Octupole correlations in $N = 88$ ^{154}Dy: Octupole vibration versus stable deformation

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We report on low-spin states of ^{154}Dy populated via the reaction ^{155}Gd (^{3}He ,4n) with a beam energy of 37.5 MeV from the Separated Sector Cyclotron at iThemba Laboratory. The AFRODITE γ-ray spectrometer was used to establish new $E1$ transitions between bands of opposite parity. The measurements broaden the $N = 88$ systematics on the relationship between the first excited positive-parity pairing isomer band and the lowest-lying negative-parity band as the nuclear quadrupole deformation decreases with increasing proton number. In a region of strong octupole correlations the data suggest that the spectroscopy of $N = 88$ nuclei is driven by stable octupole deformations and not by vibrations.

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The importance of octupole correlations in deformed nuclei has been emphasized over the past 30–40 years in many articles (including in Refs. [1–3]). The static octupole deformed nucleus shape $Y_{3,\mu}$ breaks the reflections symmetry on a plane that depends on μ [4]. For instance, in superdeformed (SD) nuclei it is the $Y_{3,2}$ collective excitation that is calculated to be lowest in energy in the SD minimum [5,6]. The ground-state bands of nuclei are rarely pure octupole deformed [4,7] but usually have a quadrupole deformation as well. For a $Y_{3,0}$ octupole shape the ground-state rotational structure will consist of interleaved natural parity states with spins and parities of $I^\pi = 0^+,1^-,2^+,3^-,4^+,5^-,\ldots$. Since the $Y_{3,0}$ octupole potential is finite at zero octupole deformation, then this lowers the positive-parity even spin states with respect to the negative-parity odd spin states. The difficulty in the experimental study of pear-shaped nuclei is a missing observable, which might be interpreted as a measure for the octupole deformation. The experimental signatures of the octupole deformed bands have been shown to include, but are not limited to, the observed low-lying bandhead of the $K^\pi = 0^-$, 1^-, 3^-, and 5^- bands in the actinide and rare-earth nuclei, the alternating parity of the even-A nuclei, and the enhanced dipole transition between the opposite parity states.. In this paper we will refer to the sequence of negative-parity states $K^\pi = 3^-$ as the octupole band.

It has been a natural understanding since the early days of the collective model [8,9] that the first excited 0^+ state in deformed even-even nuclei is a β band with vibrations along the symmetry axis. However recent improvements in technology have made available the sparse data on $K^\pi = 0_2^+$ bands. The results are mystifying [10], first—in many rare-earth-deformed nuclei, there are several excited $K^\pi = 0_2^+$ bands below the pairing gap. Second, there are huge variations in collectivity of the $K^\pi = 0_2^+$ bands in a narrow isotopic region and variations in collectivity among $K^\pi = 0_2^+$ bands in the same nucleus. Investigations using a (p,t) reaction on actinide regions revealed strong pair correlations in the lowest excited $K^\pi = 0_2^+$ bands in thorium, uranium, and plutonium [9,11]. Additionally, Sheline [12] suggests that a structural relationship between the excited $K^\pi = 0_2^+$ bands and the octupole band exist. These observations are puzzling and have intensified discussions on the nature of these $K^\pi = 0_2^+$ bands. Additional interpretations of these bands have been proposed [13–20].

Classically, the Raleigh formula [21] for the frequency of a $Y_{\lambda,\mu}$-shape vibration of a liquid drop is

$$\omega_\lambda^2 = \frac{(\lambda-1)\lambda(\lambda+2)\gamma}{\rho R^3}, \tag{1}$$

where ω_λ is the frequency of the λ pole oscillation of the drop, γ is the surface energy per unit area due to the surface tension, ρ is the density of the liquid, and R is the radius of the spherical drop. The application of this to an $A = 150$ spherical nucleus, also assuming irrotational flow, and taking the surface energy term in the Weizsäcker mass formula $\sim$18 MeV [22] gives $E_x = \hbar\omega_2 \approx 2.4$ MeV for $\lambda = 2$. For $\lambda = 3$, $\hbar\omega_3 \approx 4.6$ MeV. The Strutinski shell corrections [23] constrain the liquid drop potential quantum mechanically, and the observed greater than irrotational moments of inertia will both decrease the classical vibrational frequency further. Clearly, simple energy constraints suggest that any octupole vibration of the nucleus will be at about twice the first excited state energy of any quadrupole vibration. This was experimentally observed in $N = 88$ ^{152}Gd and ^{150}Sm [24].

Spear and Catford [25] have reviewed the measured $0_1^+ \rightarrow 3_1^-|M(E3)|^2$ transition strengths throughout the nuclear chart. They find peaks in these strengths at $N = 56$, 88, and 134 and $Z = 34$, 64, and 88. The increase in octupole collectivity is explained by the proximity of pairs of $\Delta j = 3$ single-particle

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nucleons to the Fermi surface. In the case of the nucleus $^{154}_{66}\mathrm{Dy}_{88}$ studied here, these are $i_{13/2} - f_{7/3}$ for neutrons and $h_{11/2} - d_{5/2}$ for protons. Strong octupole correlations, in the form of low-lying negative-parity bands and strong $E1$ and $E3$ transition strengths, have been observed in the lighter $N = 88$ nuclei [26–28]. We have previously studied the $N = 88$ nuclei $^{150}_{62}\mathrm{Sm}_{88}$ and $^{152}_{64}\mathrm{Gd}_{88}$ and found strong $E1$ transitions from the first excited $K^\pi = 0^+_2$ bands to the negative-parity lowest octupole bands [24]. The data suggested that pairing the $K^\pi = 0^+_2$ bands and octupole bands gives a better signature of octupole deformation than pairing the $K^\pi = 0^+_2$ ground-state bands with the octupole bands [29–33]. This finding was similar to the situation calculated by Chasman [34] in the octupole-deformed actinide region.

The excitation energy of the lowest 3^-_1 states in $N = 88$ even-even nuclei increases with proton number Z while the deformation decreases. This presents an opportunity to examine the systematics of the strong $E1$ transitions in a changing quadrupole and octupole deformation region. To this end we have populated low spin states of ^{154}Dy using the ^{155}Gd (^{3}He ,4n) reaction with a beam energy of 37.5 MeV from the Separated Sector Cyclotron at iThemba Laboratory and a 3.2-mg cm^{-2} self-supporting target. Coincident γ rays were detected with the AFRODITE spectrometer [35] array consisting of nine escape-suppressed high-purity germanium clover detectors. A total of 1.72×10^9 $\gamma\gamma$ coincidence events were collected and formed into a two-dimensional matrix RADWARE format [36] using MTSORT [37] for subsequent offline analysis.

A partial decay scheme for ^{154}Dy is shown in Fig. 1. New $E1$ transitions are shown in blue whereas other new transitions are shown in red. Unlike the situation in both $^{150}_{62}\mathrm{Sm}_{88}$ and $^{152}_{64}\mathrm{Gd}_{88}$ where the $E1$ connecting γ rays are from the $K^\pi = 0^+_2$ bands to the octupole band, in $^{154}_{66}\mathrm{Dy}_{88}$ the $E1$ connections are now from the octupole band to the $K^\pi = 0^+_2$ band. No doubt this

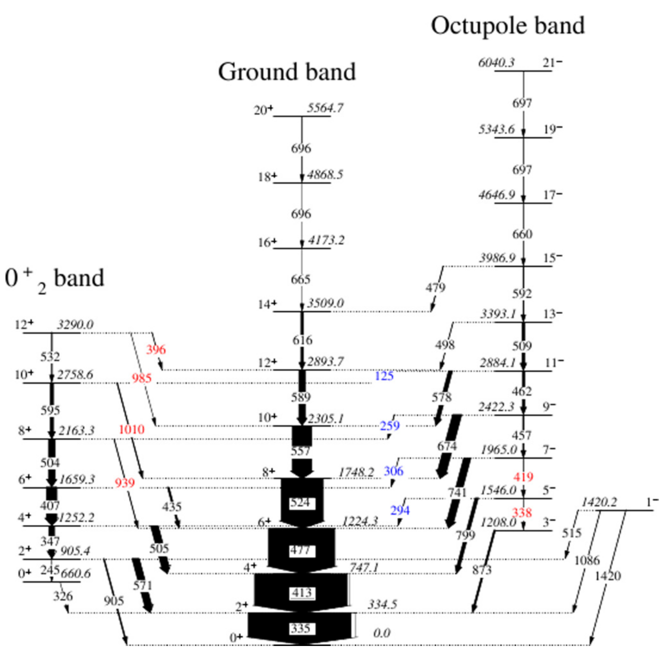

FIG. 1. Partial level scheme of ^{154}Dy obtained from the ^{155}Gd (^{3}He ,4n) reaction at 37.5 MeV. New $E1$ transitions from the octupole band to the 0^+_2 band are shown in blue, and other new transitions are shown in red. Previously published results are shown in black.

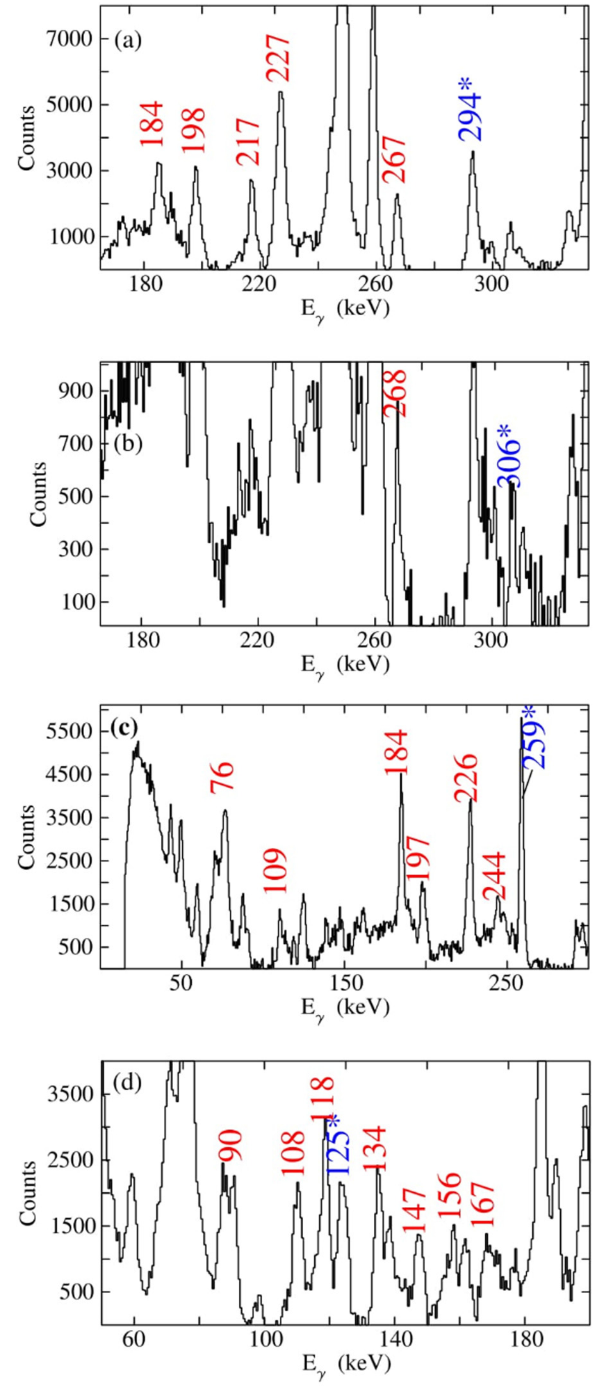

FIG. 2. Coincidence spectra illustrating the positioning of the $E1$ γ-ray transition depopulation of the octupole band feeding the 0^+_2 band (a) ^{154}Dy $\gamma\gamma$ data for the 294-keV ($5_1 \rightarrow 4^+_2$) γ ray gated by 347 keV. (b) ^{154}Dy $\gamma\gamma$ data for the 306-keV ($7^-_1 \rightarrow 6^+_2$) γ ray gated by 407 keV. (c) ^{154}Dy $\gamma\gamma$ data for the 259-keV ($9^-_1 \rightarrow 8^+_2$) γ ray gated by 504 keV. (d) ^{154}Dy $\gamma\gamma$ data for the 125-keV ($11^-_1 \rightarrow 10^+_2$) γ ray gated by 595 keV. The new $E1$ γ rays are marked with a blue asterisk.

is due to the relative excitation energies changing in ^{154}Dy. The $K^{\pi} = 0_2^+$ bandhead in ^{154}Dy has remained at a relatively low excitation energy whereas the 3_1^- level has gone up in energy (see Fig. 3 of Ref. [38]). The excitation energies of (0_2^+,3_1^-) states in ^{150}Sm, ^{152}Gd, and ^{154}Dy are (740,1071), (615,1123), and (661,1208) keV, respectively. Coincidence spectra establishing the new $E1$ transitions, blue in Fig. 1, from the octupole band to the $K^{\pi} = 0_2^+$ band are shown in Fig. 2.

There are two criteria advocated for determining any degree of octupole deformation, both measuring the interleaving of the positive- and negative-parity states. The first [5] is to divide the rotational frequency ω^- of the octupole band by the rotational frequency ω^+ of the positive-parity bands. This ratio would be equal to one if the positive-parity band is octupole deformed ($\omega^-/\omega^+ \approx 1$) and equal to $[4(I - 3) - 2]/(4I - 2)$ if the positive-parity band is not octupole deformed [39]. The rotational frequency ratio between the negative and the positive bands is defined as

$$\omega^-(I)/\omega^+(I) = 2\,\frac{E(I+1)^- - E(I-1)^-}{E(I+2)^+ - E(I-2)^+} \quad (I\,\text{even}),$$

$$\omega^-(I)/\omega^+(I) = 0.5\,\frac{E(I+2)^- - E(I-2)^-}{E(I+1)^+ - E(I-1)^+} \quad (I\,\text{odd}). \qquad (2)$$

This ratio is shown in Fig. 3(a) for pairing the negative-parity states with both the ground-state band and the $K^{\pi} = 0_2^+$ bands of ^{150}Sm, ^{152}Gd, and ^{154}Dy. It is clear that the pairing of the $K^{\pi} = 0_2^+$ bands with the negative-parity bands satisfies $\omega^-/\omega^+ \approx 1$ better than pairing with the ground-state bands.

The second test [40] takes the energy difference between the positive-parity band and the octupole band,

$$\pm\Delta E(I) = E(I)^{\mp} - \frac{E(I+1)^{\pm} + E(I-1)^{\pm}}{2}. \qquad (3)$$

Therefore $\Delta E(I) \approx 0$ if the octupole deformation is stable with spin variation. Plots of $\Delta E(I)$ are shown in Fig. 3(b), again for pairing the negative-parity states with both the ground-state band and the $K^{\pi} = 0_2^+$ bands of ^{150}Sm, ^{152}Gd, and ^{154}Dy. Not surprisingly it is again clear that the pairing of the $K^{\pi} = 0_2^+$ bands with the negative-parity bands satisfies $\Delta E(I) \approx 0$ better than pairing with the ground-state bands. In both Figs. 3(a) and 3(b) the $N = 88$ data are compared with data for the ground-state band paired with the negative-parity band in $^{224}_{88}$Ra$_{136}$, the best example of octupole deformation in the actinides [1]. The data for ^{150}Sm, ^{152}Gd [24], and ^{154}Dy are not taken past spin 10^+ because at spins at higher than 12^+ the $i_{13/2}$ neutrons are aligning causing considerable mixing of bands and perturbations in the energies of the levels.

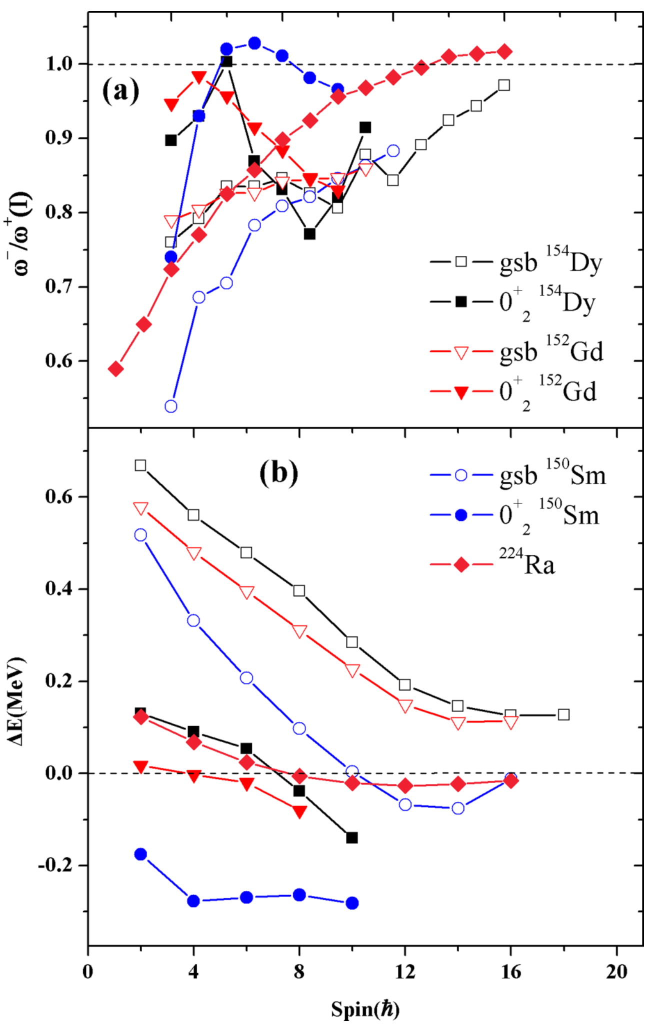

FIG. 3. (a) Frequency ratios ω^-/ω^+ Eq. (2) as a function of spin for the octupole bands divided by the ground-state bands, labeled “gsb” and for the octupole bands divided by 0_2^+ bands in ^{154}Dy and octupole-deformed ^{224}Ra [1,2,14]. For structures with permanent octupole deformation, this ratio would be 1.0. (b) Energy differences $\Delta E(I)$ Eq. (3) between negative- and positive-parity states in ^{150}Sm, ^{152}Gd, ^{154}Dy, and octupole-deformed ^{224}Ra [1,3,15]. For structures with permanent octupole deformation, this ratio would be 0.0.

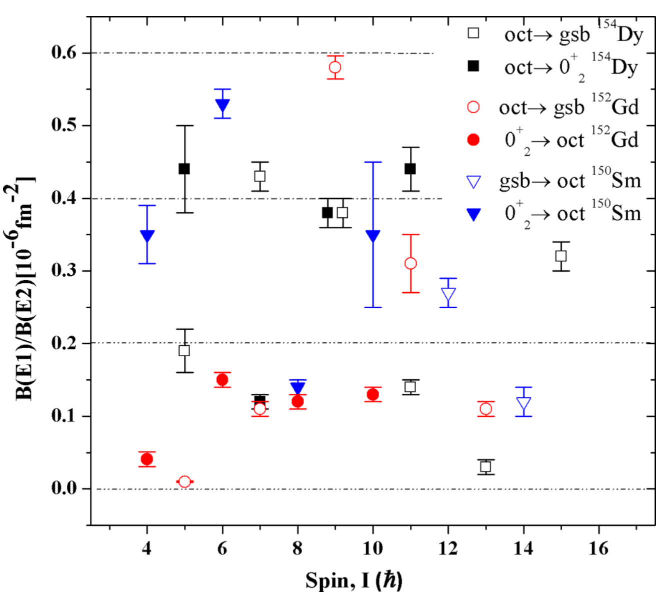

FIG. 4. Ratio of reduced transition probabilities for the out-of-band $E1$ and the in-band $E2$ transitions for ^{150}Sm, ^{152}Gd [19], and ^{154}Dy between the 0_2^+ band and the octupole band and between the octupole bands to the ground-state band.

Because an octupole shape can separate the center of charge and the center of mass in a nucleus, it is possible to have enhanced electric dipole $E1$ transitions between rotational bands [5]. The strength of these $E1$s may be determined by comparing the branching ratios of out-of-band $E1$ decays with the in-band $E2$ decays to give a ratio $B(E1)/B(E2)$ that does not depend on the actual γ-ray energies. This ratio is shown for the $E1$ transitions between octupole $\Leftrightarrow 0_2^+$ bands in ^{150}Sm, ^{152}Gd [24], and ^{154}Dy in Fig. 4 and compared with the $E1$ transitions octupole $\Leftrightarrow$ ground-state bands in the same nuclei. All these $E1$ transitions are enhanced compared to $E1$ transitions in near-spherical nuclei. These data confirm that the strong octupole correlations between the $K^\pi = 0_2^+$ bands and the lowest negative-parity band in $N = 88$ nuclei are preserved as the proton number Z increases and the quadrupole deformation decreases.

In conclusion, unique $E1$ transitions have been observed depopulating the octupole band feeding the band built on the 0_2^+ state which is opposite those observed in $N = 88$ isotopes of ^{152}Gd and ^{150}Sm [24]. Calculations that address this type of experimental data will need to include configuration-dependent pairing and octupole deformation. Chasman [34] has carried out calculations of this kind for nuclei in the actinides region. The relative strengths of the $E1$ transitions and the behavior of both 0_2^+ bands argues for, but does not prove, the Chasman interpretation [34] of the ground state being quadrupole and whereas the 0_2^+ state has an additional octupole deformation forming a simplex $s = 1$ alternating parity band with the lowest negative parity band. However, the nature of the $E1$ transitions shows that the octupole band is closely related to the band built on the 0_2^+ state than the ground-state band in ^{154}Dy. Frauendorf's tidal wave theory [40] has resemblances with the support of ^{150}Sm however less so for ^{152}Gd [24] and in ^{154}Dy. Furthermore, nuclei that exhibit such behavior at low spins ($J \sim 5$) have often been considered as the best candidates to exhibit static octupole deformation already in their ground states [1,4].

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