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Prediction and evaluation of magnetic moments in $T = 1/2$, $3/2$, and $5/2$ mirror nuclei

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The Buck-Perez analysis of mirror nuclei magnetic moments has been applied on an updated set of data for $T = 1/2, 3/2$ mirror pairs and attempted for the first time for $T = 5/2$ nuclei. The spin expectation value for mirror nuclei up to mass $A = 63$ has been reexamined. The main purpose is to test Buck-Perez analysis effectiveness as a prediction and—more importantly—an evaluation tool of magnetic moments in mirror nuclei. In this scheme, ambiguous signs of magnetic moments are resolved, evaluations of moments with multiple existing measurements have been performed, and a set of predicted values for missing moments, especially for several neutron-deficient nuclei is produced. A resolution for the case of the ^{57}Cu ground-state magnetic moment is proposed. Overall, the method seems to be promising for future evaluations and planning future measurements.

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I. INTRODUCTION

The magnetic dipole moment is an observable that is well known to offer invaluable insights to nuclear structure. As a quantum mechanical entity, the magnetic moment is a one-body operator with strong sensitivity on the orbital and spin components of the state wave function. In this framework, measurements of magnetic moments can submit nuclear models to stringent tests. Regarding mirror nuclei, experimental data on magnetic moments can check the validity of essential symmetries, such as isospin conservation.

The isospin formalism is useful in expressing the magnetic moment operator in terms of the orbital and spin components. Sugimoto [1] expressed the magnetic moment, μ, as the expectation value for the state with $M = J$, where J is the nuclear spin and M is the magnetic quantum number:

$$\mu = \left\langle \sum_i \left[\frac{1}{2}\left(1+\tau_3^i\right)\left(l_z^i + \sigma_z^i \mu_p\right) + \frac{1}{2}\left(1-\tau_3^i\right)\sigma_z^i \mu_n \right] \right\rangle_{M=J}. \tag{1}$$

In Eq. (1) μ is expressed in terms of the orbital angular momentum, l, the Pauli spin, σ, the isospin, τ, and the magnetic moments of (free) proton and neutron, μ_p and μ_n, respectively. The summation runs over different nucleons. The magnetic moment can be further expressed as a sum of an isoscalar term, μ_0, and an isovector term, μ_3:

$$\mu = \left\langle \sum_i \mu_0^i \right\rangle_J + \left\langle \sum_i \mu_3^i \right\rangle_J, \tag{2}$$

where

$$\mu_0^i = \tfrac{1}{2}\left[l_z^i + (\mu_p + \mu_n)\sigma_z^i\right],$$
$$\mu_3^i = \tfrac{1}{2}\left[\tau_3^i l_z^i + (\mu_p - \mu_n)\tau_3^i \sigma_z^i\right].$$

Assuming isospin is a good quantum number due to charge symmetry of the nuclear forces, while ignoring Coulomb effects, a mirror pair of nuclei has the expectation value $\langle \sum \mu_0 \rangle_J$ independent of T_3 and $\langle \sum \mu_3 \rangle_J$ reversing its sign from $T_3 = +T$ to $T_3 = -T$. As a consequence [2] the spin expectation value is expressed in terms of the sum moment of the mirror states:

$$\left\langle \sum \sigma_z \right\rangle = \frac{\mu(T_z = +T) + \mu(T_z = -T) - J}{\mu_p + \mu_n - 1/2} \tag{3}$$

since the total spin is $J = \langle \sum_i l_z^i \rangle + \frac{1}{2}\langle \sum_i \sigma_z^i \rangle$.

A few earlier works, mainly by Buck, Perez, and collaborators [3–5] but others as well [2,6–8], have illustrated the significance of this relation mainly due to the sensitivity of the spin expectation value to small changes in the magnetic moments of the mirror nuclei. This property offers an advantage when looking at experimental data of mirror nuclei, in particular, magnetic moments of $T = 1/2$ mirror pairs that have all been measured in mass range $A = 3$–43 and $A = 57,59$ [9].

Experimental data in the *sd* shell were analyzed by Sugimoto [1] and later by Hanna and Hugg [6]. Systematic trends in the spin expectation value as a function of the mass number for all $T = 1/2$ nuclei were noted and explored in more detail by Buck and Perez [3,4], who abandoned the extreme odd-nucleon model (Schmidt moments) and instead considered potential contributions by all the odd nucleons in forming the observed values. In that way, a linear relation between the proton magnetic moment, μ_p, and the neutron magnetic moment, μ_n, could be established [3]. Expanding this relation into a more general perspective, Buck and Perez transformed it to a relation between g factors:

$$\gamma_p = \alpha\gamma_n + \beta, \tag{4}$$

where $\gamma_{p,n} = \mu_{p,n}/J$, $\alpha = \frac{g_s^p - g_l^p}{g_s^n - g_l^n}$, $\beta = g_s^p - \alpha g_l^n$, and $g_l^p = 1, g_l^n = 0, g_s^p = +5.586, g_s^n = -3.826$.

They used Eq. (4) to fit all available experimental information to obtain a data-driven relationship in mirror pair magnetic moments. The results of the fit on $T = 1/2$ data produced α and β values that deviated from theoretical expectations significantly ($\alpha = -1.199, \beta = 1$ for the free nucleon), but not immensely. There have been several updates on the set of values over the last 30 years, as more data have been accumulated experimentally, with the most recent update in

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2008 [5]. In addition, $T = 3/2$ mirror pairs were added in the analysis and compared with the $T = 1/2$ data, hinting at a rather universal behavior of mirror pairs when examined under the Buck-Perez scheme. Shell-model calculations have also been used to predict some of the missing μ values in mirror partners [5]. It would be interesting to expand this analysis to $T = 5/2$ mirror pairs and check if mirror nuclei with large neutron-proton number differences, and very close to the proton dripline, still conform with the initial assumption of charge symmetry as in the cases of $T = 1/2$ and $3/2$. In this work, this study is undertaken for the first time.

In addition, despite evaluated data sets of nuclear magnetic moments are highly desirable, they are still very scarce [10]. In this framework, the Buck-Perez analysis can be very effective in two directions: (a) assigning the proper sign in the magnetic moment in case the sign has not been determined unambiguously in the experiment and (b) promoting the credibility of a particular measurement over competing ones in an evaluation, avoiding expensive theoretical calculations. Both directions are discussed in the present work.

II. METHODOLOGY

Sugimoto's spin expectation value as a function of mass number A of the mirror pair was reexamined and brought up to date. All data for $T = 1/2, 3/2$, and $5/2$ used in the present work have been found in IAEA's nuclear moments database that provides unevaluated data on nuclear magnetic dipole and electric quadrupole moments on a quarterly basis [9,11].

For Buck-Perez analysis, the γ_p vs γ_n data were fitted with a linear model using least squares regression. Each fit produced a set of slope, intercept, and correlation coefficient. The corresponding linear curves have been drawn in the γ_p-γ_n plots.

In case a missing sign existed in the magnetic moment of the odd-proton or odd-neutron partner in the mirror pair, an assignment was decided based on the following criterion: The corresponding point (γ_n, γ_p) should be as close as possible to the fitted curve. In all cases under consideration in this part, alternative sign assignments created points on the plots that deviated largely from what was expected. Those options were not considered further and a final evaluation of the sign was completed accordingly.

Predicted values of magnetic moments for incomplete mirror pairs, having just one of the two nuclei with a measured value, have been estimated in a similar fashion. The sign of the moment was automatically assigned without further considerations. It has to be noted, however, that all such cases produced signs that agreed with what expected for the odd-proton or odd-neutron in the nuclear shell examined.

A significant portion of the data (≈50% for $T = 1/2$) are drawn from single measurements. Weighted averages and corresponding errors have been calculated for all nuclei having multiple entries in the database. The sole exception has been ^{57}Cu, which is discussed later.

Regarding $T = 5/2$ mirror pairs, no measurements exist for any mass number to be considered. There are currently eight (8) odd-neutron nuclei that have mirror partners with a positive proton separation energy [12]. For those mirror pairs, magnetic moment predictions for the odd-proton partners have been estimated using the α and β, as estimated from the $T = 1/2$ data. Although one could argue against it, two reasons favor the present treatment: (a) there is a strong similarity between $T = 1/2$ and $T = 3/2$ linear curves with slopes and intercepts being essentially the same within statistical errors [4], and (b) charge symmetry seems to be valid for pairs of mirror nuclei, in general, as was shown in Ref. [13]. Therefore, using a linear relationship having a high correlation coefficient (see Table I) for the case of $T = 5/2$ mirror pairs does not at all seem unreasonable.

TABLE I. Least squares fit results for the slope (α), the intercept (β), and the correlation coefficient squared (R^2) for $T = 1/2$ and $3/2$ data of mirror nuclei g factors, γ_p and γ_n.

T	α	β	R^2
1/2	-1.1582 ± 0.0164	1.0344 ± 0.0266	0.996
3/2	-1.1717 ± 0.0333	1.0957 ± 0.0304	0.996

Furthermore (see full details in Table III):

(1) $T = 1/2$ mirror pairs up to $A = 63$ have at least one partner nucleus with a known experimental magnetic moment. The sole exception is the $A = 5$ pair (^{5}Li-^{5}He) that has none.
(2) Values for one partner in $T = 1/2$ pairs have been predicted for $A = 45, 49, 51, 53, 59$, and 63.
(3) Six (6) complete mirror pairs exist for $T = 3/2$, i.e., $A = 9, 13, 17, 21, 23$, and 35.
(4) Predicted moments for $T = 3/2$ correspond to $A = 25, 27, 33, 37, 39, 41, 43, 45, 47, 49, 51, 53, 57, 61$, and 63.
(5) $T = 5/2$ predictions correspond to $A = 27, 31, 43, 47, 51, 55$, and 59, all corresponding to the odd-neutron partner.

III. RESULTS AND DISCUSSION

The spin expectation values for all mirror pairs are plotted in Fig. 1. $T = 1/2$, 3/2 have been reexamined to include all recent data. The plot contains weighted values when multiple entries exist in the database.

There are eight data points for $T = 5/2$ (solid circles in Fig. 1) that have been estimated using the slope and intercept from the case of $T = 1/2$ as mentioned earlier. All spin expectation values are grouped together and fall between extreme-particle limits (solid lines in the plot). The exception of the mirror pairs for $A = 9$ and 21 that violate this trend has been discussed elsewhere [2].

The least-squares fits for the $T = 1/2$ and 3/2 γ_n-γ_p sets produce the values shown in Table I. The correlation coefficient R is very close to unity, confirming the exceptional linear trends of data, visually evident in Figs. 2 and 3. As in earlier analyses with the Buck-Perez scheme, the slopes of the straight lines are proven the same within the experimental error, $\alpha(T = 1/2) = -1.1582 \pm 0.0164$ and $\alpha(T = 3/2) = 1.1717 \pm 0.0333$. It has to be noted that the value $A = 55$ is included in the fit of $T = 1/2$ despite its marked deviation

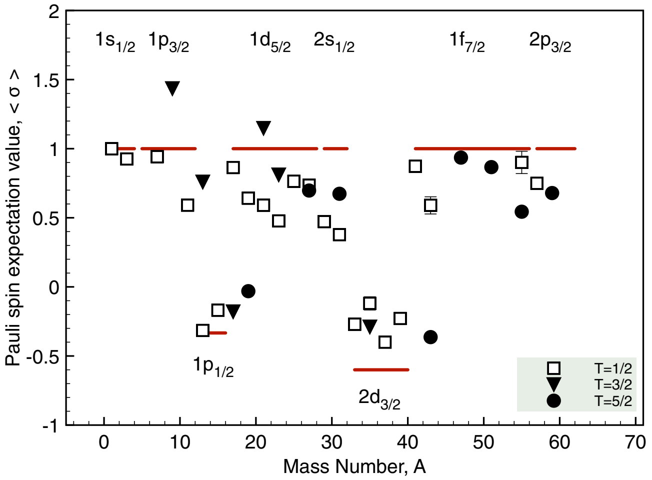

FIG. 1. Spin expectation values as a function of A for mirror nuclei pairs. $T = 1/2$ are denoted with open squares, $T = 3/2$ with triangles, and $T = 5/2$ with circles. The latter have been estimated using the linear relation established for $T = 1/2$ mirror pairs (see text for more details). Values corresponding to extreme Schmidt limits for protons and neutrons in low-lying shells (marked accordingly) are shown as solid lines.

from the straight line. This effect has been attributed to configuration mixing that influences the values of magnetic moments in ${}^{55}_{27}$Co -${}^{55}_{28}$Ni, implying a soft ^{56}Ni core, as suggested by Berryman *et al.* [2]. However, a similar trend for $A = 39$ discussed by the same authors is not evident in the present analysis.

A. Evaluation of signs in mirror nuclei

The Buck-Perez analysis can be used effectively to decide on the sign of a magnetic moment in a nucleus if the mirror partner magnetic moment is unambiguously known (see Table II). There are a few mirror pairs in which both partners present ambiguity in the sign of the magnetic moment, e.g., the $T = 3/2$ pair ^{9}Li -^{9}C. For these cases, all different sign combinations (+,−), (−,+), (−,−), and (+,+) for the mirror pair were examined to see if they fit nicely on the straight lines of the Buck-Perez scheme. Then, the best sign combination was adopted for the mirror partners. It has to be noted here that sign evaluation preceded the least-square fits presented earlier. See Table III for the complete list of sign evaluation.

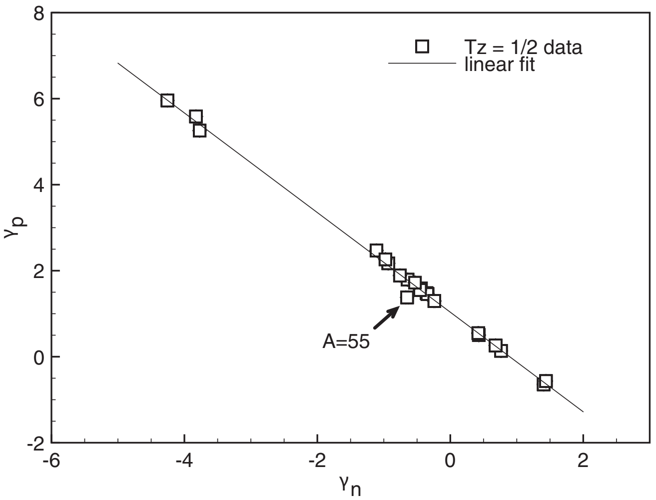

FIG. 2. Odd-proton vs odd-neutron gyromagnetic factor for $T = 1/2$ mirror nuclei. The solid line is a least square fit.

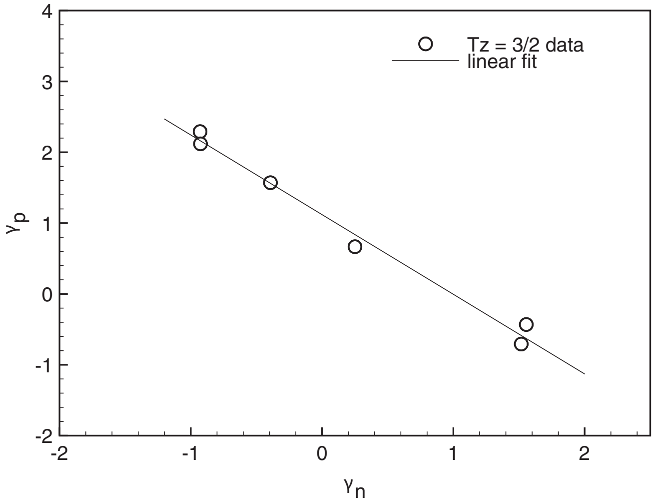

FIG. 3. Odd-proton vs odd-neutron gyromagnetic factor for $T = 3/2$ mirror nuclei. The solid line is a least square fit.

B. Evaluation of ^{57}Cu

As pointed out earlier, the value of ^{57}Cu used in the analysis was not weighted, contrary to all other mirror pairs with more than one measurements existing in literature. An evaluation of the magnetic moment has been attempted in terms of the present scheme, as two measurements for the ground-state magnetic moment in ^{57}Cu [8,14] exist, but seem to differ significantly with each other.

The result reported by Minamisono *et al.* [8], $\mu({}^{57}\text{Cu}) = 2.00(5)$, deviated significantly from both the expected single-

TABLE II. Sign evaluation in mirror nuclei using the Buck-Perez analysis. The table contains the weighted values of magnetic moments.

$T = 1/2$		$T = 3/2$	
Isotope	μ	Isotope	μ
^{13}N	-0.3222 ± 0.0004	^{9}C	-1.3914 ± 0.0005
^{23}Mg	-0.5364 ± 0.0003	^{9}Li	$+3.43680 \pm 0.00006$
^{25}Al	$+3.6455 \pm 0.0012$	^{13}O	-1.3891 ± 0.0003
^{27}Si	-0.8627 ± 0.0002	^{17}N	-0.3550 ± 0.0004
^{29}P	$+1.2348 \pm 0.0002$	^{21}F	$+3.9194 \pm 0.0012$
^{31}S	-0.48793 ± 0.00008	^{35}S	$+1.00 \pm 0.04$
^{39}Ca	$+1.02168 \pm 0.00012$	^{35}K	$+0.390 \pm 0.007$
^{43}Ti	-0.85 ± 0.02		
^{55}Ni	-0.98 ± 0.03		

TABLE III. Predicted ground-state magnetic moments for nuclei with $T = 1/2$, $3/2$, and $5/2$, based on the Buck-Perez analysis. Spin and parity values for ^{43}V and ^{59}Ge have not been deduced experimentally yet, so they have been adopted from their corresponding mirror partners.

Isotope	J^π	μ	Isotope	J^π	μ
		$T = 1/2$			
^{45}V	$7/2^-$	3.510 ± 0.027	^{49}Mn	$5/2^-$	2.035 ± 0.027
^{51}Fe	$5/2^-$	-0.849 ± 0.043	^{53}Co	$7/2^-$	1.999 ± 0.029
^{59}Zn	$3/2^-$	-0.294 ± 0.026	^{63}Ge	$3/2^-$	1.213 ± 0.026
		$T = 3/2$			
^{25}Si	$5/2^+$	-0.805 ± 0.048	^{27}P	$1/2^+$	1.029 ± 0.041
^{33}P	$1/2^+$	1.395 ± 0.059	^{37}Ca	$3/2^+$	0.819 ± 0.030
^{39}Sc	$7/2^-$	5.695 ± 0.034	^{41}Ti	$3/2^+$	1.219 ± 0.027
^{43}V	$7/2^-$	5.379 ± 0.033	^{45}Cr	$7/2^-$	-0.786 ± 0.052
^{47}Mn	$5/2^-$	3.663 ± 0.032	^{49}Fe	$7/2^-$	-0.542 ± 0.046
^{51}Co	$7/2^-$	4.929 ± 0.032	^{53}Ni	$7/2^-$	-1.023 ± 0.047
^{57}Zn	$7/2^-$	-0.756 ± 0.045	^{61}Ge	$3/2^-$	-0.397 ± 0.046
^{63}As	$3/2$	1.974 ± 0.031			
		$T = 5/2$			
^{27}S	$5/2^+$	-1.130 ± 0.060	^{31}Ar	$5/2^+$	-1.074 ± 0.060
^{43}Cr	$3/2^+$	1.199 ± 0.040	^{47}Fe	$7/2^-$	-1.485 ± 0.084
^{51}Ni	$7/2^-$	-1.320 ± 0.083	^{55}Zn	$5/2^-$	-0.762 ± 0.059
^{59}Ge	$7/2^-$	-0.869 ± 0.082			

particle prediction and the value obtained by shell-model calculations that involved a soft ^{56}Ni core and configuration mixing. Cocolios *et al.* reported a remeasurement in Ref. [14], $\mu(^{57}\mathrm{Cu}) = +2.582(7)$, which seems to perform better on those grounds. The spin expectation value was calculated for $A = 57$ considering all three possible options for the $\mu(^{57}\mathrm{Cu})$ magnetic moment combined to the one of the mirror nucleus, $\mu(^{57}_{28}\mathrm{Ni})$: (a) the result by Cocolios *et al.*, (b) the result by Minamisono *et al.*, and (c) the weighted average of both.

From the three results, exclusively for case (a) the spin expectation value is similar to the values of all other mirror pairs with masses near $A = 57$ in Fig. 1, staying inside the extreme-particle limits. In addition, the value by Cocolios *et al.* has been the only one to produce a point that was placed exactly on the straight line of $T = 1/2$ in Fig. 2. In both graphs, cases (b) and (c) gave strongly departing values. At the same time, the + sign was checked and confirmed using the criteria mentioned earlier. The case of ^{57}Cu is a good example of how Buck-Perez analysis can be used in evaluating magnetic moments of mirror nuclei. The approach seems to be handy in planning future experiments, especially on heavier masses where nuclei are near or on the proton dripline.

C. Predicted magnetic moments

Using the Buck-Perez scheme, several magnetic moments have been predicted, based on the linear regression results obtained in the present paper, provided the mirror nucleus value is known. These values are listed in full detail in Table III.

Some mirror partners, such as ^{9}B, ^{11}N and ^{15}F, ^{19}Na and ^{19}Mg, are either unbound or too short-lived to be reached experimentally; consequently they are not included in the table. In addition, ^{59}Ge, a potential candidate for $2p$ decay, has been recently found to undergo a β decay instead, with a measured half-life of $t_{1/2} = 13.3(17)$ ms [15]. This half-life is in the same order of magnitude with those from the other $T = 5/2$ nuclei listed in the same table. It has to be noted that ^{59}Ge is the heaviest bound nucleus with $T_z = -5/2$ that has a mirror partner with a known magnetic moment. Despite pushing present-day experimental techniques to their limits in regards to measuring ground-state magnetic moments in these nuclei, such an option cannot at all be discarded.

IV. CONCLUSIONS

In the present work, the Buck-Perez analysis was performed on updated sets of magnetic moments for $T = 1/2$ and $T = 3/2$ mirror nuclei. Almost identical linear trends have been resulted in both cases. Using the linear relation parameters deduced for $T = 1/2$ nuclei, the scheme was applied on eight $T = 5/2$ mirror pairs, for the first time, as a quick way to predict ground-state magnetic moments of mirror partners located very close to the proton dripline. In addition, the Buck-Perez analysis was further applied to perform an evaluation of magnetic moments in mirror nuclei and attempt resolutions on open issues, as in the case of ^{57}Cu.

Overall, the empirical Buck-Perez analysis, expressing the underlying charge symmetry of nuclear forces, seems to be effective in evaluating magnetic moments in mirror nuclei, while it proves itself as a valuable tool in planning future magnetic-moment measurements, especially for the harder-to-reach, neutron-deficient nuclei.

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