$(sd)^4$ states in ^{12,14}Be

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For suggested cluster states beginning at about 10 MeV in ^{12}Be, absolute energy and relative energy spacings agree well with calculations for the ^{8}Be $\times$ $(sd)^4$ configuration. Comparison indicates that the proposed (8^+) state at 20.9 MeV is probably 6^+. A similar calculation for ^{14}Be predicts that the lowest $(sd)^2$ and $(sd)^4$ states are rather close together in that nucleus.

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I. INTRODUCTION

In many nuclei, core excitations are important even for low-lying states. For nuclei just below ^{16}O, these correspond to excitations from the $1p$ to the $2s1d$ shell. In ^{10}Be [1,2] and ^{14}C [3,4], such excitations become relevant at about 6 MeV. However, in some nuclei, they are important even for the lowest states. For example, in ^{12}Be [5–10], about two-thirds of the ground state (g.s.) wave function has two neutrons in the sd shell; for the first 2^+ state, such a component is about 80% [11].

Up to about 6 MeV in ^{12}Be, all the known states can be described as having zero, one, or two neutrons in the sd shell. Many states of those structures remain to be identified [12,13], including 0^- and 2^- single-particle states, at least two 0^+ states, and two 2^+ states, plus perhaps more negative-parity states. The 0^- and 2^- states were not observed in the ^{10}Be(t,p) reaction [8,9] because of their un-natural parity. The 0^+ and 2^+ states referred to are expected to be weak in that reaction because of destructive interference between major components of their wave functions [14] or because their structure is more complicated than ^{10}Be(g.s.) $\times$ $\nu(sd)^2$ [13].

Very little is known about states in ^{12}Be above 6 MeV. Some evidence exists for a special set of states beginning near 10 MeV that have large cluster overlaps with α+^{8}He and/or ^{6}He + ^{6}He [15–20]. In ^{12}Be + p inelastic scattering, Korsheninnikov *et al.* [17] observed new levels of ^{12}Be at $E_x = 8.6, 10$, and $\sim$14 MeV and suggested "likely these levels have the cluster structure He + He." In the ^{9}Be(^{15}N,^{12}N) reaction at 240 MeV, Bohlen *et al.* [20] observed several strong states at high excitation. Primarily based on a $J(J+1)$ dependence of energies, they suggested J^π values of (0^+) to (8^+) (even J only) for peaks at 6.7, 7.4, 10.7, 14.6, and 21.7 MeV. Quite recently, Yang *et al.* [19] presented convincing evidence that a state at 10.3 MeV has $J^\pi = 0^+$ and exhibits an enhanced monopole transition matrix element of 7(1) fm^2 to the g.s. They performed an inelastic breakup experiment with 29-MeV/nucleon ^{12}Be incident on a carbon target. The J^π assignment was made on the basis of an angular correlation analysis in the α + ^{8}He channel. Two additional states at 12.1 and 13.6 MeV were suggested as candidates for 2^+ and 4^+ states, respectively, of the same configuration, but no J^π assignments were made for them.

Such cluster states have been predicted in a variety of theoretical models, including the generalized two-center cluster model [21–23], the generator coordinate method [24], and antisymmetrized molecular dynamics [25]. In many of these calculations, the resonances are described as α-4n-α clusters.

II. CALCULATIONS AND RESULTS

Here, I investigate whether these states can be understood in the simple shell model as having four neutrons in the sd shell, i.e., ^{8}Be $\times$ $\nu(sd)^4$ (where υ stands for neutron). Such states should exist somewhere in ^{12}Be. The simplest $\nu(sd)^4$ nucleus is ^{20}O, whose low-lying states are well described as having this structure [26,27]. Another set of states of the $(sd)^6(1p)^{-2}$ structure has also been identified. The ^{18}O(t,p) reaction [27] was instrumental in testing the ^{20}O shell-model wave functions. If these so-called cluster states in ^{12}Be indeed have the structure of $\nu(sd)^4$, we might expect their relative energies to be similar to those in ^{20}O. There should be some differences because the $1/2^+$ state is 1.78 MeV below the $5/2^+$ in ^{11}Be [28], whereas in ^{17}O [29], the $1/2^+$ is 0.87 MeV above the $5/2^+$. Thus, the s^2 configuration gains about 5 MeV from ^{20}O to ^{12}Be, relative to d^2. [Throughout, I use s and d to denote $2s_{1/2}$ and $1d_{5/2}$ respectively.] This means that the lowest 0^+,2^+, and 4^+ $\nu(sd)^4$ states in ^{12}Be would be primarily s^2d^2, whereas in ^{20}O [30], they are mostly d^4. Of course, the 6^+ state will contain no s^2 substructure in either case. This $\nu(sd)^4$ configuration has no states with J above 6. As only three 6^+ states exist in this space, they will not be very collective.

In Fig. 1, for $J^\pi = 0^+$,2^+, and 4^+, I have plotted excitation energies vs $J(J+1)$ for three sets of states: the experimental levels of ^{20}O [26], the $\nu(sd)^4$ shell-model states, and the proposed new cluster states in ^{12}Be [19]—with 10.3 MeV subtracted. The similarity in energy pattern is striking. Earlier, Freer *et al.* [15,16] had suggested higher-J cluster states, specifically (4^+),(6^+), and (8^+) at excitation energies of 13.2, 16.1, and 20.9 MeV in ^{12}Be. These were only suggested J^π's, even though the angular distribution shapes did imply J higher than 0 or 2. A later experiment by Charity *et al.* [31] did not see any of these states. Yang *et al.* [19] postulated that Freer *et al.* did not observe the lower-J states because of the lack of forward-angle data.

These suggested higher-J states are plotted in Fig. 2 along with the others. A few observations are obvious: No 6^+ state is known in ^{20}O; the shell-model 6^+ state is about 4 MeV above the suggested 6^+ state in ^{12}Be, but it is at about the same energy as the suggested 8^+. Additional states can be produced by coupling $\nu(sd)^4$ to the first 2^+ of ^{8}Be, making an

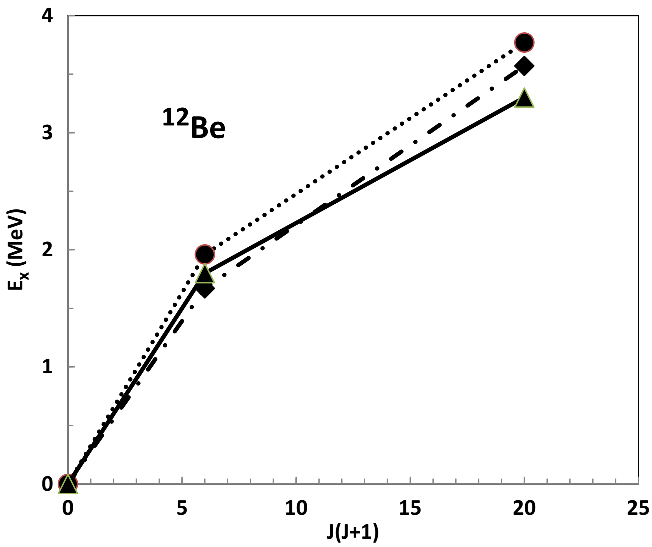

FIG. 1. For 0^+,2^+, and 4^+, excitation energies are plotted vs $J(J+1)$ for three sets of states: diamonds and dot-dashed line: experimental energies in ^{20}O; circles and dotted line: calculated shell-model energies in ^{20}O; triangles and solid line: suggested cluster states [19] in ^{12}Be after subtracting 10.3 MeV.

8^+ state and producing a 6^+ at lower energy than the one based on the g.s. of ^{8}Be. This 6^+ state is only about 1 MeV above the suggested 6^+, but a state, such as $^8\mathrm{Be}(2^+) \times \nu(sd)^4$ might not have the same decay pattern as one based on the g.s. It is probably more likely that the proposed 8^+ state is really 6^+. It would be very useful to have more definitive information on the angular momentum of the Freer resonances. Thus, I conclude that the relative energies of the proposed cluster states in ^{12}Be are in good agreement with those expected for the $\nu(sd)^4$ configuration, and I suggest that the 8^+ cluster state is 6^+. If the state at 16.1 MeV is 6^+, its configuration is probably $^8\mathrm{Be}(2^+) \times \nu(sd)^4_4$.

Now, I attempt a weak-coupling estimate of the absolute energy of the lowest $\nu(sd)^4$ state in ^{12}Be. In the weak-coupling procedure of Bansal and French [32] and Zamick [33], the energy of an m-particle n-hole state is written

$$E(mp - nh) = E(mp) + E(nh) + amn + bT_p \cdot T_h + cm_\pi n_\pi,$$

where a and c are the average nuclear and Coulomb particle-hole interaction energies and m_π and n_π are proton particles and proton holes, respectively. The isospin term is present if both the particles and the holes have nonzero isospin. For nuclei near a major shell closure, the E's are taken to be mass excesses (MEs) [34] relative to the closed-shell nucleus (^{16}O in the present case) (see Table I). The simplest version of weak coupling is not expected to work very well in the Be isotopes because the internal structure and absolute binding energy of any $\nu(sd)^n$ configuration are considerably different in O and Be nuclei. This problem can be overcome in an average way by adding a correction energy term Δ to the mass excess equation.

TABLE I. Mass excesses (MeV) [34] relevant to weak-coupling calculations.

Nucl.	ME	Nucl.	ME
^{20}O	3.796(1)	^{14}Be	39.95(13)
^{18}O	−0.783	^{12}Be	25.078(2)
^{16}O	−4.737	^{10}Be	12.607
		^{8}Be	4.942

Before addressing the $\nu(sd)^4$ states in ^{12}Be, I examine the lowest $\nu(sd)^2$ state in ^{10}Be, located at 6.18 MeV. In weak coupling, its excitation energy is given by the expression,

$$E_x[^8\mathrm{Be} \times \nu(sd)^2 \text{ in } ^{10}\mathrm{Be}] = \mathrm{ME}(^{18}\mathrm{O}) + \Delta_2 + \mathrm{ME}(^8\mathrm{Be}) - \mathrm{ME}(^{16}\mathrm{O}) - \mathrm{ME}(^{10}\mathrm{Be}) + 16a,$$

where Δ_2 is the energy correction for the $\nu(sd)^2$ configuration. Inserting the mass excesses, the right-hand side becomes $-3.71\,\mathrm{MeV} + \Delta_2 + 16a$. Equating this to 6.2 MeV leads to the result $\Delta_2 + 16a = 9.9$ MeV.

The excitation energy of the lowest $\nu(sd)^4$ state in ^{12}Be is

$$E_x[^8\mathrm{Be} \times \nu(sd)^4 \text{ in } ^{12}\mathrm{Be}] = \mathrm{ME}(^{20}\mathrm{O}) + \Delta_4 + \mathrm{ME}(^8\mathrm{Be}) - \mathrm{ME}(^{16}\mathrm{O}) - \mathrm{ME}(^{12}\mathrm{Be}) + 32a,$$

where now Δ_4 is the energy correction term for the $\nu(sd)^4$ configuration. It seems reasonable to assume $\Delta_4 \sim 2\Delta_2$ so that the right-hand side becomes $-11.60\,\mathrm{MeV} + 2(\Delta_2 + 16a)$. Using 9.9 MeV from above for the term in parentheses leads to the prediction of 8.2 MeV for the lowest $\nu(sd)^4$ state—not very different from the suggested energy of 10.3 MeV. Relaxing the condition $\Delta_4 \sim 2\Delta_2$ could improve the agreement, but a prediction that agrees with the absolute energy to within about 2 MeV is rather remarkable for such a simple model.

Before tackling the $\nu(sd)^4$ states in ^{14}Be, I address the lowest $\nu(sd)^2$ state in ^{12}Be. Well-established wave functions [6–10] have 68% $\nu(sd)^2$ and 32% p shell for the g.s. and the orthogonal linear combination for the excited 0^+ state at 2.24 MeV. With these wave functions, the $\nu(sd)^2$ energy in ^{12}Be is 0.72 MeV. Its weak-coupling excitation energy is

$$E_x[^{10}\mathrm{Be} \times \nu(sd)^2 \text{ in } ^{12}\mathrm{Be}] = \mathrm{ME}(^{18}\mathrm{O}) + \Delta_2 + \mathrm{ME}(^{10}\mathrm{Be}) - \mathrm{ME}(^{16}\mathrm{O}) - \mathrm{ME}(^{12}\mathrm{Be}) + 12a + b,$$

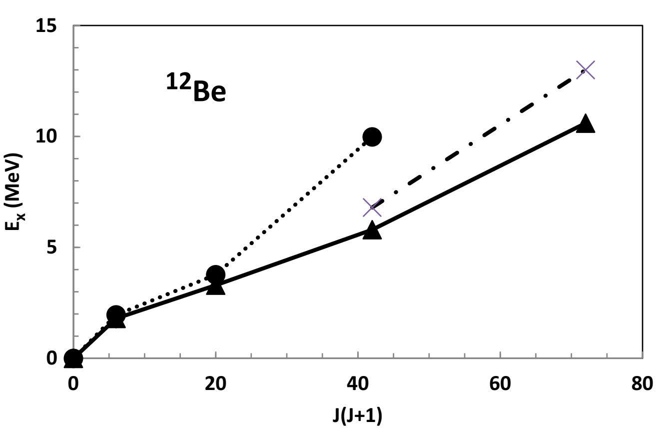

FIG. 2. Same as in Fig. 1 but with 6^+ and 8^+ [15,16] included. Also shown are 6^+ and 8^+ from $J = 2$ core, x's, and dot-dashed line.

where b is the isospin parameter that was missing in the expressions above because the core in both cases there was ^{8}Be, which has $T_h = 0$. In weak coupling, b multiplies $T_p \cdot T_h = (1/2)[T(T+1) - T_p(T_p+1) - T_h(T_h+1)]$. Equating this expression to 0.72 MeV produces the relationship $\Delta_2 + 12a + b = 9.24$ MeV.

The $\nu(sd)^4$ excitation energy in ^{14}Be is

$$E_x[^{10}\mathrm{Be} \times \nu(sd)^4 \text{ in } ^{14}\mathrm{Be}] = \mathrm{ME}(^{20}\mathrm{O}) + \Delta_4 + \mathrm{ME}(^{10}\mathrm{Be}) - \mathrm{ME}(^{16}\mathrm{O}) - \mathrm{ME}(^{14}\mathrm{Be}) + 24a + 2b.$$

The right-hand side of this expression reduces to $= -18.8\,\mathrm{MeV} + \Delta 4 + 24a + 2b$. Again assuming $\Delta_4 \sim 2\Delta_2$ and using the value of $\Delta_2 + 12a + b = 9.24$ MeV from ^{12}Be produces the result $E_x[^{10}\mathrm{Be} \times \nu(sd)^4 \text{ in } ^{14}\mathrm{Be}] = -0.3$ MeV! Recall that the prediction for the $\nu(sd)^4$ state in ^{12}Be was too low by 2.1 MeV. Applying that correction here would put the $\nu(sd)^4$ state at 1.8 MeV in ^{14}Be—still remarkably low. These calculations are only approximate, but it would appear that the lowest $\nu(sd)^4$ state in ^{14}Be may be quite close to the lowest $\nu(sd)^2$ one. Earlier [35,36], I had calculated that the g.s. of ^{14}Be was mostly $\nu(sd)^2$ and suggested that the energy of the first $\nu(sd)^4$ state was near 4 MeV.

Because the g.s. of ^{12}Be is mostly $\nu(sd)^2$, these $\nu(sd)^4$ states should be strong in the reaction ^{12}Be$(t,p)^{14}$Be (in reverse kinematics, of course), just as the ^{20}O states were strong in the ^{18}O(t,p) reaction. If the g.s. of ^{14}Be is a linear combination of $\nu(sd)^2$ and $\nu(sd)^4$ structures, three amplitudes can contribute. These will all be constructive for the g.s. but partially destructive for an excited 0^+ state.

It is an easy matter to understand how these $\nu(sd)^4$ states in ^{12}Be could have large cluster overlaps. Just consider $^8\mathrm{Be} \times \nu^4 = (\alpha \times \alpha) \times \nu^4 = \alpha \times (\alpha \times \nu^4) = \alpha \times (^8\mathrm{He})$. In ^{10}Be [2], we discovered that the so-called supercluster states could be understood as $\nu(sd)^2$ shell-model states.

III. CONCLUSIONS

To summarize: For suggested cluster states beginning at about 10 MeV in ^{12}Be, absolute energy and relative energy spacings [16,19] agree well with calculations for the $^8\mathrm{Be} \times (sd)^4$ configuration. Comparison indicates that the proposed (8^+) state at 20.9 MeV [15,16] is probably 6^+. The 2^+ core state of ^{8}Be is important for 6^+ and 8^+ states. A similar calculation for ^{14}Be predicts that the lowest $(sd)^2$ and $(sd)^4$ states are separated by only about 1.8 MeV, a smaller spacing than the 4 MeV I had estimated earlier.

APPENDIX

Here, I discuss some of the evidence that the g.s. of ^{8}Be is predominantly well described in the $(1p)^4$ shell-model space.

TABLE II. Calculated and experimental spectroscopic factors for single-nucleon transfer leading to the g.s. of ^{8}Be.

Reaction	S_{calc} [a]	S_{expt}
^{7}Li(^{3}He,d)	1.51	1.59 [b]
^{9}Be(p,d)	0.56	0.58 [c]

[a] Reference [43] in the $1p$-shell space.
[b] Reference [44].
[c] Analysis of Ref. [45] using data from Ref. [46].

Of course, this fact is necessary in order for Bansal and French [32] and Zamick [33] to be applicable.

Kanellopoulos and Wildermuth [37] demonstrated that the α-α cluster model description of ^{8}Be is equivalent to the shell-model description with four nucleons in the $1s$ shell and four in $1p$.

In discussing analysis of the β-delayed α spectra from the decay of ^{8}Li and ^{8}B [38], Warburton [39] asserted "It is found that satisfactory fits are obtained without introducing intruder states below 26-MeV excitations." For a time, Barker [40,41] obscured the issue by claiming these data required the presence of an intruder 2^+ state near 9 MeV, but that furor was short lived.

The p-shell shell-model calculations of Cohen-Kurath [42] are in good agreement with observations concerning ^{8}Be. Kumar [43] also performed $(1p)^4$ calculations and found excellent agreement with energies of nine states up to 20 MeV. He pointed out that his computed spectroscopic factors for proton stripping and neutron pickup to the g.s. of ^{8}Be were in excellent agreement with experimental values [44–46] as depicted in Table II. He also pointed out that energies of the 2^+ levels obtained by Bacher *et al.* [47] in α-α elastic scattering were in close agreement with his calculated energies. Furthermore, his calculated proton reduced widths for 2^+ levels were in agreement with those extracted [48] from $^7\mathrm{Li} + p$ data.

Arnold and Seyler [49] calculated that the lowest negative-parity state in ^{8}Be is expected near 18 MeV. Fayache *et al.* [50] in shell-model calculations with up to $4\hbar\omega$ excitations in ^{8}Be found that the lowest calculated positive-parity intruder state was near 18 MeV. Caurier *et al.* [51] performed *ab initio* shell-model calculations in model spaces consisting of up to $10\hbar\Omega$ excitations. They found that, in the $10\hbar\Omega$ space, the lowest 0^+ intruder state was near 18 MeV. They mentioned that their extrapolation suggested the energy of this intruder might stabilize at about 12 MeV. Maris *et al.*, in no-core configuration interaction calculations [52], find the lowest excited 0^+ state at ~20 MeV.

It would thus appear that the use of weak coupling with a p-shell ^{8}Be core is appropriate.

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