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Multinucleon short-range correlation model for nuclear spectral functions: Theoretical framework

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We develop a theoretical approach for nuclear spectral functions at high missing momenta and removal energies based on the multinucleon short-range correlation (SRC) model. The approach is based on the effective Feynman diagrammatic method which allows us to account for the relativistic effects important in the SRC domain. In addition to two-nucleon (2N) SRC with center of mass motion we also derive the contribution of three-nucleon SRCs to the nuclear spectral functions. The latter is modeled based on the assumption that 3N SRCs are a product of two sequential short-range nucleon-nucleon (NN) interactions. This approach allows us to express the 3N SRC part of the nuclear spectral function as a convolution of two NN SRCs. Thus the knowledge of 2N SRCs allows us to model both two- and three-nucleon SRC contributions to the spectral function. The derivations of the spectral functions are based on two theoretical frameworks for evaluating covariant Feynman diagrams: In the first, referred to as virtual nucleon approximation, we reduce Feynman diagrams to the time ordered noncovariant diagrams by evaluating nucleon spectators in the SRC at their positive energy poles, neglecting explicitly the contribution from vacuum diagrams. In the second approach, referred to as light-front approximation, we formulate the boost invariant nuclear spectral function in the light-front reference frame in which case the vacuum diagrams are generally suppressed and the bound nucleon is described by its light-front variables such as momentum fraction, transverse momentum, and invariant mass.

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I. INTRODUCTION

The knowledge of the nuclear spectral functions at high momenta of bound nucleon becomes increasingly important for further studies of nuclear QCD such as medium modification effects (EMC effects) or evolution equation of partons in nuclear medium measured at very large Q^2.

The importance of the high momentum properties of bound nucleon for nuclear EMC effects follows from the recent observations of apparent correlation between medium modification of partonic distributions and the strength of the two-nucleon short-range correlations (SRCs) in nuclei [1,2]. Concerning the QCD evolution of nuclear partonic distributions (PDFs), one expects that at very large Q^2 the knowledge of the high momentum component of the nuclear spectral function becomes important due to the contribution of quarks with momentum fractions larger than the ones provided by an isolated nucleon (i.e., partons with $x > 1$) [3,4]. The same is true for the reliable interpretation of neutrino-nuclei scatterings in which case both medium modification of PDFs as well as realistic treatment of SRCs are essential [5,6]. All these require a reasonably good understanding of the nuclear spectral functions at high momenta and removal energies of bound nucleon. With the advent of the Large Hadron Collider and expected construction of electron-ion colliders as well as several ongoing neutrino-nuclei experiments the knowledge of such spectral functions will be an important part of the theoretical interpretation of the data involving nuclear targets.

Despite impressive recent progress in *ab initio* calculations of nuclear structure (see, e.g., Ref. [7]) their relevance to the development of the spectral functions at large momenta and removal energies is rather limited. Not only the absence of relativistic effects but also the impossibility of identifying the relevant nucleon-nucleon (NN) interaction potentials makes such a program unrealistic. One way for progress is to develop theoretical models based on the short-range NN correlation approach in the description of the high momentum part of the nuclear wave function (see, e.g., [8–17]). In such an approach one will be able to take into account the empirical knowledge of SRCs acquired from different high energy scattering experiments thus reducing in some degree the theoretical uncertainty related to the description of the high momentum nucleon in the nucleus.

Our current work is such an attempt, which is based on the several phenomenological observations obtained in recent years in studies of the properties of two-nucleon (2N) SRCs [18–27]. We first develop the model describing the nuclear spectral function at large momenta and missing energies dominated by 2N SRCs with their center of mass motion generated by the mean field of the $A-2$ residual nuclear system. We then develop a theoretical framework for calculating the contribution of three-nucleon SRCs to the nuclear spectral function based on the model in which such correlations are generated by two sequential short-range NN interactions. As a result in our approach the phenomenological knowledge of the properties of NN SRCs is sufficient to calculate both the 2N and 3N SRC contribution to the nuclear spectral function. We expect the considered approach to be valid for nucleon momenta $p \geqslant k_{\rm src}$, where $k_{\rm src}$—momentum characteristic to NN SRC—is sufficiently large that NN short-range interaction can be factorized from residual mean field interaction. As a result our approach has limited validity in the transitional region of $k_F \leqslant p < k_{\rm src}$ (k_F is the Fermi momentum) where the role of the long-range correlations are more relevant.

In Sec. II we give a brief summary of recent advances in studies of the structure of NN SRCs which provides us with the phenomenology for developing the 2N and 3N SRC models of nuclear spectral functions.

Since the domain of multinucleon SRCs is characterized by relativistic momenta of the probed nucleon, special care should be given to the treatment of relativistic effects. To identify the relativistic effects, in Sec. III, we first formulate the nuclear spectral function as a quantity which is extracted in the semiexclusive high energy process whose scattering amplitude can be described through the covariant effective Feynman diagrams. The covariance here is important since it allows us to consistently trace the relativistic effects related to the propagation of the bound nucleon. We then identify the part of the covariant diagram which reproduces the nuclear spectral function. Doing so, we adopt two approaches for modeling the nuclear spectral function: virtual nucleon and light-front, general features described in Sec. III. Section IV outlines the calculation of nuclear spectral functions based on the effective Feynman diagrammatic method, identifying diagrams corresponding to the mean field, 2N SRC with center of mass motion and 3N SRC contributions.

In Secs. V and VI we present the detailed derivation of the spectral functions within virtual nucleon and light-front approximations. In Sec. VII we discuss briefly the set of parameters which will be used for numerical estimates of the spectral functions to be presented in Ref. [28]. Section VIII summarizes the results. This work represents the theoretical foundation and derivation of spectral functions; the follow-up paper [28] will present numerical estimates and parametrizations that can be used in practical calculations of different nuclear processes.

II. PHENOMENOLOGY OF TWO NUCLEON SHORT-RANGE CORRELATIONS IN NUCLEI

Recent experimental studies of high energy eA and pA processes [18–21,23,26,29,30] resulted in a significant progress in understanding the dynamics of 2N SRCs in nuclei. The series of electron-nucleus inclusive scattering experiments [18,19,23] confirmed the prediction [31,32] of the scaling for the ratios of inclusive cross section of a nucleus to the deuteron cross section in the kinematic region dominated by the scattering from the bound nucleons with momenta $p > k_F \sim 250$ MeV/c. Within the 2N SRC model, these ratios allowed us to extract the parameter $a_2(A,Z)$ which characterizes the probability of finding 2N SRC in the nucleus relative to the deuteron.

High energy semi-inclusive experiments [20,21] allowed us for the first time to probe the isospin composition of the 2N SRCs, observing strong (by factor of 20) dominance of the pn SRCs in nuclei, as compared to the pp and nn correlations, for internal momentum range of $\sim$250–650 MeV/c. This observation is understood [20,33,34] based on the dominance of the tensor forces in the NN interaction at this momentum range corresponding to the average nucleon separations of $\sim$1.1 fm. The tensor interaction projects the NN SRC part of the wave function to the isosinglet–relative angular momentum, $L = 2$, state, almost identical to the high momentum part of the D-wave component of the deuteron wave function. As a result pp and nn components of the NN SRC are strongly suppressed since they are dominated by the central NN potential with relative angular momentum $L = 0$.

Based on the observation of the strong dominance of pn SRCs in Refs. [35,36] it was predicted that single proton or neutron momentum distributions in the 2N SRC domain are inverse proportional to their relative fractions in nuclei. This prediction is in agreement with the results of variational Monte Carlo calculation of momentum distributions of light nuclei [37] as well as for medium to heavy nuclei based on the SRC model calculations of Ref. [15]. The recent finding of the pn dominance in heavy nuclei (up to ^{208}Pb) [26] validates the universality of the above prediction for the whole spectrum of atomic nuclei. The inverse proportionality of the high momentum component to the relative fraction of the proton or neutron is important for asymmetric nuclei and they need to be included in the modeling of nuclear spectral functions in the 2N SRC region.

The pn dominance in the SRC region and its relation to the high momentum part of the deuteron wave function makes the studies of the deuteron structure at large internal momenta a very important part for the SRC studies in nuclei. In this respect the recent experiments [38,39] and planned new measurements [40] of high energy exclusive electrodisintegration of the deuteron opens up new possibilities in the extraction of the deuteron momentum distribution at very large momenta. The measured distributions can then be utilized in the calculation of the nuclear spectral functions in the multinucleon SRC region.

Finally, another progress relevant to the SRC studies was the extraction of the center of mass momentum distribution of 2N SRCs from the data on triple coincidence scattering in $A(p,ppn)X$ [41] and $A(e,e',pn)X$ [42,43] reactions. The Gaussian form and the width of the extracted distributions were in a good agreement with the predictions made in Ref. [10], which were based on the estimate of the mean kinetic energy of the NN pair in the shell-model description of the nuclei. Similar results have been also obtained within the correlated wave function method of Ref. [44].

As it will be elaborated in the text, all above discussed results will provide us with the necessary empirical input for modeling nuclear spectral function in the SRC region.

III. FORMULATION OF NUCLEAR SPECTRAL FUNCTION

Our approach in the definition of nuclear spectral functions is based on identifying a nuclear "observable" which can be extracted from the cross section of the large momentum ($\gg M_N$, the nucleon mass) transfer semi-inclusive $h + A \to h' + N + (A-1)^*$ reaction in which the N can be unambiguously identified as a struck nucleon carrying almost all the energy and momentum transferred to the nucleus by the probe h. The reaction is specifically chosen to be semi-inclusive that allows us, in the approximation in which no final state interactions are considered, to relate the missing momentum and energy of the reaction to the properties of bound nucleon in the nucleus. With above conditions satisfied the extracted observable, referred to as a nuclear spectral function, represents a joint probability of finding bound nucleon in the nucleus with given missing momentum and energy.

In Refs. [45–47] we developed an effective Feynman diagrammatic approach for calculation of the $h + A \to h' + N +$

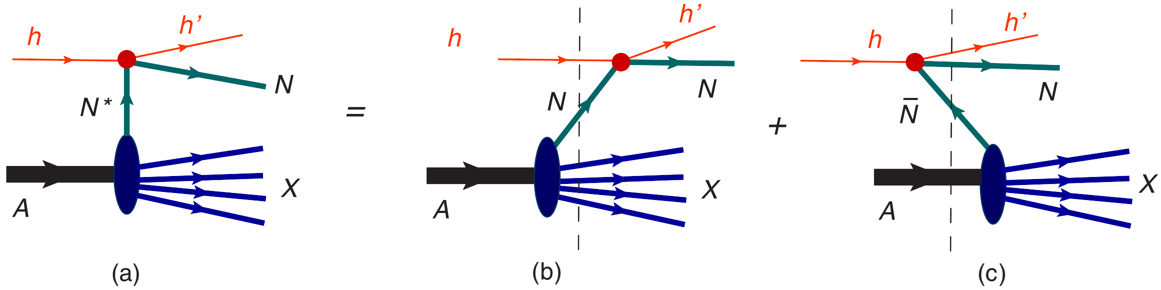

FIG. 1. Representation of the covariant Feynman amplitude through the sum of the time ordered amplitudes. Panel (b) corresponds to the scenario in which first, the bound nucleon is resolved in the nucleus which interacts with the incoming probe h. In panel (c), initially, the incoming probe produces $\bar{N}N$ pair with $\bar{N}$ being subsequently absorbed in the nucleus.

$(A-1)^*$ reactions. In this approach the covariant Feynman scattering amplitude is expressed through the effective nuclear vertices, vertices related to the scattering of the probe h with the bound nucleon, as well as vertices related to the final-state NN interactions. The nuclear vertices with the propagator of bound nucleon cannot be associated *a priori* with the single nucleon wave function of the nucleus, since they contain negative energy components which are related to the vacuum fluctuations rather than the probability amplitude of finding nucleon with given momentum in the nucleus. This problem is illustrated in the diagrammatic representation of the reaction shown in Fig. 1, in which the covariant diagram (a) is a sum of two noncovariant time ordered scattering diagrams (b) and (c). Here, for the calculation of the Lorentz invariant amplitude of Fig. 1(a) one can use the Feynman diagrammatic rules given in Ref. [47]. However the nuclear spectral function can only be formulated for the diagram of Fig. 1(b), where the time ordering is such that it first exposes the nucleus as being composed of a bound nucleon and residual nucleus, followed by an interaction of the incoming probe h off the bound nucleon. The other time ordering [Fig. 1(c)] presents a very different scenario of the scattering in which the probe produces a $\bar{N}N$ pair with subsequent absorption of the $\bar{N}$ in the nucleus. The latter is usually referred to as a Z graph and is not related to the nuclear spectral function. It is worth noting that the Z-graph contribution is a purely relativistic effect and does not appear in the nonrelativistic formulation of the nuclear spectral function. Its contribution however increases with an increase of the momentum of the bound nucleon (see, e.g., Ref. [8]).

The above discussion indicates that while defining the nuclear spectral function is straightforward in the nonrelativistic domain (no Z-graph contribution), its definition becomes increasingly ambiguous with an increase of bound nucleon momentum. This ambiguity is reflected in the lack of uniqueness in defining the nuclear spectral function in the domain where one expects to probe SRCs.

In the present work we consider two approaches in defining the nuclear spectral function from the covariant scattering amplitude. In the first approach we neglect the Z-graph contribution considering only the positive energy pole for the nucleon propagators in the nucleus. The energy and momentum conservation in this case requires the interacting nucleon to be virtual which renders certain ambiguity in treating the propagator of the bound nucleon. The approach we follow is to recover the energy and momentum of the interacting nucleon from kinematic parameters of on-shell spectators (see Ref. [48] for general discussion of the spectator model of relativistic bound states). The advantage of this approximation is that the spectral function is expressed through the nuclear wave function defined in the rest frame of the nucleus which in principle can be calculated using the conventional NN potentials. One shortcoming of the approximation is that while it satisfies baryonic number conservation, the momentum sum rule is not satisfied reflecting the virtual nature of the probed nucleon in the nucleus. We will refer to this approach as virtual-nucleon (VN) approximation [49,50].

In the second approach the nuclear spectral function is defined on the light front which corresponds to a reference frame in which the nucleus has infinite momentum. In this approach, referred to as light-front (LF) approximation, the Z-graph contribution is kinematically suppressed[1] and as a result the invariant sum of the two light-cone time ordered amplitudes in Fig. 1 is equal to the contribution from the graph of Fig. 1(b) only (see, e.g., Refs. [8,51–53]). This situation allows us to define the boost invariant LF spectral function in which the probed nucleon is the constituent of the nucleus with given light-front momentum fraction, transverse momentum, and invariant mass. It is worth noting that the LF approximation satisfies both baryonic and momentum sum rules, thus providing a better framework for studies of the effects associated with nuclear medium modification of interacting particles. Our approach is field-theoretical in which Feynman diagrams are constructed with effective interaction vertices and the spectral functions are extracted from the imaginary part of the covariant forward scattering nuclear amplitude. Another approach, in LF approximation, is the construction of the nuclear spectral function based on the relativistic Hamiltonian dynamics representing the interaction of fixed number on-mass shell constituents [54].

IV. DIAGRAMMATIC METHOD OF CALCULATION OF NUCLEAR SPECTRAL FUNCTION

In both VN and LF approximations we can use the diagrammatic approach of Ref. [47] to calculate the spectral

[1]This statement is exact for a specific component of the interacting nucleon spinor which in electromagnetic interaction constitutes the "good" component of the nuclear electromagnetic current.

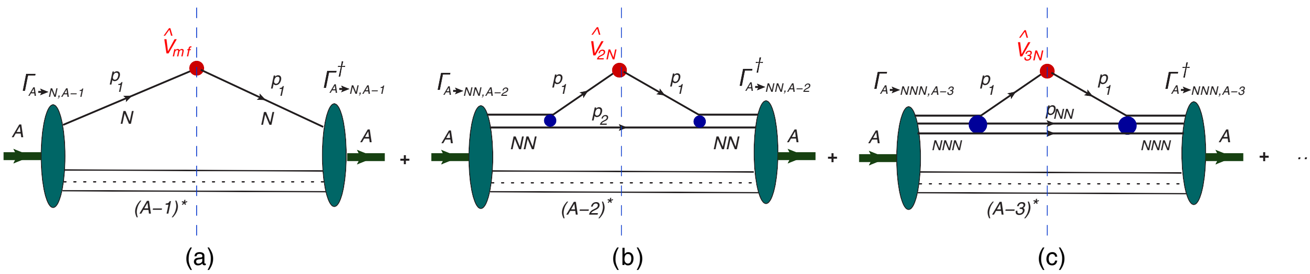

FIG. 2. Expansion of the nuclear spectral function into the contributions of mean field (a), 2N (b), and 3N (c) SRCs. For each case the initial nuclear transition vertices are different, corresponding to transition of $A \to N, A-1$; $A \to NN, A-1$; and $A \to NNN, A-3$ for the mean field, 2N, and 3N SRCs respectively. The NN (b) and NNN (c) labels identify 2N and 3N SRCs with effective NN and NNN vertices elaborated in the text.

functions. For this we identify the effective interaction vertices $\hat{V}$ such that the imaginary part of the covariant forward scattering nuclear amplitude will reduce to the nuclear spectral function either in VN or LF approximations. The specific form of these vertices can be established by considering amplitude of Fig. 1(b), taking into account the kinematics of mean field, 2N and 3N SRC scattering within VN and LF approximations, with subsequent factorization of the scattering factors related to the external probe h. As a result the $\hat{V}$ vertices will be different for mean field, 2N and 3N SRCs. They will also depend on the VN or LF approximations used to calculate the scattering amplitude.

In applying the diagrammatic approach one can express the forward nuclear scattering amplitude as a sum of the mean field and multinucleon SRC contributions as presented in Fig. 2, with (a), (b), and (c) corresponding to the contributions from mean field, 2N, and 3N short-range correlations.

Since the mean field contribution is dominated by the momenta of interacting nucleon below the characteristic Fermi momentum k_F, one can approximate the corresponding spectral function to the result following from nonrelativistic calculation. In this case both VN and LF approximations are expected to give very close results.

For 2N SRCs, the momenta of probed nucleon is $k_F < p \leqslant 600$–700 MeV/c and the nonrelativistic approximation is increasingly invalid. Currently there is a rather robust phenomenology on 2N SRCs in nuclei [18–21,23,24,26], which should be taken into account in the calculation of the 2N SRC contribution to the nuclear spectral function.

Finally, Fig. 2(c), corresponds to 3N SRCs. Currently, there are a few rather contradictory experimental evidences on 3N SRCs [19,23,55] and the first high-energy dedicated studies are expected in the near future [56]. In the present work we develop a model for 3N SRC which is based on the assumption that 3N SRCs are a result of the sequential short-range NN interactions. The final result represents a convolution of two 2N SRCs. In this way just the knowledge of 2N SRCs will be sufficient to account for both two- and three-nucleon short-range correlation contributions to the nuclear spectral function.

In the calculation we apply the effective Feynman rules [47] to the covariant forward scattering amplitudes corresponding to mean field, two-, and three-nucleon SRC contributions (Fig. 2) separately. Then, within VN or LF approximation, we estimate the loop integrals through the on-mass shell conditions of intermediate states. Doing so we absorb nuclear to nucleon transition vertices into the definition of nuclear wave functions. Such a definition is based on the identification of the interaction diagrams for the bound sates with the corresponding equations for the bound state wave function. For example in the nonrelativistic limit, the interaction diagrams for the bound state, calculated based on effective Feynman diagrammatic rules, is identified with the Lippmann-Schwinger equation [57,58] in the nonrelativistic limit. In the relativistic case, similar identifications are made with Bethe-Salpeter type [48] (for VN approximation) or Weinberg type [51] (for LF approximation) equations for the relativistic bound state wave function.

In the following calculations we express the covariant forward scattering amplitude, A, in the following form:

$$A = A^{MF} + A^{2N} + A^{3N} + \cdots, \tag{1}$$

where A^{MF}, A^{2N}, and A^{3N} correspond to the contributions from the diagrams of Figs. 2(a)–2(c) respectively, and then consider each contribution separately.

A. Mean field contribution

In the mean field approximation the probed nucleon interacts with the nuclear field induced by the $A-1$ residual system. In such approximation the spectral function corresponds to the nuclear configuration in which the residual nuclear system is identified as a coherent $A-1$ state with excitation energy in the order of *tens* of MeV.

Applying the effective Feynman rules to the diagram of Fig. 2(a) corresponding to the mean field contribution of nuclear spectral function one obtains

$$\begin{aligned} \mathrm{Im}A^{MF} = -\mathrm{Im}\int \chi_A^{s_A,\dagger}\Gamma^{\dagger}_{A\to N,A-1}\frac{\not{p}_1+M_N}{p_1^2-M_N^2}\hat{V}^{MF}\frac{\not{p}_1+M_N}{p_1^2-M_N^2} \\ \times\left[\frac{G_{A-1}(p_{A-1},\alpha)}{p_{A-1}^2-M_{A-1}^2+i\varepsilon}\right]^{on}\Gamma_{A\to N,A-1}\chi_A^{s_A}\frac{d^4p_{A-1}}{i(2\pi)^4}, \end{aligned} \tag{2}$$

where M_N and M_{A-1} are the masses of nucleon and residual $A-1$ nuclear system, χ_A is the nuclear spin wave function, $\Gamma_{A\to N,A-1}$ represents the covariant vertex of $A \to N + (A-1)$ transition, G_{A-1} describes the propagation of the $A-1$

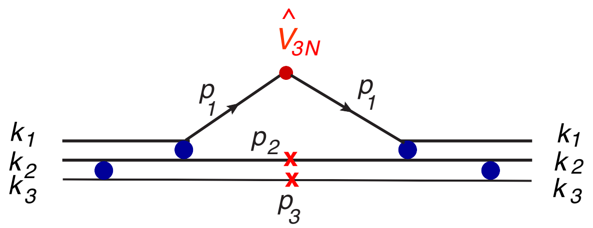

FIG. 3. Diagram corresponding to 3N SRC contribution to the spectral function.

residual nucleus in the intermediate state having an excitation α. The label $[\ldots]^{on}$ indicates that one estimates the cut diagram in which the residual nuclear system is on mass shell.

B. Two-nucleon src contribution

In the two-nucleon SRC model one assumes that the intermediate nuclear state consists of two correlated fast ($>k_F$) nucleons and a slow ($<k_F$) coherent $A-2$ nuclear system. The corresponding Feynman diagram is presented in Fig. 2(b), for which using the same diagrammatic rules [47] one obtains

$$\mathrm{Im}A^{2N} = \mathrm{Im}\int \chi_A^{s_A,\dagger}\Gamma^{\dagger}_{A\to NN,A-2}\frac{G(p_{NN})}{p_{NN}^2-M_{NN}^2}\Gamma^{\dagger}_{NN\to NN}\frac{\not p_1+M_N}{p_1^2-M_N^2}\hat{V}^{2N}\frac{\not p_1+M_N}{p_1^2-M_N^2}\left[\frac{\not p_2+M_N}{p_2^2-M_N^2+i\varepsilon}\right]^{on}$$
$$\times\Gamma_{NN\to NN}\frac{G(p_{NN})}{p_{NN}^2-M_{NN}^2}\left[\frac{G_{A-2}(p_{A-2})}{p_{A-2}^2-M_{A-2}^2+i\varepsilon}\right]^{on}\Gamma_{A\to NN,A-2}\chi_A^{s_A}\frac{d^4p_2}{i(2\pi)^4}\frac{d^4p_{A-2}}{i(2\pi)^4}, \quad (3)$$

where M_{NN} is the mass of the 2N SRC system, $\Gamma_{A\to NN,A-2}$ now describes the transition of the nucleus A to the NN SRC and coherent $A-2$ residual state, while the $\Gamma_{NN\to NN}$ vertex describes the short-range NN interaction that generates two-nucleon correlation in the spectral function.

C. Three-nucleon SRC contribution in collinear approximation

The spectral function due to 3N short-range correlations is described in Fig. 2(c) in which the intermediate state consists of three fast ($>k_F$) nucleons and a slow ($<k_F$) coherent $A-3$ residual system. The dynamics of 3N SRCs allow more complex interactions than that of 2N SRCs. One of the complexities is the irreducible three-nucleon forces that cannot be described by NN interaction only. Such interactions may contain inelastic transitions such as $NN\to N\Delta$. As our early studies demonstrate [33,59] irreducible three-nucleon forces predominantly contribute at very large magnitudes of missing energy characteristic to the Δ excitations $\sim$300 MeV$/c$. Thus for spectral functions for which the missing energy does not exceed the Δ resonance threshold $\sim M_\Delta - M_N$, one can consider the contributions of $NN\to NN$ interactions only. In the present work we will follow the two sequential NN short-range interaction scenario of the generation of 3N SRCs. In this approximation, the 3N SRC contribution to the spectral function can be represented through the diagram of Fig. 3. Here we factored out the low momentum residual $A-3$ system from consideration. This is justified by the fact that much larger momenta are involved in the 3N SRCs as compared to the one in the 2N SRCs discussed in the previous section. As a result the effects due to center of mass motion of the $A-3$ system are neglected. The present approximation assumes that the initial three collinear nucleons undergo two short-range NN interactions generating one nucleon with much larger momenta than the other two. The collinear approximation is commonly used in the calculation of the quark structure function of the nucleon in the valence quark region. In this respect, our calculations for the LF approximation are analytically similar to the QCD calculation of the nucleon structure function. Here one assumes that the momentum fractions of 3N SRCs carried by each initial nucleon is unity and their transverse momenta are neglected. Within VN approximation the collinear approximation assumes that the initial momenta of the three nucleons are much smaller than k_{src}—momenta characteristic to NN SRC—and therefore can be neglected. Note that the momenta of all three nucleons in the intermediate state of the scattering in Fig. 3 should exceed the nuclear Fermi momentum k_F to satisfy the three-particle–three-hole condition in the Fermi distribution of nucleons in the nucleus.

Using the effective Feynman diagrammatic rules for the diagram of Fig. 3 one obtains

$$\mathrm{Im}A^{3N} = \mathrm{Im}\int \bar u(k_1,\lambda_1)\bar u(k_2,\lambda_2)\bar u(k_3,\lambda_3)\Gamma^{\dagger}_{NN\to NN}\frac{\not p_{2'}+M_N}{p_{2'}^2-M_N^2}\Gamma^{\dagger}_{NN\to NN}\frac{\not p_1+M_N}{p_1^2-M_N^2}\hat V^{3N}\frac{\not p_1+M_N}{p_1^2-M_N^2}$$
$$\times\left[\frac{\not p_2+M_N}{p_2^2-M_N^2+i\varepsilon}\right]^{on}\Gamma_{NN\to NN}\frac{\not p_{2'}+M_N}{p_{2'}^2-M_N^2}\left[\frac{\not p_3+M_N}{p_3^2-M_N^2+i\varepsilon}\right]^{on}\Gamma_{NN\to NN}u(k_1,\lambda_1)u(k_2,\lambda_2)u(k_3,\lambda_3)\frac{d^4p_3}{i(2\pi)^4}\frac{d^4p_2}{i(2\pi)^4}, \quad (4)$$

where "2′" labels the intermediate state of the nucleon 2 after the first short-range NN interaction, λ_i is the spin of the ith nucleon, and the $\Gamma_{NN\to NN}$ is the same short-range NN interaction vertex included in Eq. (3). Note that there are several other 3N SRC diagrams which differ from that of Fig. 3 by the ordering of the two sequential NN short-range interactions. In collinear approximation these diagrams result in the same analytic form both in VN and LF approximations (see, e.g., Ref. [3]), thus their contribution can be absorbed in the definition of the parameter n_{3N}^N [see Eq. (36)], which defines the contribution of the norm of the 3N SRCs to the total normalization of the nuclear wave function.

D. Models of calculation

To calculate the spectral functions from the forward scattering amplitudes in Eqs. (2)–(4) one needs to define the effective vertices $\hat{V}$ which identify the bound nucleon in the mean field, 2N, and 3N SRCs, as well as to define the poles at which the cut propagators of the intermediate states are estimated. Both depend on the approximation used to reduce the covariant diagrams to the time ordered diagrams which allow an introduction of the nuclear spectral function. In the following section we will derive these spectral functions from the covariant forward scattering amplitudes A^{MF}, A^{2N}, and A^{3N} within VN and LF approximations.

V. SPECTRAL FUNCTION IN VIRTUAL NUCLEON APPROXIMATION

Our first approach is VN approximation which describes the nucleus in the laboratory frame treating an interacting bound nucleon as a virtual particle while spectators are put on their mass shells. In VN approximation the spectral function, $S_A^N(p,E_m)$ defines the joint probability of finding a nucleon in the nucleus with momentum p and removal energy E_m defined as

$$E_m = E_{A-1} + M_N - M_A - \frac{p^2}{2M_{A-1}}, \tag{5}$$

where E_{A-1} and M_{A-1} are the energy and the mass of the residual $A-1$ nuclear system. Note that in the above expression we followed the conventional definition of E_m, in which the nonrelativistic expression for the kinetic energy of the $A-1$ system is subtracted. However in our calculation the kinetic energy of the $(A-1)$ system depends on the mean field, 2N SRC or 3N SRC picture of nuclear wave function. For each particular case, the kinetic energy of the $(A-1)$ system is accordingly defined in the text.

The normalization condition for the spectral function can be fixed from the condition of the conservation of baryonic number of the nucleus in hadron-nucleus scattering [61] or from the condition for the charge form factor of nucleus at vanishing momentum transfer, $F_A(0) = Z$ [49], which yields

$$\sum_{N=1}^{A} \int S_A^N(p,E_m)\alpha d^3p dE_m = A, \tag{6}$$

where α is the ratio of the flux factors of the (external probe)–(bound nucleon) and (external probe)–(nucleus) systems, which in high momentum limit of the probe (hadron or virtual photon) yields

$$\alpha = \frac{E_N + p_z}{M_A/A} = A\frac{p_+}{p_{A+}}. \tag{7}$$

Here, p_+ and p_{A+} are the light front longitudinal momenta of the nucleon and nucleus respectively, E_N is the energy of the bound nucleon, and the z direction is defined opposite to the direction of the incoming probe.

Following the decomposition of Fig. 2 we consider the mean field, 2N, and 3N SRC contributions to the nuclear spectral function separately. In VN approximation the cut diagrams of Figs. 2 and 3 will be evaluated at positive energy poles of the spectator residual system. For the mean field contribution it corresponds to the positive energy pole of the coherent $A-1$ system. For the case of 2N SRC it corresponds to the positive energy poles of the correlated nucleon and $A-2$ system, whereas for the 3N SRC case these are positive energy poles of the two correlated nucleons and $A-3$ system.

A. Mean field contribution

In the mean field approximation the missing momentum $\mathbf{p_m} = -\mathbf{p} \equiv -\mathbf{p_1}$ and missing energy E_m characterizes the total momentum and excitation energy of the residual $A-1$ system. In the nuclear shell model E_m also defines the energy needed to remove the nucleon from the particular nuclear shell. For such a situation we can define the effective vertex $\hat{V}_{MF}$ in Eq. (2) as

$$\hat{V}^{MF} = i\bar{a}(p_1,s_1)\delta^3(p_1 + p_{A-1})\delta(E_m - E_\alpha)a(p_1,s_1), \tag{8}$$

where E_α is the characteristic energy of the given nuclear shell. The creation and annihilation operators are defined in such a way that

$$a(p_1,s_1)(\not{p}_1 + M_N) = \bar{u}(p_1,s_1) \quad \text{and}$$
$$(\not{p}_1 + M_N)\bar{a}(p_1,s_1) = u(p_1,s_1). \tag{9}$$

Hereafter the $a(p_1,s_1)$ and $\bar{a}\ (p_1,s_1)$ represent the annihilation and creation operators in the Dirac space. We follow the convention (see, e.g., Ref. [60]) for which the product of the annihilation operator, the nucleon propagator projected to the positive energy state, and the nuclear transition vertex produces the Fock component of the nuclear wave function. Note that this definition is different from the conventional definition (see, e.g., Ref. [16]) in which the annihilation operator acts on the nuclear wave function to produce nucleon-hole states. However the final results in both approaches are similar in the nonrelativistic limit.

Next, in Eq. (2) we take the integral by d^0p_{A-1} through the positive energy pole of the propagator of the $A-1$ state:

$$\frac{dp_{A-1}^0}{p_{A-1}^2 - M_{A-1}^2 + i\varepsilon} = -\frac{2\pi i}{2E_{A-1}}\bigg|_{E_{A-1}=\sqrt{M_{A-1}^2+p_{A-1}^2}}, \tag{10}$$

and for the on-shell $(A-1)$ spectator state we use the sum rule

$$G_{A-1}(p_{A-1},\alpha) = \sum_{s_{A-1}} \chi_{A-1}(p_{A-1},s_{A-1},E_\alpha)\chi_{A-1}^\dagger(p_{A-1},s_{A-1},E_\alpha), \tag{11}$$

where χ_{A-1} is the spin wave function of the residual $(A-1)$ nucleus. Note that in relativistic treatment the spin wave functions are momentum dependent as indicated in the argument of χ_{A-1}. Such a momentum dependence is also accounted for the spin wave function of other particles discussed in the text. The above relations allow us to introduce the single nucleon wave function for the given nuclear shell E_α, $\psi_{N/A}$ in the form

$$\psi^{s_A}_{N/A}(p_1,s_1,p_{A-1},s_{A-1},E_\alpha) = \frac{\bar{u}(p_1,s_1)\chi^\dagger_{A-1}(p_{A-1},s_{A-1},E_\alpha)\Gamma_{A\to N,A-1}\chi^{s_A}_A}{(M_N^2-p_1^2)\sqrt{(2\pi)^3 2E_{A-1}}}, \tag{12}$$

which, inserting into Eq. (2) and taking the d^3p_{A-1} integration through the $\delta^3(p_1+p_{A-1})$, results in the mean field nuclear spectral function $S^N_{A,MF}$ in the VN approximation:

$$S^N_{A,MF}(p_1,E_m) = \sum_\alpha \sum_{s_1,s_{A-1}} \left|\psi^{s_A}_{N/A}(p_1,s_1,p_{A-1},s_{A-1},E_\alpha)\right|^2 \times \delta(E_m - E_\alpha), \tag{13}$$

which defines the joint probability of finding a nucleon in the mean field of the nucleus with momentum p and removal energy E_m.

For numerical estimates of the above spectral function we note that in the mean field approximation, the substantial strength of the VN wave function $\psi_{N/A}$ comes from the momentum range of $p_1 \leqslant k_F$. Therefore in this case the nonrelativistic approximation is valid, which allows us to approximate this wave function by the nonrelativistic wave function obtained from the conventional mean field calculations of single nucleon wave functions. Additionally, in the nonrelativistic limit $\alpha \approx 1 + \frac{p_{1,z}}{M_A/A}$, and from Eq. (6) one observes that the $\frac{p_{1,z}}{M_A/A}$ part does not contribute to the integral resulting in the condition for nonrelativistic normalization:

$$\int S^N_{A,MF}(p,E_m)dE_m d^3p = n^N_{MF}, \tag{14}$$

where n^N_{MF} is the norm of the mean field contribution of nucleon N to the total normalization of the nuclear spectral function.

B. Two-nucleon short-range correlations

We consider now Eq. (3) with the effective vertex $\hat{V}_{2N}$ identified as

$$\hat{V}_{2N} = i\bar{a}(p_1,s_1)\delta^3(p_1+p_2+p_{A-2})\delta(E_m - E^{2N}_m)a(p_1,s_1), \tag{15}$$

where $\bar{a}(p_1,s_1)$ and $a(p_1,s_1)$ are creation and annihilation operators of nucleon with four-momentum p_1 and spin s_1 satisfying the relations of Eq. (9).

The magnitude of E^{2N}_m follows from the NN correlation model (in which the correlated NN pair has a total momentum $-\mathbf{p_{A-2}}$ in the mean field of $A-2$ residual nuclear system) according to which

$$E^{2N}_m = E^{(2)}_{\rm thr} + T_{A-2} + T_2 - T_{A-1} = E^{(2)}_{\rm thr} + E_{A-2} - M_{A-2} + T_2 - \frac{p_1^2}{2M_{A-1}}, \tag{16}$$

where $E^{(2)}_{\rm thr}$ is the threshold energy needed to remove two nucleons from the nucleus. In numerical evaluations, we estimate it as $E^{(2)}_{\rm thr} \approx 2M_N + M_{A-2} - M_A$. Furthermore, T_2 and T_{A-2} are kinetic energies of the correlated nucleon 2 and the residual $(A-2)$ nucleus respectively, that add up to the actual kinetic energy of the $A-1$ residual system in the 2N SRC model. The additional subtraction of the $\frac{p_1^2}{2M_{A-1}}$ term follows from the definition of missing energy in Eq. (5).

According to VN prescription we perform integrations by dp^0_2 and dp^0_{A-2} in Eq. (3) through the positive energy poles of the propagators of 2 and $A-2$ particles. This yields

$$\frac{dp^0_2}{p_2^2 - M_N^2 + i\varepsilon} = -\frac{2\pi i}{2E_2}\bigg|_{E_2=\sqrt{M_N^2+p_2^2}}, \quad \frac{dp^0_{A-2}}{p^2_{A-2} - M^2_{A-2} + i\varepsilon} = -\frac{2\pi i}{2E_{A-2}}\bigg|_{E_{A-2}=\sqrt{M^2_{A-2}+p^2_{A-2}}}. \tag{17}$$

Since 2 and $A-2$ are now on mass shell, we can write the sum rule relations for the numerator of their propagators as

$$\not{p}_2 + M_N = \sum_{s2} u(p_2,s_2)\bar{u}(p_2,s_2), \quad G(p_{A-2}) = \sum_{s_{A-2}} \chi_{A-2}(p_{A-2},s_{A-2})\chi^\dagger_{A-2}(p_{A-2},s_{A-2}), \tag{18}$$

where χ_{A-2} is the spin wave function of the $A-2$ nucleus. The s_2 and s_{A-2} are the spin projections of the nucleon 2 and $A-2$ nucleus respectively.

In the 2N SRC model we assume that the center of mass momentum of the NN SRC is small which justifies the use of the on-mass-shell sum rule condition:

$$G(p_{NN}) = \sum_{s_{NN}} \chi_{NN}(p_{NN},s_{NN})\chi^\dagger_{NN}(p_{NN},s_{NN}), \tag{19}$$

where χ_{NN} is the spin wave function and s_{NN} is the projection of the total spin of the NN correlation with the three-momentum, $\mathbf{p_{NN}} = \mathbf{p_A} - \mathbf{p_{A-2}}$. As one will see below, the above equation allows us to introduce the wave function of the center of mass motion of the 2N SRC thus decoupling the center of mass and relative motions of the NN correlation.

Inserting Eqs. (9) and (17)–(19) in Eq. (3) reduces the latter to the NN SRC part of the nuclear spectral function, for which one obtains

$$S^N_{A,2N}(p_1,E_m) = \sum_{s_2,s_{A-2},s_{NN},s'_{NN}} \int \chi^{s_A,\dagger}_A \Gamma^\dagger_{A\to NN,A-2} \frac{\chi_{NN}(p_{NN},s_{NN})\chi^\dagger_{NN}(p_{NN},s_{NN})}{p^2_{NN} - M^2_{NN}} \Gamma^\dagger_{NN\to NN} \times \frac{u(p_1,s_1)}{p_1^2 - M_N^2}\delta^3(p_1+p_2+p_{A-2})\delta(E_m - E^{2N}_m)\frac{\bar{u}(p_1,s_1)}{p_1^2 - M_N^2}$$

$$
\times \frac{u(p_2,s_2)\bar{u}(p_2,s_2)}{2E_2}\Gamma_{NN\to NN}\frac{\chi_{NN}(p_{NN},s_{NN})\chi^{\dagger}_{NN}(p_{NN},s_{NN})}{p^2_{NN}-M^2_{NN}}
$$

$$
\times \frac{\chi_{A-2}(p_{A-2},s_{A-2})\chi^{\dagger}_{A-2}(p_{A-2},s_{A-2})}{2E_{A-2}}\Gamma_{A\to NN,A-2}\chi^{s_A}_{A}\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_{A-2}}{(2\pi)^3}, \tag{20}
$$

which defines the joint probability of finding a nucleon in NN SRC with momentum p_1 and removal energy E_m.

Now we introduce $A \to (NN) + (A-2)$ and $(NN) \to N + N$ transition wave functions defined in the rest frame of the nucleus A and the 2N SRC respectively (see, e.g., Ref. [47]):

$$
\psi^{s_A}_{CM}(p_{NN},s_{NN},p_{A-2},s_{A-2}) = \frac{\chi^{\dagger}_{NN}(p_{NN},s_{NN})\chi^{\dagger}_{A-2}(p_{A-2},s_{A-2})\Gamma_{A\to NN,A-2}\chi^{s_A}_{A}}{\left(M^2_{NN}-p^2_{NN}\right)\sqrt{2E_{A-2}(2\pi)^3}},
$$

$$
\psi^{s_{NN}}_{NN}(p_1,s_1,p_2,s_2) = \frac{\bar{u}(p_1,s_1)\bar{u}(p_2,s_2)\Gamma_{NN\to NN}\chi_{NN}(p_{NN},s_{NN})}{\left(M^2_{N}-p^2_{1}\right)\sqrt{2E_2(2\pi)^3}}, \tag{21}
$$

which allows us to present the 2N SRC part of the nuclear spectral function in the following form:

$$
S^N_{A,2N}(p_1,E_m) = \sum_{s_1,s_2,s_{A-2},s_{NN},s'_{NN}} \int \psi^{s_A,\dagger}_{CM}(p_{NN},s'_{NN},p_{A-2},s_{A-2})\psi^{s'_{NN},\dagger}_{NN}(p_1,s_1,p_2,s_2)
$$

$$
\times \psi^{s_A}_{CM}(p_{NN},s_{NN},p_{A-2},s_{A-2})\psi^{s_{NN}}_{NN}(p_1,s_1,p_2,s_2)\delta^3(p_1+p_2+p_{A-2})\delta\left(E_m-E^{2N}_m\right)d^3p_2d^3p_{A-2}. \tag{22}
$$

We use $\mathbf{p_{NN}} = -\mathbf{p_{A-2}}$ and integrate by d^3p_2 through $\delta^3(p_1+p_2+p_{A-2})$. Furthermore based on the 2N SRC model in which the wave function of the relative motion is dominated by the pn component with spin 1 with the low momentum CM wave function being in the S state, one can perform the summation by s_{A-2} resulting in $\delta_{s_{NN},s'_{NN}}$. Using the latter relation one obtains

$$
S^N_{A,2N}(p_1,E_m) = \sum_{s_1,s_2,s_{A-2},s_{NN}} \int \left|\psi^{s_A}_{CM}(p_{NN},s_{NN},s_{A-2})\right|^2\left|\psi^{s_{NN}}_{NN}(p_1,s_1,p_2,s_2)\right|^2\delta\left(E_m-E^{2N}_m\right)d^3p_{NN}, \tag{23}
$$

where $\mathbf{p_2} = \mathbf{p_{NN}} - \mathbf{p_1}$. The above expression is simplified further by introducing effective momentum distribution of the nucleon in the NN SRC, n_{NN}, as well as distribution of the center of mass momentum of the NN correlation, n_{CM}, which results in

$$
S^N_{A,2N}(p_1,E_m) = \int n_{CM}(p_{NN})n_{NN}(p_{\rm rel})\delta\left(E_m-E^{2N}_m\right)d^3p_{NN}, \tag{24}
$$

where $\mathbf{p}_{\rm rel} = \frac{\mathbf{p_1}-\mathbf{p_2}}{2}$.

The normalization of this spectral function should be related to the total probability of finding a nucleon in such a correlation. This can be defined from the normalization condition of Eq. (6) which yields

$$
\int S^N_{A,2N}(p_1,E_m)\alpha_1 dE_m d^3p_1 = n^N_{2N}, \tag{25}
$$

where for the 2N SRC model $\alpha \equiv \alpha_1 = \frac{M_N-E_m-T_{A-1}+p_{1,z}}{M_A/A}$ and n^N_{2N} is the norm of the 2N SRC contribution of nucleon N in the total normalization of the nuclear wave function.

As it follows from Eq. (24), given the relative and center of mass momentum distributions of the NN correlations we can numerically calculate the 2N SRC part of the nuclear spectral function. Since it is assumed that the center of mass momenta of the NN SRCs are small, for n_{CM} we use the distribution obtained in Ref. [10] through the overlap of two Fermi momentum distributions which results in the simple Gaussian distribution:

$$
n_{CM}(p_{NN}) = N_0(A)e^{-\beta(A)p^2_{NN}} \tag{26}
$$

normalized to unity: $\int n_{CM}(p_{NN})d^3p_{NN} = 1$. The parameter $\beta(A)$ is estimated from the nuclear mean field distribution, while $N_0(A)$ is found from the normalization condition. The relative momentum distribution of the NN SRC, $n_{NN}(p_{\rm rel})$, can be modeled according to Refs. [35,36], where the high momentum strength of the nucleon momentum distribution is predicted to be inverse proportional to the relative fraction of the nucleon in the nucleus. Such a distribution is in agreement with the recently observed dominance of pn SRCs [20,21,26] and can be expressed in the form

$$
n^N_{NN}(p_{\rm rel}) = \frac{a_2(A,Z)}{(2x_N)^\gamma}\frac{n_d(p_{\rm rel})\Theta(p_{\rm rel}-k_{\rm src})}{\frac{M_N-E_m-T_{A-1}}{M_A/A}}, \tag{27}
$$

where $x_N = N/A$ with N being the number of protons (Z) and neutrons $(A-Z)$ in the nucleus A, the parameter $a_2(A,Z)$ is related to the probability of finding 2N SRC in the nucleus A relative to the deuteron, and γ is a free parameter $\gamma \lesssim 1$. The $n_d(p_{\rm rel})$ is the high momentum distribution in the deuteron, and $k_{\rm src} \gtrsim k_F$ is the momentum threshold at which a NN system with such relative momentum can be considered in the short-range correlation. The factor $\frac{M_N - E_m - T_{A-1}}{M_A/A}$ is the generalization of the normalization scheme of [61] which enforces the normalization condition of Eq. (6). The normalization of the above defined distribution, $\int n_{NN}^N(p)d^3p = n_{2N}^N$, defines the contribution of the 2N SRCs to the total norm of the momentum distribution for the nucleon N.

It is worth mentioning that in the nonrelativistic limit, and assuming an equal 2N SRC contributions from proton and neutron, $n_{NN}^N(p_{\rm rel}) = a_2(A)n_d(p_{\rm rel})$, the expression in Eq. (24) reduces to the "NN SRC-CM motion" model of Ciofi-Simula [10].

C. Three-nucleon short-range correlations

For the 3N SRC model in collinear approximation we consider the covariant amplitude of Eq. (4) in which the effective vertex $\hat{V}_{3N}$ is defined as

$$\hat{V}_{3N} = i\bar{a}(p_1,s_1)\delta^3(p_1+p_2+p_3)\delta\big(E_m - E_m^{3N}\big)a(p_1,s_1), \tag{28}$$

where $\bar{a}(p_1,s_1)$ and $a(p_1,s_1)$ are the creation and annihilation operators defined in Eq. (9).

The magnitude of E_m^{3N} is calculated based on the considered 3N SRC model in which the recoil nuclear system consists of two fast nucleons and a slow $A-3$ nuclear system whose excitation energy is neglected. In this case

$$E_m^{3N} = E_{\rm thr}^{(3)} + T_3 + T_2 - \frac{p_1^2}{2M_{A-1}}, \tag{29}$$

where $E_{\rm thr}^{(3)}$ is the threshold energy needed to remove three nucleons from the nucleus. Similar to the case of 2N SRC, we estimate it as $E_{\rm thr}^{(3)} \approx 3M_N + M_{A-3} - M_A$. Within the considered 3N SRC model the kinetic energy of residual nucleus is due to kinetic energies of correlated spectator nucleons T_2 and T_3 which are treated relativistically. Here, as in the case of 2N SRC, the additional subtraction of the $\frac{p_1^2}{2M_{A-1}}$ follows from the definition of missing energy in Eq. (5).

According to VN prescription we take integrations by dp_2^0 and dp_3^0 in Eq. (4), at the positive energy poles of propagators of 2 and 3 particles, i.e.,

$$\frac{d^0 p_{2,3}}{p_{2,3}^2 - M_N^2 + i\varepsilon} = -\frac{2\pi i}{2E_{2,3}}\bigg|_{E_{2,3}=\sqrt{M_N^2+p_{2,3}^2}}. \tag{30}$$

Using this and the relations of Eq. (9), as well as assuming the sum rule relations [similar to Eq. (18)] for the spinors of $2'$ intermediate state, from Eq. (4) one obtains for the 3N SRC contribution to the nuclear spectral function

$$\begin{aligned} S_{A,3N}^N(p_1,E_m) = &\sum_{s_{2'},\tilde{s}_{2'},s_2,s_3}\int \bar{u}(k_1,\lambda_1)\bar{u}(k_2,\lambda_2)\bar{u}(k_3,\lambda_3)\Gamma_{NN\to NN}^{\dagger}\frac{u(p_{2'},s_{2'})\bar{u}(p_{2'},s_{2'})}{p_{2'}^2 - M_N^2} \\ &\times \Gamma_{NN\to NN}^{\dagger}\frac{u(p_1,s_1)}{p_1^2-M_N^2}\delta^3(p_1+p_2+p_3)\delta\big(E_m - E_m^{3N}\big)\frac{\bar{u}(p_1,s_1)}{p_1^2-M_N^2}\frac{u(p_2,s_2)\bar{u}(p_2,s_2)}{2E_2} \\ &\times \Gamma_{NN\to NN}\frac{u(p_{2'},\tilde{s}_{2'})\bar{u}(p_{2'},\tilde{s}_{2'})}{p_{2'}^2-M_N^2}\frac{u(p_3,s_3)\bar{u}(p_3,s_3)}{2E_3}\Gamma_{NN\to NN}u(k_1,\lambda_1)u(k_2,\lambda_2)u(k_3,\lambda_3)\frac{d^3p_3}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}, \end{aligned} \tag{31}$$

which defines the joint probability of finding a nucleon in 3N SRC with momentum p_1 and removal energy E_m.

Introducing $2N$ SRC wave functions in analogy with Eq. (21),

$$\psi_{NN}(p_1,s_1,p_2,s_2;\,p_{1i},s_{1i},p_{2i},s_{2i}) = \frac{\bar{u}(p_1,s_1)\bar{u}(p_2,s_2)\Gamma_{NN\to NN}u(p_{1i},s_{1i})u(p_{2i},s_{2i})}{\big(M_N^2-p_1^2\big)\sqrt{2}\sqrt{2E_2(2\pi)^3}}, \tag{32}$$

where subscript "i" indicates incoming nucleons with their spin projections, Eq. (31) can be expressed as follows:

$$\begin{aligned} S_{A,3N}^N(p_1,E_m) = &\sum_{s_{2'},\tilde{s}_{2'},s_2,s_3}\int \psi_{NN}^{\dagger}(p_1,s_1,p_2,s_2;k_1,\lambda_1,p_{2'},s_{2'})\psi_{NN}^{\dagger}(p_{2'},s_{2'},p_3,s_3;k_2,\lambda_2,k_3,\lambda_3) \\ &\times \psi_{NN}(p_1,s_1;\,p_2,s_2;k_1,\lambda_1,p_{2'},s_{2'})\psi_{NN}(p_{2'},\tilde{s}_{2'};\,p_3,s_3;k_2,\lambda_2,k_3,\lambda_3) \\ &\times \delta^3(p_1+p_2+p_3)\delta\big(E_m-E_m^{3N}\big)d^3p_3d^3p_2. \end{aligned} \tag{33}$$

Based on the assumption that NN SRC is dominated by spin-1 short-range pn configuration, and using similar arguments as in Eq. (23), one can sum over the polarizations of 2 and 3 particles resulting in $s_2 = s_{2'} = \tilde{s}_{2'}$. Then, taking the d^3p_2 integration through the $\delta^3(p_1+p_2+p_3)$ function, one obtains

$$S_{A,3N}^N(p_1,E_m) = \sum_{s_1,s_2,s_3}\int |\psi_{NN}(p_{2'},s_2,p_3,s_3;k_2,\lambda_2,k_3,\lambda_3)|^2|\psi_{NN}(p_1,s_1;\,p_2,s_2;k_1,\lambda_1,p_{2'},s_2)|^2\delta\big(E_m-E_m^{3N}\big)d^3p_3. \tag{34}$$

The above expression can be represented in a more simple form by noticing that the NN correlation wave functions depend on their relative momenta and we sum over the final and average by all possible initial polarization configurations. This yields

$$S^N_{A,3N}(p_1,E_m) = \int n_{NN}(p_{2',3})n_{NN}(p_{12})\delta\big(E_m - E^{3N}_m\big)d^3p_3, \tag{35}$$

where $\mathbf{p_{12}} = \frac{\mathbf{p_1}-\mathbf{p_2}}{2} = \mathbf{p_1} + \frac{\mathbf{p_3}}{2}$ and $\mathbf{p_{2',3}} = \frac{\mathbf{p_{2'}}-\mathbf{p_3}}{2} \approx -\mathbf{p_3}$. The normalization condition for the 3N SRC spectral function is defined as follows:

$$\int S^N_{A,3N}(p_1,E_m)\alpha_1 d^3p_1 dE_m = n^N_{3N}, \tag{36}$$

where n^N_{3N} is the norm of the 3N SRC contributing to the total normalization of the nuclear wave function for the given nucleon N.

Within the model of pn dominance of two-nucleon SRCs one predicts that the 3N SRCs are generated predominantly through the two sequential short-range pn interactions. As a result our model of 3N SRCs predicts that the overall probability of finding such correlations is proportional to the factor $a_2^2(A,Z)$, where $a_2(A,Z)$ is defined in Eq. (27). Using the relations similar to Eq. (27), one approximates

$$n_{NN}(p_{2',3})n_{NN}(p_{12}) = a_2^2(A,Z)C(A,Z)\frac{n_d(p_{2',3})n_d(p_{12})}{\frac{M_N-E_m-T_{A-1}}{M_A/A}}\Theta(p_{2',3}-k_{\rm src})\Theta(p_{12}-k_{\rm src}), \tag{37}$$

where $k_{\rm src} > k_F$ is the relative momentum threshold at which the NN system can be considered as a short-range correlation. Here $C(A,Z)$ is a function which accounts for the effects associated with the isospin structure of two-nucleon recoil system. Namely, in the collinear approximation two recoil nucleons emerge with small relative momenta (or invariant mass). In Ref. [33] it was demonstrated that the NN system with small relative momenta is strongly dominated in the isosinglet pn channel. This situation introduces an additional restriction on the isospin composition of the 3N SRCs, in which the recoil NN system predominately consists of a pn pair. For example, one direct consequence of such dynamics is that high momentum neutrons in ^{3}He nucleus cannot be generated in 3N SRCs while protons can.

VI. SPECTRAL FUNCTION IN LIGHT-FRONT APPROXIMATION

The nuclear spectral function on the light front was formulated in Ref. [31] however its calculation from the first principles is impossible due to the lack of the knowledge of nuclear light-front wave functions. The current work uses two assumptions which allow us to obtain calculable LF nuclear spectral functions. The first assumption is that the nuclear mean field contribution to the light-front spectral function corresponds to the nonrelativistic limit of the momentum and missing energy of a bound nucleon. As a result the mean field part of the LF spectral function can be related to the mean field contribution of conventional nuclear spectral function discussed in the previous section. The second assumption is that the dynamics of the LF spectral function in the high momentum domain is defined mainly by the pn interaction at short distances. Thus to obtain the calculable spectral function in relativistic domain one will need only a LF model for the deuteron wave function at short distances.

Before we proceed with the above approach we first define the kinematic parameters that characterize the bound nucleon in the light front as well as the sum rules that the light-front nuclear spectral function should satisfy.

In defining the light-front nuclear spectral function the primary requirement is that it is a Lorentz boost invariant function in the direction of the large CM momentum of the nucleus p_A. To satisfy this condition we require that the bound nucleon N is described by a light-front "+" momentum fraction $\alpha_N = A\frac{p_{N+}}{p_{A+}}$, transverse (to $\mathbf{p_A}$) momentum $\mathbf{p_{N\perp}}$ and invariant mass $\tilde{M}_N^2 = p_{N-}p_{N+} - p_{N,\perp}^2$. As it will be shown below $\tilde{M}_N$ is related to the excitation energy of the residual nucleus. For future derivations it is useful to present the invariant phase space of bound nucleon, d^4p_N, through these light-front variables as follows:

$$d^4p_N = \frac{1}{2}dp_{N-},dp_{N+}d^2p_{N,\perp} = \frac{d\alpha_N}{2\alpha_N}d^2p_{N,\perp}d\tilde{M}_N^2. \tag{38}$$

After identifying the kinematic variables describing the bound nucleon, one now defines the light-front nuclear spectral function, $P_A(\alpha_N,p_{N,\perp},\tilde{M}_N^2)$, as a joint probability of finding a bound nucleon in the nucleus with light-front momentum fraction α_N, transverse momentum $\mathbf{p_{N,\perp}}$, and invariant mass $\tilde{M}_N^2$. The normalization condition for such spectral functions is defined from the requirements of baryonic number and total light-front momentum conservations [31,61]:

$$\sum_{N=1}^{A}\int P^N_A\big(\alpha_N,p_{N,\perp},\tilde{M}_N^2\big)\frac{d\alpha_N}{2\alpha_N}d^2p_{N,\perp}d\tilde{M}_N^2 = A,$$
$$\sum_{N=1}^{A}\int \alpha_N P^N_A\big(\alpha_N,p_{N,\perp},\tilde{M}_N^2\big)\frac{d\alpha_N}{2\alpha_N}d^2p_{N,\perp}d\tilde{M}_N^2 = A, \tag{39}$$

where the second relation is exact if one assumes that all the momentum in the nucleus is carried by the constituent nucleons. From Eq. (39) one deduces the relation between the LF spectral function and the light-front density matrix, $\rho^N_A(\alpha_N,p_{N,\perp})$, in the form

$$\rho^N_A(\alpha_N,p_{N,\perp}) = \int P^N_A\big(\alpha_N,p_{N,\perp},\tilde{M}_N^2\big)\frac{1}{2}d\tilde{M}_N^2. \tag{40}$$

To proceed with the derivations, similar to the VN approximation, we follow the decomposition of Fig. 2 considering mean field, 2N, and 3N SRC contributions separately. In LF approximation the cut diagrams of Figs. 2 and 3 will be evaluated at the positive light-front ("−" component) energy poles of the spectator residual system. For the mean field contribution the spectator residual system consists of $A-1$ nucleus, while for the SRC case it consists of one or two on-energy-shell nucleons correlated with the bound nucleon, as well as $A-2$ and $A-3$ uncorrelated nuclear systems for 2N and 3N correlations respectively.

A. Mean field approximation

To calculate the light-front nuclear spectral function in mean field approximation one needs in principle to start with Eq. (2) and proceed by evaluating the integral at the pole of the "minus" component of four-momentum of the $A-1$ residual nucleus. Such an integration will express the spectral function through the unknown light-front mean field wave function of the nucleus.

We adopt a different approach in which one uses the fact that the mean field nuclear spectral function is dominating at small momenta and missing energies of bound nucleon, for which the nonrelativistic limit of light-front approximation is well justified. Then we need only to relate the mean field light-front spectral function $P^N_{A,MF}(\alpha_1,p_{1,\perp},\tilde{M}^2_N)$ to the above discussed VN mean field spectral function, $S^N_{A,MF}(E_m,p)$ in the nonrelativistic limit.[2] The relation between $P^N_{A,MF}$ and $S^N_{A,MF}(E_m,p)$ can be found by using the normalization condition:

$$\int P^N_{A,MF}\big(\alpha_1,p_{1,\perp},\tilde{M}^2_N\big)\frac{d\alpha_1}{2\alpha_1}d^2p_{1,\perp}d\tilde{M}^2_N = \int S^N_{A,MF}(E_m,p)dE_m d^3p_1 = n^N_{MF}, \tag{41}$$

where we need to relate the LF phase space to $dE_m d^3p$. For this, we use the relation between the total energy of the $A-1$ nucleus and missing energy E_m in the mean field approximation:

$$E_{A-1} = \sqrt{M^2_{A-1}+p^2_{A-1}} = M_A - M_N + E_m + \frac{p_1^2}{2M^0_{A-1}}, \tag{42}$$

where the last part of the equation follows from the definition of the missing energy E_m [Eq. (5)] which is inherently nonrelativistic. In the above expression M_{A-1} is the mass of the $A-1$ residual nucleus which can be in the excited state, while M^0_{A-1} represents the ground state mass of the residual nucleus. With the above equation one obtains for α_1

$$\alpha_1 = A - \frac{E_{A-1}-p_{1,z}}{M_A/A} \tag{43}$$

and for $\tilde{M}^2_N$

$$\tilde{M}^2_N = \frac{\alpha_1}{A}\left(M^2_A - \frac{M^2_{A-1}+p^2_{1,\perp}}{(A-\alpha_1)/A}\right) - p^2_{1,\perp}. \tag{44}$$

These relations allow us to relate

$$d\tilde{M}^2_N d\alpha_1 = 2\alpha_1 dE_m dp_{1,z}. \tag{45}$$

This, together with Eq. (38), results in $d^4p_1 = \frac{d\alpha_1}{2\alpha_1}d^2p_{1,\perp}d\tilde{M}^2_N = dE_m d^3p_1$, which substituting in Eq. (41) allows us to obtain for the mean field approximation

$$P^N_{A,MF}\big(\alpha_1,p_{1,\perp},\tilde{M}^2_N\big) = S^N_{A,MF}(E_m,p_1), \tag{46}$$

where α_1 and $\tilde{M}^2_N$ are expressed through E_m and $\mathbf{p_1}$ according to Eqs. (42)–(44). Note that the above equation is valid for up to the overall normalization factor, since VN and LF approximations result in different normalizations for the mean field contribution to the spectral function. (For more details see Sec. VII.)

B. Two-nucleon short-range correlations

To calculate 2N SRC contribution to the light-front spectral function $P^N_{A,2N}(\alpha_1,p_{1,\perp},\tilde{M}^2_N)$ we start with Eq. (3), with the vertex operator defined as follows (see also [3]):

$$\hat{V}_{2N} = i\bar{a}(p_1,s_1)2\alpha_1^2\delta(\alpha_1+\alpha_2+\alpha_{A-2}-A) \times \delta^2(p_{1,\perp}+p_{2,\perp}+p_{A-2,\perp})\delta\big(\tilde{M}^2_N-\tilde{M}^{(2N),2}_N\big)a(p_1,s_1), \tag{47}$$

where $(\alpha_2,p_{2,\perp})$, $(\alpha_{A-2},p_{A-2,\perp})$ are light-front momentum fractions and transverse momenta of correlated second nucleon and residual $(A-2)$ nucleus. In the considered 2N SRC model,

$$\begin{aligned}\tilde{M}^{(2N),2}_N &= p_{1+}(p_{A-}-p_{2-}-p_{A-2,-}) - p^2_{1,\perp}\\ &= \frac{\alpha_1}{A}\left(M^2_A - A\frac{M^2_N+(p_{A-2,\perp}-p_{1\perp})^2}{A-\alpha_1-\alpha_{A-2}} - A\frac{M^2_{A-2}+p^2_{A-2,\perp}}{\alpha_{A-2}}\right) - p^2_{1\perp},\end{aligned} \tag{48}$$

where we consider the reference frame with the z axis in $\vec{p}_A$ direction. To proceed, in Eq. (3) we treat the nucleon 2 and residual nucleus '$A-2$ on light-front energy shells. This is achieved by integrating their respective − components through the positive pole value of the propagators, provided that their + components are large and positive. For the 2 nucleon the integration is performed as follows:

$$\begin{aligned}\frac{d^4p_2}{p_2^2-M^2_N+i\epsilon} &= \frac{\frac{1}{2}dp_{2-}dp_{2+}d^2p_{2,\perp}}{p_{2+}p_{2-}-p^2_{2,\perp}+M^2_N+i\epsilon}\\ &= \frac{\frac{1}{2}dp_{2-}dp_{2+}d^2p_{2,\perp}}{p_{2+}\big(p_{2-}-\frac{p^2_{2,\perp}+M^2_N}{p_{2+}}+i\epsilon\big)}\\ &= -i\pi\frac{d\alpha_2}{\alpha_2}d^2p_{2,\perp}\bigg|_{p_{2-}=\frac{p^2_{2,\perp}+M^2_N}{p_{2+}}}.\end{aligned} \tag{49}$$

[2]Note that hereafter we will identify α_N and $p_{N,\perp}$ with α_1 and $p_{1,\perp}$ respectively, giving the subscript 1to the bound nucleon.

Similarly for the $A-2$ residual nucleus,

$$\frac{d^4 p_{A-2}}{p_{A-2}^2 - M_{A-2}^2 + i\epsilon} = -i\pi \frac{d\alpha_{A-2}}{\alpha_{A-2}} d^2 p_{A-2,\perp}\bigg|_{p_{A-2,-}=\frac{p_{A-2,\perp}^2+M_{A-2}^2}{p_{A-2,+}}}. \tag{50}$$

Note that the above integrations project the intermediate state to the positive light-front energy state thus excluding the contribution from the Z graph of Fig. 1(c). The Z diagram in this scheme will be suppressed by the inverse power of large + component of the nucleon's four-momentum (see, e.g., [51]). With the diminished contribution from the Z graph Eq. (3) will result in the light-front spectral function $P_{A,2N}^N(\alpha_1, p_{1,\perp}, \tilde{M}_N^2)$ of the 2N SRC.

The on-shell conditions for the nucleon 2 and residual nucleus $A-2$ allows us to use the sum rule relations

$$\not{p}_2 + M_N = \sum_{s2} u(p_2,s_2)\bar{u}(p_2,s_2) \text{ and } G(p_{A-2}) = \sum_{s_{A-2}} \chi_{A-2}(p_{A-2},s_{A-2})\chi_{A-2}^\dagger(p_{A-2},s_{A-2}). \tag{51}$$

Using also the nonrelativistic limit for the center of mass motion of 2N SRC, $k_{CM} \ll M_{NN}$ [for k_{CM} see Eq. (65) below], we approximate

$$G(p_{NN},s_{NN}) \approx \sum_{s_{NN}} \chi_{NN}(p_{NN},s_{NN})\chi_{NN}^\dagger(p_{NN},s_{NN}), \tag{52}$$

where χ_{NN} is the spin wave function of the center of mass motion of the 2N SRC. Using also the relations $a(p_1,s_1)(\not{p}_1 + M_N) = \bar{u}(p_1,s_1)$ and $(\not{p}_1 + M_N)\bar{a}(p_1,s_1) = u(p_1,s_1)$ for the light-front spectral function, from Eq. (3) one obtains

$$\begin{aligned}
P_{A,2N}^N(\alpha_1, p_{1,\perp}, \tilde{M}_N^2) = &\sum_{s_2,s_{NN},s_{A-2}} \int \chi_A^{s_A,\dagger}\Gamma_{A\to NN,A-2}^\dagger \chi_{A-2}(p_{A-2},s_{A-2}) \\
&\times \frac{\chi_{NN}(p_{NN},s_{NN})\chi_{NN}^\dagger(p_{NN},s_{NN})}{p_{NN}^2 - M_{NN}^2}\Gamma_{NN\to NN}^\dagger \frac{u(p_1,s_1)u(p_2,s_2)}{p_1^2 - M_N^2} \\
&\times \left[2\alpha_1^2\delta(\alpha_1+\alpha_2+\alpha_{A-2}-A)\delta^2(p_{1,\perp}+p_{2,\perp}+p_{A-2,\perp})\delta\left(\tilde{M}_N^2 - \tilde{M}_N^{(2N),2}\right)\right]\frac{\bar{u}(p_1,s_1)\bar{u}(p_2,s_2)}{p_1^2 - M_N^2} \\
&\times \Gamma_{NN\to NN}\frac{\chi_{NN}(p_{NN},s_{NN})\chi_{NN}^\dagger(p_{NN},s_{NN})}{p_{NN}^2 - M_{NN}^2}\chi_{A-2}^\dagger(p_{A-2},s_{A-2})\Gamma_{A\to NN,A-2}\chi_A^{s_A} \\
&\times \frac{d\alpha_2}{\alpha_2}\frac{d^2 p_{2,\perp}}{2(2\pi)^3}\frac{d\alpha_{A-2}}{\alpha_{A-2}}\frac{d^2 p_{A-2,\perp}}{2(2\pi)^3}.
\end{aligned} \tag{53}$$

Light-front wave function of the NN SRC. We now focus on the following combination in Eq. (53):

$$\frac{\bar{u}(p_1,s_1)\bar{u}(p_2,s_2)}{p_1^2 - M_N^2}\Gamma_{NN\to NN}\cdot\chi_{NN}(p_{NN},s_{NN}), \tag{54}$$

which enters in Eq. (53) in a direct and complex-conjugated form. For the propagator in Eq. (54), using light-front momentum and energy conservation at the $\Gamma_{NN\to NN}$ vertex, one obtains

$$\begin{aligned}
p_1^2 - M_N^2 &= (p_{NN}-p_2)^2 - M_N^2 = (p_{NN,+}-p_{2,+})\left(p_{NN,-} - p_{2,-} - \frac{M_N^2 + p_{1\perp}^2}{p_{NN,+}-p_{2,+}}\right) \\
&= \alpha_1(p_{A+}/A)\left(\frac{M_{NN}^2 + p_{NN,\perp}^2}{\alpha_{NN}p_{A+}/A} - \frac{M_N^2 + p_{2,\perp}^2}{\alpha_2 p_{A+}/A} - \frac{M_N^2 + p_{1,\perp}^2}{\alpha_1 p_{A+}/A}\right),
\end{aligned} \tag{55}$$

where in the last part of the equation we used the on-shell conditions for the nucleon 2 ($p_{2-} = \frac{M_N^2 + p_{2,\perp}^2}{p_{2+}}$) and the condition $k_{CM} \ll M_{NN}$, which justifies the approximation, $p_{NN-} \approx \frac{M_{NN}^2 + p_{NN,\perp}^2}{p_{NN+}}$. Equation (55) can be further simplified using relations $\alpha_1 + \alpha_2 = \alpha_{NN}$ and $p_{1\perp} + p_{2\perp} = p_{NN,\perp}$ yielding

$$p_1^2 - M_N^2 = \frac{\alpha_1}{\alpha_{NN}}\left(M_{NN}^2 - \frac{\alpha_{NN}^2}{\alpha_1\alpha_2}\left[M_N^2 + \left(p_{1\perp}^2 - \frac{\alpha_1}{\alpha_{NN}}p_{NN,\perp}\right)^2\right]\right). \tag{56}$$

The above propagator can be completely expressed though the internal momenta of the NN system by introducing the momentum fraction of the 2N SRC carried by the nucleon 1, β_1, and the relative transverse momentum, $k_{1,\perp}$, as follows:

$$\beta_1 = 2 - \beta_2 = \frac{2\alpha_1}{\alpha_{NN}} \quad \text{and} \quad k_{1,\perp} = p_{1,\perp} - \frac{\beta_1}{2}p_{NN,\perp}. \tag{57}$$

With these definitions Eq. (54) can be written as

$$\frac{\bar{u}(p_1,s_1)\bar{u}(p_2,s_2)}{\beta_1\frac{1}{2}\left[M_{NN}^2-\frac{4}{\beta_1(2-\beta_1)}\left(M_N^2+k_{1,\perp}^2\right)\right]}\Gamma_{NN\to NN}\cdot\chi_{NN}(p_{NN},s_{NN}), \tag{58}$$

where one observes that the term in the denominator which is subtracted from M_{NN}^2 represents the invariant energy of the NN system, s_{NN}. This allows us to introduce the relative momentum k_1 in the NN system, which is invariant with respect to the Lorentz boost in the $\mathbf{p_{NN}}$ direction, in the form

$$s_{NN}=\frac{4}{\beta_1(2-\beta_1)}\left(M_N^2+k_{1,\perp}^2\right)=4\left(M_N^2+k_1^2\right). \tag{59}$$

The above defined relative momentum k_1 will be used to set a momentum scale for the 2N SRCs, requiring $k_1\geqslant k_{\text{src}}$ similar to Eq. (24).

The expression in Eq. (58) can be presented in a more compact form if one introduces the light-front wave function of the NN SRC [3,8] in the form

$$\psi_{NN}^{s_{NN}}(\beta_1,k_{1,\perp},s_1,s_2)=-\frac{1}{\sqrt{2(2\pi)^3}}\frac{\bar{u}(p_1,s_1)\bar{u}(p_2,s_2)\Gamma_{NN\to NN}\cdot\chi_{NN}(p_{NN},s_{NN})}{\frac{1}{2}\left[M_{NN}^2-4\left(M_N^2+k_1^2\right)\right]}, \tag{60}$$

where χ_{NN} represents the spin wave function of the NN pair emerging from the nuclear vertex $\Gamma_{A,NN,A-2}$. With this definition for Eq. (54) one obtains

$$\frac{\bar{u}(p_1,s_1)\bar{u}(p_2,s_2)}{p_1^2-M_N^2}\Gamma_{NN\to NN}\cdot\chi_{NN}(p_{NN},s_{NN})=-\frac{\sqrt{2(2\pi)^3}}{\beta_1}\psi_{NN}^{s_{NN}}(\beta_1,k_{1,\perp},s_1,s_2). \tag{61}$$

Light-front wave function of the NN $(A-2)$ system. Next we consider the combination that defines the wave function of the NN $(A-2)$ system that will describe the motion of the center of mass of the NN correlation:

$$\frac{\chi_{NN}^\dagger(p_{NN},s_{NN})\chi_{A-2}^\dagger(p_{A-2},s_{A-2})}{p_{NN}^2-M_{NN}^2}\Gamma_{A\to NN,A-2}\chi_A. \tag{62}$$

We first elaborate on the propagator of the NN system using the on-shell conditions for the initial A and residual $A-2$ nucleus:

$$p_{NN}^2-M_{NN}^2=p_{NN+}\left(p_{A-}-p_{A-2,-}-\frac{M_{NN}^2+p_{NN,\perp}^2}{p_{NN,+}}\right)=\frac{\alpha_{NN}}{A}\left(M_A^2-s_{NN,A-2}\right), \tag{63}$$

where in the last part of the equation we used the on-energy shell relations $p_{A-}=\frac{M_A^2+p_{A,\perp}^2}{p_{A,+}}$, $p_{A-2,-}=\frac{M_{A-2}^2+p_{A-2,\perp}^2}{p_{A-2,+}}$, the definition, $\alpha_{NN}=\frac{Ap_{NN,+}}{p_{A+}}$, as well as introduced the invariant energy of the $NN-(A-2)$ system as follows:

$$s_{NN,A-2}=A^2\frac{\left[M_{NN}^2+\frac{\alpha_{NN}}{A}\left(M_{A-2}^2-M_{NN}^2\right)+\left(p_{NN,\perp}-\frac{\alpha_{NN}}{A}p_{A,\perp}\right)^2\right]}{\alpha_{NN}(A-\alpha_{NN})}. \tag{64}$$

This invariant energy can be used to estimate the relative three-momentum in the $NN-(A-2)$ system:

$$k_{CM}=\frac{\sqrt{[s_{NN,A-2}-(M_{NN}+M_{A-2})^2)(s_{NN,A-2}-(M_{NN}-M_{A-2})^2]}}{2\sqrt{s_{NN,A-2}}}, \tag{65}$$

where, $\mathbf{k_{CM,\perp}}=\mathbf{p_{NN,\perp}}$. Note that $\mathbf{k_{CM}}$ can be used to calculate the light-front momentum fraction of the NN pair as follows:

$$\alpha_{NN}=\frac{A(E_{NN}+k_{CM,z})}{E_{NN}+E_{A-2}}, \tag{66}$$

where $E_{NN}=\sqrt{M_{NN}^2+k_{CM}^2}$ and $E_{A-2}=\sqrt{M_{A-2}^2+k_{CM}^2}$.

With the above definitions, similar to Eq. (60), one introduces the light-front wave function of the $NN-(A-2)$ system:

$$\psi_{CM}^{s_A}(\alpha_{NN},k_{NN,\perp},s_{NN},s_{A-2})=-\frac{1}{\sqrt{\frac{A-2}{2}}}\frac{1}{\sqrt{2(2\pi)^3}}\frac{\chi_{NN}^\dagger(p_{NN},s_{NN})\chi_{A-2}^\dagger(p_{A-2},s_{A-2})\Gamma_{A\to NN,A-2}\chi_A^{s_A}}{\frac{2}{A}\left[M_A^2-s_{NN,A-2}\right]}. \tag{67}$$

The coefficients in the above equation are chosen such that in the nonrelativistic limit, $k_{CM}\ll M_{NN}$,

$$\psi_{CM}^{s_A}(\alpha_{NN},k_{NN,\perp},s_{NN},s_{A-2})\approx\psi_{CM}^{NR}(k_{CM})2M_N. \tag{68}$$

Substituting Eqs. (67) and (63) in Eq. (62) one obtains

$$\frac{\chi^{\dagger}_{NN}(p_{NN},s_{NN})\chi^{\dagger}_{A-2}(p_{A-2},s_{A-2})}{p^2_{NN}-M^2_{NN}}\Gamma_{A\to NN,A-2}\chi_A = -\frac{\sqrt{\frac{A-2}{2}}\sqrt{2(2\pi)^3}}{\alpha_{NN}/2}\psi^{s_A}_{CM}(\alpha_{NN},k_{NN,\perp},s_{NN},s_{A-2}). \tag{69}$$

Using Eqs. (61) and (69) in Eq. (53), for the 2N SRC unpolarized light-front spectral function one obtains

$$P^N_{A,2N}(\alpha_1,p_{1,\perp},\tilde{M}^2_N) = \frac{A-2}{2}\sum_{s_2,s_{NN},s_{A-2}}\int \psi^{s_A,\dagger}_{CM}(\alpha_{NN},k_{NN,\perp},s_{NN},s_{A-2})$$
$$\times\, \psi^{s_{NN},\dagger}_{NN}(\beta_1,k_{1,\perp},s_1,s_2)\big[2\delta(\alpha_1+\alpha_2+\alpha_{A-2}-A)\delta^2(p_{1,\perp}+p_{2,\perp}+p_{A-2,\perp})\delta\big(\tilde{M}^2_N-\tilde{M}^{(2N),2}_N\big)\big]$$
$$\times\, \psi^{s_A}_{CM}(\alpha_{NN},k_{NN,\perp},s_{NN},s_{A-2})\psi^{s_{NN}}_{NN}(\beta_1,k_{1,\perp},s_1,s_2)\frac{d\beta_2}{\beta_2}d^2p_{2,\perp}\frac{d\alpha_{A-2}}{\alpha_{A-2}}d^2p_{A-2,\perp}. \tag{70}$$

To obtain the spin averaged spectral function, using similar arguments as in Eq. (23), the above equation can be diagonalized in terms of spin projections and simplified further by introducing spin averaged density matrices for the relative motion in the 2N SRC (Refs. [3,8]):

$$\rho^N_{NN}(\beta_1,k_{1,\perp}) = \frac{1}{2}\frac{1}{2s_{NN}+1}\sum_{s_{NN},s_1,s_2}\frac{\left|\psi^{s_{NN}}_{NN}(\beta_1,k_{1,\perp},s_1,s_2)\right|^2}{2-\beta_1}, \tag{71}$$

and for the center of mass motion of the 2N SRC:

$$\rho_{CM}(\alpha_{NN},k_{NN,\perp}) = \frac{1}{2}\frac{A-2}{2s_A+1}\sum_{s_{NN},s_{A-2}}\frac{\left|\psi^{s_A}_{CM}(\alpha_{NN},k_{NN,\perp},s_{NN},s_{A-2})\right|^2}{A-\alpha_{NN}}. \tag{72}$$

With the above definitions, for the 2N SRC unpolarized light-front spectral function one obtains

$$P^N_{A,2N}(\alpha_1,p_{1,\perp},\tilde{M}^2_N) = \frac{1}{2}\int \rho^N_{NN}(\beta_1,k_{1,\perp})\rho_{CM}(\alpha_{NN},k_{NN,\perp})2\delta(\alpha_1+\alpha_2-\alpha_{NN})$$
$$\times\, \delta^2(p_{1,\perp}+p_{2,\perp}-p_{NN,\perp})\delta\big(\tilde{M}^2_N-\tilde{M}^{(2N),2}_N\big)d\beta_2 d^2p_{2,\perp}d\alpha_{NN}d^2p_{NN,\perp}. \tag{73}$$

The normalization conditions for the above introduced density matrices are defined from the sum-rule conditions of Eqs. (39) and (40). For the density matrices of NN SRC, ρ^N_{NN}, the normalization conditions to satisfy the baryonic and momentum sum rules [8] yield

$$\int \rho^N_{NN}(\beta,k_\perp)\frac{d\beta}{\beta}d^2k_\perp = \int \rho^N_{NN}(\beta,k_\perp)\beta\frac{d\beta}{\beta}d^2k_\perp = n^N_{2N}, \tag{74}$$

where n^N_{2N} is the contribution of the 2N SRCs to the total norm of the spectral function. Similar to Eq. (27) we can model the light-front density matrix of the 2N SRC through the high momentum part of the light-front density matrix of the deuteron $\rho_d(\beta_1,k_{1,\perp})$ in the form

$$\rho^N_{NN}(\beta_1,k_{1,\perp}) = \frac{a_2(A,Z)}{(2x_N)^\gamma}\rho_d(\beta_1,k_{1,\perp})\Theta(k_1-k_{\rm src}), \tag{75}$$

where k_1 is defined in Eq. (59), and $a_2(A,Z)$ in Eq. (27). In the second part of the current work [28] we will discuss the specific models for $\rho_d(\beta_1,k_{1,\perp})$ which will allow us to perform numerical estimates.

For the light-front density function of the center of mass motion the conditions of Eq. (39) require the following normalization relations:

$$\int \rho_{CM}(\alpha_{NN},k_{NN,\perp})\frac{d\alpha_{NN}}{\alpha_{NN}}d^2k_{NN,\perp} = 1 \;\; \text{and} \;\; \int \rho_{CM}(\alpha_{NN},k_{NN,\perp})\alpha_{NN}\frac{d\alpha_{NN}}{\alpha_{NN}}d^2k_{NN,\perp} = 2. \tag{76}$$

Since in the considered 2N SRC model the CM motion is nonrelativistic ($k_{NN} \ll 2M_N$), we can use the momentum distribution used in VN approximation [Eq. (26)] which can be related to ρ_{CM} as follows:

$$\rho_{CM}(\alpha_{NN},k_{NN,\perp}) = \frac{E_{NN}E_{A-2}}{(E_{NN}+E_{A-2})/A}\frac{n_{CM}(k_{CM})}{A-\alpha_{NN}} \approx 2M_N n_{CM}(k_{CM}), \tag{77}$$

where n_{CM} is defined in Eq. (26). Note that for the "middle" form of the ρ_{CM} the first normalization condition of Eq. (76) is exact while the second one is approximate satisfying it only in the nonrelativistic limit (last part of the equation).

Finally, it is worth mentioning that in the nonrelativistic limit of the density matrix of 2N SRC, $\rho(\beta_1,k_{1,\perp}) \approx n_{NN}(p_1)M_N$ and Eq. (73), similar to VN approximation, reduces to the SRC model of spectral function of Ref. [10].

C. Three-nucleon short-range correlation model

To calculate the contribution of the 3N SRCs to the light-front spectral function we adopt the collinear approach discussed in Sec. IV C. In this approach the assumption of the total momentum of 3N SRCs being negligible imposes several kinematic restrictions on the light-front momenta of nucleons in the correlation. The vertex operator $\hat{V}_{3N}$ entering in Eq. (4) takes into account these kinematic restrictions in the following form:

$$\hat{V}_{3N} = i\bar{a}(p_1,s_1)2\alpha_1^2\delta(\alpha_1+\alpha_2+\alpha_3-3)\delta^2(p_{1,\perp}+p_{2,\perp}+p_{3,\perp})\delta\big(\tilde{M}_N^2-\tilde{M}_N^{(3N),2}\big)a(p_1,s_1), \tag{78}$$

where in the considered 3N SRC model

$$\tilde{M}_N^{(3N),2} = p_{1+}(p_{A-}-p_{2-}-p_{3-}-p_{A-3,-})-p_{1,\perp}^2 = \frac{\alpha_1}{3}\left(M_{3N}^2-\frac{M_N^2+p_{2,\perp}^2}{\alpha_2/3}-\frac{M_N^2+p_{3,\perp}^2}{\alpha_3/3}\right)-p_{1,\perp}^2, \tag{79}$$

with the mass of the 3N SRC defined as

$$M_{3N}^2 = \frac{3}{A}M_A^2 - 3\frac{M_{A-3}^2}{\alpha_{A-3}}. \tag{80}$$

The following derivation is in many ways similar to the one in the previous section. We first substitute the vertex function V_{3N} into Eq. (4), expressing four-dimensional differentials through the light-front momenta. Then we estimate the $dp_{2,-}$ and $dp_{3,-}$ integrals at the pole values of the propagators of 2 and 3 nucleons corresponding to the positive values of their + components as follows:

$$\frac{d^4p_{2/3}}{p_{2/3}^2-M_N^2+i\epsilon} = -i(2\pi)\frac{dp_{2/3,+}d^2p_{2/3,\perp}}{2p_{2/3,+}} = -i\pi\frac{d\alpha_{2/3}}{\alpha_{2/3}}d^2p_{2/3,\perp}\Big|_{p_{2/3-}} = \frac{M_N^2+p_{2/3,\perp}^2}{p_{2/3+}}. \tag{81}$$

The above integrations allow us to use the on-mass-shell sum rule relations for the numerators of the propagators of 2 and 3 nucleons $\not{p}+M_N=\sum_s u(p,s)\bar{u}(p,s)$. Using a similar approximate relation for the $2'$ propagator which represents the nucleon between consecutive NN interaction vertices, as well as the properties of creation, $\bar{a}(p_1,s_1)$, and annihilation $a(p_1,s_1)$ operators, for the 3N SRC light-front spectral function one obtains from Eq. (4)

$$\begin{aligned}P_{A,3N}^N\big(\alpha_1,p_{1,\perp},\tilde{M}_N^2\big) = &\sum_{s_2,s_3,s_{2'},\tilde{s}_{2'}}\int \bar{u}(k_1)\bar{u}(k_2)\bar{u}(k_3)\Gamma^\dagger_{NN\to NN}\frac{u(p_{2'},\tilde{s}_{2'})\bar{u}(p_{2'},\tilde{s}_{2'})}{p_{2'}^2-M_N^2}\Gamma^\dagger_{NN\to NN}\frac{u(p_1,s_1)}{p_1^2-M_N^2}u(p_2,s_2)\\ &\times\big[2\alpha_1^2\delta(\alpha_1+\alpha_2+\alpha_3-3)\delta^2(p_{1,\perp}+p_{2,\perp}+p_{3,\perp})\delta\big(\tilde{M}_N^2-\tilde{M}_N^{(3N),2}\big)\big]\bar{u}(p_2,s_2)\frac{\bar{u}(p_1,s_1)}{p_1^2-M_N^2}\Gamma_{NN\to NN}\\ &\times\frac{u(p_{2'},s_{2'})\bar{u}(p_{2'},s_{2'})}{p_{2'}^2-M_N^2}u(p_3,s_3)\bar{u}(p_3,s_3)\Gamma^\dagger_{NN\to NN}u(k_1)u(k_2)u(k_3)\frac{d\alpha_2}{\alpha_2}\frac{d^2p_{2,\perp}}{2(2\pi)^3}\frac{d\alpha_3}{\alpha_3}\frac{d^2p_{3,\perp}}{2(2\pi)^3},\end{aligned} \tag{82}$$

where we suppress the spin notations of the initial and final collinear nucleons for simplicity of expressions.

Next we consider the following combination from the above expression:

$$\frac{\bar{u}(p_1,s_1)\bar{u}(p_2,s_2)\Gamma_{NN\to NN}u(p_{2'},s_{2'})u(k_1)}{p_1^2-M_N^2}. \tag{83}$$

Here the denominator, similar to the previous section, can be expressed through the relative light-front momentum variables:

$$p_1^2-M_N^2 = \frac{\beta_1}{2}\left[M_{12}^2-\frac{4\big[M_N^2+(p_{1,\perp}-\frac{\beta_1}{2}p_{12,\perp})^2\big]}{(2-\beta_1)\beta_1}\right], \tag{84}$$

where we also applied the kinematic conditions following from the collinear approximation:

$$\begin{aligned}M_{12}^2 &= (k_1+k_2+k_3-p_3)^2 \approx M_{3N}^2\Big(1-\frac{\alpha_3}{3}\Big)-3\frac{M_N^2+p_{3,\perp}^2}{\alpha_3}+M_N^2,\\ p_{12,\perp} &\approx -p_{3,\perp} \quad\text{and}\quad \beta_1 = \frac{2\alpha_1}{\alpha_{12}} = \frac{2\alpha_1}{3-\alpha_3},\end{aligned} \tag{85}$$

with $\alpha_{12}=\alpha_1+\alpha_2=3\frac{p_{1+}+p_{2+}}{k_{1+}+k_{2+}+k_{3+}}$.

Using Eq. (84) one introduces the light-front wave function of NN SRC similar to Eq. (60):

$$\psi_{NN}(\beta_1,\tilde{k}_{1,\perp},s_1,s_2) = -\frac{1}{\sqrt{2(2\pi)^3}}\frac{\bar{u}(p_1,s_1)\bar{u}(p_2,s_2)\Gamma_{NN\to NN}u(p_{2'},s_{2'})u(k_1)}{\frac{1}{2}\big[M_{12}^2-4\frac{M_N^2+\tilde{k}_{1,\perp}^2}{\beta_1(2-\beta_1)}\big]}, \tag{86}$$

where $\tilde{\mathbf{k}}_{1,\perp} = \mathbf{p}_{1,\perp} - \frac{\beta_1}{2}\mathbf{p}_{12,\perp}$ and we also define the relative momentum in the NN center of mass frame as

$$\tilde{k}_1^2 = \frac{M_N^2 + \tilde{k}_{1,\perp}^2}{\beta_1(2-\beta_1)} - M_N^2. \tag{87}$$

With the above definitions for Eq. (83) one obtains

$$\frac{\bar{u}(p_1,s_1)\bar{u}(p_2,s_2)\Gamma_{NN\to NN}u(p_{2'},s_{2'})u(k_1)}{p_1^2 - M_N^2} = \sqrt{2(2\pi)^3}\frac{\psi_{NN}(\beta_1,\tilde{k}_{1,\perp},s_1,s_2)}{\beta_1}. \tag{88}$$

For the second NN SRC contribution in Eq. (82) we consider the term

$$\frac{\bar{u}(p_{2'},s_{2'})\bar{u}(p_3,s_3)\Gamma_{NN\to NN}u(k_2)u(k_3)}{p_{2'}^2 - M_N^2}, \tag{89}$$

for which the denominator, similar to Eq. (84), can be represented in the form

$$p_{2'}^2 - M_N^2 = \frac{2-\beta_3}{2}\left[M_{23}^2 - 4\frac{M_N^2 + p_{3,\perp}^2}{\beta_3(2-\beta_3)}\right], \tag{90}$$

where the several relations below follow from the collinear approximation:

$$M_{23}^2 = (k_2+k_3)^2 \approx 4M_N^2; \quad p_{2',\perp} \approx -p_{3,\perp}; \quad \alpha_{2'3} = \frac{p_{2',+} + p_{3+}}{k_{1+}+k_{2+}+k_{3+}} \approx 2; \quad \beta_{2'} = \alpha_{2'3} - \beta_3 \approx 2 - \beta_3, \tag{91}$$

and the relative momentum in the NN center of mass frame can be defined as

$$\tilde{k}_3^2 = \frac{M_N^2 + p_{3,\perp}^2}{\beta_3(2-\beta_3)} - M_N^2. \tag{92}$$

Equations (90) and (91) allow us to use the definition of the NN SRC wave function of Eq. (86) with the replacements of $M_{12} \to M_{23}$, $\beta_1 \to 2-\beta_3 = \beta_{2'}$, $\tilde{k}_{1,\perp} \to -p_{3,\perp}$ to describe the wave function of the second NN correlation. This results in the following expression for Eq. (89):

$$\frac{\bar{u}(p_{2'},s_{2'})\bar{u}(p_3,s_3)\Gamma_{NN\to NN}u(k_2)u(k_3)}{p_{2'}^2 - M_N^2} = \sqrt{2(2\pi)^3}\frac{\psi_{NN}(\beta_{2'},p_{3,\perp},s_{2'},s_3)}{2-\beta_3}. \tag{93}$$

Using Eqs. (88) and (93) in Eq. (82) for the 3N SRC light-front spectral function one arrives at

$$\begin{aligned}P_{A,3N}^N(\alpha_1,p_{1,\perp},\tilde{M}_N^2) = \sum_{s_1,s_2,s_3,s_{2'}}\int &\frac{\psi_{NN}^\dagger(\beta_{2'},p_{3,\perp},s_{2'},s_3)}{2-\beta_3}\frac{\psi_{NN}^\dagger(\beta_1,\tilde{k}_{1,\perp},s_1,s_2)}{\beta_1}\\ &\times\left[2\alpha_1^2\delta(\alpha_1+\alpha_2+\alpha_3-3)\delta^2(p_{1,\perp}+p_{2,\perp}+p_{3,\perp})\delta\left(\tilde{M}_N^2 - \tilde{M}_N^{(3N),2}\right)\right]\\ &\times\frac{\psi_{NN}(\beta_1,\tilde{k}_{1,\perp},s_1,s_2)}{\beta_1}\frac{\psi_{NN}(\beta_{2'},p_{3,\perp},s_{2'},s_3)}{2-\beta_3}\frac{d\alpha_2}{\alpha_2}d^2p_{2,\perp}\frac{d\alpha_3}{\alpha_3}d^2p_{3,\perp}.\end{aligned} \tag{94}$$

In the above expression, the polarizations are summed similar to the one in VN approximation [Eq. (23)] which allows us to express the unpolarized spectral function in the form of the convolution of two NN light-front density matrices, as those defined in Eq. (71), as follows:

$$\begin{aligned}P_{A,3N}^N(\alpha_1,p_{1,\perp},\tilde{M}_N^2) = \int &\frac{3-\alpha_3}{2(2-\alpha_3)^2}\rho_{NN}(\beta_{2'},p_{3,\perp})\rho_{NN}(\beta_1,\tilde{k}_{1,\perp})2\delta(\alpha_1+\alpha_2+\alpha_3-3)\\ &\times\delta^2(p_{1,\perp}+p_{2,\perp}+p_{3,\perp})\delta\left(\tilde{M}_N^2 - \tilde{M}_N^{(3N),2}\right)d\alpha_2 d^2p_{2,\perp}d\alpha_3 d^2p_{3,\perp},\end{aligned} \tag{95}$$

where within collinear approximation $\beta_3 = \alpha_3$, $\beta_1 = \frac{2\alpha_1}{3-\alpha_3}$ as well as $\tilde{k}_{1,\perp} = \mathbf{p}_{1,\perp} + \frac{\beta_1}{2}\mathbf{p}_{3,\perp}$.

In the above expression, similar to Eq. (37) for VN approximation, the product of the two density matrices is expressed through the product of high momentum parts of the deuteron density matrices in the form

$$\rho_{NN}(\beta_{2'},p_{3,\perp})\rho_{NN}(\beta_1,\tilde{k}_{1,\perp}) = a_2^2(A,Z)C(A,Z)\rho_d(\beta_{2'},p_{3,\perp})\Theta(\tilde{k}_1 - k_{\rm src})\rho_d(\beta_1,\tilde{k}_{1,\perp})\Theta(\tilde{k}_3 - k_{\rm src}), \tag{96}$$

where $\tilde{k}_1$ and $\tilde{k}_3$ are defined in Eqs. (87) and (92) respectively. The factors $a_2(A,Z)$ and $C(A,Z)$ are the same as in the case of 3N SRCs within VA approximation.

VII. RANGE ON VALIDITY AND OVERVIEW OF PARAMETERS ENTERING THE MODEL

Here we discuss briefly the range of validity and set of parameters which will be used for numerical estimates of the spectral functions to be presented in Ref. [28].

The main assumption on which our models are based is the dominance of NN SRC in the nuclear dynamics for internal momenta $p \gtrsim k_{\rm src}$. Next major assumption is the dominance of the isosinglet *pn* component in the NN SRC. The empirical evidence of the dominance of NN SRCs was accumulated during the last several decades (see, e.g., [18,19,23,29–32]) in high energy electro- and hadroproduction reactions. Recent triple-coincident experiments [20,21,26] indicated that the *pn* dominance in the nucleon-nucleon SRC persists for up to the heavy nuclei such as $A = 208$. With this one expects that our model should be valid for the wide range of atomic nuclei.

The most important parameter that defines the strength of 2N SRC is $a_2(A,Z)$. Within the short-range correlation framework, this parameter can be extracted from the ratios of the cross sections of high momentum transfer inclusive electronuclear scattering off nuclei *A* and the deuteron [31,32]. Recent measurements at Jefferson Lab [18,19,23] provided the magnitudes of $a_2(A,Z)$ for a rather wide spectrum of atomic nuclei.

In addition to $a_2(A,Z)$, the two other parameters $k_{\rm src}$ and γ define the momentum distribution of NN SRC in Eqs. (27) and (75). The value of $k_{\rm src}$ is defined from the condition that it is sufficiently large for mean field contribution to be insignificant, as well as close to the threshold value for which *pn* dominance is observed empirically [20,21,26]. Another condition in defining $k_{\rm src}$ is the onset of the dominance of the *d*-wave contribution in the high momentum part of the deuteron wave function. With these conditions we evaluate $k_{\rm src} \sim 300 - 350$ MeV/c. In the current model we neglected the contribution of isotriplet NN SRCs. To account for the effects due to these correlations we introduce the parameter γ. Based on the experimental observation [26] that in the 2N SRC regions *pn* dominates by almost a factor of 20 for a wide range of nuclei (up to $A = 208$) we take $\gamma \approx 0.8 - 0.9$.

For the width of the center of mass distribution, $\beta(A)$ in Eq. (26), we use the estimates based on the convolution of the mean field distribution of two independent nucleons according to Ref. [10]. The parameter N_0 in Eq. (26) is defined from the normalization condition.

For the case of 3N SRCs, the only additional parameter needed to define the spectral function is the suppression factor $C(A,Z)$ in Eqs. (37) and (96). This factor accounts for the suppression of the 3N configurations with two identical spectators like *pp* and *nn* pairs. It affects only the distribution of the minority component in the asymmetric nucleus. For example, according to the considered model, the neutron cannot be generated from 3N SRC in the ^{3}He nucleus, since it will produce two "parallel" protons in the final state. For parametrization of these effects we use the expression $C(A,Z) = 1 - 3(y/A)$ for the minority component, where $y = |1 - 2\frac{Z}{A}|$ is the asymmetry parameter, and $C(A,Z) = 1$ for the majority component.

Note that the above discussed parameters are independent of the use of the VN or LF approximations. Therefore one can achieve further refinement in their values for lightest nuclei ($A \leqslant 12$) by considering the nonrelativistic limit of our approximations and comparing them with the results from *ab initio* calculations like one based on the variational Monte Carlo methods [37].

With the parameters for 2N and 3N SRCs fixed one can calculate the normalization factors n_{2N}^N and n_{3N}^N within VN and LF approximations. Note that these factors will be model dependent since the 2N and 3N momentum distributions predicted in VN and LF approximations are different. Once these normalizations are calculated one can estimate the norm of the mean field distributions from the relation $n_{MF}^N = 1 - n_{2N}^N - n_{3N}^N$. Thus the estimates for the normalization of mean field distributions will be indirectly VN or LF model dependent.

VIII. SUMMARY

Based on the NN short-range correlation picture of the high-momentum component of nuclear wave function we developed a model for the nuclear spectral functions in the domain of large momentum and removal energy of bound nucleon in the nucleus. Our main focus is in treating the relativistic effects which are important for the bound nucleon momenta exceeding characteristic Fermi momentum, k_F, in the nucleus. The relativistic effects in this work are treated based on the effective Feynman diagrammatic approach, in which one starts with Lorentz covariant amplitudes reducing them to the nuclear spectral functions that allows us to trace the relativistic effects entering in these functions. One of the main ambiguities related to the treatment of the relativistic effects is the account for the vacuum fluctuations (Z graphs) which ultimately alter the definition of the spectral function as a probability of finding a bound nucleon in the nucleus with the given momentum and removal energy. We employed two: virtual nucleon and light-front approaches in treating the relativistic effects.

The results for the 2N SRC model within VN [Eq. (24)] and LF [Eq. (73)] approximations agree with the 2N SRC (with center of mass motion) model of Ref. [10] in the nonrelativistic limit. Our results represent an attempt to account for the relativistic effects in the domain of 2N SRCs with center of mass motion of the NN pair.

We extended both approaches to calculate also the contributions from three-nucleon short-range correlations. Derivations in this case are based on the collinear approach in which one assumes negligible center of mass momentum for the residual or uncorrelated $(A-3)$ nuclear system. The derived spectral functions within VN [Eq. (35)] and LF [Eq. (95)] approximations represent results for 3N SRC contribution to the nuclear spectral functions.

The main property of the obtained spectral functions is that to describe them quantitatively in a high momentum domain one needs only the knowledge of the high momentum deuteron wave function either in the laboratory frame (for VN approximation) or on the light front (for LF approximation). In the follow-up work [28] we will present the quantitative studies of the properties of nuclear spectral functions based on specific models of the deuteron wave functions.

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[1] L. B. Weinstein, E. Piasetzky, D. W. Higinbotham, J. Gomez, O. Hen, and R. Shneor, Phys. Rev. Lett. **106**, 052301 (2011).
[2] O. Hen, E. Piasetzky, and L. B. Weinstein, Phys. Rev. C **85**, 047301 (2012).
[3] A. J. Freese, M. M. Sargsian, and M. I. Strikman, Eur. Phys. J. C **75**, 534 (2015).
[4] A. J. Freese and M. M. Sargsian, arXiv:1511.06044v1 (2015).
[5] G. P. Zeller *et al.* (NuTeV Collaboration), Phys. Rev. Lett. **88**, 091802 (2002); **90**, 239902 (2003).
[6] M. Sargsian, Neutrino interactions in the nuclear environment, APS April Meeting 2013, April 13–16, 2013 (unpublished).
[7] G. R. Jansen, J. Engel, G. Hagen, P. Navratil, and A. Signoracci, Phys. Rev. Lett. **113**, 142502 (2014).
[8] L. L. Frankfurt and M. I. Strikman, Phys. Rep. **76**, 215 (1981).
[9] C. Ciofi degli Atti, S. Simula, L. L. Frankfurt, and M. I. Strikman, Phys. Rev. C **44**, R7(R) (1991).
[10] C. Ciofi degli Atti and S. Simula, Phys. Rev. C **53**, 1689 (1996).
[11] M. Vanhalst, J. Ryckebusch, and W. Cosyn, Phys. Rev. C **86**, 044619 (2012).
[12] M. Alvioli, C. Ciofi degli Atti, L. P. Kaptari, C. B. Mezzetti, and H. Morita, Phys. Rev. C **87**, 034603 (2013).
[13] M. Alvioli, C. CiofiDegli Atti, L. P. Kaptari, C. B. Mezzetti, and H. Morita, Int. J. Mod. Phys. E **22**, 1330021 (2013).
[14] A. Rios, A. Polls, and W. H. Dickhoff, Phys. Rev. C **89**, 044303 (2014).
[15] J. Ryckebusch, W. Cosyn, and M. Vanhalst, J. Phys. G **42**, 055104 (2015).
[16] C. Ciofi degli Atti, Phys. Rep. **590**, 1 (2015).
[17] T. Neff, H. Feldmeier, and W. Horiuchi, Phys. Rev. C **92**, 024003 (2015).
[18] K. S. Egiyan *et al.* (CLAS Collaboration), Phys. Rev. C **68**, 014313 (2003).
[19] K. S. Egiyan *et al.* (CLAS Collaboration), Phys. Rev. Lett. **96**, 082501 (2006).
[20] E. Piasetzky, M. Sargsian, L. Frankfurt, M. Strikman, and J. W. Watson, Phys. Rev. Lett. **97**, 162504 (2006).
[21] R. Subedi, R. Shneor, P. Monaghan, B. D. Anderson, K. Aniol, J. Annand, J. Arrington, H. Benaoum *et al.*, Science **320**, 1476 (2008).
[22] L. Frankfurt, M. Sargsian, and M. Strikman, Int. J. Mod. Phys. A **23**, 2991 (2008).
[23] N. Fomin, J. Arrington, R. Asaturyan, F. Benmokhtar, W. Boeglin, P. Bosted, A. Bruell, M. H. S. Bukhari *et al.*, Phys. Rev. Lett. **108**, 092502 (2012).
[24] J. Arrington, D. W. Higinbotham, G. Rosner, and M. Sargsian, Prog. Part. Nucl. Phys. **67**, 898 (2012).
[25] C. Colle, O. Hen, W. Cosyn, I. Korover, E. Piasetzky, J. Ryckebusch, and L. B. Weinstein, Phys. Rev. C **92**, 024604 (2015).
[26] O. Hen, M. Sargsian, L. B. Weinstein, E. Piasetzky, H. Hakobyan, D. W. Higinbotham, M. Braverman, W. K. Brooks *et al.*, Science **346**, 614 (2014).
[27] O. Hen, L. B. Weinstein, E. Piasetzky, G. A. Miller, M. M. Sargsian, and Y. Sagi, Phys. Rev. C **92**, 045205 (2015).
[28] O. Artiles and M. Sargsian (unpublished).
[29] J. L. S. Aclander *et al.*, Phys. Lett. B **453**, 211 (1999).
[30] I. Yaron, L. Frankfurt, E. Piasetzky, M. Sargsian, and M. Strikman, Phys. Rev. C **66**, 024601 (2002).
[31] L. L. Frankfurt and M. I. Strikman, Phys. Rep. **160**, 235 (1988).
[32] L. L. Frankfurt, M. I. Strikman, D. B. Day, and M. Sargsyan, Phys. Rev. C **48**, 2451 (1993).
[33] M. M. Sargsian, T. V. Abrahamyan, M. I. Strikman, and L. L. Frankfurt, Phys. Rev. C **71**, 044615 (2005).
[34] R. Schiavilla, R. B. Wiringa, S. C. Pieper, and J. Carlson, Phys. Rev. Lett. **98**, 132501 (2007).
[35] M. M. Sargsian, Phys. Rev. C **89**, 034305 (2014).
[36] M. McGauley and M. M. Sargsian, arXiv:1102.3973v3 (2012).
[37] R. B. Wiringa, R. Schiavilla, S. C. Pieper, and J. Carlson, Phys. Rev. C **89**, 024305 (2014).
[38] W. U. Boeglin *et al.* (Hall A Collaboration), Phys. Rev. Lett. **107**, 262501 (2011).
[39] W. Boeglin and M. Sargsian, Int. J. Mod. Phys. E **24**, 1530003 (2015).
[40] W. U. Boeglin *et al.*, arXiv:1410.6770v1 (2014).
[41] A. Tang, J. W. Watson, J. Aclander, J. Alster, G. Asryan, Y. Averichev, D. Barton, V. Baturin, N. Bukhtoyarova, A. Carroll, S. Gushue, S. Heppelmann, A. Leksanov, Y. Makdisi, A. Malki, E. Minina, I. Navon, H. Nicholson, A. Ogawa, Y. Panebratsev, E. Piasetzky, A. Schetkovsky, S. Shimanskiy, and D. Zhalov, Phys Rev. Lett. **90**, 042301 (2003).
[42] R. Shneor *et al.* (Jefferson Lab Hall A Collaboration), Phys. Rev. Lett. **99**, 072501 (2007).
[43] I. Korover *et al.* (Jefferson Lab Hall A Collaboration), Phys. Rev. Lett. **113**, 022501 (2014).
[44] C. Colle, W. Cosyn, J. Ryckebusch, and M. Vanhalst, Phys. Rev. C **89**, 024603 (2014).
[45] L. L. Frankfurt, M. M. Sargsian, and M. I. Strikman, Phys. Rev. C **56**, 1124 (1997).
[46] L. Frankfurt, M. Strikman, G. Piller, and M. Sargsian, Nucl. Phys. A **631**, 502 (1998).
[47] M. M. Sargsian, Int. J. Mod. Phys. E **10**, 405 (2001).
[48] F. Gross, Phys. Rev. C **26**, 2203 (1982).
[49] M. M. Sargsian, Phys. Rev. C **82**, 014612 (2010).
[50] W. Cosyn and M. Sargsian, Phys. Rev. C **84**, 014601 (2011).
[51] S. Weinberg, Phys. Rev. **150**, 1313 (1966).
[52] L. Frankfurt and M. Strikman, in *Modern Topics in Electron Scattering*, edited by B. Frois and I. Sick (World Scientific, Singapore, 1991), pp. 645–694.
[53] G. A. Miller, Prog. Part. Nucl. Phys. **45**, 83 (2000).
[54] S. Scopetta, A. Del Dotto, L. Kaptari, E. Pace, M. Rinaldi, and G. Salm, Few Body Syst. **56**, 425 (2015).

[55] D. W. Higinbotham and O. Hen, Phys. Rev. Lett. **114**, 169201 (2015).
[56] J. Arrington, D. Day, N. Fomin, and P. Solvignon-Slifer (unpublished).
[57] V. N. Gribov, Zh. Eksp. Teor. Fiz. **57**, 1306 (1969) [Sov. Phys. JETP **30**, 709 (1970)].
[58] L. Bertocchi, Nuovo Cimento A **11**, 45 (1972).
[59] M. M. Sargsian, T. V. Abrahamyan, M. I. Strikman, and L. L. Frankfurt, Phys. Rev. C **71**, 044614 (2005).
[60] S. J. Brodsky, H. C. Pauli, and S. S. Pinsky, Phys. Rep. **301**, 299 (1998).
[61] L. L. Frankfurt and M. I. Strikman, Phys. Lett. B **183**, 254 (1987).