Dynamics of the tri-nuclear system at spontaneous fission of ^{252}Cf

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To describe the dynamics of ternary fission of ^{252}Cf an equation of motion of the tri-nuclear system was obtained and it was solved numerically. The fission of the ^{70}Ni + ^{50}Ca + ^{132}Sn channel was chosen as one of the more probable channels of true ternary fission of ^{252}Cf. The collinearity of ternary fission was checked by analyzing the results of the equation of motion. The results show that if initially all nuclei are placed collinearly (potential energy of this position is the smallest) and the component of the middle fragment's initial velocity which is perpendicular to this line is zero, then ternary fission is collinear, otherwise noncollinear ternary fission takes place.

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I. INTRODUCTION

The interest to study ternary fission appeared [1] after the discovery of binary fission of heavy nuclei by various authors. Ternary fission is one of the oldest problems of the nuclear reaction but still it has topicality. Diehl and Greiner [2] tried to explain ternary fission within the framework of the liquid drop model. They mainly calculated the potential energy for the prolate and oblate configurations of the ternary system. Also, within the three-center shell model, Degheidy and Maruhn [3] generalized the phenomenological shell model based on the harmonic oscillator potential to systems with three clusters. They showed that the centers of nuclei may be in arbitrary geometrical configurations and nuclei may have different masses. The experimental work [4] was dedicated to the study of ternary fission of the ^{252}Cf nucleus with an eight $\Delta E \times E$ particle telescope. In this experiment, mainly, light charged particles (like He, Be, B, etc.) were observed in coincidence with γ emission.

The experimental group FOBOS (in the Flerov Laboratory of Nuclear Reactions of the Joint Institute for Nuclear Research, Dubna, Russia) made an effort to study the unusual mode of ternary fission: collinear cluster tripartition [5–7]. Studies of the spontaneous fission products of ^{252}Cf in coincidence with the emitted neutrons have been performed in two missing-mass experiments [5,7]. These experiments demonstrated a new mode for the ternary fission process as a collinear cluster tripartition (CCT). At first, in the CCT mode, the masses of nuclei are comparable, and the nuclei are especially in clusters, i.e., with a magic number of mass (or charge). Next, the collinearity of the momenta of the ternary fission fragments is proved by the fact that the two detectors registering Sn-like and Ni-like fragments are placed on the opposite sides from the fissioning source ^{252}Cf such that the angle between them is 180°. The probability of the yield of Ni and Sn nuclei observed was approximately 10^3 times less then the one of binary fission. The authors of Refs. [8,9] concluded that the middle fragment is a calcium nucleus. This mode of ternary fission differs from the usual ternary fission which is binary fission with the emission of light fragments (He, Li, Be, etc.) as the third (middle) nucleus in the perpendicular plane to the fission axis.

Moreover some theoretical works dedicated to this kind of ternary fission have been published [8–16]. In Ref. [12] the ternary fission of ^{252}Cf was studied through the potential energy surfaces for two different arrangements in a collinear configuration, and the authors concluded that true ternary fission (with almost equal fragment mass) is energetically possible due to the minima in the fragmentation potential energy and high Q values. Also, in this method it is shown that collinear geometry with the lightest fragment between two heavier nuclei is expected to give the highest probabilities in the decay.

In our previous works [9,15], we studied the possible channels of true ternary fission. In Ref. [7] it was shown that a more possible channel of ternary fission in the ^{252}Cf(sf) reaction is ^{70}Ni + ^{50}Ca + ^{132}Sn which was theoretically proven in Ref. [9]. Our experience from the previous works [9,16] leads to the interesting question, how does a tri-nuclear system evaluate during its decay? Because in those works it was not proven that the momenta of fission products are collinear. In the present work we decided to study the dynamical change of the relative distance between nuclei and their velocities. So, the main aim of the current work is to study the dynamics of fission of the ^{70}Ni + ^{50}Ca + ^{132}Sn system; in other words, to check whether ternary fission is collinear.

Certainly, to get information about dynamics, an equation of motion should be solved. Results of solution of the equation of motion depend on the initial conditions. The dependance of the result on the initial condition is studied in detail to find the collinear flying of the ternary fission products. Thus from results it will be easy to know what initial condition leads to collinear fission.

II. MODEL

The theoretical model is based on the formation of the tri-nuclear system (TNS). The TNS is a system that has three interacting nuclei [9,16], and its interaction is studied on

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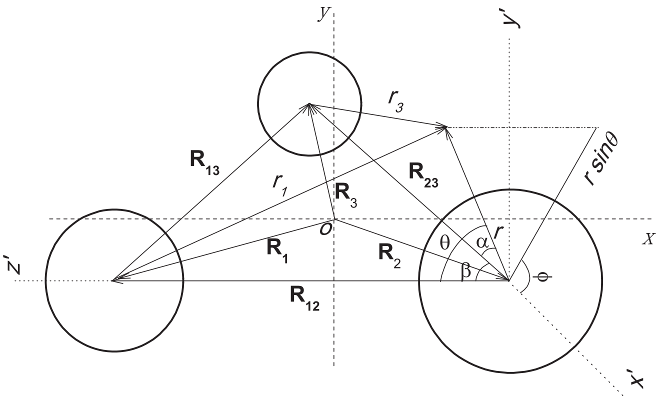

FIG. 1. Point ($\mathbf{R}_k$) and relative ($\mathbf{R}_{ij}$) vectors of the tri-nuclear system. The point O (origin) corresponds to the center of mass.

the basis of the dinuclear system model [17–19]. The stage proceeding to formation of the TNS is not studied. It is assumed that the system is formed, and any ternary fission of heavy nuclei passes through the TNS stage.

The main task is obtaining the classical Lagrange equations of motion, and solving them. First of all, the Lagrangian is $L = T - V$, where $T = \frac{1}{2}\sum_{i=1}^{3} m_i \dot{\mathbf{R}}_i^2$ is the kinetic energy of the system and V is the total interaction potential between fragments. The following system of equations can be written from Fig. 1:

$$\begin{aligned} \mathbf{R}_{12} &= \mathbf{R}_1 - \mathbf{R}_2, \\ \mathbf{R}_{13} &= \mathbf{R}_3 - \mathbf{R}_1, \\ \mathbf{R}_{23} &= \mathbf{R}_3 - \mathbf{R}_2, \end{aligned} \tag{1}$$

where $\mathbf{R}_k$ ($k = 1,2,3$) are point vectors of nuclei and the magnitude of a vector $\mathbf{R}_{ij}$ is the relative distance between the ith and jth nuclei.

It is clear that any kind of fission process occurs in one plane, i.e., it can be chosen to be the two-dimensional space where fission fragments move. So any $\mathbf{R}_i$ vector can be described only with x and y components (R_{ix} and R_{iy}) in the Cartesian system. Correspondingly, velocities are defined as $\upsilon_{ix} = \dot{R}_{ix}$ and $\upsilon_{iy} = \dot{R}_{iy}$, therefore, the kinetic energy can be written as

$$T = \frac{1}{2}\sum_{i=1}^{3} m_i\left(\upsilon_{ix}^2 + \upsilon_{iy}^2\right). \tag{2}$$

A. Lagrange equation of motion

In the framework of the classical Lagrange formalism, three equations of motion for the x variable and three for the y variable can be obtained:

$$\begin{aligned} \frac{d}{dt}\frac{\partial T}{\partial \upsilon_{ix}} - \frac{\partial T}{\partial R_{ix}} &= -\frac{\partial V}{\partial R_{ix}}, \\ \frac{d}{dt}\frac{\partial T}{\partial \upsilon_{iy}} - \frac{\partial T}{\partial R_{iy}} &= -\frac{\partial V}{\partial R_{iy}}. \end{aligned}$$

It is clear that the kinetic energy does not depend on a distance R_i, i.e., $\frac{\partial T}{\partial R_{ix}} = \frac{\partial T}{\partial R_{iy}} = 0$. Therefore,

$$m_i \dot{\upsilon}_{ix} = -\frac{\partial V}{\partial R_{ix}}, \tag{3}$$

$$m_i \dot{\upsilon}_{iy} = -\frac{\partial V}{\partial R_{iy}}. \tag{4}$$

The magnitude of an $\mathbf{R}_{ij}$ vector is $R_{ij} = \sqrt{R_{ijx}^2 + R_{ijy}^2}$. Potential energy V depends only on relative distance R_{ij} (or R_{ik}). So

$$\begin{aligned} \frac{\partial V}{\partial R_{ix}} &= \frac{\partial V}{\partial R_{ijx}}\frac{\partial R_{ijx}}{\partial R_{ix}} + \frac{\partial V}{\partial R_{ikx}}\frac{\partial R_{ikx}}{\partial R_{ix}} \\ &= \frac{\partial V}{\partial R_{ij}}\frac{\partial R_{ij}}{\partial R_{ijx}}\frac{\partial R_{ijx}}{\partial R_{ix}} + \frac{\partial V}{\partial R_{ik}}\frac{\partial R_{ik}}{\partial R_{ikx}}\frac{\partial R_{ikx}}{\partial R_{ix}}. \end{aligned} \tag{5}$$

It can be noted that $R_{ij} = R_{ji}$ and $R_{ik} = R_{ki}$. If Eq. (3) is written for each nucleus, then using Eq. (5) the following equations will be obtained:

$$\begin{aligned} m_1\dot{\upsilon}_{1x} &= -\frac{R_{12x}}{R_{12}}\frac{\partial V}{\partial R_{12}} + \frac{R_{13x}}{R_{13}}\frac{\partial V}{\partial R_{13}}, \\ m_2\dot{\upsilon}_{2x} &= \frac{R_{12x}}{R_{12}}\frac{\partial V}{\partial R_{12}} + \frac{R_{23x}}{R_{23}}\frac{\partial V}{\partial R_{23}}, \\ m_3\dot{\upsilon}_{3x} &= -\frac{R_{23x}}{R_{23}}\frac{\partial V}{\partial R_{23}} - \frac{R_{13x}}{R_{13}}\frac{\partial V}{\partial R_{13}}. \end{aligned} \tag{6}$$

The relation between R_i (or R_{ix}) and R_{ij} (or R_{ijx}) is found from the system of equations (1). Three symmetric equations can be obtained for the y component:

$$\begin{aligned} m_1\dot{\upsilon}_{1y} &= -\frac{R_{12y}}{R_{12}}\frac{\partial V}{\partial R_{12}} + \frac{R_{13y}}{R_{13}}\frac{\partial V}{\partial R_{13}}, \\ m_2\dot{\upsilon}_{2y} &= \frac{R_{12y}}{R_{12}}\frac{\partial V}{\partial R_{12}} + \frac{R_{23y}}{R_{23}}\frac{\partial V}{\partial R_{23}}, \\ m_3\dot{\upsilon}_{3y} &= -\frac{R_{23y}}{R_{23}}\frac{\partial V}{\partial R_{23}} - \frac{R_{13y}}{R_{13}}\frac{\partial V}{\partial R_{13}}. \end{aligned} \tag{7}$$

Taking into account the conservation law of linear momentum $\sum_{i=1}^{3} m_i \upsilon_{ix} = 0$ (since $\mathbf{P}_{\text{c.m.}} = 0$ for the spontaneous fission of ^{252}Cf) one of the equations in formulas (6) and (7) can be skipped. It means that υ_{3x} and R_{3x} are found as

$$\begin{aligned} \upsilon_{3x} &= -\frac{m_1\upsilon_{1x} + m_2\upsilon_{2x}}{m_3}, \\ R_{3x} &= -\frac{m_1 R_{1x} + m_2 R_{2x}}{m_3}. \end{aligned} \tag{8}$$

Because the origin is placed at the center of mass, there is no "const" term in the definition of R_{3x}. Equations for the y components are similar to the last equation.

B. Derivative of total interaction potential

It is clear from the Eqs. (6)–(8) that the dynamics of motion strongly depends on the derivative of the total interaction potential. The total interaction potential consists of two parts: Coulomb and nuclear

$$V = V_C + V_{\text{nuc}}. \tag{9}$$

There are three interacting nuclei, so there are three terms on each part. To calculate the nuclear part, the double folding procedure is used:

$$V_C(R_{12},R_{23},R_{13}) = e^2 \sum_{i<j}^{3} \frac{Z_i Z_j}{R_{ij}}, \quad (10)$$

$$V_{\rm nuc}(R_{12},R_{23},R_{13}) = \int \sum_{i<j}^{3} \rho_i(r_i) f_{ij}(r_i,r_j)\rho_j(r_j)\, d\mathbf{r}. \quad (11)$$

The following formulas are necessary to calculate the nuclear part:

$$f_{ij}(r_i,r_j) = C\left[f_{\rm in} + (f_{\rm ex} - f_{\rm in})\frac{\rho_0 - (\rho_i + \rho_j)}{\rho_0}\right],$$

$$\rho_i(r_i) = \frac{\rho_0}{1 + \exp\left[\frac{r_i - R_{0i}}{a}\right]},$$

$$r_1(R_{12}) = \sqrt{r^2 + R_{12}^2 - 2rR_{12}\cos\theta},$$

$$r_2 = r,$$

$$r_3(R_{12},R_{23},R_{13}) = \sqrt{r^2 + R_{23}^2 - 2rR_{23}\cos\alpha},$$

$$\cos\alpha = \cos\theta\cos\beta + \sin\theta\sin\beta\sin\phi,$$

$$\cos\beta = \frac{R_{12}^2 + R_{23}^2 - R_{13}^2}{2R_{12}R_{23}}.$$

Here, r_i is the radial distance of the ith nucleus (see Fig. 1); r, θ, and ϕ are variables of the spherical coordinate system; $R_{0i} = r_0 A^{1/3}$ is the radius of the ith spherical nucleus; $r_0 = 1.16$ fm is the radius parameter; $\rho_0 = 0.17$ fm^{-3} is the density parameter; $a = 0.54$ fm is the diffuseness parameter; $C = 300$ MeV fm^3, $f_{\rm in} = 0.09$, and $f_{\rm ex} = -2.59$ are constants of the interaction potential; and f_{ij} is the effective nuclear-nuclear force, which is taken from Ref. [20].

By formula (9) the derivative of the total interaction potential is found with the two terms

$$\frac{\partial V}{\partial R_{ij}} = -e^2\frac{Z_i Z_j}{R_{ij}^2} + \frac{\partial V_{\rm nuc}}{\partial R_{ij}},$$

$$\frac{\partial V_{\rm nuc}}{\partial R_{ij}} = \int (F_{12} + F_{23} + F_{13})\mathrm{d}\mathbf{r},$$

$$F_{12} = \rho_2\left[f_{12} - \frac{\rho_1}{\rho_0}C(f_{\rm ex} - f_{\rm in})\right]\frac{\partial\rho_1}{\partial R_{ij}},$$

$$F_{23} = \rho_2\left[f_{23} - \frac{\rho_3}{\rho_0}C(f_{\rm ex} - f_{\rm in})\right]\frac{\partial\rho_3}{\partial R_{ij}},$$

$$F_{13} = \rho_3\left[f_{13} - \frac{\rho_1}{\rho_0}C(f_{\rm ex} - f_{\rm in})\right]\frac{\partial\rho_1}{\partial R_{ij}} + \rho_1\left[f_{13} - \frac{\rho_3}{\rho_0}C(f_{\rm ex} - f_{\rm in})\right]\frac{\partial\rho_3}{\partial R_{ij}},$$

$$\frac{\partial\rho_1}{\partial R_{ij}} = \frac{\rho_1(\rho_1 - \rho_0)}{a\rho_0}\frac{\partial r_1}{\partial R_{ij}},$$

$$\frac{\partial\rho_3}{\partial R_{ij}} = \frac{\rho_3(\rho_3 - \rho_0)}{a\rho_0}\frac{\partial r_3}{\partial R_{ij}}.$$

Derivatives $\frac{\partial r_1}{\partial R_{ij}}$ and $\frac{\partial r_3}{\partial R_{ij}}$ are calculated as follows:

$$\frac{\partial r_1}{\partial R_{12}} = \frac{R_{12} - r\cos\theta}{r_1},$$

$$\frac{\partial r_1}{\partial R_{23}} = \frac{\partial r_1}{\partial R_{13}} = 0,$$

$$\frac{\partial r_3}{\partial R_{12}} = (R_{23}\cos\beta - R_{12})h(r),$$

$$\frac{\partial r_3}{\partial R_{23}} = \frac{R_{23} - r\cos\alpha}{r_3} - (R_{23} - R_{12}\cos\beta)h(r),$$

$$\frac{\partial r_3}{\partial R_{13}} = R_{13}h(r),$$

where $h(r) = \frac{r}{R_{12}r_3}(\cos\theta - \cot\beta\sin\theta\sin\phi)$. Be reminded that the integration (11) is provided in the (x',y',z') system, and if $\phi = \pi/2$ then $\theta = \alpha + \beta$ (see Fig. 1).

III. RESULTS OF CALCULATION

As mentioned above the channel for spontaneous ternary fission of the ^{252}Cf nucleus is chosen as ^{70}Ni + ^{50}Ca + ^{132}Sn. ^{70}Ni is the first nucleus (placed left side), ^{132}Sn is the second nucleus (placed right side), and ^{50}Ca is the third one (placed in the middle) in Fig. 1. The collinearity of the momenta of the tripartition is determined by the dynamics of the middle fragment ^{50}Ca since the heavier fragment ^{132}Sn is separated

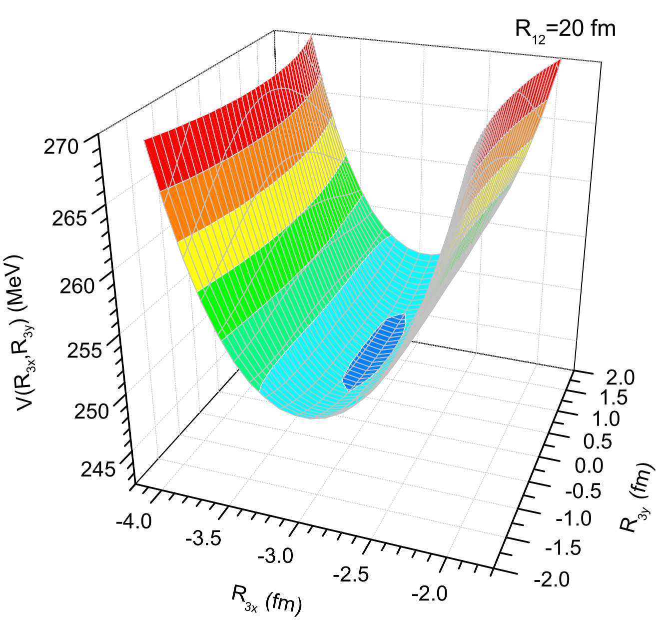

FIG. 2. Total interaction potential as the function of R_{3x} and R_{3y} when $R_{12} = 20$ fm.

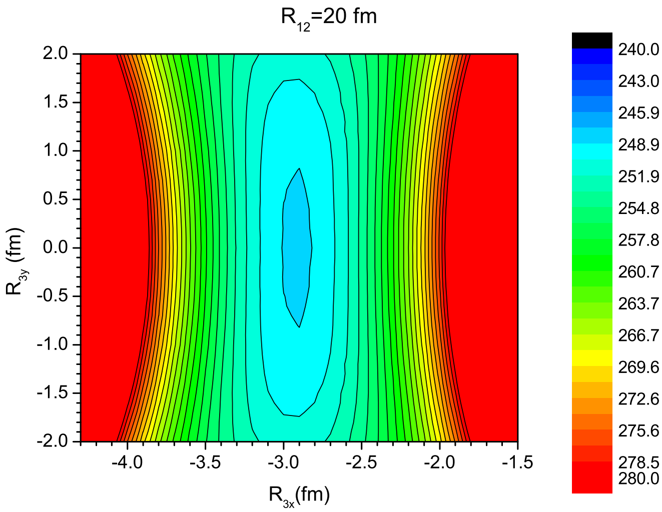

FIG. 3. Contour plot of Fig. 2.

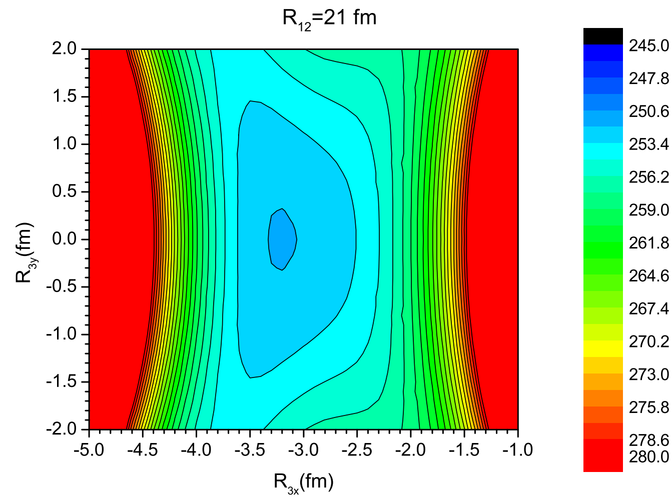

FIG. 5. Same as Fig. 3, but for $R_{12} = 21$ fm.

first and then the middle fragment separates from ^{70}Ni. This sequence of ternary fission was discussed in the Ref. [9] and it is confirmed by the solution of dynamical equations in this work.

It is interesting to discuss how the total interaction potential looks as a function of R_{3x} and R_{3y}. It is shown in Figs. 2–7 for different values of R_{12} (relative distance between Ni and Sn nuclei). The origin (which is not shown) corresponds to the center of mass. There is a local minimum at point $R_{3x} = -2.9$ fm and $R_{3y} = 0$ fm (see Figs. 2 and 4). By increasing R_{12} the minimum goes to the left (to the side of the Ni nucleus), but starting from $R_{12} = 22$ fm this minimum point is transferred to a saddle point.

In the first case it is considered that initially all nuclei are placed in one line, which means $R_{1y}(t = 0) = R_{2y}(t = 0) = R_{3y}(t = 0) = 0$, since the energy of the collinear configuration in the prescission state is the smallest, and x coordinates of that nuclei (or relative distance between nuclei) correspond to the local minimum in Fig. 2, i.e., $R_{1x}(t = 0) = -12.3$ fm, $R_{2x}(t = 0) = 7.7$ fm, and $R_{3x}(t = 0) = -2.9$ fm. Both components (x and y) of initial velocities of the three nuclei are zero. In other words, formation of fragments of the TNS is so slow that fragments have zero (or too small) velocities. Nevertheless, the assumption that all initial velocities are zero means that there is no net force which acts on the nuclei in the equilibrium state. Results of calculation of the equations of motion (6) and (7) [together with Eq. (8)] with the initial conditions mentioned above are shown in Fig. 8. It is shown that from the beginning, the Sn nucleus is going to break up from the Ni+Ca system, and then at $t \approx 13.5 \times 10^{-22}$ s the Ni+Ca system has decayed. Moreover an important result has been obtained that the third nucleus (Ca) almost does not change its coordinate, because its velocity is about zero. It means that detecting the middle nucleus (Ca) is almost impossible in an experiment. This conclusion proves the assumption made in our previous

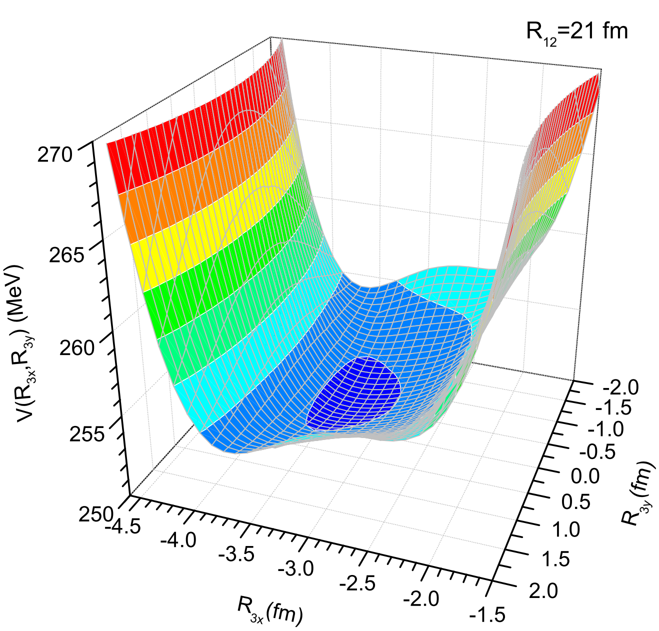

FIG. 4. Same as Fig. 2, but for $R_{12} = 21$ fm.

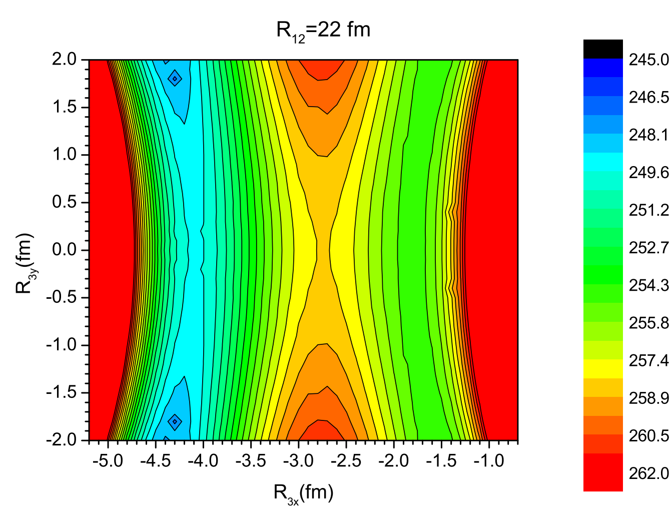

FIG. 6. Same as Fig. 3 but for $R_{12} = 22$ fm.

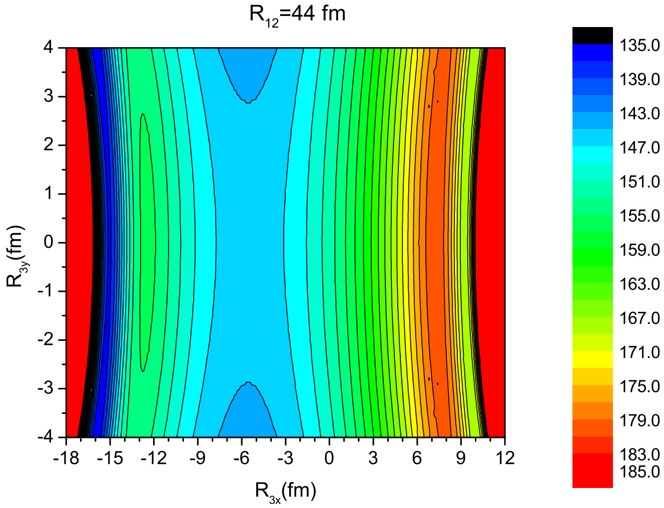

FIG. 7. Same as Fig. 3 but for $R_{12} = 34$ fm.

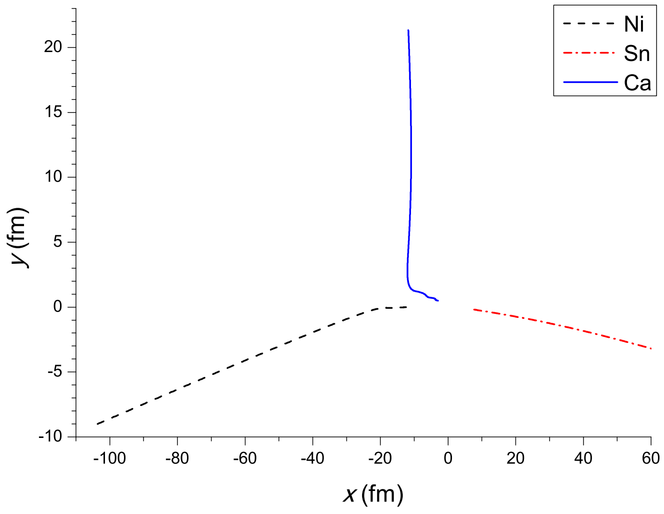

FIG. 9. Trajectories of three decaying nuclei when $R_{3y}(t=0) = 0.5$ fm and the same initial velocities as in Fig. 8.

paper [9]. Only this condition leads to collinear fission of the tri-nuclear system.

In the second case, the velocities of all nuclei are zero, but the middle nucleus (Ca) is placed a little bit upper, i.e., $R_{3y}(t=0) = 0.5$ fm, $R_{1x}(t=0) = -12.3$ fm, $R_{2x}(t=0) = 7.7$ fm, $R_{3x}(t=0) = -2.9$ fm, and and $R_{1y}(t=0) = R_{2y}(t=0) = 0$. Results of the calculation are shown in Fig. 9. It is clear from the figure that the deviation of the location of the calcium nucleus at 0.5 fm on the y axis from the origin is enough to get noncollinear fission. Moreover, the sequence of the noncollinear fission is similar to the one of the collinear fission: at first Sn is separated from Ni+Ca, then the Ni+Ca system is broken up. It is interesting that the decay time of the tri-nuclear system is $t \approx 13.2 \times 10^{-22}$ s, which is almost the same as the time of collinear ternary fission.

In the third case, the initial location of all the nuclei are the same as in the first case, i.e., $R_{1y}(t=0) = R_{2y}(t=0) = R_{3y}(t=0) = 0$, $R_{1x}(t=0) = -12.3$ fm, $R_{2x}(t=0) = 7.7$ fm, and $R_{3x}(t=0) = -2.9$ fm. But the initial velocity of the middle fragment is $\upsilon_{3y}(t=0) = 0.1$ cm/ns, and the other initial velocities are zero: $\upsilon_{1x}(t=0) = \upsilon_{2x}(t=0) = \upsilon_{3x}(t=0) = \upsilon_{1y}(t=0) = \upsilon_{2y}(t=0) = 0$. Figure 10 shows that if the y component of the initial velocity of the Ca nucleus is not zero, ternary fission will be noncollinear. Also, it should be noted that in this case the decay time of the TNS is $t \approx 13.4 \times 10^{-22}$ s, which is almost the same as the time in the previous cases. Comparing Figs. 9 and 10, we see that the final paths of all the fragments are similar in spite of differences in the initial conditions of the second and third cases.

From the figures it can be concluded that there is collinear fission only when all three nuclei are located in one line ($R_{iy} = 0$) and there is not a y component of the initial velocity of the middle fragment ($\upsilon_{3y} = 0$).

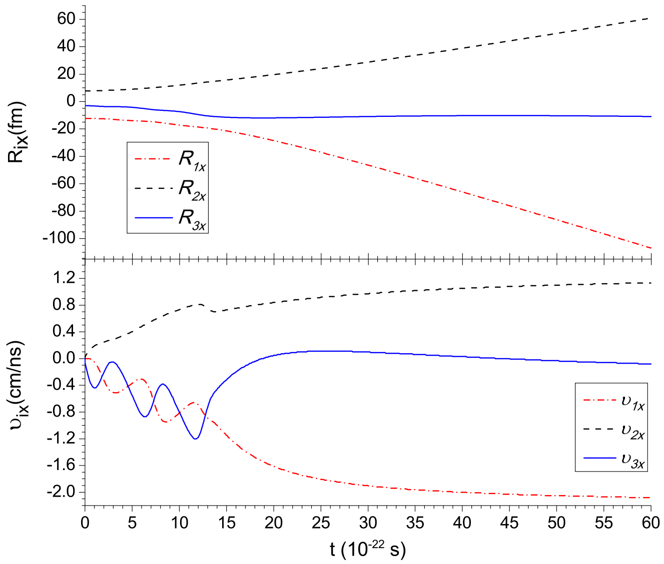

FIG. 8. x component of coordinates (upper) and velocities (lower) of three nuclei as functions of time.

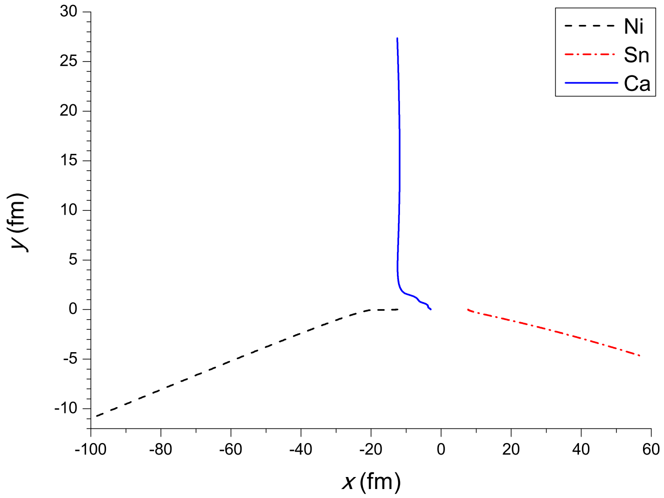

FIG. 10. Trajectories of three decaying nuclei when $\upsilon_{3y}(t=0) = 0.1$ cm/ns and the same initial coordinates as in Fig. 8.

IV. SUMMARY

We conclude that if in the prescission stage all nuclei are placed collinearly which corresponds to the minimum in the potential energy surface and there is no the net force on the third nucleus (Ca) on the y axis (or y component if its initial velocity is zero), then the tri-nuclear system can be broken up collinearly. This theoretical result proves the experimental results of the collinear cluster tripartition in Ref. [7]. The experiment shows that collinear ternary fission can be observed. Therefore, in the framework of the TNS model the initial condition which leads to collinear fission has a place in nature.

From the comparison of the potential energy surfaces in the Figs. 2–7 it can be concluded that as R_{12} (relative distance between Ni and Sn nuclei) increases, the minimum at the point when $R_{3x} = -2.9$ fm and $R_{3y} = 0$ fm in Fig. 2 disappears, and instead of this minimum the saddle point emerges (see Fig. 7). It means the TNS with a value of R_{12} higher then 22 fm is an unstable system.

Moreover, from Figs. 9 and 10 it can be concluded that noncollinear ternary fission occurs in the following initial conditions: the deviation in y axis of location from the origin of the middle (Ca) nucleus or the difference from zero of the y component of the velocity of that nucleus.

Nevertheless, it is interesting that in all cases the decay time of the TNS has nearly the same value. It means that time does not depend on the initial conditions. This is because of the sequence of the fission: first, the Sn nucleus is separated from the Ni+Ca system, and then Ni is decayed from the Ca nucleus.

Because the collinearity of the ternary fission depends on initial conditions, the probability (or weight) of each initial condition's population is an open question which will be studied in future investigations.

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