

# Nucleon-nucleon interaction with one-pion exchange and instanton-induced interactions

C. S. Vanamali\* and K. B. Vijaya Kumar†

*Department of Physics, Mangalore University, Mangalagangothri 574199, India*

(Received 22 June 2016; published 11 November 2016)

Singlet ( $^1S_0$ ) and triplet ( $^3S_1$ ) nucleon-nucleon potentials are obtained in the framework of the SU(2) nonrelativistic quark model using the resonating-group method in the Born-Oppenheimer approximation. The full Hamiltonian used in the investigation includes the kinetic energy, two-body confinement potential, one-gluon-exchange potential (OGEP), one-pion exchange potential (OPEP), and instanton induced interaction (III), which includes the effect of quark exchange between the nucleons. The contribution of the OGEP, III, and OPEP to the nucleon-nucleon adiabatic potential is discussed.

DOI: [10.1103/PhysRevC.94.054002](https://doi.org/10.1103/PhysRevC.94.054002)

## I. INTRODUCTION

The nucleon-nucleon (N-N) interaction has remained one of the formidable challenges in nuclear physics since its discovery by Rutherford and subsequent confirmation that the nucleon is made up of quarks. The N-N interaction is conventionally explained by the exchange of various mesons [1,2]. With the advent of QCD, and its acceptance as the theory of strong interaction, attempts have been made since the 1980s to explain the N-N interaction from QCD. Since the exact form of confinement is not known from QCD various phenomenological quark models, both relativistic and nonrelativistic, have been developed to explain the N-N interaction [3–16].

Despite the enormous progress made in the understanding of the N-N interaction, the understanding of the dynamics of the short-range interaction is still unsatisfactory. The short-range N-N interaction is repulsive in nature, which is in fact crucial for the stability of the nucleus. Quark model Hamiltonians usually consist of the kinetic energy, one-gluon exchange potential (OGEP), one-pion exchange potential (OPEP), and confinement potential. There are models which have tried to explain the N-N interaction using the kinetic energy, OGEP, and confinement potential in the framework of quark models. In all these models, short-range repulsion is entirely attributed to the exchange part of the color magnetic interaction [15,16]. The pioneering work on short-range repulsion was carried out by Neudatchin *et al.* [17]. In the framework of nonrelativistic quark models (NRQM), Oka *et al.* [18] have tried to explain the interaction by including the instanton induced interaction (III) and eliminating the OPEP. The III also has a color magnetic term and hence short-range repulsion is attributed to the exchange part of the color magnetic interaction of both the OGEP and the III [18]. The reasons for the inclusion of the III are that (i) a large value of the strong-coupling constant ( $\alpha_s \sim 1.6$ ) is required to reproduce the baryon spectrum in the phenomenological model, and (ii) for N- $\Delta$  and  $\pi$ - $\rho$  splitting, there is a sizable contribution from the nonperturbative  $q\bar{q}$

condensate, which is evident from the lattice QCD simulation in the quenched approximation [19].

The understanding of the N-N interaction has important implications for nuclear physics, particle physics, and astrophysics, and at present neither experimental nor theoretical analysis is satisfactory as far as the short-range interaction is concerned. Vinh Mau *et al.*, have solved the Schrödinger wave equation and computed all the observables for p-p scattering below threshold pion production and have investigated the effect of quark degrees of freedom on N-N observables in the framework of the NRQM and have come to the conclusion that the description of the N-N interaction at short distances by quark models depends on the additional terms added for the medium- and long-range forces [20]. The experimental data are as old as 20 years, which has severely hampered the progress in the theoretical sector. But with the advent of the J-PARC [21], PANDA [22], NICA [23], and HIAF [25] projects, the situation is likely to change [24]. Also, there are attempts to calculate the nuclear potential from lattice QCD by utilizing the Nambu-Bethe-Salpeter (NBS) wave functions, which were obtained from the four-point nuclear correlator. The Schrödinger wave equations were solved using the NBS wave functions and the nuclear potentials were obtained as an independent nonlocal interaction kernel which reproduces the qualitative features of the  $^1S_0$  and  $^3S_1$  states. The details can be found in Refs. [26,27].

The OPEP has to be included to obtain the partially conserved axial current, as it is important to know the contribution of the OPEP to the N-N potential [28]. In the framework of the NRQM, the pion is treated as an elementary field which couples to quarks with a strength which reproduces the experimental  $\pi$ -N coupling strength at zero momentum transfer [29]. In an alternative approach, a suitable form factor with a cutoff mass  $\Lambda$  is introduced to take care of the effect of the inner structure of pions at short range [30]. In the earlier versions of quark cluster models, the long- and medium-range parts of the N-N potential were introduced phenomenologically [31–33]. The long- and medium-range parts are dominated by the simple  $q\bar{q}$  pair exchange potential. The  $q\bar{q}$  pair is a color singlet cluster with pseudoscalar or vector meson quantum numbers. The obtained potentials have many characteristics of the conventional OBEP. In order to include the mesonic degrees of freedom, Fujiwara and Hecht

\*vanamalishastry@gmail.com

†kbvijayakumar@yahoo.com

incorporated the  $q\bar{q}$  excitations in the nonrelativistic model of the N-N interaction. The calculated nucleon-meson coupling constants are listed in Refs. [3,34]. Three of the coupling constants,  $f_{NN\eta}$ ,  $f_{NN\omega}$ , and  $f_{NN\rho}$  agree with the observed values of the coupling constants. But the calculated  $f_{NN\pi}$  is one-third of the observed  $f_{NN\pi}$  coupling. Hence, its contribution in the OBEP is small [34]. If pion is treated as a single  $q\bar{q}$  pair, it is not possible to obtain simultaneously proper values for both  $f_\pi$  and the charge radius  $\langle r_\pi^2 \rangle$ . If  $f_\pi$  is reproduced, then  $\langle r_\pi^2 \rangle$  turns out to be very large [35]. Hence, just the inclusion of the  $q\bar{q}$  pair will not suffice to produce long-range attraction; the introduction of the  $(3q)(q\bar{q})^2$  components leads to potentials with the characteristics of the conventional  $\sigma$  meson exchange potential and gives the additional medium-range attraction needed to bind the deuteron. But the identification of the scalar mesons is a long-standing problem, since scalar mesons have a very large decay width and hence cause an overlap between the resonances and the background. The problem is further compounded by the large non- $q\bar{q}$  scalar objects like glue balls and multiquark states. The PDG lists the scalar meson  $f_0(500)$  with  $I^G(0^+)$  and  $J^{PC}(0^{++})$  as a possible candidate for the  $\sigma$  meson suggested in the linear sigma model. Full details on scalar mesons below 2 GeV are given by the PDG [36]. Further,  $q\bar{q}$  excitations lead to an attractive part in the 0.8- to 1.5-fm range. However, this attraction is too weak to bind the deuteron and to reproduce the low-energy  $S$ -wave scattering parameters [34].

The inclusion of the  $q\bar{q}$  excitation with the quantum numbers of  $\omega$  does not lead to short-range repulsion given by the  $\omega$  meson exchange of the OBEP [37]. Although both  $\rho$  and the  $\omega$  coupling constants fall into the range of values extracted from the nucleon scattering data, the predicted value for the  $\pi$  meson is too weak [34,38,39]. Hence, the simple  $q\bar{q}$  cluster with the pion quantum numbers cannot be expected to give a realistic picture of the pion. It should be noted that in our present model the  $q\bar{q}$  pair creation and annihilation cannot be incorporated, as it is a nonrelativistic model and hence no double-counting occurs in adding the OPEP. Further, the present model is compatible with the nucleon quark-core r.m.s. radii of  $\sim 0.6$  fm. Hence, a picture of the nucleon of core radii of about 0.6 fm and a pion of small size coupling to quarks would help to clarify why the long-range N-N potential could be accurately described by the OPEP, which is of vital importance for nuclear physics.

Instantons were introduced in relation to the  $U_A(1)$  problem and their role was pointed out by t'Hooft by deriving effective interactions by coupling of the instantons and light quarks, whose strength of interaction depends on the instanton density, which was estimated from the gluon condensate of the QCD vacuum [40–42]. It was argued that the NRQM should include the III as a short-range nonperturbative gluon effect [43], where it was shown that in the  $p$ -wave sector of the N-N interaction, there is cancellation between the OGEP and the III. Also, lattice QCD suggests that the QCD vacuum contains instantons and its density is consistent with the gluon condensate expected from QCD sum rules [44]. It is well known that chiral symmetry is dynamically broken by the instanton vacuum and massless quarks are transformed into constituent quarks, which acquire mass as a function of momentum. Also, the

diquark correlations may induce phase transition of the QCD vacuum at high density into a color superconducting phase. Further, within the framework of the NRQM and relativistic quark models the III explains the  $\pi - \eta$  mass difference. There are models, both relativistic and nonrelativistic, employed to explain hadron spectra and baryon-baryon interaction with either the OGEP or the III. As has been pointed out [18] in quark models with only the OGEP, to reproduce the hyperfine splitting  $\alpha_s$  must be roughly twice as large as that naively expected in QCD. With the typical  $\alpha_s \sim 1.6$ , it is hard to justify the perturbative truncation of multigluon exchanges. In some quark models the OGEP has been completely eliminated and attempts have been made to explain the baryon spectrum and N-N interaction only with the III. We think it is not proper to eliminate the OGEP completely for light quarks, as it is consistent with asymptotic freedom. Also, the III vanishes for heavy quarks, and hence in heavy-quark spectra the OGEP is the only source of interaction which can explain the mass splitting. Hence, a consistent model has to incorporate both the OGEP and the III.

The basic aim of the present investigation is to make a detailed study of the contribution of the OGEP, III, and OPEP to the N-N adiabatic potential to the  $^1S_0$  and  $^3S_1$  states in the framework of the NRQM. One of the aims of the present study is to test whether the strong-coupling constant  $\alpha_s$  can be treated as a perturbative effect and to obtain a consistent set of parameters which reproduce both the singlet and the triplet N-N potentials. The adiabatic N-N potentials have been obtained using the Born-Oppenheimer approximation [4].

The paper is organized as follows: in Sec. II, we review the NRQM. In Sec. III a brief description of the resonating-group method (RGM) is given. The results of the calculations are presented in Sec. IV. Conclusions are given in Sec. V.

## II. NONRELATIVISTIC QUARK MODEL

The full Hamiltonian used in the investigation is

$$H = K + V_{\text{int}} + V_{\text{Conf}} - K_{\text{CM}}, \quad (1)$$

where  $K$  is the kinetic energy,  $V_{\text{int}}$  is the interaction potential term,  $V_{\text{Conf}}$  is the harmonic confinement potential, and  $K_{\text{CM}}$  is the kinetic energy of the center of mass,

$$K = \sum_{i=1}^6 \frac{\mathbf{p}_i^2}{2m_i}, \quad K_{\text{CM}} = \frac{\mathbf{P}^2}{12m_i},$$

where  $m_i$  and  $\mathbf{p}_i$  are the mass and the momentum of the  $i$ th quark and  $\mathbf{P}$  is the momentum of the center of mass. The interaction potential is

$$V_{\text{int}} = V_{\text{OGEP}} + V_{\text{III}} + V_{\text{OPEP}},$$

where

$$V_{\text{OGEP}} = \frac{\alpha_s}{4} \sum_{i<j} \left( \frac{1}{r_{ij}} - \frac{\pi}{m_q^2} \left( 1 + \frac{2}{3} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right) \delta(\mathbf{r}_i - \mathbf{r}_j) \right) \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j. \quad (2)$$

The first two terms are the color electric terms and the third term is the color magnetic interaction leading to the hyperfine

splitting. Here,  $\lambda_i$  and  $\lambda_j$  are the generators of the color SU(3) group for the  $i$ th and the  $j$ th quarks,  $\sigma_i$  is the Pauli spin operator,  $\alpha_s$  is the strong-coupling constant,

$$V_{\text{III}} = - \sum_{i < j} \frac{1}{2} W_{ij} (1 - P_{ij}) \left[ 1 - \frac{1}{5} (\sigma_i \cdot \sigma_j) \right] \delta(\mathbf{r}_i - \mathbf{r}_j), \quad (3)$$

$$V_{\text{OPEP}} = \frac{f_Q^2}{3} \sum_{i < j} \frac{e^{-m_\pi r_{ij}}}{r_{ij}} (\sigma_i \cdot \sigma_j) (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j), \quad (4)$$

$$V_{\text{Conf}} = \sum_{i < j} a_c r_{ij}^2. \quad (5)$$

In the above expressions,  $r_{ij}$  is the separation between quarks,  $m_q$  is the mass of the quark,  $\boldsymbol{\tau}_i$  is the isospin of the  $i$ th quark,  $m_\pi$  is the mass of the pion, and  $a_c$  is the confinement strength.  $f_Q$  is the OPEP strength parameter and is related to the pion-nucleon coupling constant by the relation  $f_Q^2 = \frac{f_{\pi NN}^2}{4\pi}$  [28]. Since chiral symmetry breaking in the quark models specifies the coupling of pions with quarks, our model incorporates the spontaneous breaking of chiral symmetry, which is the most important among the dynamics of low-energy QCD and hence justifies qualitatively, the nonrelativistic model of hadrons.

In the framework of the SU(2) NRQM, the expression for  $V_{\text{III}}$  reduces to

$$V_{\text{III}} = -\frac{1}{2} W \sum_{i < j} \left[ \frac{16}{15} + \frac{2}{5} \lambda_i \cdot \lambda_j + \frac{1}{10} \sigma_i \cdot \sigma_j \lambda_i \cdot \lambda_j \right] \times \delta(\mathbf{r}_i - \mathbf{r}_j), \quad (6)$$

where  $W$  is the strength of the III potential. The corresponding expression in the SU(3) NRQM can be found in Ref. [18].

### III. RESONATING-GROUP METHOD

The N-N interaction can arise only if the quarks are exchanged between the nucleons. If the quarks are not exchanged between the nucleons, then there is no N-N interaction, since the matrix element  $\lambda_i \cdot \lambda_j$  vanishes when the  $i$ th quark is in one nucleon and the  $j$ th quark is in the other nucleon as a consequence of the Wigner-Eckart theorem. Hence, the N-N interaction arises when one constructs a completely antisymmetric wave function for the six-quark system, in which case exchange terms arising solely from antisymmetrization do not vanish. The advantage of the RGM formulation is that it employs a totally antisymmetric wave function and correctly treats the motion of the center of mass. We have employed the RGM to solve the equation [4]

$$\langle \psi | (H - E) A | \psi \rangle = 0 \quad (7)$$

to get the energy ( $E$ ) of the interacting nucleons. Here,  $H$  is the Hamiltonian,  $\psi$  is the wave function of the nucleons, and  $A$  is the antisymmetrization operator,

$$A = \frac{1}{10} (1 - 9P_{36}^{\text{OSTC}}), \quad (8)$$

where  $P_{36}^{\text{OSTC}}$  is the permutation operator for quarks 3 and 6 and OSTC stands for orbital, spin, isospin, and color, respectively. Thus, the  $P_{36}^{\text{OSTC}}$  operator exchanges the orbital, spin, isospin, and color quantum numbers of quarks 3 and 6.

This can be visualized in the following manner. When the nucleons are sufficiently close, their wave functions overlap to a considerable extent. When this happens, any one or more quarks belonging to one nucleon can be exchanged with an equal number of quarks of the other nucleon. Three-quark exchange results in the interchange of the nucleons themselves; two-quark exchange can be thought of as one-quark exchange plus an interchange of nucleons. Since quarks are identical indistinguishable particles, only the number of quarks exchanged needs to be considered and there are nine ways of exchanging one pair of quarks between two nucleons. Hence the factor of 9 in the operator.

The antisymmetrization operator splits each term in the Hamiltonian into two parts: the direct part and the exchange part. The direct part corresponds to the interaction without exchange of quarks and the exchange part corresponds to the interaction with exchange of one quark between the nucleons. At asymptotic distances, the exchange part of the interaction vanishes since the overlap of wave functions is absent.

The wave function  $\psi$  includes the orbital [ $\phi(\mathbf{r}_i)$ ], spin ( $S$ ), isospin ( $T$ ), and color ( $C$ ) wave functions. The harmonic oscillator wave function is chosen as the the orbital wave function,

$$\phi(\mathbf{r}_i) = \frac{1}{(\pi b^2)^{3/4}} \exp\left(-\frac{1}{2b^2} \left(\mathbf{r}_i - \frac{\mathbf{s}_I}{2}\right)^2\right), \quad (9)$$

where  $b$  is the oscillator size parameter and  $\mathbf{s}_I$  is the generator coordinate.

Using Eq. (7), we calculate three kernels: (a) the normalization kernel

$$\langle \psi | A | \psi \rangle, \quad (10)$$

(b) the kinetic energy kernel

$$\langle \psi | K A | \psi \rangle, \quad (11)$$

and (c) the potential energy kernel

$$\langle \psi | (V_{\text{int}} + V_{\text{Conf}}) A | \psi \rangle. \quad (12)$$

The energy is given by

$$E = \frac{\langle \psi | H A | \psi \rangle_l}{\langle \psi | A | \psi \rangle_l}. \quad (13)$$

The subscript  $l$  indicates that the quantities have been projected to the angular momentum state  $l$ .

In the present formalism, each nucleon is treated as a cluster of three quarks and the two-nucleon system is denoted as cluster A and cluster B, respectively. The basic idea in the RGM is to express the total wave function of the system in terms of the antisymmetric product of single-particle wave functions. The total wave function of the six-quark system is

$$\psi_{\text{TOT}}(\xi_A, \xi_B, \mathbf{R}_{AB}) = \mathbf{A}[\phi_A(\xi_A)\phi_B(\xi_B)\chi(\mathbf{R}_{AB})], \quad (14)$$

where  $\phi_A$  and  $\phi_B$  are the internal wave functions of clusters A and B, respectively,  $\chi$  is the relative wave function between the two clusters, and  $\mathbf{A}$  is the total anti-symmetric operator of the six-quark system. To separate the total wave function into the form given above, the following choice of coordinates

TABLE I. List of parameters.

$b$ (fm)	0.6
$\alpha_s$	0.713
$W$ (MeV fm <sup>3</sup> )	67.67
$a_c$ (MeV fm <sup>-2</sup> )	40.5
$m_q$ (MeV)	300.0
$m_\pi$ (MeV)	140.0
$f_q^2$	12.6

is made [4,16]:

$$\begin{aligned}\xi_1 &= \mathbf{r}_1 - \mathbf{r}_2, & \xi_2 &= \mathbf{r}_3 - \frac{\mathbf{r}_1 + \mathbf{r}_2}{2}, \\ \mathbf{R}_A &= \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3), \\ \xi_3 &= \mathbf{r}_4 - \mathbf{r}_5, & \xi_4 &= \mathbf{r}_6 - \frac{\mathbf{r}_4 + \mathbf{r}_5}{2}, \\ \mathbf{R}_B &= \frac{1}{3}(\mathbf{r}_4 + \mathbf{r}_5 + \mathbf{r}_6), \\ \mathbf{R}_{AB} &= \mathbf{R}_A - \mathbf{R}_B, & \mathbf{R}_G &= \frac{1}{2}(\mathbf{R}_A + \mathbf{R}_B).\end{aligned}$$

Here  $\mathbf{r}_i$  is the coordinate of the  $i$ th quark, the coordinates  $\xi_A = (\xi_1, \xi_2)$  and  $\xi_B = (\xi_3, \xi_4)$  are the internal coordinates of the two clusters A and B, respectively,  $\mathbf{R}_{AB}$  is the relative coordinate between the two clusters, and  $\mathbf{R}_G$  is the center-of-mass coordinate of the total system.

Since the Hamiltonian is translationally invariant,  $L_{ij}^l$  can be written as [4,16]

$$\begin{aligned}L_{ij}^l &= \int \left[ \phi_A^{+\text{SM}}\left(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \frac{S_I}{2}\right) \phi_B^{+\text{SM}}\left(\mathbf{r}_4, \mathbf{r}_5, \mathbf{r}_6, \frac{-S_I}{2}\right) Y_{lm}^*(\hat{s}_I) \right] \\ &\times (H - E) A \left[ \phi_A^{\text{SM}}\left(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \frac{S_J}{2}\right) \phi_B^{\text{SM}}\left(\mathbf{r}_4, \mathbf{r}_5, \mathbf{r}_6, \frac{-S_J}{2}\right) \right. \\ &\left. \times Y_{lm}(\hat{s}_J) \right] \prod_k d^3\mathbf{r}_k d\hat{s}_I d\hat{s}_J.\end{aligned}$$

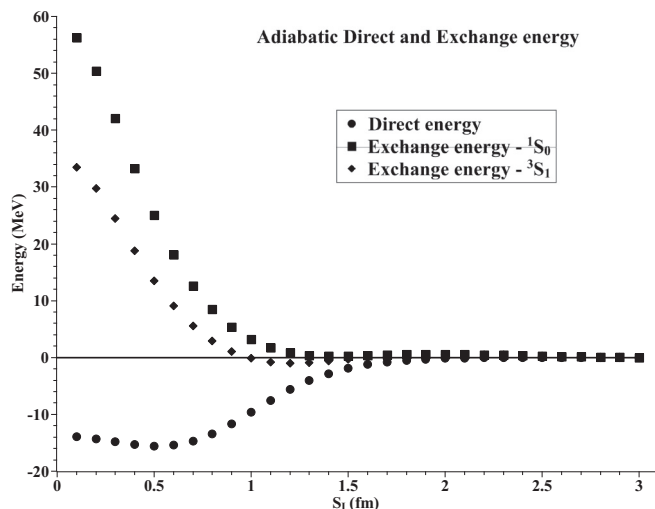


FIG. 1. Adiabatic direct and exchange potentials.

To take into account all possible interactions between the quarks, we have to consider seven types of operators for the potential  $V_{ij}$  in the Hamiltonian. They are  $V_{12\text{DR}}$ ,  $V_{36\text{DR}}$ ,  $V_{12\text{EX}}$ ,  $V_{13\text{EX}}$ ,  $V_{16\text{EX}}$ ,  $V_{14\text{EX}}$ , and  $V_{36\text{EX}}$ , where DR stands for the direct part of the quark interaction between quark  $i$  and quark  $j$  and EX stands for the corresponding exchange part.

#### IV. RESULTS AND DISCUSSION

There are seven parameters in our model: the masses of the quarks ( $m_q$ ), the confinement strength ( $a_c$ ), the harmonic oscillator size parameter ( $b$ ), the quark-gluon coupling constant ( $\alpha_s$ ), the instanton coupling coefficient ( $W$ ), the mass of the pion ( $m_\pi$ ), and the quark-pion coupling constant  $f_q^2$ . The coupling constants  $\alpha_s$  and  $W$  are fixed by the  $N$ - $\Delta$  mass splitting which comes from the color magnetic term of the OGEP and III and  $f_q^2$  is fixed from the Goldberger-Treiman relation [28]. Though the masses of the quarks are usually estimated from analysis of the baryon spectrum, there are uncertainties in the determination of the masses of the light quarks, as the bounds obtained from Feynman-Hellman theorem [45] are not very accurate for light quarks and a better estimation of quark masses is obtained from the determination of the magnetic moments for  $u$  and  $d$  quarks, which is about 330 MeV [46,47]. We have chosen the value of the oscillator size parameter to be 0.6 fm, which is consistent with the experimental results of the charge distribution of the nucleons and the axial charge distribution [3]. As in any constituent quark model there are theoretical uncertainties in the values of the parameters. For example, there is slight uncertainty in the value of the the oscillator size parameter, since the diagonalization of the matrix elements in a larger harmonic oscillator basis lowers the value of  $b$  due to the contribution from the off-diagonal elements [48]. The parameters used in the model are listed in Table I.

Figure 1 is a plot of the direct and exchange parts of the Hamiltonian in the adiabatic limit. The exchange parts of the

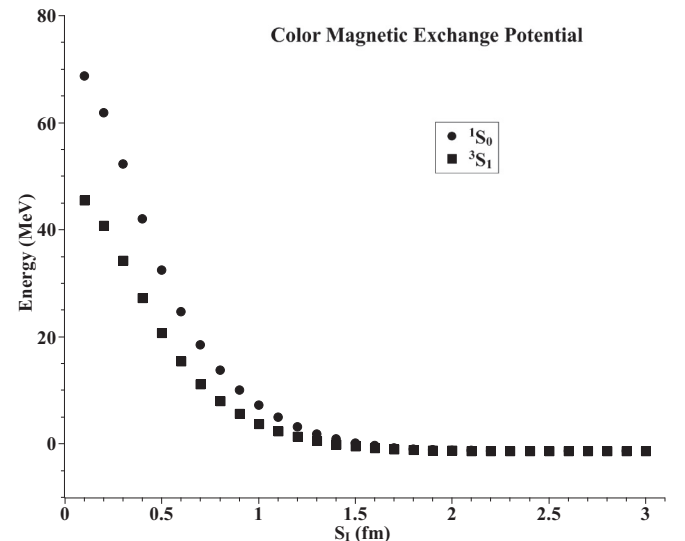


FIG. 2. Color magnetic exchange potential.

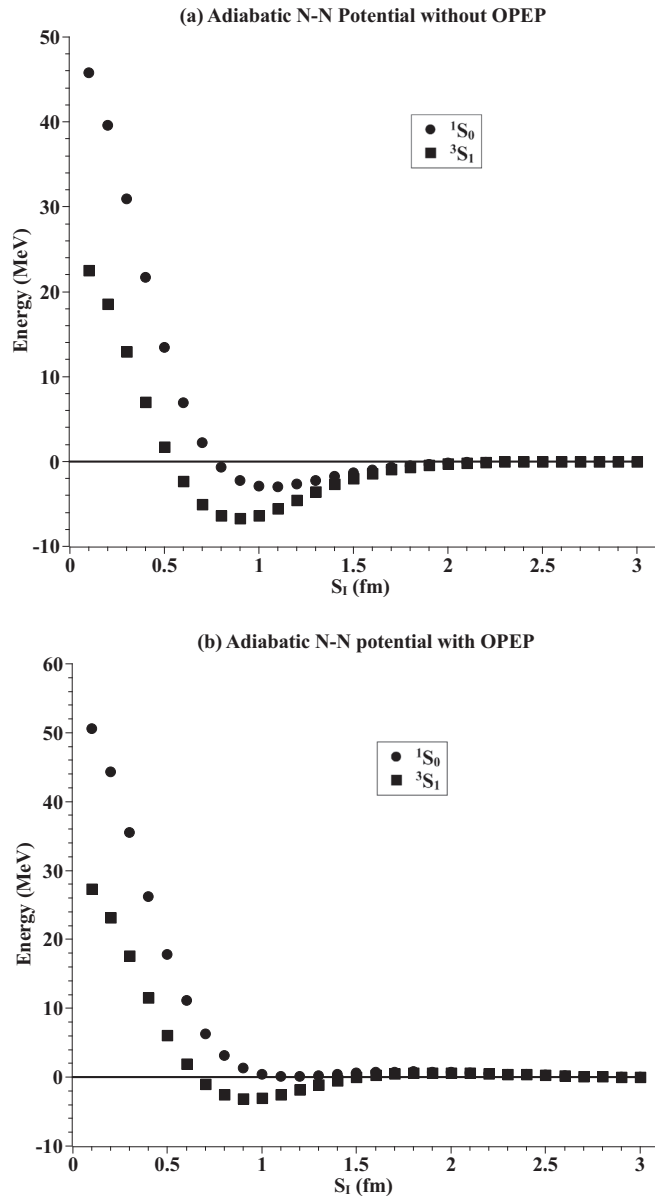


FIG. 3. Adiabatic potential for singlet and triplet states (a) without OPEP and (b) with OPEP.

potentials of the  $^1S_0$  and  $^3S_1$  states are repulsive in the short range. The exchange potential of the  $^1S_0$  state is completely repulsive and that of the  $^3S_1$  state shows a small attraction in the intermediate range. Figure 2 is a plot of the exchange part

of the color magnetic interaction in the adiabatic limit. There is a substantial repulsive contribution to the adiabatic potential at short range to both the singlet and the triplet  $S$  states, which is consistent with the established results [3,11,14]. It should be noted that the color electric term does not contribute to the N-N interaction. Since the radial matrix elements are the same for the  $2(0s)^3$  configuration and for the  $(0S)^6$  configuration, the energy difference between the  $2(0s)^3$  and the  $(0S)^6$  configurations must come from the expectation value of  $\lambda_i \cdot \lambda_j$ . But the expectation value of  $\lambda_i \cdot \lambda_j$  depends only on the number of quarks. Hence, the color electric elements of the OGEP, III, and confinement term do not contribute to the N-N adiabatic potential. For the color magnetic part the expectation value of  $\lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j$  for the  $2(0s)^3$  configuration and for the  $(0S)^6$  configuration does not vanish and the color magnetic part provides short-range repulsion [3].

The first term in the III interaction [Eq. (6)] due to the antisymmetrization operator gives direct and exchange interactions and corresponds to the color singlet exchange. The adiabatic potential due to the color singlet is attractive in the short range. The entire result of our work is summarized in Fig. 3, which gives plots of the adiabatic N-N potential with and without the OPEP. In the presence of the OPEP, attraction in the intermediate range for the  $^1S_0$  state vanishes. Short-range repulsion is larger for the singlet state than the triplet state. This difference is entirely due to the color magnetic part of the OGEP and III.

## V. SUMMARY AND CONCLUSIONS

In this work, the well-known RGM technique in the framework of the NRQM has been employed to obtain the N-N adiabatic potential for the  $^1S_0$  and  $^3S_1$  states using the Born-Oppenheimer approximation. The aim is to understand the role played by the OGEP, III, and OPEP to the adiabatic potential. To calculate the N-N matrix elements six-quark antisymmetric wave functions are constructed. Only exchange terms of the N-N interaction contribute to the N-N adiabatic potential. The adiabatic potentials obtained in the Born-Oppenheimer approximation for the  $^1S_0$  and  $^3S_1$  states have a qualitative similarity to the phenomenological potentials. The short-range repulsion arises from the kinetic energy and exchange terms of the color magnetic terms of the OGEP and III. The OPEP provides state-independent repulsion and contributes substantially to the N-N adiabatic potential. The color octet term of the III provides short-range attraction. The color electric terms do not contribute to the N-N potential. The contribution from the color magnetic terms to the short-range repulsion is significant.

- [1] A. de Rujula, H. Georgi, and S. L. Glashow, *Phys. Rev. D* **12**, 147 (1975).  
 [2] R. Machleidt, K. Holinde, and Ch. Elster, *Phys. Rep.* **149**, 1 (1987).  
 [3] K. Shimizu, *Rep. Prog. Phys.* **52**, 1 (1989).  
 [4] M. Oka and K. Yazaki, *Prog. Theor. Phys.* **66**, 556 (1981).

- [5] W. Weise, in *International Review of Nuclear Physics*, edited by T. T. S. Kuo and E. Osnes, Vol. 1 (World Scientific, Singapore, 1984), p. 58.  
 [6] C. E. De Tar, *Phys. Rev. D* **17**, 302 (1978).  
 [7] D. A. Liberman, *Phys. Rev. D* **16**, 1542 (1977).  
 [8] K. Maltman and N. Isgur, *Phys. Rev. Lett.* **50**, 1827 (1983).

- [9] C. S. Warke and R. Shanker, *Phys. Rev. C* **21**, 2643 (1980).
- [10] F. Stancu, S. Pepin, and L. Y. Glozman, *Phys. Rev. C* **56**, 2779 (1997).
- [11] F. Myhrer and J. Wroldnen, *Rev. Mod. Phys.* **60**, 629 (1988).
- [12] A. Manohar and H. Georgi, *Nucl. Phys. B* **234**, 189 (1984).
- [13] Z. Zhang, A. Faessler, U. Straub, and L. Ya Glozman, *Nucl. Phys. A* **578**, 573 (1994).
- [14] A. Valcarce, A. Buchmann, F. Fernandez, and A. Faessler, *Phys. Rev. C* **50**, 2246 (1994).
- [15] S. B. Khadkikar and K. B. Vijaya Kumar, *Phys. Lett. B* **254**, 320 (1991).
- [16] K. B. Vijaya Kumar and S. B. Khadkikar, *Nucl. Phys. A* **556**, 396 (1993).
- [17] V. G. Neudatchin, F. Yu Smirnov, and R. Tamagaki, *Prog. Theor. Phys.* **58**, 1072 (1977).
- [18] M. Oka and S. Takeuchi, *Phys. Rev. Lett.* **63**, 1780 (1989).
- [19] M. Fukugita, Y. Oyanagi, and A. Ukawa, *Phys. Rev. D* **36**, 824 (1987); P. Bacilieri *et al.*, *Phys. Lett. B* **214**, 115 (1988).
- [20] R. Vinh Mau, C. Semay, B. Loiseau, and M. Lacombe, *Phys. Rev. Lett.* **67**, 1392 (1991).
- [21] S. Kumano, *Nucl. Phys. A* **782**, 442 (2007).
- [22] W. Erni *et al.* (PANDA Collaboration), arXiv:0903.3905 [hep-ex].
- [23] A. N. Sissakian *et al.* (NICA Collaboration), *J. Phys. G* **36**, 064069 (2009).
- [24] M. Naghdi, *Phys. Part. Nucl.* **45**, 924 (2014).
- [25] J. C. Yang *et al.*, *Nucl. Instrum. Methods Phys. Res. Sec. B* **317**, 263 (2013).
- [26] K. Murano, N. Ishii, S. Aoki, and T. Hatsuda, in *Proceedings of the 27th International Symposium on Lattice field theory (Lattice 2009), Beijing, China* (SISSA, Trieste, Italy, 2009), p. 126; in *Proceedings of the 28th International Symposium on Lattice field theory (Lattice 2010), Villasimius, Italy* (SISSA, Trieste, Italy, 2010), p. 150; K. Murano *et al.* (HAL-QCD Collaboration), *Prog. Theor. Phys.* **125**, 1225 (2011).
- [27] N. Ishii *et al.*, in *Proceedings of the 27th International Symposium on Lattice field theory (Lattice 2009), Beijing, China* (SISSA, Trieste, Italy, 2009), p. 019; *Few Body Syst.* **49**, 269 (2011).
- [28] M. L. Goldberger and S. B. Treiman, *Phys. Rev.* **110**, 1178 (1958).
- [29] K. Shimizu, *Phys. Lett. B* **148**, 418 (1984).
- [30] F. Fernandez and E. Oset, *Nucl. Phys. A* **455**, 720 (1986).
- [31] M. Oka and K. Yazaki, *Phys. Lett. B* **90**, 41 (1980).
- [32] A. Faessler and F. Fernandez, *Phys. Lett. B* **124**, 145 (1983).
- [33] A. Faessler, F. Fernandez, G. Lubeck, and K. Shimizu, *Phys. Lett. B* **112**, 201 (1982).
- [34] Y. Fujiwara and K. T. Hecht, *Nucl. Phys. A* **444**, 541 (1985); *Phys. Lett. B* **171**, 17 (1986); *Nucl. Phys. A* **451**, 625 (1986); **456**, 669 (1986); **462**, 621 (1987).
- [35] R. van Royen and V. Weisskopf, *Nuovo Cimento* **50**, 617 (1967).
- [36] K. A. Olive *et al.*, *Chin. Phys. C* **38**, 090001 (2014).
- [37] K. Yazaki, *Nucl. Phys. B* **279**, 81 (1987); *Prog. Theor. Phys. Suppl.* **91**, 146 (1987).
- [38] I. Duck, *Phys. Lett. B* **77**, 223 (1978).
- [39] Y. W. Yu and Z. Y. Zhang, *Nucl. Phys. A* **426**, 557 (1984).
- [40] G. t'Hooft, *Phys. Rev. D* **14**, 3432 (1976).
- [41] M. A. Shifman, A. I. Vainshtet, and V. I. Zakharov, *Nucl. Phys. B* **163**, 46 (1980).
- [42] E. V. Shuryak, *Phys. Rep.* **115**, 151 (1984).
- [43] S. Takeuchi, *Phys. Rev. D* **53**, 6619 (1996).
- [44] M. C. Chu, J. M. Grandy, S. Huang, and J. W. Negele, *Phys. Rev. D* **49**, 6039 (1994).
- [45] R. A. Bertlmann and S. Ono, *Z. Phys. C - Particles and Fields* **8**, 271 (1981).
- [46] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, *Hadron Transitions in the Quark Model* (Gordon and Breach Science, Philadelphia, 1988), p. 187.
- [47] E. Oset, T. Tegen, and W. Weise, *Nucl. Phys. A* **426**, 456 (1984).
- [48] Bhavyashri, K. B. Vijaya Kumar, S. Sarangi, and S.-G. Zhou, *J. Phys. G* **31**, 981 (2005).