig. 1, we show the longitudinal structure functions by using Eqs. (8) and (23). The solid (red) curves are the longitudinal structure function from the (e,e') reaction and the dashed lines (black) are from the NC neutrino scattering. The magnitudes are comparable with each other because the longitudinal term in the neutrino scattering contains only the electric form factor G_E from Eq. (12) in Ref. [14]. The peak positions of the solid lines shift to the right about 30 MeV. The shift of the peak is due to the effect of the Coulomb distortion of the electrons from the target nucleus. This is similar to our previous results [25,28].

While the role of F_1 and F_2 is well known, the contribution of the axial form factor G_A is relatively uncertain. To study the role of the axial form factor, we calculate the longitudinal structure function by removing the axial term in Eq. (19). The dotted lines (blue) are the results from turning off the axial term of the NC reaction, as shown in Fig. 1. Namely, they result from the isoscalar vector current in the electromagnetic current including the strangeness contribution. The position of the peak is not changed but the magnitude is reduced about 2.5 times. Therefore, the main contribution to the longitudinal structure function in the neutrino NC scattering comes from the axial current. But the amount of the contribution becomes smaller with the larger energy transfer and higher three-momentum transfer.

Figure 2 shows the transverse functions, and the explanation of the curves is the same as in Fig. 1. The positions of the peaks shift to higher energy transfer by about 10 MeV, and the difference of magnitude increases with higher momentum transfer. The magnitude of the transverse structure functions is much larger than that of the longitudinal structure functions in the neutrino scattering while the magnitudes of both structure functions in the electron scattering are comparable. The reason for the above phenomenology is the following: For the case of the electron scattering, the magnitude in Eq. (2) is similar to that in Eq. (3). For the NC neutrino scattering, the longitudinal term in Eq. (14) consists of N^0 minus $\frac{\omega}{q}N^z$ because the gauge invariance was not constrained to the neutrino scattering. This additional contribution reduces the longitudinal magnitude of the neutrino scattering to that for the electron scattering. But the transverse term is the same as that of the electron scattering, which reveals the large contribution from the axial part. Therefore the magnitude of the transverse structure function is larger than that of the longitudinal term in the NC neutrino scattering.

The dotted lines (blue) are the results without the axial vector term of the NC reaction. Surprisingly, the contributions

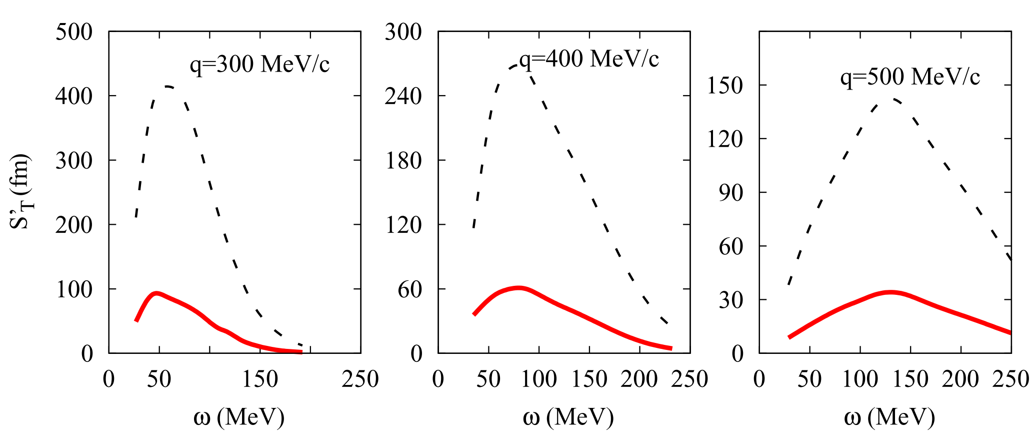

FIG. 3. The transverse-interference structure functions in terms of the energy transfer by keeping the momentum transfer. The solid curves are the results for the NC neutrino scattering and the dashed lines are for the CC neutrino scattering.

of the F_1 and F_2 terms are negligible on the transverse structure function. This comes from the fact that the isoscalar vector current in the neutrino CC scattering should be small. From the full result, the axial vector term turns out to be most important in the NC neutrino scattering.

The transverse-interference structure functions for the NC and CC reactions are extracted from Eq. (24) as shown in Fig. 3. The solid curves are the results for the NC neutrino scattering and the dashed lines are for the CC neutrino scattering. The function extracted from the CC reaction is very much larger than that from the NC reaction because of the contribution of the induced pseudoscalar term. Among the three structure functions, the contribution of the transverse function is biggest in the NC reaction. If one switches off the axial term in the electroweak current, the transverse-interference term is exactly zero because the term is proportional to the axial form factor according to Eq. (11) in Ref. [14].

IV. SUMMARY AND CONCLUSION

In this work, we extract the structure functions of the inclusive (e,e'), (ν,ν'), $(\bar{\nu},\bar{\nu}')$, (ν_e,e^-), and $(\bar{\nu}_e,e^+)$ reactions from the theoretical cross sections calculated within a single-particle model generated by the $\sigma - \omega$ model. Although the structure functions can be obtained theoretically, it shows a possible method to extract the functions experimentally. To extract the functions, we use the Rosenbluth separation method by keeping the momentum transfer and the scattering angle fixed. The magnitude of the transverse structure function from the neutrino scattering is much larger than that of the corresponding functions from the electron scattering because of the axial current, while the magnitudes of the longitudinal functions are comparable with each other. This behavior originates from the gauge invariance imposed on the electron scattering, which was not exploited in the neutrino scattering and reduced the large longitudinal structure function by the axial current to that for the electron scattering.

The magnitude of the transverse-interference structure functions from the CC reaction is much larger than that from the NC reaction because of inclusion of the pseudoscalar term. The peak shifts to higher energy transfer because of the Coulomb distortion of the electrons. Furthermore, we learn that the contribution of the transverse structure function is the largest, owing to the axial part in the weak current. According to our previous works [17,18], the difference of the magnitude for the cross sections between the electron and the neutrino scattering is about the order of 7, but for the structure functions it is less than the order of 1.

In conclusion, we show the difference between the electromagnetic current operator and the electroweak current operator. We learn that the axial and pseudoscalar terms play an important role in the neutrino scattering. To investigate precisely the structure functions due to the electroweak current, more experimental data are needed.

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