Timescale of non-exponential decay for the nuclear β-decay process in a decoherence model

A. Ray* and A. K. Sikdar
Variable Energy Cyclotron Centre, HBNI, 1/AF Bidhannagar, Kolkata-700064, India
(Received 10 June 2016; revised manuscript received 5 October 2016; published 30 November 2016)

The quantum-mechanical time evolution of an isolated β-unstable nuclear state in the context of certain models predicts that the square of the amplitude of the initial undecayed state becomes an approximately exponential function of time on a timescale on the order of 10^{-22}–10^{-21} s. It was argued that a measurement process required to distinguish between the parent and the daughter nuclear states in such a short time would destroy the characteristics of the long-lived β-unstable nuclear state, thus fundamentally restricting the observability of β decay on a short timescale. Since the interaction of the nuclear state with the environment is almost inevitable, we have obtained the timescale of initial non-exponential decay for the nuclear β decay from an estimation of quantum decoherence time considering the atom of the decaying nucleus as a quantum detector. It has been found from such considerations that a decoherence timescale of β decay is on the order of 10^{-16}–10^{-15} s and the decay should remain reversible and non-exponential on this timescale. The possibilities of observing the effect of non-exponential decay in nuclear systems have been discussed.

DOI: 10.1103/PhysRevC.94.055503

I. INTRODUCTION

The exponential decay law is the hallmark of all radioactive decays studied so far. However, according to quantum mechanics, there should certainly be deviations from the exponential decay law at early times [1–5]. It is important to know how long the non-exponential decay would persist so that experiments could be performed to observe and study the time range of non-exponential decay for different systems. However, the estimation of the timescale of non-exponential decay is not straightforward. Quantum mechanics predicts that the time evolution of an isolated unstable state produces a superposition of an initial undecayed state and decayed states at any finite time [1–3]. The quantum-mechanical time evolution of an unstable isolated compound state having an energy form factor of the form of a Lorentzian-like term multiplied by a threshold factor to take care of low-energy behavior predicts that the square of the amplitude of the undecayed state is a non-exponential function of time at an early time and at a later time, becomes an approximately exponential function of time and follows a power-law behavior in the long-time limit [1–3]. In the case of the time evolution of an isolated β-unstable nuclear state, the square of the amplitude of the parent state becomes an approximate exponential function of time after $\sim 10^{-22}$–10^{-21} s in certain model calculations [6–9], and this result generally is interpreted as the onset of exponential decay in a very short time.

However, no decay could be conceived without a measurement process, and the unstable systems would almost always interact with their surroundings. So many authors considered [1,10–13] the onset of the exponential decay as a result of the interaction between the unstable system and the environment. First of all, the initial unstable compound state cannot be formed without a measurement process. An unstable compound state has to be formed initially by a measurement process [1] that would determine whether the fragments forming the compound state are sufficiently close to each other. Moreover, since the time-evolved state would always remain in a linear superposition of an initial undecayed state and a decayed state, an interaction with the environment or a measurement process is required to differentiate between the initial undecayed state and the decayed state and obtain the classical results [1,10,11]. The coupling between the quantum system and its environment leads to a decoherence process [1,2,10,11] resulting in the loss of quantum coherence between the initial state and the decayed state and the onset of the exponential decay of the system. The decoherence process leads to environment-induced superselection or einselection of the pointer states retaining correlations with the rest of the universe [11]. In this paper, we will examine the timescales of non-exponential decay obtained from the time evolution of an isolated β-unstable nuclear state versus decoherence time obtained from the interaction of the β-unstable nuclear state with its host atom in the light of the available experimental data.

II. NON-EXPONENTIAL DECAY

Let us consider the time evolution of a β-unstable state ($|\psi_{\text{undecayed}}\rangle$) from the time $t = 0$ to time t. A β-unstable state could be considered as an unstable bound state of all the β decay products at $t = 0$. Let ($|\psi_{\text{decayed}}(t)\rangle$) denote the β-decayed state (unbound state of decay products) at time t. The time evolution should produce the following superposition of an undecayed initial state and a β-decayed state at any finite time t:

$$e^{-(iHt/\hbar)}|\psi_{\text{undecayed}}(t=0)\rangle = \alpha(t)|\psi_{\text{decayed}}(t)\rangle + \beta(t)|\psi_{\text{undecayed}}(t=0)\rangle, \quad (1)$$

where H is the Hamiltonian of the system. $\alpha(t)$ and $\beta(t)$ are time-dependent complex coefficients. $\hbar = \frac{h}{2\pi}$, where h is Planck's constant. At any time t (when decay could be defined), ($|\psi_{\text{decayed}}(t)\rangle$) satisfies the orthogonality condition $\langle\psi_{\text{undecayed}}(t=0)|\psi_{\text{decayed}}(t)\rangle = 0$. It can be shown [1,2] that

$$\beta(t+t') = \beta(t)\beta(t') + \alpha(t)\langle\psi_{\text{undecayed}}(t=0)| \times \exp\left(-\frac{iHt'}{\hbar}\right)|\psi_{\text{decayed}}(t)\rangle. \quad (2)$$

*ray@vecc.gov.in; amlanray2016@gmail.com

The second term in Eq. (2) is the so-called memory term or reformation amplitude indicating the possibility of regenerating the initial-state ($|\psi_{\text{undecayed}}(t=0)\rangle$) from the decay products, which modifies the classical exponential decay represented solely by the first term. We get $\beta(t) \propto \exp(-\lambda t)$ with $\text{Re}(\lambda) > 0$, only when the second term, i.e., the reformation amplitude of the unstable state from the decayed state becomes zero. However, the reformation amplitude can never be zero [1] from the solution of the time-dependent Schrödinger equation unless the Hamiltonian is unbounded from below, i.e., its eigenvalues could go down to $-\infty$ or the decay is forbidden. The result implies that in quantum mechanics, a decay process is in general time reversible, as expected. Following Khalfin [3], we can write $\beta(t) = \int e^{-iEt/\hbar}\omega(E)dE$ where E is the energy eigenvalues of the Hamiltonian and $\omega(E)$ is energy distribution density of the initial state that does not change with time. Since the Hamiltonian H must be bounded from below, the energy eigenvalue E should have a lower bound say $E_{\text{min}}(E_{\text{min}} > -\infty)$. So following Ref. [1], $\omega(E)$ will be a Lorentzian-like term multiplied by a threshold factor to take care of the behavior as $E \to E_{\text{min}}$. Then for the ιth relative partial wave, we can write [1] $\omega(E) = \frac{\lambda}{2\pi}\frac{(E-E_{\text{min}})^{l+1/2}g(E)}{(E-E_R)^2+1/4\lambda^2}$, where E_R and λ are resonance energy and width of the state respectively and $g(E)$ is a continuous function of energy. It can be shown [1] that for large t,

$$\beta(t) \sim C\ \exp\left(-\frac{iE_Rt}{\hbar}\right)\exp\left(-\frac{1}{2}\frac{\lambda t}{\hbar}\right) + D\exp\left(-\frac{iE_{\text{min}}t}{\hbar}\right)t^{-(l+3/2)}\cdots, \tag{3}$$

where C and D depend on E_R, λ, and $g(E)$. From Eq. (3), we see that at an intermediate time, $|\beta(t)|^2$ would approximately follow an exponential behavior with respect to time with lifetime $\tau = 1/\lambda$ and in the limit of very long t, it would follow a power-law behavior. Khalfin showed [3] that $|\beta(t)|^2$ would dominantly be an exponential function of time when

$$\frac{\lambda t}{\hbar}e^{-\lambda t/\hbar} \gg \left(\frac{\lambda}{E_R}\right)^2\cdots. \tag{4}$$

Winter [4] and Dicus *et al.* [5] performed numerical simulations for the tunneling of a sinusoidal wave function from a specific type of barrier and obtained that the decay of the initial wave function becomes approximately exponential (within a few percent) after about one exponential lifetime. Applying Eq. (4), it was found [8] that for nuclear decays with half-lives ($\tau_{1/2}$) of about 10 h, the exponential decay would start from a time $t \gg 10^{-55}\tau_{1/2}$ after the start of the time evolution. In the case of decay of ^{60}Co ($\tau_{1/2} = 5.2$ yr), it was obtained [7,14] (considering the time evolution of an isolated state in the context of certain models) that the non-exponential component would be equal to the exponential component at a time $t \sim 10^{-29}\tau_{1/2}$, i.e., on the order of 10^{-21} s after the start of the time evolution. In the case of the decay of ^{40}K ($\tau_{1/2} = 4.5 \times 10^9$ yr), a similar calculation [15] predicted that the non-exponential and exponential components would be comparable at a time $t \sim 10^{-38}\tau_{1/2}$ after the start of time evolution.

Avignone [9] considered an isolated γ-emitting nucleus and performed first-order perturbation calculations to determine the time dependence of the transition rate from the initial to the final nuclear eigenstate of the unperturbed Hamiltonian. The transition rate is essentially the product of the square of the γ-ray matrix element (connecting the initial and final nuclear eigenstates) and the phase space factor. The time dependence of the transition rate arises from the time-dependent part of the stationary nuclear eigenstates. It was found [9] that this time-dependent transition rate became approximately independent of time after $\sim 5 \times 10^{-22}$ s and this time ($\sim 5 \times 10^{-22}$ s) was taken as the non-exponential timescale of γ-emission decay. It was assumed [9] that the non-exponential timescale for the β-decay process would be similar. This calculation implicitly assumes that the emission of a γ-ray photon from the nucleus is an irreversible process and there is no reformation amplitude [1,2] of recombining the γ-ray photon with the residual nucleus after the γ-ray photon leaves the nucleus. With these assumptions, Avignone obtained [9] a very short timescale ($\sim 5 \times 10^{-22}$ s) for the onset of an approximate exponential decay.

So far, we have assumed that the unstable quantum system is isolated and evolves undisturbed from the preparation time $t = 0$ to the time when a measurement was performed to determine whether the system has decayed or not. However, a measurement is required to determine whether an unstable system comprising two or more fragments sufficiently close to one another has been formed and define a new $t = 0$ instant that is different from the instant when a scattering reaction to form the compound system was performed because the measurement process would take a certain amount of time. The time evolution of the compound unstable state is meaningful from this new $t = 0$ instant as defined by the measurement process [1]. In order to determine whether a β decay has occurred within a timescale on the order of 10^{-22} s from the start of the time evolution of the β-unstable nuclear state, another measurement has to be performed to distinguish between the undecayed and the decayed states within this timescale. According to the energy-time uncertainty principle, such a measurement would increase the width of the β-emitting nuclear state to about 6 MeV, i.e., many orders of magnitude larger than its typical natural width, thus destroying the characteristics of the β-unstable nuclear state and fundamentally restricting the observability of β decay on a short timescale [12]. So it is not possible to observe the non-exponential β decay on a timescale on the order of 10^{-22} s, even in principle and hence, this timescale should have no physical significance for observing non-exponential β-decay. Moreover, in all practical situations, the unstable quantum system would invariably interact with their surroundings, and such interactions would be equivalent to repeated measurements modifying the decay rate [1,12]. So the interaction of the environment with the quantum state should be considered [1,11–13] in any realistic formalism. In the case of interaction of the decaying state with the environment [1], the reformation amplitude is suppressed and made zero as a result of the interactions of the quantum system with the environment. The measurement process takes a certain amount of time to distinguish between the undecayed and the decayed

states. As long as, it is not possible to distinguish between the undecayed state and the decayed state in principle, the quantum state would remain in a superposition of undecayed and decayed states, and the reformation amplitude would remain nonzero. Hence, the condition for the start of the exponential decay and the classical description of the process is the loss of the quantum coherence between $(|\psi_{\text{undecayed}}(t=0)\rangle)$ and $(|\psi_{\text{decayed}}(t)\rangle)$ as a result of the interaction of the system with the environment. So, the quantum decoherence time would naturally depend on the surrounding environment that interacts with the decaying state. In principle, the interaction of a quantum state with another state (that we call the detector state) should still produce a superposition of the combined states, and they would interact with the environment eventually producing classical correlations [10,11].

III. QUANTUM DECOHERENCE PROCESS

We consider the quantum decoherence process for the nuclear β decay as a two-step process [10,11,16]. In the first step, the entire atom containing the β-particle emitting nucleus acts as a quantum detector and records the event. As a result, after a certain time, the nuclear system couples with the atomic detector system to produce a fully correlated pure nuclear-atom state [10,11]. The diagonal elements of the corresponding density matrix are generally real numbers, and the off-diagonal elements contain complex numbers expressing purely quantum correlations. In the second step, by considering the interaction of the environment with the pure nuclear-atom state, one gets a reduced density matrix by tracing over the environment [10,11,16], and this reduced density matrix contains only classical correlations, thus indicating complete loss of quantum coherence. As the β-particle emission from the parent nucleus takes place, the electrons of the valence orbital of the concerned atom undergo dramatic rearrangement and the chemical property of the atom changes, whereas only minor adjustments take place in the inner atomic orbitals. Since the electronic valence orbitals define the chemistry of the element, valence states of the parent and daughter elements would be considered as orthogonal pointer states [1,10,11] of the atomic detector for the β-decay process. Let $|\psi_{\text{undecayed}}\rangle$ denote the β-unstable state at $t=0$ and $|\psi_{\text{decayed}}(t)\rangle$ denote the β-decayed state (unbound state of decay products) at time t. At any instant t, there would be a superposition of $|\psi_{\text{undecayed}}\rangle$ and $|\psi_{\text{decayed}}(t)\rangle$ because of the time evolution of the initial-state $|\psi_{\text{undecayed}}\rangle$ as shown in Eq. (1). Let $|A\rangle$ denote the atomic state that would act as a detector. We assume that as a result of β emission, primarily the valence state of the atom would be affected. Following Ref. [11], at any time t, the interaction of the undecayed nuclear state with the detector state (atomic state) is given by

$$|\psi_{\text{undecayed}}\rangle|A\rangle \rightarrow |\psi_{\text{undecayed}}\rangle|A_{\text{VP}}\rangle, \tag{5}$$

where $|A_{\text{VP}}\rangle$ denotes the valence electronic state of the parent element.

We consider the nuclear transition time as instantaneous compared to the electronic rearrangement time of the valence electronic orbitals. Considering the change in energy of the electronic valence state of ^{60}Co following β decay, the minimum time (t_V) required to distinguish between the valence orbitals of parent Co and daughter Ni is on the order of 10^{-16} s. Then for $t > t_V$,

$$|\psi_{\text{decayed}}\rangle|A\rangle \rightarrow |\psi_{\text{decayed}}\rangle|A_{\text{VD}}\rangle, \tag{6}$$

where $|A_{\text{VD}}\rangle$ denotes the valence electronic state of the daughter element.

We have the orthogonality condition $\langle A_{\text{VD}}|A_{\text{VP}}\rangle = 0$.

So using Eq. (1), for time $t > t_V$, the combined nucleus-atom state evolves [10,11,16] into a correlated state,

$$|\psi_{\text{correlated}}(t)\rangle = \alpha(t)|\psi_{\text{decayed}}\rangle|A_{\text{VD}}\rangle + \beta(t)|\psi_{\text{undecayed}}\rangle|A_{\text{VP}}\rangle. \tag{7}$$

The corresponding density matrix of the pure state $|\psi_{\text{correlated}}(t)\rangle$ is

$$\begin{aligned}\rho^c(t) &= |\psi_{\text{correlated}}(t)\rangle\langle\psi_{\text{correlated}}(t)| \\ &= |\alpha|^2|\psi_{\text{decayed}}\rangle\langle\psi_{\text{decayed}}||A_{\text{VD}}\rangle\langle A_{\text{VD}}| \\ &\quad + \alpha^*\beta|\psi_{\text{undecayed}}\rangle\langle\psi_{\text{decayed}}||A_{\text{VP}}\rangle\langle A_{\text{VD}}| \\ &\quad + \alpha\beta^*|\psi_{\text{decayed}}\rangle\langle\psi_{\text{undecayed}}||A_{\text{VD}}\rangle\langle A_{\text{VP}}| \\ &\quad + |\beta|^2|\psi_{\text{undecayed}}\rangle\langle\psi_{\text{undecayed}}||A_{\text{VP}}\rangle\langle A_{\text{VP}}|. \end{aligned} \tag{8}$$

Considering the interaction of the correlated state $|\psi_{\text{correlated}}(t)\rangle$ with the environment and tracing over the environment [10,11], only the stable classical correlations are retained, and the reduced density matrix at time $t > t_V$ is given by

$$\begin{aligned}\rho^r(t) &= |\alpha(t)|^2|\psi_{\text{decayed}}\rangle\langle\psi_{\text{decayed}}||A_{\text{VD}}\rangle\langle A_{\text{VD}}| \\ &\quad + |\beta(t)|^2|\psi_{\text{undecayed}}\rangle\langle\psi_{\text{undecayed}}||A_{\text{VP}}\rangle\langle A_{\text{VP}}|. \end{aligned} \tag{9}$$

The reduced density matrix contains only classical correlations implying that the system would be either in the undecayed parent state or in the decayed daughter state. The states $|\psi_{\text{undecayed}}\rangle|A_{\text{VP}}\rangle$ and $|\psi_{\text{decayed}}\rangle|A_{\text{VD}}\rangle$ are the einselected pointer basis as they remain stable in the interaction with the environment and survive the decoherence process [10,11,16]. At a time $t < t_V$, the interaction of the decayed state with the atom will not produce the stable valence state of the daughter element, and the unobservable configurations, such as the electronic configuration of the parent element, hosting the daughter nucleus would be eliminated by tracing over the environment. Since the interaction with the environment would only retain the stable classical states, the time (t_V) required to distinguish between the valence orbitals of the parent and daughter atoms following a β-particle emission from the parent nucleus could be considered as the minimum time required for decoherence. So, decoherence time for the β-decay process is $>t_V$, i.e., on the order of 10^{-16} s. Considering the interaction of the atom with the environment, the time required to select out the stable valence states of the element should be on the order of chemical bond formation time, i.e., $\sim 10^{-15}$ s [17]. Hence, the decoherence time for the β-decay process should be on the order of 10^{-16}–10^{-15} s. It means that a minimum time on the order of 10^{-16}–10^{-15}s is required to distinguish between the two pointer states—parent and daughter elements, and the non-exponential decay should persist during this time (10^{-16}–10^{-15}s). So, our estimated decoherence time for the

nuclear β decay is many orders of magnitude longer than the non-exponential timescale (10^{-22}–10^{-21} s) [6–9] expected from the time evolution of an isolated β-unstable nuclear state.

IV. EXPERIMENTAL RESULTS

In the case of radioactive decay, no deviation from the exponential decay law has been seen so far. Norman *et al.* [14,15] carried out high precision tests of the decay law of radioactive nuclei at an early time and did not find any deviation from the exponential decay law down to $10^{-4}\tau_{1/2}$ time for ^{60}Co (half-life ≈ 5.3 yr) and $10^{-10}\tau_{1/2}$ time for ^{40}K (half-life $\approx 1.25 \times 10^{9}$ yr) nuclei. Although Norman *et al.* [14,15] monitored the decay curve at an early time that is a very small fraction of the half-life of the radioactive nucleus, they actually started monitoring the decay curve a few hours after the formation of the radioactive nuclei. Hence, clearly, it is not expected that Norman *et al.* [14,15] would find any deviation from the exponential decay law on the basis of the estimated timescales for the onset of the exponential decay obtained earlier. The observations of Norman *et al.* appear to be consistent with the timescales obtained from both the time evolution of an isolated nuclear state [6–9] as well as with our decoherence model where the interaction with the environment causes the onset of the exponential decay. The timescales for non-exponential decay obtained either from the considerations of the time evolution of an isolated nuclear state or the decoherence model are too short to be measured by the β-decay counting experiments performed so far [14,15], and so those experiments cannot distinguish between these two theoretical timescales for non-exponential decay, although the two timescales differ from each other by many orders of magnitude.

So the question is whether there is any way to come to a more definitive conclusion regarding the timescale of the expected initial non-exponential decay for the nuclear β decays. In this context, it could be useful to discuss the tunneling of ultracold sodium atoms from an atom trap where non-exponential decay and associated phenomena had been seen. About 10^5 sodium atoms were trapped and cooled in a periodic optical potential created by a standing wave of light [18]. The Gaussian width of the energy distribution of the sodium atoms was about 3×10^{-9} eV [18,19]. An accelerating potential of the form $V = V_0 \cos[2k_L x - K_L a t^2]$, where V_0, a, x, t, and k_L are well depth, acceleration, position in laboratory frame, time, and laser wave number, respectively, was applied and the tunneling of the sodium atoms from the trapped states to the continuum took place. The survival probability of sodium atoms in the trap was measured as a function of the duration of the applied acceleration. It was found that the survival probability followed a non-exponential curve as a function of the duration of the acceleration time for about 5 to 6 μs and then followed an exponential curve. The tunneling process took place as a result of exchange of energy between the reservoir and the trapped atoms. The energy was taken out from the reservoir to accelerate the trapped atoms and the very low-energy sodium atoms tunneled out and gave back the energy to the reservoir. The applied acceleration separated out the trapped and tunneled out sodium atoms by their energies, and so the trapped and tunneled out sodium atoms could be distinguished. As long as the trapped and tunneled out sodium atoms could not be resolved within a certain time window by their energy difference, even in principle (i.e., by applying the energy-time uncertainty principle), the coupling between the sodium atom and the reservoir remained reversible, and the decay was non-exponential within that time window. If an acceleration of 7000 m/s^2 is applied for 1 μs, then the energy difference between the tunneled out atoms and the trapped atoms would be about 6×10^{-12} eV and from the energy-time uncertainty principle, a minimum time of about 100 μs would be required to distinguish between the trapped atom and the tunneled out atom. So, the trapped and tunneled out atoms could not be resolved within 1 μs, and it was necessary to apply the acceleration for 5 to 6 μs so that the energy difference between the trapped and the tunneled out sodium atoms became sufficiently large to distinguish them within a time period of 5 to 6 μs. Kofman and Kurizki [12,13] included the effect of an external reservoir in the analysis of the transition rate between the eigenstates of a Hamiltonian in the framework of a time-dependent Schrödinger equation and found that the decay process becomes irreversible and classical on a timescale when the decayed and undecayed states could unambiguously be distinguished as a result of interaction with the environment. Fischer *et al.* [19] performed a similar experiment using ultracold sodium atoms in a magneto-optical trap but interrupted the accelerating potential after every 1 μs for a long time (of about 50 μs) and found that the tunneling of the trapped atoms was severely inhibited (quantum Zeno effect). The results implied that the acceleration for 1 μs was not sufficient to distinguish between the trapped atom and the tunneled atom within this short time period of 1 μs and a new $t = 0$ time was defined after each long interruption followed by a slow non-exponential decay. Hence, the results [18,19] of the tunneling of ultracold sodium atoms from a magneto-optical trap clearly demonstrate the importance of the interactions between the atoms and the environment for the onset of exponential decay.

Let us now try to perform a similar analysis in the case of the β-decay process. Suppose a nuclear reaction is performed to produce a β-unstable nucleus at $t = 0$ instant. However this instant has no physical significance. A measurement process has to be performed to determine whether a β-unstable nuclear state with the required characteristics has been formed. The measurement process actually projects out [1] a wave function with the required localization characteristics of the compound state and defines a new $t = 0$ instant when the time evolution of the compound state could be considered. Then another measurement process has to be performed to distinguish between the parent state and the daughter state and determine whether a decay has occurred. As discussed before, the timescale of non-exponential decay appears to be on the order of 10^{-22}–10^{-21} s in certain models [6–9] if the β-unstable nuclear state is evolving without any interaction with the environment. However, in order to see β decay within 10^{-22}–10^{-21} s, a measurement has to be performed within this timescale to distinguish between the parent and the daughter nuclear states. The corresponding energy uncertainty or spread of the width of the β-unstable nuclear state would be on the order of several million electron volts, thus effectively

destroying the characteristics of the long-lived β-unstable nuclear state [12]. The energy spread could result in the daughter nuclear state at a higher energy than the parent nuclear state, thus energetically forbidding the decay [12]. Hence, in the case of the decay of a β-unstable nucleus, such as ^{60}Co ($\tau_{1/2} \approx 5.3$ yr), it is not possible to observe non-exponential decay on the timescale of 10^{-22} s, even in principle, and so this timescale should have no physical significance for observing the non-exponential β-decay process.

On the other hand, the nuclear state would invariably couple with the corresponding host atom. So, in the case of estimating the decoherence time of the nuclear β decay, we have considered the host atomic system as a detector and the interaction of the atomic system with the environment produces classical correlations [11]. The emitted high-energy ($\sim$MeV) β-particle and antineutrino or the recoiled daughter nucleus do not immediately lose their energies in the vicinity of the nucleus and travel much longer distances before depositing their energies. However, the presence of the daughter nucleus would dramatically alter the valence state of the initial parent atom and its chemical properties, thus unambiguously recording a β-decay process. So the valence states of the parent and daughter elements have been considered as orthogonal pointer states [1,11]. Since there would be significant overlaps between the wave functions of the inner orbitals of parent and daughter elements, and they do not interact with the environment, the inner electronic orbitals cannot be orthogonal pointer states [1,11]. The time required to distinguish between the two pointer states, namely, the parent and daughter elements that would maintain their identities after interaction with the environment, has been taken as the non-exponential decay time in analogy [11,18,19] with the tunneling of ultracold sodium atoms from a magneto-optical trap. Repeated measurements to distinguish between the valence electronic states of parent and daughter elements on the timescale of 10^{-16}–10^{-15} s would modify the exponential lifetime of a β-unstable nucleus slightly ($\approx$1%) as discussed by Fonda *et al.* [1] and Kofman and Kurizki [12]. However, the measurements on the atomic valence states should not significantly alter the characteristics of the nuclear state, and so the observation of β decay on the timescale of 10^{-16}–10^{-15} s is possible in principle. Hence, β decay should remain reversible and non-exponential on the timescale on the order of 10^{-16}–10^{-15} s, i.e., the minimum time required to distinguish between the pointer states (parent and daughter elements).

We have considered that an atom records a β-decay process when its valence electrons rearrange to a new valence state corresponding to a new element as a result of β-particle emission from the nucleus. However what would happen if the atom containing the β-particle emitting nucleus is stripped of all electrons or contains only the inner orbital electrons that would not undergo very significant changes due to β decay from the nucleus? In such a situation, the atom does not have orthogonal pointer states that could distinguish between the parent and the daughter nuclear states and hence, the atom hosting the nucleus cannot be considered as a detector. So the β-decayed state would have to interact with more distant detectors, and decoherence time would be longer. However, in the case of a β-decay process, decoherence time would generally remain many orders of magnitude shorter than the timescale of exponential decay, and it would not be feasible to directly probe non-exponential decay. Considering repeated measurements of the nuclear charge by the orbiting valence electron inhibiting the time evolution of the decaying nuclear state, Fonda *et al.* [1] obtained that the measured nuclear lifetime would be slightly longer ($\sim$1%) than the theoretical lifetime. Since the theoretical lifetime is generally not known very accurately, it is difficult to perform this comparison. However, if the atom containing the β unstable nucleus is subjected to a high pressure to substantially reduce the energy difference between the valence states of parent and daughter atoms, it would take a longer time to distinguish between the parent and daughter elements resulting in a shorter nuclear lifetime under compression. So, applying high pressure on the atom containing the β-unstable nucleus, one might study the effect of quantum decoherence time on the β-decay process. In the case of proton or α emission from a medium-to-heavy mass nucleus, the valence electronic state of the corresponding atom could be considered as a pointer state similar to the β-decay process. The expected α emission time of highly excited ($E_X > 20$ MeV) α-particle-emitting states from medium mass nuclei, such as produced by the fusion of ^{28}Si $+^{164,167,170}$Er [20] could be on the order of 10^{-20} s, whereas the decoherence time would be on the order of 10^{-16} s based on the time required to distinguish between the valence states of the parent and daughter atoms. Hence, if one performs an α-particle-K x-ray coincidence measurement and could obtain characteristic K x-ray lines of a Pb atom produced by a Si + Er reaction, then that would indicate that the compound nuclei have survived much longer than 10^{-20} s because the lifetime of the K vacancy in a Pb atom is on the order of 10^{-18} s, implying the possible presence of a long non-exponential decay time.

V. SUMMARY

To summarize, we have estimated the quantum decoherence time of nuclear β-decay processes by considering the interaction of the undecayed and decayed nuclear states with the atomic electronic orbitals. Quantum decoherence time depends on the environment surrounding the decaying state because the coherence between the undecayed and the decayed states is lost as a result of interaction with the environment. Our estimate of the quantum decoherence time of the β-decay process ($\sim 10^{-16}$–10^{-15} s) is many orders of magnitude longer than the estimates of the timescale of non-exponential decay (on the order of 10^{-22}–10^{-21} s) obtained from the time evolution of an isolated β-decaying nuclear state [6–9] but consistent with the observations of Norman *et al.* [14,15]. We have argued that it would not be possible to observe β decay on the timescale of 10^{-22} s even in principle because the measurement process in such a short time would increase the width of the long-lived β-unstable nuclear state by many orders of magnitude compared to its natural width, thus, effectively destroying the state and fundamentally restricting the observability of the β decay in a very short time. The question of observing non-exponential decay has been discussed in the context of tunneling of ultracold sodium atoms from a magneto-optical trap. We have discussed experiments that could see the effect of the quantum docoherence time in radioactive decays.

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