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Hadronic weak charges and parity-violating forward Compton scattering

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Background: Parity-violating elastic electron-nucleon scattering at low momentum transfer allows one to access the nucleon's weak charge, the vector coupling of the Z-boson to the nucleon. In the Standard Model and at tree level, the weak charge of the proton is related to the weak mixing angle and accidentally suppressed, $Q_W^{p,\,\mathrm{tree}} = 1 - 4\sin^2\theta_W \approx 0.07$. Modern experiments aim at extracting Q_W^p at $\sim$1 % accuracy. Similarly, parity nonconservation in atoms allows to access the weak charge of atomic nuclei.

Purpose: We consider a novel class of radiative corrections due to the exchange of two photons, with parity violation in the hadronic/nuclear system. These corrections are prone to long-range interactions and may affect the extraction of $\sin^2\theta_W$ from the experimental data at the relevant level of precision.

Methods: The two-photon exchange contribution to the parity-violating electron-proton scattering amplitude is studied in the framework of forward dispersion relations. We address the general properties of the parity-violating forward Compton scattering amplitude and use relativistic chiral perturbation theory to provide the first field-theoretical proof that it obeys a superconvergence relation.

Results: We show that the significance of this new correction increases with the beam energy in parity-violating electron scattering, but the superconvergence relation protects the formal definition of the weak charge as a limit at zero-momentum transfer and zero energy. We evaluate the new correction in a hadronic model with pion loops and the $\Delta(1232)$ resonance, supplemented with a high-energy contribution. For the kinematic conditions of existing and upcoming experiments we show that two-photon exchange corrections with hadronic or nuclear parity violation do not pose a problem for the interpretation of the data in terms of the weak mixing angle at the present level of accuracy.

Conclusions: Two-photon exchange in presence of hadronic or nuclear parity violation gives rise to long-range parity-violating interactions. Depending on the kinematic conditions and precision goal, this novel correction may affect the extraction of weak charges from experiments with atoms and electron scattering.

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I. INTRODUCTION

Experimental studies of parity-violating (PV) neutral current interactions offer a possibility for a precise determination of the parameters of the Standard Model (SM) and constrain possible contributions of physics beyond the Standard Model (BSM) [1]. Of particular interest is parity-violating electron scattering (PVES) with electron beams with energies of a few hundred MeV to a few GeV at low-momentum transfer, and PV interactions of atomic electrons with atomic nuclei. The weak charge of the proton, the vector coupling of the neutral Z boson to the proton, which is accidentally suppressed in SM, $Q_W^p \approx 0.07$, was pointed out to be a sensitive probe of BSM [2]. A precise measurement of this quantity with elastic PVES at a low-momentum transfer is the subject of the Q-Weak experiment at Jefferson Lab [3] and at Mainz [4] with the new MESA facility. An interpretation of these experiments in favor or disfavor of a BSM signal requires a precise account of SM radiative corrections of order $O(\alpha)$, with $\alpha \approx 1/137$ the fine structure constant. The original analysis of radiative corrections to the weak charges was tailored for atomic PV [5,6], but was updated in Ref. [7] for the PVES case. More recently, Ref. [8] pointed out an additional, dispersion γZ-box correction that exhibits a steep energy dependence: While absent in the conditions of atomic PV experiments, it was shown to reach several percent of Q_W^p in PVES. This contribution was actively studied by several groups [9–17].

The PV γZ-box correction arises from the generalized γZ-interference Compton scattering on a hadronic target. In this work we study a novel effect: the contribution of the parity-violating electromagnetic Compton process to the elastic PV electron-proton (or electron-nucleus) scattering amplitude via two-photon exchange. The source of parity violation in a purely electromagnetic reaction can be hadronic parity-violating interactions or admixtures of levels of opposite parity in an atom or a nucleus. This contribution was not studied before in the context of PVES. We provide estimates for this effect in the kinematics of the upcoming experiments.

This article is organized as follows. In Sec. II we define the context and the formalism in which the PV two-photon exchange is studied and sketch the mechanism that can lead to an enhancement. Section III considers the contribution of the nucleon anapole moment to the weak charge. In Sec. IV we derive a sum rule for the leading logarithmic term in the low-momentum transfer expansion, originating from real PV Compton scattering amplitude. The properties of

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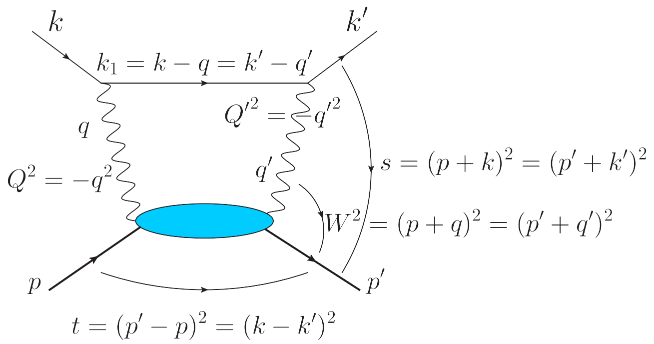

FIG. 1. The two-boson exchange diagram ($\gamma\gamma$ or γZ) with the relevant kinematic variables.

this amplitude, most notably the superconvergence relation are considered in Sec. V. We prove the superconvergence relation in relativistic chiral perturbation theory and construct a self-consistent model of PV Compton amplitude in Sec. VI. Finally, we present results and discuss their consequences for running and upcoming PVES and atomic PV experiments in Sec. VII. We provide technical details of the calculation in Appendix A.

II. GENERAL FRAMEWORK

We consider the elastic scattering process $e(k) + N(p) \to e(k') + N(p')$. It will be helpful to use the electron energy E defined in the laboratory system, i.e., in the rest frame of the target nucleon. The total energy squared in the ep system is then given by $s = (p+k)^2 = M^2 + 2ME$ and the momentum transfer by $t = (k-k')^2 = (p'-p)^2 < 0$. Where possible, we will neglect the electron mass m_e, but keep the nucleon mass M, i.e., $m_e^2 \ll M^2, s$. The scattering regime corresponds to the range $s \geqslant (M+m_e)^2 \approx M^2$ and $-(s-M^2)^2/s \leqslant t \leqslant 0$.

As a starting point we recapitulate the calculation of the γZ box in the limit of forward scattering. Here we have to evaluate a loop integral with intermediate nuclear or hadronic states with arbitrary mass W. The relevant kinematic variables are shown in Fig. 1. We parametrize the momentum of the virtual boson q^μ in the laboratory frame by $q^\mu = (\nu, \vec{q})$ with the help of the usual variable $\nu = (pq)/M$. We then have $\vec{q}^{\,2} = \nu^2 + Q^2$ and $W^2 = (p+q)^2 = M^2 + 2M\nu - Q^2$. The hadronic mass W takes its minimal value at the pion production threshold $W_\pi^2 = (M+m_\pi)^2$.

Following Refs. [8,11] we write for the forward nucleon or nuclear spin-independent amplitude (i.e., in the limit $t=0$ and $Q'^2 = Q^2$),

$$\begin{aligned}\mathrm{Im}\, T_{\gamma Z}(E, t=0)\\ = -\frac{e^2 G_F}{\sqrt{2}} \int \frac{d^4 k_1}{(2\pi)^4} 2\pi \delta\big(k_1^2 - m_e^2\big) \frac{2\pi W_{\gamma Z}^{\mu\nu} L_{\mu\nu}^{\gamma Z}}{Q^2\big(1 + Q^2/M_Z^2\big)}. \quad (1)\end{aligned}$$

Here G_F is the Fermi constant and the hadronic tensor is given by

$$\begin{aligned} W_{\gamma Z}^{\mu\nu} = &\left(-g^{\mu\nu} - \frac{q^\mu q^\nu}{Q^2}\right) F_1^{\gamma Z} \\ &+ \frac{1}{(pq)}\left(p + \frac{(pq)}{Q^2} q\right)^\mu \left(p + \frac{(pq)}{Q^2} q\right)^\nu F_2^{\gamma Z} \\ &+ \frac{i\varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(pq)} F_3^{\gamma Z}. \quad (2)\end{aligned}$$

The structure functions $F_i^{\gamma Z}$ are functions of the Lorentz scalars Q^2 and ν. G_F is the Fermi constant; M_Z the mass of the Z boson. The leptonic tensor is given by

$$L_{\mu\nu}^{\gamma Z} = \bar{u}(k)\gamma_\mu(\not{k}_1 + m_e)\gamma_\nu\big(g_V^e - g_A^e \gamma_5\big)u(k). \quad (3)$$

In the SM and at tree level, the weak and axial electron charges are $g_V^e = -1 + 4\sin^2\theta_W$ and $g_A^e = -1$, respectively. Performing the tensor contraction and working out the Dirac algebra one can separate the result into a vector and an axial-vector part,

$$\begin{aligned}\mathrm{Im}\,\Box_{\gamma Z}^V(E,0) = &\,\alpha \int_{W_\pi^2}^{s} \frac{dW^2}{(2ME)^2} \int_0^{Q_{\max}^2} \frac{dQ^2}{1 + Q^2/M_Z^2} \\ &\times \left[F_1^{\gamma Z} + \frac{s\big(Q_{\max}^2 - Q^2\big)}{(W^2 - M^2 + Q^2)Q^2} F_2^{\gamma Z}\right], \\ \mathrm{Im}\,\Box_{\gamma Z}^A(E,0) = &\,\alpha \int_{W_\pi^2}^{s} \frac{dW^2}{(2ME)^2} \int_0^{Q_{\max}^2} \frac{dQ^2}{1 + Q^2/M_Z^2} \\ &\times \left[\frac{2(s - M^2)}{W^2 - M^2 + Q^2} - 1\right] F_3^{\gamma Z}, \quad (4)\end{aligned}$$

which are combined to give the full γZ-box correction as

$$\mathrm{Im}\, T_{\gamma Z} = -\frac{G_F}{\sqrt{2}} \bar{u}\not{p}\gamma_5 u\big[g_A^e \mathrm{Im}\,\Box_{\gamma Z}^V + g_V^e \mathrm{Im}\,\Box_{\gamma Z}^A\big]. \quad (5)$$

The superscripts V and A indicate the vector and axial-vector Z coupling to the nucleon, respectively. We note that the on-shell condition for the intermediate electron required in the calculation of the imaginary part of the box graph limits the maximal value of the photon's virtuality to $Q_{\max}^2 = (s - M^2)(s - W^2)/s$ for a fixed value of W^2 which, in turn, may vary between W_π^2 and s.

Because of the finite threshold for pion production, $W_\pi = M + m_\pi > M$, the above integrals do not contain IR (soft photon) singularities. Nevertheless, collinear singularities may occur, if the nearly massless electron is emitting a real energetic photon. However, the analysis of the above equations shows that for the γZ box such singularities are absent because the structure function $F_2^{\gamma Z}$ vanishes at the real photon point, i.e., at $Q^2 = 0$.

The real parts of the corrections $\Box_{\gamma Z}^{A,V}$ are reconstructed using forward dispersion relations,

$$\begin{aligned}\mathrm{Re}\,\Box_{\gamma Z}^V(E,0) &= \frac{2E}{\pi} \mathcal{P} \int_{E_\pi}^\infty \frac{dE'}{E'^2 - E^2} \mathrm{Im}\Box_{\gamma Z}^V(E',0), \\ \mathrm{Re}\,\Box_{\gamma Z}^A(E,0) &= \frac{2}{\pi} \mathcal{P} \int_{E_\pi}^\infty \frac{dE' E'}{E'^2 - E^2} \mathrm{Im}\Box_{\gamma Z}^A(E',0), \quad (6)\end{aligned}$$

where $\mathcal{P}$ in front of the integrals stands for the principal value prescription. The corrections $\Box_{\gamma Z}^{V,A}$ have been extensively studied in the literature.

In the present work we are interested in assessing a similar correction that is associated with the exchange of two photons between the electron and the nucleon or nucleus, while parity violation occurs in the hadronic-nuclear system. Quite straightforwardly, we obtain in the forward limit,

$$\mathrm{Im}\, T_{\gamma\gamma}^{\rm PV} = e^4 \int \frac{d^4k_1}{(2\pi)^4} 2\pi\delta\big(k_1^2 - m_e^2\big) \frac{2\pi\, {}^{\rm PV}W_{\gamma\gamma}^{\mu\nu} L_{\mu\nu}^{\gamma\gamma}}{Q^4}, \quad (7)$$

with $L_{\mu\nu}^{\gamma\gamma} = \bar{u}(k)\gamma_\mu(\not{k}_1 + m_e)\gamma_\nu u(k)$. The leptonic tensor will contain an antisymmetric, spin-dependent part for the case of polarized electron scattering. The PV hadronic spin-independent forward Compton tensor has only one term,

$${}^{\rm PV}W_{\gamma\gamma}^{\mu\nu} = \frac{i\varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(pq)} F_3^{\gamma\gamma}. \quad (8)$$

Contributions to the structure function $F_3^{\gamma\gamma}$ can arise because of the interference of the PV and parity-conserving (PC) $\gamma N\Delta$ interaction, in the presence of PV πNN couplings, or from a mixing of two closely lying nuclear levels of equal spin but opposite parity. We define the box correction according to[1]

$$T_{\gamma\gamma}^{\rm PV} = e^2 \bar{u}\not{p}\gamma_5 u\, \Box_{\gamma\gamma}^{\rm PV}. \quad (9)$$

It can immediately be seen that the forward PV 2γ box will contain a collinear singularity because of the fact that there is an extra photon propagator compared with the case of $\Box_{\gamma Z}^{A}$,

$$\mathrm{Im}\,\Box_{\gamma\gamma}^{\rm PV}(E) = \alpha \int_{W_\pi^2}^{s} \frac{dW^2}{2(2ME)^2} \int_{Q^2_{\rm min}}^{Q^2_{\rm max}} \frac{dQ^2}{Q^2} \times \left[\frac{2(s-M^2)}{W^2 - M^2 + Q^2} - 1\right] F_3^{\gamma\gamma}, \quad (10)$$

where the upper and lower limits of the integral over Q^2 are

$$Q^2_{\rm max} \approx \frac{(s-W^2)(s-M^2)}{s}, \qquad Q^2_{\rm min} = \frac{m_e^2(W^2-M^2)^2}{s Q^2_{\rm max}}, \quad (11)$$

and m_e^2 can be neglected in the expression for $Q^2_{\rm max}$. The leading contribution is finite because of the finiteness of the electron mass and a finite threshold, $W \geqslant M + m_\pi$, separating the excited hadronic states from the ground state,

$$\sim \int_{Q^2_{\rm min}}^{Q^2_{\rm max}} \frac{dQ^2}{Q^2} = \ln \frac{(s-M^2)^2(s-W^2)^2}{m_e^2 s(W^2-M^2)^2}, \quad (12)$$

but possibly large because it contains a logarithm of the electron mass.

If the intermediate state is the ground state, i.e., for $W = M$, an infrared divergence does not appear because the elastic contribution to $F_3^{\gamma\gamma}$ vanishes for real photons. We address this elastic contribution in detail in the following section.

III. ELASTIC CONTRIBUTION: ANAPOLE MOMENT

To calculate the box-graph contribution with a proton in the intermediate state, we start with a study of Compton scattering. PV can appear in Compton scattering from an explicit PV term in the Lagrangian of the form,

$$\mathcal{L}_{\rm PV} = ie\, a_0 \partial_\mu F^{\mu\nu} \bar{N}\gamma_\nu\gamma_5 N. \quad (13)$$

The origin of this term lies in electroweak corrections at the single quark level (thus calculable at one loop in the SM), as well as multiquark contributions. These latter give rise to the anapole moment, the main source of the uncertainty in the value of a_0. We can identify a_0 with a correction to the axial charge of the proton G_A, which appears when a process with a charged lepton is compared with the corresponding neutrino process according to

$$a_0 = \frac{G_F}{8\pi\alpha\sqrt{2}} g_V^e(0)\, \delta G_A^{ep}, \quad (14)$$

where the weak charge of the electron $g_V^e(0) = -(1 - 4\sin^2\theta_W(0)) \approx -0.0712(7)$ will be taken at zero-momentum transfer in the $\overline{\rm MS}$ scheme. The axial charge of the proton G_A^{ep} can be found from the recent analysis in Refs. [18–20],

$$G_A^{ep}(Q^2) = G_a(Q^2)\Big[G_A\big(1 + R_A^{T=1}\big) + \frac{3F - D}{2} R_A^{T=0} + \Delta s\big(1 + R_A^{(0)}\big)\Big] \equiv G_a(Q^2)\big[G_A + \delta G_A^{ep}\big]. \quad (15)$$

The value of the axial charge, $G_A = -1.2701(25)$, is known from the free neutron β decay [21]. The baryon octet parameters F and D can be obtained from neutron and hyperon β decays with the assumption of SU(3) symmetry, $3F - D = 0.58(12)$. $\Delta s = -0.07(6)$ is the strange quark contribution to the nucleon spin, and can be deduced from polarized deep inelastic scattering data assuming that its Q^2 dependence from DGLAP evolution can be neglected [19]. The radiative corrections to the isovector, isoscalar, and SU(3) singlet hadronic axial vector amplitudes, respectively, are $R_A^{T=1} = -0.258(340)$, $R_A^{T=0} = -0.239(200)$, $R_A^{(0)} = -0.55(55)$ [18]. These quantities arise from several sources: Alongside the so-called one-quark contribution which corresponds to the one-loop renormalization of the Standard Model electron-quark couplings C_{2q} [22], multiquark effects, such as the anapole moment, and coherent strong interaction mechanisms contribute. Combining these numbers and adding errors in quadrature gives $\delta G_A^{ep} = 0.23(43)$, corresponding to a shift and uncertainty of the modulus of G_A by $-18(35)\%$. This

[1]Note that because of the normalization by the electromagnetic coupling e^2, the quantity $\Box_{\gamma\gamma}^{\rm PV}$ has dimension 1/energy^2, while $\Box_{\gamma Z}^{V,A}$, normalized by G_F, is dimensionless.

leads to

$$a_0 = -(0.74 \pm 1.38) \times 10^{-6}\,\mathrm{GeV}^{-2}. \tag{16}$$

The Q^2-dependent axial form factor is assumed to follow a dipole form,

$$G_a(Q^2) = \frac{1}{\left(1 + Q^2/M_A^2\right)^2}, \tag{17}$$

where $M_A \sim 1.02$ GeV, consistent with the world PVES data [19].

Interference of the PV vertex derived from Eq. (13) with the PC electromagnetic vertex,

$$\Gamma^\mu_{\mathrm{em}}(q) = F_1(Q^2)\gamma^\mu + F_2(Q^2) i\sigma^{\mu\beta}\frac{q_\beta}{2M}, \tag{18}$$

with the Dirac and Pauli form factors $F_{1,2}$ leads to the following expression for the elastic contribution to the PV structure function $F_3^{\gamma\gamma}$,

$$F_3^{\gamma\gamma} = a_0 G_a(Q^2) G_M(Q^2)\, 2M\nu Q^2\, \delta(2M\nu - Q^2), \tag{19}$$

where $G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$ is the nucleon magnetic form factor. Inserting this expression for $F_3^{\gamma\gamma}$ into Eq. (10) leads to the elastic contribution to $\mathrm{Im}\,\Box^{\mathrm{PV,\,el}}$,

$$\mathrm{Im}\,\Box^{\mathrm{PV,\,el}}_{\gamma\gamma}(E,t=0) = \frac{4\pi\alpha^2}{2ME} a_0 \int_0^{\frac{4M^2E^2}{M^2+2ME}} dQ^2 G_M(Q^2) G_a(Q^2)\left(2 - \frac{Q^2}{2ME}\right). \tag{20}$$

The real part is obtained from a forward dispersion relation,

$$\begin{aligned}\mathrm{Re}\,\Box^{\mathrm{PV,\,el}}_{\gamma\gamma}(E,t=0) &= \frac{2}{\pi}\mathcal{P}\int_0^\infty \frac{E'dE'}{E'^2 - E^2}\mathrm{Im}\,\Box^{\mathrm{PV}}_{\gamma\gamma}(E',t=0)\\ &= \frac{4\alpha^2 a_0}{ME}\int_0^\infty dQ^2 G_M(Q^2) G_a(Q^2)\left[\ln\left|\frac{E+E_Q}{E-E_Q}\right| + \frac{Q^2}{2ME}\ln\left|1 - \frac{E^2}{E_Q^2}\right|\right].\end{aligned} \tag{21}$$

We have changed the order of integration and performed the integral over E' analytically, using the abbreviation $E_Q = (Q^2 + \sqrt{Q^2(Q^2+4M^2)})/(4M)$. This result is infrared finite and analogous to the expression for the elastic contribution to $\Box^A_{\gamma Z}$.

The effect of including $\Box^{\mathrm{PV,\,el}}_{\gamma\gamma}$ along with $\Box^{A,\,\mathrm{el}}_{\gamma Z}$ can easily be obtained from the latter by a shift of the proton's axial charge $G_A \to G_A + \delta G_A^{ep}$. This leads to a reduction of $\Box^{A,\,\mathrm{el}}_{\gamma Z}$ by 18%, accompanied by an uncertainty of 44% of the corrected value of $G_A + \delta G_A^{ep}$. We show the correction of the effective weak charge of the proton resulting from these box-graph contributions as a function of the electron energy in Fig. 3. The discussion above shows in a transparent way how this uncertainty originates from uncertainties in the data. This is one of the important results of this work.

Our result can be compared to previous evaluations of the elastic contribution to the $\Box^A_{\gamma Z}$ correction. In Refs. [5,6], this correction was evaluated at $E = 0$ and applied in an analysis of PV in atoms. The result was adopted without further modification for PVES in Ref. [7]. The authors of Ref. [23] observed, however, a considerable energy dependence of $\Box^A_{\gamma Z}$, as is visible in the energy behavior of the black curve of Fig. 3. Their result corresponds to the one-loop accuracy: Upon cutting the left graph of Fig. 2, the subgraphs corresponding to the Z^0 and γ exchanges are taken at tree level. The parameters of the SM that serve as input for a one-loop calculation may be significantly modified when one-loop effects are added on top of the tree-level amplitudes. We note here that the inclusion of such higher-order corrections formally exceeds the one-loop accuracy, yet the choice to include one-loop corrections in the determination of the values of SM parameters is often made. This does not pose a problem per se because once the full two-loop result is obtained, the respective two-loop corrections included in the one-loop result can be removed to avoid double counting.

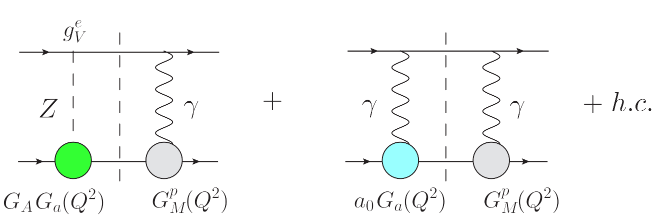

FIG. 2. A schematic representation of the elastic contribution to the imaginary part of the two-boson exchange correction to the elastic PVES amplitude. The left and right parts show the γZ and PV $\gamma\gamma$ contributions, respectively. The vertical dashed line cutting through the diagrams indicates that the intermediate *ep* state is on-shell.

Recently, Blunden *et al.* [12] proposed such a prescription taking into account the one-loop running of $\sin^2\theta_W$ and α. This results in a smaller value of g_V^e and a reduction of the previous result of Marciano and Sirlin [5,6] by 17%. Note that because of the presence of nucleon form factors, the loop integral is only sensitive to $g_V^e(Q^2)$ at $Q^2 \lesssim 1$ GeV^2 where the scale dependence is negligible, $g_V^e(Q^2) \approx$ const. Blunden *et al.*'s result is fairly well represented by the red curve in Fig. 3. The choice made in Ref. [12] is not unique but is a viable one, as explained above.

Another possible choice would be to use the full one-loop result for the elastic PVES amplitude, i.e., for the left side of the box diagrams shown in Fig. 2. This would include the tree-level diagram, the running of $\sin^2\theta_W$ and of α, plus further terms, most notably the WW- and ZZ-box graphs, and finally, the PV γNN vertex, also formally a one-loop effect. From the point of view of dispersion relations this choice is more natural: If we decide to partially include two-loop effects at least to the elastic box, this can be achieved by using the full one-loop result for the PV elastic ep-scattering amplitude inside the box. This is the choice that we pursue here. Numerically, the WW and ZZ boxes are known to largely cancel the effect of the running of $\sin^2\theta_W$ in the product $g_V^e G_A$ [through $g_V^e(M_Z) \to g_V^e(0)$]. In turn, the additional contribution from the induced PV γNN vertex leads to a suppression of the proton's axial charge, $G_A \to G_A + \delta G_A^{ep}$. As a result, our central value (18% reduction with respect to Marciano and Sirlin's result at $E = 0$) is very close to that of Ref. [12], but allows for a data-driven estimate of the uncertainty of our calculation. This is the main reason for our proposal to include these effects in the one-loop calculation. Unfortunately, hadronic PV effects are largely unconstrained, and this leads to an increased uncertainty represented by the shaded area in Fig. 3. Future electron scattering and atomic PV experiments may help taming this uncertainty.

The actual kinematics of the P2 experiment at MESA will not be at forward scattering but at scattering angles $\sim 25^\circ$, yet at a very low-momentum transfer $-t \sim 0.005$ GeV^2. The correction from this finite momentum transfer is expected to be of the order $Q^2 R_M^2/3$, with R_M standing for the relevant nucleon size (magnetic or axial). With the magnetic radius $R_M \approx 0.77$ fm the effect of such a finite size correction would be a few percent relative to the result for forward scattering. This is quite comfortably within the large $\sim$44% uncertainty from δG_A. We will address the explicit t dependence of the elastic contribution in upcoming work.

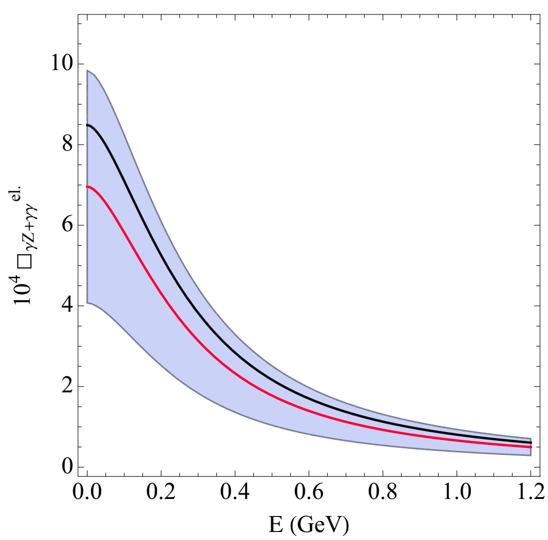

FIG. 3. Correction $\Box^{\rm el}_{\gamma Z+\gamma\gamma} = \Box^{A,\rm el}_{\gamma Z} + \Box^{\rm PV,el}_{\gamma\gamma}$ to the effective weak charge of the proton in units of 10^{-4} in the exact forward limit and as a function of the electron energy in GeV. The black curve shows the result for $\Box^{A,\rm el}_{\gamma Z}$ with $G_A = -1.2701$. The red curve is our new central value obtained from the sum $\Box^{A,\rm el}_{\gamma Z} + \Box^{PV,\rm el}_{\gamma\gamma}$. The shaded region corresponds to the uncertainty from the proton's anapole moment.

We end this section with a comment regarding the contribution of the nuclear anapole moment to the nuclear weak charge via two-photon exchange. This can be obtained by evaluating Eq. (21) at $E = 0$ and in the limit of a heavy nuclear mass M. If we assume the nuclear form factors to only depend on the nuclear size R roughly as $G(Q^2) \sim \exp(-R^2Q^2/6)$ and employ the definition of Eq. (14), we arrive at

$$\delta Q_W^{\rm Nucl} \sim -\frac{4\sqrt{3}}{\sqrt{\pi}}\frac{Z\alpha}{MR}\frac{g_V^e}{g_A^e}\mu_N\,\delta G_A^{\rm anapole}, \tag{22}$$

where μ_N is the nuclear magnetic moment in units of the nuclear magneton and $\delta G_A^{\rm anapole}$ is the contribution of the nuclear anapole moment to the nuclear axial charge normalized to the axial charge from the exchange of a Z-boson $G_A^{\rm NC}$. For a numerical estimate, e.g., for the case of ^{133}Cs consisting of 55 protons and 78 neutrons, $G_A^{\rm NC} \approx 55g_A^p + 78g_A^n \approx -23G_A \approx 29$. Unlike for a single nucleon where the anapole moment may reduce the axial charge by some 30%, for nuclei it is expected to dominate over the standard Z exchange by an order of magnitude [24], so we assume $\delta G_A^{^{133}\rm Cs,\,anapole} \sim 300$ for the sake of a rough estimate. Putting numbers together, we arrive at the naïve expectation $\delta Q_W^{\rm Nucl} \lesssim 10^{-3}$. This contribution can be safely neglected.

IV. INELASTIC CONTRIBUTION

To account for inelastic contributions and correctly calculate the leading t behavior of the box graph at low t, we follow the method laid out in Refs. [25,26] where the 2γ-exchange correction in the parity-conserving case was considered. The method consists of taking the form of the hadronic tensor in the exact (i.e., nonforward) form. The PV tensor $\sim\epsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{2(pq)}F_3^{\gamma\gamma}$ is extended beyond the forward limit in the following gauge-invariant form:

$$^{\rm PV}\widetilde{W}^{\mu\nu}_{\gamma\gamma} = \frac{1}{4(Pq)^2}[(Pq)i\varepsilon^{\mu\nu\alpha\beta}P_\alpha(q+q')_\beta - P^\nu i\varepsilon^{\mu\alpha\beta\gamma}q_\alpha q'_\beta P_\gamma - P^\mu i\varepsilon^{\nu\alpha\beta\gamma}q_\alpha q'_\beta P_\gamma]\widetilde{F}_3^{\gamma\gamma}, \tag{23}$$

where $P = (p+p')/2$, and $p'(q')$ stand for the final nucleon (photon) momenta, respectively, with $t = (q-q')^2 = (p'-p)^2$. In the forward limit, i.e., for $q' = q$, $P = p$, this tensor reduces to the forward one, Eq. (8). After tensor contraction, the off-forward result for the box correction reads

$${\rm Im}\,\Box^{\rm PV}_{\gamma\gamma}(E,t) = \frac{\alpha}{2\pi}\int\frac{E_1^{\rm cm}dE_1^{\rm cm}}{s-M^2}\int\frac{d\Omega}{Q^2Q'^2} \times\left[\frac{Q^2+Q'^2}{2} + \frac{E_1^{\rm cm}t}{2E^{\rm cm}}\right]\frac{(P,k+k_1)}{(Pq)}\widetilde{F}_3^{\gamma\gamma}(\nu,Q^2,Q'^2,t). \tag{24}$$

The center-of-mass (c.m.) energies are given by $E^{\rm cm} = (s-M^2)/(2\sqrt{s})$ and $E_1^{\rm cm} = (s-W^2)/2\sqrt{s}$, respectively. The nonforward amplitude $\widetilde{F}_3^{\gamma\gamma}(\nu,Q^2,Q'^2,t)$ is assumed to be an analytic function of t, thus

$$\widetilde{F}_3^{\gamma\gamma}(\nu,Q^2,Q'^2,t)\big|_{t\to 0} = F_3^{\gamma\gamma}(\nu,Q^2) + O(t), \tag{25}$$

where we used the fact that inside the loop $Q^2 = Q'^2$ for $t = 0$. Furthermore, also analyticity in Q^2 at $Q^2 = 0$ is assumed,

$$F_3^{\gamma\gamma}(\nu,Q^2)\big|_{Q^2\to 0} = F_3^{\gamma\gamma}(\nu,0) + O(Q^2). \tag{26}$$

For the leading t-behavior associated with $F_3^{\gamma\gamma}(\nu,0)$ under the integral, i.e., keeping terms $\sim \ln|t|$ and t-independent terms, we obtain

$$\mathrm{Im}\,\square_{\gamma\gamma}^{\mathrm{PV}}(E,t) = \alpha\int_{E_\pi}^{E}\frac{d\omega F_3^{\gamma\gamma}(\omega,0)}{4ME^2}\left\{\left[\frac{2E}{\omega}-1\right]\ln\frac{(s-M^2)^2}{-st} - \frac{2E}{\omega}\ln\left[1+\frac{(s-M^2)(s-W^2)}{s(W^2-M^2)}\right]\right\}, \tag{27}$$

where $E_\pi = [(M+M_\pi)^2 - M^2]/2M$ denotes the pion production threshold. The real part of the box is obtained from a dispersion relation at fixed $t \approx 0$, analogous to that for $\square_{\gamma Z}^{A}$ in Eq. (6). Changing the order of integration, the integral over E' can be carried out analytically,

$$\mathrm{Re}\,\square_{\gamma\gamma}^{\mathrm{PV}}(E,t) = \frac{\alpha}{\pi M}\int_{E_\pi}^{\infty}\frac{d\omega}{\omega^2}F_3^{\gamma\gamma}(\omega,0)G(E,\omega,t), \tag{28}$$

with the auxiliary function G given by

$$\begin{aligned} G(E,\omega,t) = {}& \frac{\omega}{2E}\ln\left|\frac{E+\omega}{E-\omega}\right|\ln\left(\frac{4E^2}{|t|\left(1+\frac{2E^2}{M\omega}\right)}\right) + \frac{\omega^2}{4E^2}\ln\left|1-\frac{E^2}{\omega^2}\right|\ln\left(\frac{2M\omega}{|t|}\right)\\ &-\frac{\omega^2}{4E^2}\left[\mathrm{Li}_2\left(\frac{1+\frac{M}{2\omega}}{1-\frac{M}{2E}}\right) - \mathrm{Li}_2\left(\frac{1}{1-\frac{M}{2E}}\right) + \mathrm{Li}_2\left(\frac{1+\frac{M}{2\omega}}{1+\frac{M}{2E}}\right) - \mathrm{Li}_2\left(\frac{1}{1+\frac{M}{2E}}\right) + \frac{1}{2}\mathrm{Li}_2\left(\frac{E^2}{\omega^2}\right)\right]\\ &+\frac{\omega}{2E}\mathrm{Re}\left[\mathrm{Li}_2\left(\frac{1+\frac{E}{\omega}}{1+i\sqrt{\frac{2E^2}{M\omega}}}\right) + \mathrm{Li}_2\left(\frac{1+\frac{E}{\omega}}{1-i\sqrt{\frac{2E^2}{M\omega}}}\right) - \mathrm{Li}_2\left(\frac{1-\frac{E}{\omega}}{1+i\sqrt{\frac{2E^2}{M\omega}}}\right) - \mathrm{Li}_2\left(\frac{1-\frac{E}{\omega}}{1-i\sqrt{\frac{2E^2}{M\omega}}}\right)\right]. \end{aligned} \tag{29}$$

In the limit of vanishing electron energy, $E \to 0$, the box correction can be cast in a more elegant form,

$$\mathrm{Re}\,\square_{\gamma\gamma}^{\mathrm{PV}}(0,t) = \frac{3\alpha}{4\pi M}\int_{E_\pi}^{\infty}\frac{d\omega}{\omega^2}F_3^{\gamma\gamma}(\omega,0)\left[\ln\left[\frac{4\omega^2}{-t\left(1+\frac{2\omega}{M}\right)}\right] + \frac{7}{3} - \frac{4\omega^2}{3M^2}\left[\ln\left(1+\frac{M}{2\omega}\right) - \frac{M}{2\omega}\right] - \frac{8}{3}\sqrt{\frac{2\omega}{M}}\arctan\sqrt{\frac{M}{2\omega}}\right]. \tag{30}$$

We see that the collinear divergence from the loop integral gives rise to terms $\sim \ln(4E_\pi^2/|t|)$.

V. GENERAL PROPERTIES OF THE PV REAL COMPTON AMPLITUDE

Analyzing Eq. (30) obtained in the previous section, we notice that the inclusion of this correction in the analysis of PVES at low-momentum transfer is in conflict with the conventional definition of the weak charge. Usually the polarization asymmetry measured in a PVES experiment modified to include the energy-dependent dispersive box-graph corrections $\square_{\gamma Z}(E)$ [11] is used to define the weak charge by writing

$$Q_W^p = \lim_{E,t\to 0}\frac{A^{\mathrm{PV}}_{\mathrm{measured}}}{A_0^{\mathrm{PV}}}. \tag{31}$$

However, Eq. (30) signals the appearance of a new term $\sim \ln(|t|)$ in the one-loop expression:

$$\begin{aligned} A^{\mathrm{PV}} = A_0^{\mathrm{PV}}\bigg[&Q_W^{p,\,1\text{-loop}} + tB(t) + \mathrm{Re}\,\square_{\gamma Z}(E)\\ &- \frac{4\sqrt{2}\pi\alpha}{G_F}\mathrm{Re}\,\square_{\gamma\gamma}^{\mathrm{PV}}(E,t)\bigg], \end{aligned} \tag{32}$$

and the singular logarithmic t-dependent term in Eq. (30) does not vanish in the zero-energy limit. This would represent a general setback for the formalism of extracting the weak charge from PVES, because the presence of such a term, no matter small or large, would prevent one from connecting the measured asymmetry at a finite value of t to the tree-level coupling defined at $t = 0$. Even though the apparent divergence is regularized by a finite electron mass (this is in fact a collinear, not an infrared divergence), the presence of this correction would have serious consequences not only for the analysis of PVES, but also for atomic PV experiments. We take a step back and consider the general properties of the PV forward real Compton scattering amplitude to prove that this catastrophic scenario is not realized.

The dispersion representation for the real part of the forward PV real Compton amplitude T_3 generically reads

$$T_3^{\gamma\gamma}(\nu,0) = \frac{2\nu}{\pi}\int_{\nu_{\mathrm{thr}}}^{\infty}\frac{d\nu'}{\nu'^2-\nu^2}F_3^{\gamma\gamma}(\nu',0), \tag{33}$$

where ν_{thr} is the inelastic threshold, e.g., the pion production threshold for a nucleon target, or a nuclear excitation threshold for atomic nuclei. The above dispersion relation is a consequence of Lorentz and gauge invariance, crossing symmetry, and the high-energy asymptotic behavior of $F_3^{\gamma\gamma}(\nu\to\infty) <$

$C\nu^d$ with $d < 1$. On the other hand, the low-energy expansion (LEX) of the amplitude $T_3^{\gamma\gamma}(\nu,0)$ at $\nu \to 0$ starts at $O(\nu^3)$. The Lagrangian density that corresponds to the tensor in Eq. (23) reads

$$\partial_\alpha \bar{N}\gamma^\beta N F^{\alpha\mu}\widetilde{F}_{\beta\mu}, \tag{34}$$

but Eq. (23) contains a conventional $\sim 1/\nu^2$ prefactor introduced to comply with the definition of the inelastic PV structure function $F_3^{\gamma Z,\gamma\gamma}$. We consider now the low-energy behavior of an amplitude accompanying the operator in Eq. (34). Because PV does not occur when real photons couple to an on-shell nucleon (the anapole moment requires virtual photons), this amplitude cannot have negative powers of energy. Secondly, the operator in Eq. (34) is odd under photon crossing $q \leftrightarrow -q'$, so also the amplitude multiplying it has to be an odd function of ν. Together with the conventional $1/\nu^2$ pre-factor this leads to the requirement that $T_3^{\gamma\gamma}(\nu \to 0,0) = O(\nu^3)$. This implies a superconvergence relation

$$\int_{\nu_{\rm thr}}^{\infty} \frac{d\nu}{\nu^2} F_3^{\gamma\gamma}(\nu,0) = 0\,, \tag{35}$$

which is simply a consequence of the fact that the linear term in the LEX of $T_3^{\gamma\gamma}$ vanishes. The superconvergence relation Eq. (35) has been stated already some time ago in the literature [27,28].

This property of $T_3^{\gamma\gamma}$ and $F_3^{\gamma\gamma}$ is of great importance for model estimates of the PV 2γ-box. First of all, analyzing the forward limit of $\Box_{\rm PV}^{\gamma\gamma}$ we notice that the coefficient multiplying the divergent $\ln t$ term has to vanish at $E = 0$ according to the superconvergence relation. This means that the definition of the weak charge in the limit $E \to 0, t \to 0$ is safe, and radiative corrections only modify it by constant contributions which can be calculated and removed from the measured observable. This said, the logarithmic t-behaviour will still be present at finite energies and may be non-negligible compared to the precision of relevant PVES experiments. Also nuclear resonance contributions to $F_3^{\gamma\gamma}$ need to be studied to understand whether they are relevant, or irrelevant, for the analysis of atomic PV experiments in terms of nuclear weak charges.

With this in mind we proceed with a study of the superconvergence relation of Eq. (35) in a consistent, relativistic field theory framework.

VI. SUPERCONVERGENCE RELATION FOR $T_3^{\gamma\gamma}$ IN RELATIVISTIC CHPT

An analog of the superconvergence relation of Eq. (35) is the Gerasimov-Drell-Hearn (GDH) sum rule for the parity-conserving spin-dependent amplitude, that relates the value of the anomalous magnetic moment of a fermion to an integral over its excitation spectrum [29,30]. The validity of this sum rule has been checked for an electron in perturbation theory in QED, to order $O(\alpha)$ in Refs. [31,32] and $O(\alpha^3)$ in Ref. [32]. Recently, a proof of the GDH sum rule for the nucleon was provided in relativistic Chiral Perturbation Theory (ChPT) [33].

Note that in the heavy-baryon version of ChPT (HBChPT) that uses an additional $1/M$ expansion (M is the nucleon mass), the sum rule does not hold [34], because the heavy-baryon approximation alters the high-energy behavior of the cross sections. It should come as no surprise that a check of the superconvergence relation of Eq. (35) in Ref. [28] using the HBChPT results of Refs. [35,36] had a negative outcome. Therefore we proceed in the next section with a proof of the superconvergence relation of Eq. (35) in relativistic ChPT.

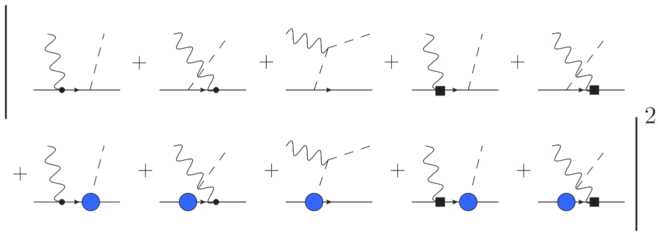

FIG. 4. Tree-level Feynman diagrams in BChPT needed to calculate the integrand of Eq. (35). Small black dots indicate the photon coupling to the nucleon charge, solid black squares denote couplings to the anomalous magnetic moment, and large blue circles describe the PV πNN coupling.

A. Baryon χPT

The relevant part of the PC πN Lagrangean is given by [37]

$$\mathcal{L}_{\pi N}^{\rm PC} = \frac{g_A}{2f_\pi}\bar{N}\tau^a \,\partial\!\!\!/\pi^a\gamma_5 N = -g_{\pi NN}\bar{N}\tau^a\gamma_5 N\pi^a, \tag{36}$$

with π^a denoting the pion field, a vector in the isospin space with isospin index a, τ^a the isospin matrix, and the isodoublet of nucleon bi-spinors $N = \binom{p}{n}$. The Goldberger-Treiman relation $g_{\pi NN} = g_A(M/f_\pi)$ was used in the right part of Eq. (36). The pseudoscalar coupling is obtained from the pseudovector one by means of a chiral rotation of the nucleon field [37] and is fully equivalent to the usual ChPT. As a consequence of the field redefinition the contact coupling $\gamma\pi NN$ is relegated to a higher order in the chiral expansion. This leads to a reduction of the number of diagrams in the lowest order calculation.

At lowest order, the PV pion-nucleon coupling has no derivatives and is given by [20,38]

$$\mathcal{L}_{\pi N}^{\rm PV} = \frac{h_\pi^1}{\sqrt{2}}\bar{N}[\vec{\tau}\times\vec{\pi}]^3 N = -ih_\pi^1(\bar{n}\pi^+ p - \bar{p}\pi^- n). \tag{37}$$

All further terms are of higher order in the chiral expansion. Finally, the nucleon electromagnetic interaction contains terms determined by the charge and the anomalous magnetic moment,

$$\mathcal{L}_{\gamma N} = ie\bar{N}\left[\frac{1+\tau_3}{2}\gamma^\mu A_\mu + (\kappa_S + \tau_3\kappa_V)\frac{i\sigma^{\mu\nu}F_{\mu\nu}}{4M}\right]N, \tag{38}$$

with $\kappa_{S,V} = \frac{1}{2}(\kappa^p \pm \kappa^n)$ the isoscalar and isovector combinations of the proton and neutron anomalous magnetic moments, A^μ the electromagnetic field, and $F^{\mu\nu}$ the electromagnetic field-strength tensor.

The Feynman diagrams that contribute are displayed in Fig. 4. Details of the calculation of the pion production

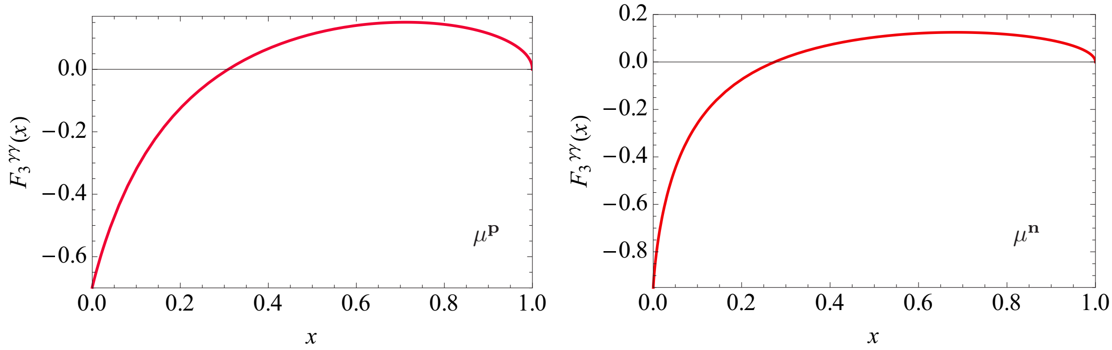

FIG. 5. The contributions of the terms proportional to the anomalous magnetic moments of the proton $\mu^p = 1 + \kappa^p$ (left) and of the neutron $\mu^n = \kappa^n$ (right) to the integrand of Eq. (41) in arbitrary units.

contribution to the PV structure function $F_3^{\gamma\gamma}$ are given in the appendix. Here we display the final result:

$$\begin{aligned} F_3^{\gamma\gamma}(\nu,t=0) &= -\frac{g_{\pi NN} h_\pi^1 q_\pi}{2\sqrt{2}\pi^2\sqrt{s}}\left\{(1+\kappa^p)\left[\frac{E'}{\sqrt{s}} - \frac{E_\pi}{q} + \frac{m_\pi^2}{2qq_\pi}\ln\frac{E_\pi+q_\pi}{E_\pi-q_\pi}\right]\right. \\ &\quad -\kappa^n\left[-\frac{E}{q} + \frac{m_\pi^2}{2qq_\pi}\ln\frac{E_\pi+q_\pi}{E_\pi-q_\pi} + \frac{M^2}{2qq_\pi}\ln\frac{E'+q_\pi}{E'-q_\pi}\right] \\ &\quad \left. -\frac{qE'}{2M^2}\kappa_V\kappa_S + (\kappa^n)^2\frac{s-M^2}{4M^2}\left[-\frac{E'}{q} + \frac{M^2}{2qq_\pi}\ln\frac{E'+q_\pi}{E'-q_\pi}\right]\right\}, \end{aligned} \tag{39}$$

with $s = M^2 + 2M\nu$. The kinematic variables are defined in the c.m. frame of the γp initial state as

$$E = \frac{s+M^2}{2\sqrt{s}}, \quad E' = \frac{s+M^2-m_\pi^2}{2\sqrt{s}}, \quad q = \frac{s-M^2}{2\sqrt{s}}, \quad E_\pi = \frac{s-M^2+m_\pi^2}{2\sqrt{s}}, \tag{40}$$

and the magnitude of the three vector of the pion is $q_\pi = \sqrt{E_\pi^2 - m_\pi^2}$. We are now in a position to check whether the superconvergence relation, rewritten in terms of the dimensionless variable $x = E_\pi/\nu \in (0,1]$,

$$\int_0^1 dx\, F_3^{\gamma\gamma}(E_\pi/x, t=0) = 0, \tag{41}$$

holds. The result of Eq. (39) contains three terms: the proton charge (as part of the full magnetic moment of the proton), and linear and quadratic terms in the anomalous magnetic moments. Numerical integration leads to exactly zero for the first two terms. This cancellation is illustrated in Fig. 5 for the terms linear in μ_p (left) and μ_n (right): The total area under the curve is zero. We did not try to prove Eq. (41) analytically, but we find that numerically the cancellation is obtained to any desired precision.

The terms in Eq. (39) which are quadratic in the anomalous magnetic moment do not obey the superconvergence relation because $F_3^{\gamma\gamma}[\kappa_V\kappa_S, (\kappa^n)^2](\nu \to \infty) \sim \nu$ and the respective integral diverges. This result is also not quite unexpected because the magnetic coupling contains a derivative that affects the high-energy behavior. Moreover, in ChPT the anomalous magnetic moment scales as $g_{\pi NN}^2$, so the problematic terms are proportional to $\sim g_{\pi NN}^5$. However, our tree-level calculation of the sum rule integral is not complete at this order and we expect that missing higher-order terms should restore the superconvergence relation.

B. Model for $F_3^{\gamma\gamma}$ with pions, Δ, and a high-energy background

For numerical estimates of the effect of $\Box_{\gamma\gamma}^{\rm PV}$ on the extraction of the weak charge from PVES experiments, we can use the result of Eq. (39) where we only keep the terms linear in magnetic moments. This is self-consistent in terms of the superconvergence relation and is expected to provide the dominant contribution at low energies. To extend the model of $F_3^{\gamma\gamma}$ to higher energies, we include the Δ isobar. In addition, to ensure validity of the superconvergence relation and provide reasonable estimates for energies beyond the Δ region we also include a simple Regge-like background. The following chiral effective Lagrangian terms for the PC and PV interaction of the Δ are used [39–41]:

$$\begin{aligned} \mathcal{L}_{\rm PC}^{\gamma N\Delta} &= i\sqrt{\frac{3}{2}}\frac{eg_M}{M(M+M_\Delta)}\bar{\Delta}_\alpha\tau_3 N p_{\Delta\beta}\widetilde{F}^{\alpha\beta}, \\ \mathcal{L}_{\rm PV}^{\gamma N\Delta} &= i\frac{e}{\Lambda_\chi}[d_\Delta^+\bar{\Delta}_\alpha^+\gamma_\beta\, p + d_\Delta^-\bar{\Delta}_\alpha^-\gamma_\beta\, n]F^{\alpha\beta}, \end{aligned} \tag{42}$$

with $M_\Delta = 1.232$ GeV. The PC magnetic $\gamma N\Delta$ coupling, $g_M = 3.03$, is taken from Ref. [41]. The chiral symmetry breaking scale is taken as $\Lambda_\chi = 1$ GeV. In what follows we assume purely isovector coupling constants $d_\Delta^+ = -d_\Delta^-$. A measurement by the G0 collaboration [42] with π^- production on a deuteron target has found $|d_\Delta^-| = (3.1 \pm 9.1) \times 10^{-7}$. A straightforward calculation leads to

$$\begin{aligned} F_{3\,\Delta}^{\gamma\gamma}(\omega,0) &= \sqrt{\frac{2}{3}}\frac{2g_M d_\Delta^+}{\pi\Lambda_\chi(M+M_\Delta)}(W^2-M^2)^2 \\ &\quad \times {\rm Im}\frac{1}{W^2-M_\Delta^2-iW\Gamma_\Delta(W)}. \end{aligned} \tag{43}$$

The energy-dependent width $\Gamma_\Delta(W)$ from the dominant $\Delta \to \pi + N$ decay channel depends on the three-momentum of the pion as $\sim q_\pi^{2\ell+1}(W)$, with $\ell = 1$ the orbital momentum of the

TABLE I. The corrections to the proton's weak charge from PV $\gamma\gamma$ box graphs for three PVES experiments. Line two contains the elastic contribution from the proton intermediate state, line three from $N\pi$ intermediate states, and the following three lines contain the Δ plus high-energy contribution with various options for the parameter λ. The central value in the last line is obtained by summing the various contributions and averaging over the explored range of λ. The first uncertainty reflects the one from the spread in λ; the second error is obtained by adding the uncertainties from the elastic, π, and Δ + HE contributions in quadrature.

	Contribution	P2@MESA	Qweak	MOLLER
	Elastic	$-(1.0 \pm 2.0) \times 10^{-4}$	$-(1.2 \pm 2.2) \times 10^{-5}$	$-(3 \pm 5) \times 10^{-7}$
	π	$-(2.0 \pm 2.0) \times 10^{-5}$	$-(5.5 \pm 5.5) \times 10^{-5}$	$-(2.8 \pm 2.8) \times 10^{-5}$
δQ_W^p	Δ + HE ($\lambda = 0.5$)	$-(0.67 \pm 2.0) \times 10^{-4}$	$-(1.3 \pm 3.8) \times 10^{-4}$	$-(1.1 \pm 3.3) \times 10^{-4}$
	Δ + HE ($\lambda = 0$)	$-(0.4 \pm 1.2) \times 10^{-4}$	$-(1.1 \pm 3.3) \times 10^{-4}$	$-(0.5 \pm 1.4) \times 10^{-4}$
	Δ + HE ($\lambda = -0.5$)	$-(0.32 \pm 0.93) \times 10^{-4}$	$-(1.1 \pm 3.3) \times 10^{-4}$	$-(0.2 \pm 0.6) \times 10^{-4}$
	Total	$-(1.7 \pm 0.3 \pm 2.5) \times 10^{-4}$	$-(1.9 \pm 0.1 \pm 3.6) \times 10^{-4}$	$-(0.9 \pm 0.5 \pm 1.8) \times 10^{-4}$

pion in the p wave. Then, we obtain

$$\Gamma_\Delta(W) = \Gamma_\Delta\left[\frac{q_\pi(W)}{q_\pi(M_\Delta)}\right]^3, \tag{44}$$

and $\Gamma_\Delta = 120$ MeV the total width of the Δ resonance. In the zero-width limit one obtains

$$F_{3\,\Delta}^{\gamma\gamma}(\omega,0)\big|_{\Gamma_\Delta\to 0} = \sqrt{\frac{2}{3}}\frac{4Mg_M d_\Delta^+}{\Lambda_\chi(M+M_\Delta)}\omega_\Delta^2\delta(\omega-\omega_\Delta), \tag{45}$$

with $\omega_\Delta = (M_\Delta^2 - M^2)/(2M)$. It is seen that the Δ contribution alone does not obey the superconvergence relation. For this reason we will assume a rather generic high-energy contribution,

$$F_{3\,\mathrm{HE}}^{\gamma\gamma}(\omega,0) = C_\lambda(\Lambda)\left(\frac{\omega}{\Lambda}\right)^\lambda\Theta(\omega-\Lambda), \tag{46}$$

with the Heaviside Θ function switching the high-energy contribution on above a scale $\Lambda \sim 1$ GeV, and the power $\lambda < 1$ (the Pomeron cannot contribute to the PV amplitude, and only meson trajectories with $\lambda \lesssim 1/2$ are viable) such that the integral in the superconvergence relation converges. We do not know what the power behavior should be and will explore the range $-1/2 \leqslant \lambda \leqslant 1/2$. From the requirement,

$$\int_{\omega_\pi}^\infty \frac{d\omega}{\omega^2}\left[F_{3\,\Delta}^{\gamma\gamma}(\omega,0) + F_{3\,\mathrm{HE}}^{\gamma\gamma}(\omega,0)\right] = 0, \tag{47}$$

we obtain a simple constraint on the normalization C_λ:

$$C_\lambda(\Lambda) = -\sqrt{\frac{2}{3}}\frac{4Mg_M d_\Delta^+\Lambda}{\Lambda_\chi(M+M_\Delta)}(1-\lambda). \tag{48}$$

Finally, we will use

$$F_3^{\gamma\gamma} = F_{3\,\pi}^{\gamma\gamma} + F_{3\,\Delta}^{\gamma\gamma} + F_{3\,\mathrm{HE}}^{\gamma\gamma} \tag{49}$$

for numerical estimates as input in Eq. (28). This model is exploratory because of the lack of certainty about the high-energy behavior of the PV structure function. Nonetheless it is constructed in such a way as to obey the very general constraints imposed by symmetries and analyticity. Moreover it uses the (very uncertain) available experimental information on the strength of the hadronic PV interaction. Thus it can be used for reasonable numerical estimates of the PV two-photon exchange effect on the effective weak charge of the proton in the kinematics of relevant experiments.

VII. RESULTS AND DISCUSSION

Measurements of the proton's weak charge are planned within three PVES experiments using different kinematical conditions. The Qweak experiment at JLab [3] uses an electron beam with energy $E = 1.165$ GeV and momentum transfer $t = -0.022$ GeV^2; the P2 experiment at MESA [4] will be performed at the lower energy $E = 155$ MeV and $t = -0.0045$ GeV^2; the MOLLER experiment [43] will capitalize on the 12-GeV JLab upgrade with the high electron energy of $E = 11$ GeV, and the momentum transfer $t = -0.0056$ GeV^2. While the main focus of the latter experiment is Møller (elastic ee) scattering, elastic ep scattering will also be measured.

The model of $F_3^{\gamma\gamma}$ specified in Eqs. (39), (45), (46), and (48), obeying the superconvergence relation, can now be used for numerical estimates by evaluating the integral of Eq. (28) with the auxiliary function $G(E,\omega,t)$ given in Eq. (29). The object of interest is

$$\delta Q_W^p(E,t) = -\frac{4\sqrt{2}\pi\alpha}{G_F}\mathrm{Re}\,\Box_{\gamma\gamma}^{\mathrm{PV}}(E,t), \tag{50}$$

to be compared with the SM result for the proton's weak charge at one loop accuracy,

$$Q_W^p = 0.0713(8). \tag{51}$$

The precision of the SM prediction for Q_W^p, 8×10^{-4}, sets the relevant scale for the contributions from the PV two-photon exchange.

The most precise existing experimental determination of the nuclear weak charge was obtained from atomic PV in Cesium-113 atoms [44,45],

$$Q_W(^{113}\mathrm{Cs}) = -72.58(29)_{\mathrm{exp}}(32)_{\mathrm{th}}, \tag{52}$$

to be compared to the SM expectation,

$$Q_W^{\mathrm{SM}}(^{113}\mathrm{Cs}) = -73.23(2). \tag{53}$$

For the estimate of the PV two-photon exchange in this case we use the $E = 0$ limit of Eq. (29) resulting in Eq. (30). In that latter equation, the t dependence can be neglected as a direct consequence of the superconvergence relation, and the result

TABLE II. Same as in Table I for the ^{113}Cs nucleus.

	Contribution	^{113}Cs
$\delta Q_W(^{113}\mathrm{Cs})$	Elastic	$-(2.0 \pm 3.9) \times 10^{-2}$
	π	$-(3.3 \pm 3.3) \times 10^{-3}$
	Δ + HE ($\lambda = 0.5$)	$-(8 \pm 24) \times 10^{-3}$
	Δ + HE ($\lambda = 0$)	$-(5 \pm 15) \times 10^{-3}$
	Δ + HE ($\lambda = -0.5$)	$-(4 \pm 12) \times 10^{-3}$
	Total	$-(3.0 \pm 4.3) \times 10^{-2}$

is finite. Keeping in mind that there are large uncertainties associated with the model, we do not attempt to take into account nuclear effects and simply assume the isoscalar PV two-photon exchange contribution on a single nucleon to scale with the atomic number, $A = 113$ in the case of cesium.

Our results are compiled in Tables I and II. We find that the PV two-photon exchange correction does not affect the experimental extraction of the weak mixing angle, neither from PVES experiments (see Table I), nor from atomic PV experiments (see Table II) at the currently achievable accuracy. Possible nuclear resonance contributions with PV are expected to be more important for atomic experiments [46,47]. For example, in Ref. [47] a P-odd polarizability of an atom was discussed and a number of mechanisms that can enhance its effect were considered, e.g., the presence of nearly degenerate levels of opposite parity, leading to an enhancement of several orders of magnitude over the naive estimates of the effect. Our work demonstrates that such resonant contributions have to obey the superconvergence relation of Eq. (35) for the structure function $F_3^{\gamma\gamma}$ also in the nuclear range. It is plausible to assume that from the scale separation between nuclear and hadronic contributions, to a good extent the cancellation in Eq. (35) should occur in the nuclear and the hadronic range independently. This observation may serve as a more rigorous basis for implementing the enhancement mechanisms addressed in Ref. [47].

In summary, we have studied a novel correction to the weak charges from hadronic PV effects entering via two-photon exchange. Although such a correction is potentially enhanced by large logarithms, we could demonstrate that an inclusion of this correction does not influence the extraction of the weak charge from the experimental observables in either PVES or atomic PV experiments. This conclusion is a direct consequence of a general property of the PV electromagnetic inelastic structure function $F_3^{\gamma\gamma}$, i.e., a superconvergence relation that requires that a certain energy-weighted integral of this function over the inelastic spectrum should vanish exactly. This property was pointed out in the literature before, but it is for the first time that we were able to formally prove it in a relativistic field theory calculation in the first nonvanishing order of Baryon Chiral Perturbation Theory. Capitalizing on this proof, we constructed a minimal self-consistent model for the structure function $F_3^{\gamma\gamma}$, in the hadronic energy range incorporating the hadronic PV couplings h_π^1 and d_Δ, and complemented by a hypothetic high-energy contribution necessary to obey the superconvergence relation. In addition, we considered the effect of the nucleon anapole moment that also affects the nucleon weak charge via the two-photon exchange mechanism. Using all available information on the values and uncertainties of h_π^1, d_Δ, and the proton's anapole moment, we were able to demonstrate that at the currently viable level of experimental accuracy these effects are under control and do not affect the experimental determination of nuclear and the proton's weak charges. We also pointed out that possible resonant enhancements of long-range parity-nonconserving interactions in nuclei, atoms, and molecules, proposed earlier in the literature, are also subject to at least a partial cancellation because of the superconvergence relation that has to hold in the nuclear energy range, as well.

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APPENDIX: $F_3^{\gamma\gamma}$ AT ONE-LOOP LEVEL IN BχPT

In this Appendix we provide details for the calculation of the PV structure function $F_3^{\gamma\gamma}$ at the one-loop level in BχPT. This is done by relating the forward amplitude T_3 to the forward Compton helicity amplitudes $T_{\lambda_\gamma h,\lambda_\gamma h}$ as

$$T_3 = \frac{1}{2\pi e^2}\frac{1}{2}\sum_h [T_{1h,1h} - T_{-1h,-1h}], \tag{A1}$$

with $h = \pm 1/2$ the nucleon helicity in the initial and final state, and $\lambda_\gamma = \pm 1$ the photon helicity. Only amplitudes conserving both nucleon and photon helicities survive in the forward limit. The conventional factor $1/(2\pi e^2)$ reflects the normalization of the structure function $F_3^{\gamma\gamma}$ as defined in Eqs. (7) and (8). From unitarity we obtain the imaginary part of the forward Compton helicity amplitudes as

$$\mathrm{Im}\, T_{\lambda h,\lambda h} = \frac{q_\pi}{16\pi\sqrt{s}}\int_{-1}^{1} d\cos\theta \sum_{h'} \left|T_{h',\lambda h}^{\gamma N\to\pi N}\right|^2, \tag{A2}$$

where the unitarity relation was evaluated in the c.m. frame, in which the pion three-momentum is $q_\pi = \sqrt{[s-(M+m_\pi)^2][s-(M-m_\pi)^2]/4s}$. Thus, to obtain the forward Compton amplitude at one-loop level we need to calculate the pion photoproduction helicity amplitudes at tree level, square them, and integrate over the pion angles.

For the photon moving along the positive z direction, photon polarization vectors are

$$\epsilon_\lambda^\mu = -\frac{\lambda}{\sqrt{2}}\begin{pmatrix}0\\1\\i\lambda\\0\end{pmatrix}, \tag{A3}$$

and the nucleon helicity-dependent spinors are

$$N_h = \sqrt{E+M}\begin{bmatrix} \chi_h \\ 2h\frac{q}{E+M}\chi_h \end{bmatrix},$$

$$N'_{h'} = \sqrt{E'+M}\begin{bmatrix} \chi'_{h'} \\ 2h'\frac{q_\pi}{E'+M}\chi'_{h'} \end{bmatrix}, \qquad \text{(A4)}$$

where the Pauli spinors for $\vec{p}=(0,0,-q)$ and $\vec{p}\,'=(-q_\pi\sin\theta,0,-q_\pi\cos\theta)$ are given by

$$\chi_{\frac12} = \begin{pmatrix}0\\1\end{pmatrix},\quad \chi_{-\frac12} = \begin{pmatrix}-1\\0\end{pmatrix},$$
$$\chi'_{\frac12} = \begin{pmatrix}-\sin\frac{\theta}{2}\\ \cos\frac{\theta}{2}\end{pmatrix},\quad \chi'_{-\frac12} = \begin{pmatrix}-\cos\frac{\theta}{2}\\ -\sin\frac{\theta}{2}\end{pmatrix}. \qquad \text{(A5)}$$

A straightforward calculation of c.m. helicity amplitudes for π^+ production on the proton target gives

$$T_{\frac12,1\frac12} = \mathcal{N}e\sqrt{2}q_\pi\sin\frac{\theta}{2}\left[\frac{C_1^+\kappa_V\sqrt{2}g_{\pi NN} - C_2^+\kappa_S h_\pi^1}{M} - \left[\frac{\mu^p}{s-M^2}+\frac{\mu^n}{u-M^2}\right]\left(C_3^-\sqrt{2}g_{\pi NN}+C_4^- h_\pi^1\right)\right] - T_{\frac12,-1\frac12},$$

$$T_{-\frac12,-1-\frac12} = \mathcal{N}e\sqrt{2}q_\pi\sin\frac{\theta}{2}\left[\frac{C_1^+\kappa_V\sqrt{2}g_{\pi NN} + C_2^+\kappa_S h_\pi^1}{M} - \left[\frac{\mu^p}{s-M^2}+\frac{\mu^n}{u-M^2}\right]\left(C_3^-\sqrt{2}g_{\pi NN}-C_4^- h_\pi^1\right)\right] - T_{-\frac12,1-\frac12},$$

$$T_{-\frac12,1\frac12} = \mathcal{N}e\sqrt{2}q_\pi\cos\frac{\theta}{2}\left[\frac{C_1^-\kappa_V\sqrt{2}g_{\pi NN} - C_2^-\kappa_S h_\pi^1}{M} - \left[\frac{\mu^p}{s-M^2}+\frac{\mu^n}{u-M^2}\right]\left(C_3^+\sqrt{2}g_{\pi NN}+C_4^+ h_\pi^1\right)\right] - T_{-\frac12,-1\frac12},$$

$$T_{\frac12,-1-\frac12} = -\mathcal{N}e\sqrt{2}q_\pi\cos\frac{\theta}{2}\left[\frac{C_1^-\kappa_V\sqrt{2}g_{\pi NN} + C_2^-\kappa_S h_\pi^1}{M} - \left[\frac{\mu^p}{s-M^2}+\frac{\mu^n}{u-M^2}\right]\left(C_3^+\sqrt{2}g_{\pi NN}-C_4^+ h_\pi^1\right)\right] - T_{\frac12,1-\frac12},$$

$$T_{\frac12,-1\frac12} = -\mathcal{N}e\sqrt{2}q_\pi\sin\theta\cos\frac{\theta}{2}\left[\frac{C_1^- h_\pi^1 + C_2^-\sqrt{2}g_{\pi NN}}{t-m_\pi^2} - \frac{\kappa^n}{2M}\frac{C_3^+ h_\pi^1 + C_4^+\sqrt{2}g_{\pi NN}}{u-M^2}\right],$$

$$T_{-\frac12,1-\frac12} = -\mathcal{N}e\sqrt{2}q_\pi\sin\theta\cos\frac{\theta}{2}\left[\frac{-C_1^- h_\pi^1 + C_2^-\sqrt{2}g_{\pi NN}}{t-m_\pi^2} - \frac{\kappa^n}{2M}\frac{-C_3^+ h_\pi^1 + C_4^+\sqrt{2}g_{\pi NN}}{u-M^2}\right],$$

$$T_{-\frac12,-1\frac12} = \mathcal{N}e\sqrt{2}q_\pi\sin\theta\sin\frac{\theta}{2}\left[\frac{C_1^+ h_\pi^1 + C_2^+\sqrt{2}g_{\pi NN}}{t-m_\pi^2} - \frac{\kappa^n}{2M}\frac{C_3^- h_\pi^1 + C_4^-\sqrt{2}g_{\pi NN}}{u-M^2}\right],$$

$$T_{\frac12,1-\frac12} = -\mathcal{N}e\sqrt{2}q_\pi\sin\theta\sin\frac{\theta}{2}\left[\frac{-C_1^+ h_\pi^1 + C_2^+\sqrt{2}g_{\pi NN}}{t-m_\pi^2} - \frac{\kappa^n}{2M}\frac{-C_3^- h_\pi^1 + C_4^-\sqrt{2}g_{\pi NN}}{u-M^2}\right]. \qquad \text{(A6)}$$

The c.m. frame quantities E, E', q, and E_π are given in Eq. (40), and in terms of them the Mandelstam invariants read

$$u - M^2 = -2q(E' + q_\pi\cos\theta),\quad t - m_\pi^2 = -2q(E_\pi - q_\pi\cos\theta). \qquad \text{(A7)}$$

Furthermore,

$$\mathcal{N} = \sqrt{(E+M)(E'+M)},\quad C_1^\pm = 1 \pm \frac{q}{E+M}\frac{q_\pi}{E'+M},\quad C_2^\pm = \frac{q}{E+M} \pm \frac{q_\pi}{E'+M},$$
$$C_3^\pm = \sqrt{s} - M \pm (\sqrt{s}+M)\frac{q}{E+M}\frac{q_\pi}{E'+M},\quad C_4^\pm = (\sqrt{s}+M)\frac{q}{E+M} \pm (\sqrt{s}-M)\frac{q_\pi}{E'+M}. \qquad \text{(A8)}$$

Combining these results we find

$$\frac12\sum_{h,h'}\left[|T_{h',1h}|^2 - |T_{h',-1h}|^2\right] = 4\sqrt{2}e^2 g_{\pi NN}h_\pi^1\left[\left(\mu^p + \frac{s-M^2}{u-M^2}\mu^n\right)\left(\frac{2q_\pi^2\sin^2\theta}{t-m_\pi^2} - \frac{u-M^2}{s-M^2}\right)\right.$$
$$\left. + (\mu^n)^2\frac{(s-M^2)q_\pi^2\sin^2\theta}{2M^2(u-M^2)} - \frac{\kappa_S\kappa_V}{2M^2}(qE' - Eq_\pi\cos\theta)\right]. \qquad \text{(A9)}$$

Upon integrating over the pion phase space we finally obtain

$$F_3^{\gamma\gamma}(s,t=0) = -\frac{g_{\pi NN} h_\pi^1 q_\pi}{2\sqrt{2}\pi^2\sqrt{s}} \left\{ \mu^p \left[\frac{E'}{\sqrt{s}} - \frac{E_\pi}{q} + \frac{m_\pi^2}{2qq_\pi} \ln \frac{E_\pi + q_\pi}{E_\pi - q_\pi} \right] - \mu^n \left[-\frac{E}{q} + \frac{m_\pi^2}{2qq_\pi} \ln \frac{E_\pi + q_\pi}{E_\pi - q_\pi} + \frac{M^2}{2qq_\pi} \ln \frac{E' + q_\pi}{E' - q_\pi} \right] - \frac{qE'}{2M^2}\kappa_V\kappa_S + (\mu^n)^2 \frac{s - M^2}{4M^2} \left[-\frac{E'}{q} + \frac{M^2}{2qq_\pi} \ln \frac{E' + q_\pi}{E' - q_\pi} \right] \right\}. \tag{A10}$$

[1] J. Erler, C. J. Horowitz, S. Mantry, and P. A. Souder, Ann. Rev. Nucl. Part. Sci. **64**, 269 (2014).
[2] J. Erler, A. Kurylov, and M. J. Ramsey-Musolf, Phys. Rev. D **68**, 016006 (2003).
[3] D. Androic *et al.* (Qweak Collaboration), Phys. Rev. Lett. **111**, 141803 (2013).
[4] D. Becker, K. Gerz, S. Baunack, K. Kumar, and F. E. Maas, PoS Bormio **2013**, 024 (2013).
[5] W. J. Marciano and A. Sirlin, Phys. Rev. D **27**, 552 (1983).
[6] W. J. Marciano and A. Sirlin, Phys. Rev. D **29**, 75 (1984); **31**, 213(E) (1985).
[7] M. J. Ramsey-Musolf, Phys. Rev. C **60**, 015501 (1999).
[8] M. Gorchtein and C. J. Horowitz, Phys. Rev. Lett. **102**, 091806 (2009).
[9] A. Sibirtsev, P. G. Blunden, W. Melnitchouk, and A. W. Thomas, Phys. Rev. D **82**, 013011 (2010).
[10] B. C. Rislow and C. E. Carlson, Phys. Rev. D **83**, 113007 (2011).
[11] M. Gorchtein, C. J. Horowitz, and M. J. Ramsey-Musolf, Phys. Rev. C **84**, 015502 (2011).
[12] P. G. Blunden, W. Melnitchouk, and A. W. Thomas, Phys. Rev. Lett. **107**, 081801 (2011).
[13] C. E. Carlson and B. C. Rislow, Phys. Rev. D **85**, 073002 (2012).
[14] B. C. Rislow and C. E. Carlson, Phys. Rev. D **88**, 013018 (2013).
[15] N. L. Hall, P. G. Blunden, W. Melnitchouk, A. W. Thomas, and R. D. Young, Phys. Rev. D **88**, 013011 (2013).
[16] M. Gorchtein, H. Spiesberger, and X. Zhang, Phys. Lett. B **752**, 135 (2016).
[17] N. L. Hall, P. G. Blunden, W. Melnitchouk, A. W. Thomas, and R. D. Young, Phys. Lett. B **753**, 221 (2016).
[18] S. L. Zhu, S. J. Puglia, B. R. Holstein, and M. J. Ramsey-Musolf, Phys. Rev. D **62**, 033008 (2000).
[19] J. Liu, R. D. McKeown, and M. J. Ramsey-Musolf, Phys. Rev. C **76**, 025202 (2007).
[20] D. B. Kaplan and M. J. Savage, Nucl. Phys. A **556**, 653 (1993); **570**, 833 (1994); **580**, 679 (1994).
[21] K. A. Olive *et al.* (Particle Data Group), Chin. Phys. C **38**, 090001 (2014).
[22] M. J. Musolf and B. R. Holstein, Phys. Lett. B **242**, 461 (1990).
[23] H. Q. Zhou, C. W. Kao, S. N. Yang, and K. Nagata, Phys. Rev. C **81**, 035208 (2010).
[24] V. V. Flambaum, I. B. Khriplovich, and O. P. Sushkov, Phys. Lett. B **146**, 367 (1984).
[25] M. Gorchtein, Phys. Rev. C **90**, 052201 (2014).
[26] M. Gorchtein, Phys. Lett. B **644**, 322 (2007).
[27] L. Lukaszuk, Nucl. Phys. A **709**, 289 (2002).
[28] K. Kurek and L. Lukaszuk, Phys. Rev. C **70**, 065204 (2004).
[29] S. B. Gerasimov, Yad. Fiz. **2**, 598 (1965) [Sov. J. Nucl. Phys. **2**, 430 (1966)].
[30] S. D. Drell and A. C. Hearn, Phys. Rev. Lett. **16**, 908 (1966).
[31] W. Y. Tsai, L. L. Deraad, and K. A. Milton, Phys. Rev. D **6**, 1428 (1972); **11**, 703(E) (1975).
[32] D. A. Dicus and R. Vega, Phys. Lett. B **501**, 44 (2001).
[33] B. R. Holstein, V. Pascalutsa, and M. Vanderhaeghen, Phys. Rev. D **72**, 094014 (2005).
[34] V. Bernard, Prog. Part. Nucl. Phys. **60**, 82 (2008).
[35] P. F. Bedaque and M. J. Savage, Phys. Rev. C **62**, 018501 (2000).
[36] J. W. Chen, T. D. Cohen, and C. W. Kao, Phys. Rev. C **64**, 055206 (2001).
[37] V. Lensky and V. Pascalutsa, Eur. Phys. J. C **65**, 195 (2010).
[38] B. Desplanques, J. F. Donoghue, and B. R. Holstein, Annals Phys. **124**, 449 (1980).
[39] S. L. Zhu, C. M. Maekawa, B. R. Holstein, and M. J. Ramsey-Musolf, Phys. Rev. Lett. **87**, 201802 (2001).
[40] S. L. Zhu, C. M. Maekawa, G. Sacco, B. R. Holstein, and M. J. Ramsey-Musolf, Phys. Rev. D **65**, 033001 (2001).
[41] V. Pascalutsa, M. Vanderhaeghen, and S. N. Yang, Phys. Rept. **437**, 125 (2007).
[42] D. Androic *et al.* (G0 Collaboration), Phys. Rev. Lett. **108**, 122002 (2012).
[43] J. Mammei (MOLLER Collaboration), Nuovo Cim. C **035N04**, 203 (2012).
[44] C. S. Wood, S. C. Bennett, D. Cho, B. P. Masterson, J. L. Roberts, C. E. Tanner, and C. E. Wieman, Science **275**, 1759 (1997).
[45] V. A. Dzuba, J. C. Berengut, V. V. Flambaum, and B. Roberts, Phys. Rev. Lett. **109**, 203003 (2012).
[46] I. B. Khriplovich and O. L. Zhizhimov, Zh. Eksp. Teor. Fiz. **82**, 1026 (1982) [Sov. Phys. JETP **55**, 601 (1982)].
[47] V. V. Flambaum, Phys. Rev. A **45**, 6174 (1992).
[48] D. Binosi, J. Collins, C. Kaufhold, and L. Theussl, Comput. Phys. Commun. **180**, 1709 (2009).