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Applicability of the continuum-discretized coupled-channels method to the deuteron breakup at low energies

Kazuyuki Ogata* and Kazuki Yoshida

Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki 567-0047, Japan

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We reexamine the deuteron elastic breakup cross sections on ^{12}C and ^{10}Be at low incident energies, for which a serious discrepancy between the continuum-discretized coupled-channels (CDCC) method and the Faddeev-Alt-Grassberger-Sandhas (FAGS) theory was pointed out. We show the closed channels neglected in the preceding study affect significantly the breakup cross section calculated with the CDCC method, resulting in good agreement with the result of FAGS theory.

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Projectile breakup reactions have played a major role in studying the structure of loosely bound nuclei [1]. Such a reaction contains at least three particles in the final state. Thus, one may say that the accurate description of the three-body breakup process is a minimum requirement for nuclear reaction theories. It is well known that the Faddeev theory [2] or, alternatively, the Alt-Grassberger-Sandhas (AGS) theory [3] gives the exact solution to such a three-body scattering problem. However, the continuum-discretized coupled-channels (CDCC) method [4–6] has widely been applied with high success to projectile breakup reactions at various incident energies. The theoretical foundation of the CDCC method was given in Refs. [7,8] in connection with the distorted-wave Faddeev formalism [9]. Quite recently [10], invention of the treatment of the Coulomb interaction made the Faddeev-AGS (FAGS) theory applicable to various three-body breakup reactions, and the results of the FAGS theory have directly been compared with those of the CDCC method. In many cases the two give very similar cross sections, which validates the CDCC method as an effective three-body reaction model, as predicted in Refs. [7,8].

In a systematic comparison [11] between FAGS theory and the CDCC method, however, it was shown that at high incident energies E_d of a deuteron, (d,p) transfer cross sections calculated with the CDCC method somewhat deviate from those calculated with FAGS theory, i.e., the exact cross sections. More seriously, at E_d below about 20 MeV, the deuteron elastic breakup cross sections obtained with the CDCC method overshoot those of FAGS theory by about a factor of 3 at most. The latter finding can particularly be a striking indication of the limitation of the CDCC method, suggesting that at low incident energies one has to rely on a more elaborate reaction model or exact FAGS theory for describing even elastic breakup processes. In Ref. [11], however, the so-called closed channels (see below) were not included. As mentioned in the literature, e.g., Refs. [5,8], inclusion of closed channels is crucial for quantitative discussion of observables, at low incident energies in particular. This was numerically confirmed in Ref. [12] for a one-dimensional scattering problem, and in Ref. [13] for the scattering of ^{11}Be. There exist several indications of the importance of closed channels also for transfer reactions [14–16]. Under the circumstances, in the present study, we revisit the problem reported on the low-energy elastic breakup cross sections for ^{10}Be$(d,pn)^{10}$Be at $E_d = 21$ MeV and ^{12}C$(d,pn)^{12}$C at $E_d = 12$ MeV, and discuss more in detail the convergence of CDCC results, putting emphasis on the closed channels.

We give a brief review on the CDCC method; for more details, see, e.g., Refs. [4–6]. We describe the deuteron elastic breakup with the target nucleus A, on the basis of a $p + n + A$ three-body model. We do not explicitly take into account the excitation of A during the breakup process. We neglect also the intrinsic spin of each of the three particles, following Ref. [11]. In the CDCC method the total three-body wave function for the total angular momentum J and its projection M is expanded in terms of the complete set of the projectile wave function $\{\phi\}$:

$$\Psi^{JM}(\boldsymbol{r},\boldsymbol{R}) = \sum_{i=0}^{i_{\max}} \sum_{\ell=0}^{\ell_{\max}} \sum_{L=|J-\ell|}^{J+\ell} \phi_{i\ell}(r)\chi_c(R) \times [i^\ell Y_\ell(\hat{\boldsymbol{r}}) \otimes i^L Y_L(\hat{\boldsymbol{R}})]_{JM}, \tag{1}$$

where $\boldsymbol{r}$ $(\boldsymbol{R})$ is the coordinate of p (the center of mass of d) relative to n (A); i is the energy index and $i = 0$ represents the ground state of d. The orbital angular momenta corresponding to $\boldsymbol{r}$ and $\boldsymbol{R}$ are denoted by ℓ and L, respectively; Y_{lm} is the spherical harmonics. We put the channel indices of the scattering wave χ altogether in c, i.e., $c = \{J,i,\ell,L\}$. In the derivation of Eq. (1) we discretized the p-n continua with the so-called momentum-bin average method:

$$\phi_{i\ell}(r) = \frac{1}{\sqrt{\Delta k}} \int_{k_i}^{k_i+\Delta k} dk\, \varphi_{k,\ell}(r), \tag{2}$$

where $k_i = (i-1)\Delta k$ and φ_k is the partial wave of the p-n scattering wave function under a p-n interaction V_{pn}, with k the absolute value of the asymptotic relative momentum. The discretized p-n energy of the ith state $(i > 0)$ is given by [4]

$$\hat{\epsilon}_i = \frac{\hbar^2}{2\mu_{pn}}\left[\frac{\Delta k}{12} + \frac{(2k_i + \Delta k)^2}{4}\right],$$

where μ_{pn} is the p-n reduced mass. The size Δk of the momentum bin, the maximum linear momentum $k_{\max} = i_{\max}\Delta k$

*kazuyuki@rcnp.osaka-u.ac.jp

(in the unit of $\hbar$), and $\ell_{\max}$ are key values for determining the reaction model space of the CDCC method.

The asymptotic form of χ_c is given by

$$\chi_c \to U^{(-)}_{L,\eta_i}(K_i R)\delta_{cc_0} - \sqrt{K_0/K_i}\; S_{cc_0} U^{(+)}_{L,\eta_i}(K_i R) \quad (3)$$

for $E_i > 0$, and

$$\chi_c \to -S_{cc_0} W_{-\eta_i, L+1/2}(-2i K_i R) \quad (4)$$

for $E_i \leqslant 0$, where $E_i = E - \hat{\epsilon}_i$ and $K_i = \sqrt{2\mu E_i}/\hbar$; c_0 represents the incident channel. $U^{(-)}_{L,\eta_i}$ ($U^{(+)}_{L,\eta_i}$) is the incoming (outgoing) Coulomb wave function with the Sommerfeld parameter η_i and $W_{-\eta_i,L+1/2}$ is the Whittaker function. Channels having $E_i > 0$ and $E_i \leqslant 0$ are called open channels and closed channels, respectively. S_{cc_0} for open channels are scattering matrix elements, with which physics observables are calculated in a standard manner. However, S_{cc_0} for closed channels are not related to observables, at least directly. It is obvious, however, that the closed channels can affect the breakup observables through mainly continuum-continuum couplings [8].

In the CDCC calculation shown below, we disregard the intrinsic spins of p and n as mentioned, and also the Coulomb

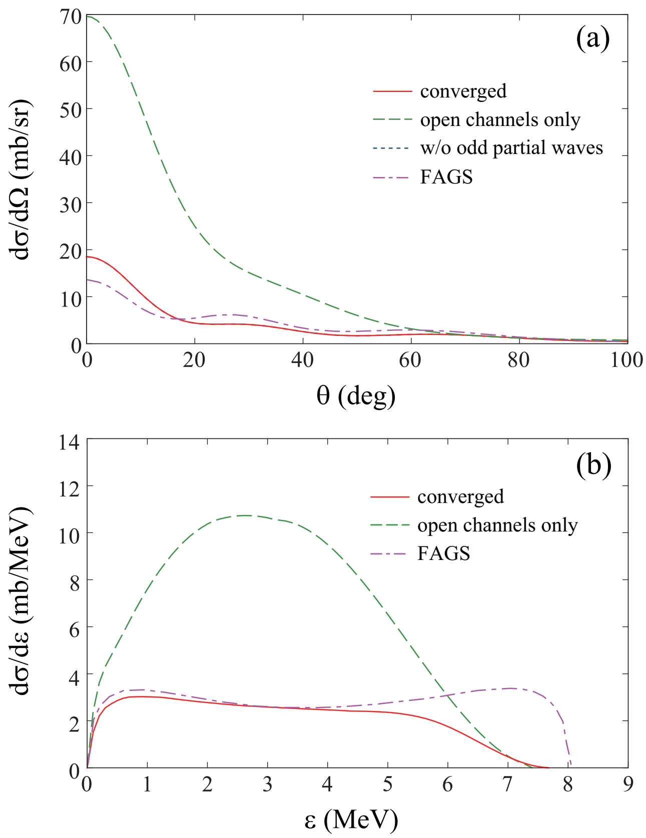

FIG. 1. (a) Angular distribution and (b) breakup energy distribution of the elastic breakup cross section for $^{12}\mathrm{C}(d,pn)^{12}\mathrm{C}$ at $E_d = 12$ MeV. The solid, dashed, and dash-dotted lines in each panel show the converged CDCC result, the result of the CDCC method calculated with including only the open channels, and the result of FAGS theory taken from Ref. [11], respectively. The dotted line in (a) is the same as the solid line but omitting the odd partial waves between p and n.

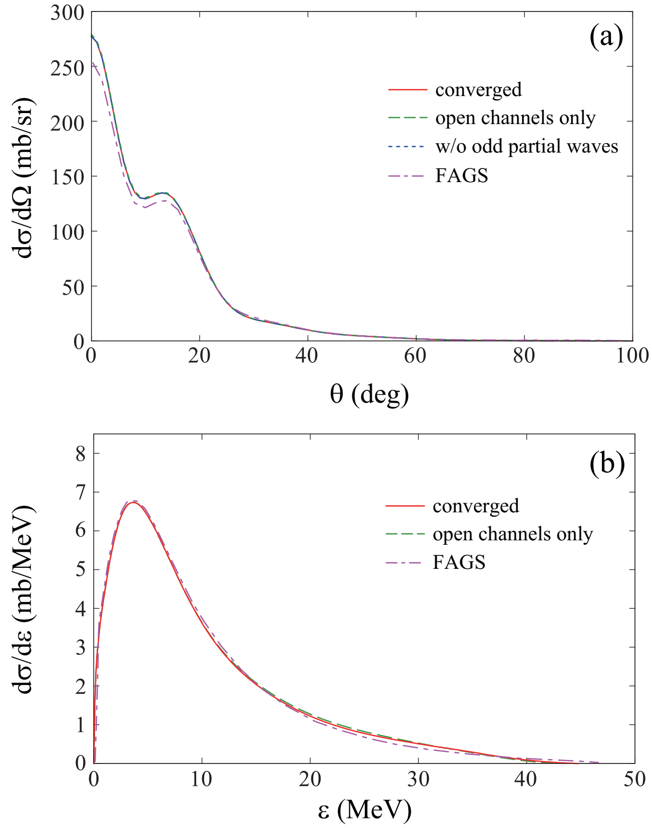

FIG. 2. Same as Fig. 1 but at $E_d = 56$ MeV.

breakup. For V_{pn}, we adopt the one-range Gaussian interaction of Ref. [17], and for the nucleon-nucleus optical potential, we employ the CH89 global potential [18]. These are the same model settings as in Ref. [11]. We use $\Delta k = 0.05$ fm^{-1} and $\ell_{\max} = 8$ for all the calculations shown below. As for $k_{\max}$, we take 0.9 fm^{-1} for $^{12}\mathrm{C}(d,pn)^{12}\mathrm{C}$ at $E_d = 12$ MeV (Fig. 1) and 1.1 fm^{-1} for other two reactions (Figs. 2 and 3). We checked the convergence of the breakup cross sections by further increasing the model space, and thereby convergence with 98% accuracy was confirmed. In the multipole expansion of the nucleon-nucleus optical potential, we take the multipolarities λ up to 16; it turned out that the multipoles for $\lambda > 8$ have no effect on the results shown below.

Figure 1(a) shows the angular distribution of the deuteron breakup cross section on ^{12}C at $E_d = 12$ MeV integrated over the p-n breakup energy ϵ. The horizontal axis is the scattering angle θ of the center of mass of the p-n system. The solid line is the converged result of the CDCC method that agrees well with the result of FAGS theory (dash-dotted line) taken from Fig. 9(a) of Ref. [11]. The dashed line in Fig. 1(a) is the CDCC result calculated with including open channels only, as in Ref. [11], which seems to be inside the hatched band in Fig. 9(a) of Ref. [11]. One sees in Fig. 1(a) a significant reduction of the cross section due to the coupling with the closed channels. Although still a small difference remains between the converged CDCC result in the present study and the FAGS results in Ref. [11], we conclude that the severe overshooting problem of the CDCC method pointed out in

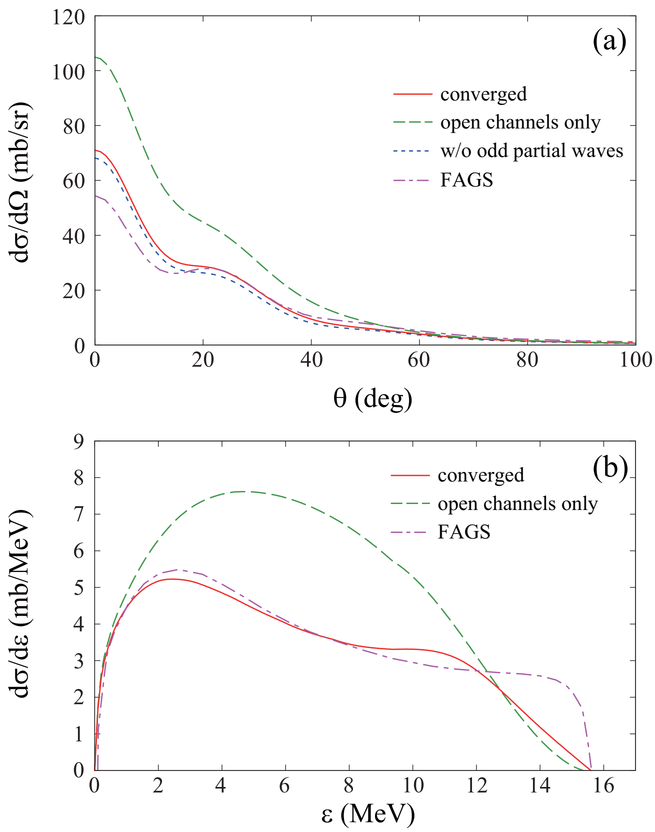

FIG. 3. Same as Fig. 1 but for ^{10}Be $(d,pn)^{10}$Be at $E_d = 21$ MeV.

Ref. [11] is mainly due to the lack of the closed channels in the CDCC calculation. The dotted line in Fig. 1(a) shows the converged CDCC result including only the even partial waves of ℓ, which perfectly agrees with the solid line. This is due to the neglect of the Coulomb breakup and to the small difference between the p-^{12}C and n-^{12}C potentials. This fact allows one to neglect the odd partial waves in the CDCC result, at least in some cases, which makes the comparison between the CDCC and FAGS results much easier, although in reality we always have the Coulomb breakup effect. Figure 1(b) is the p-n breakup energy distribution, with θ integrated. The features of the results are the same as in Fig. 1(a). The disagreement found in the high ϵ region will need further investigation.

Next we show in Fig. 2 the results for ^{12}C at $E_d = 56$ MeV. For this reaction, no significant difference between the CDCC and FAGS results was reported in Ref. [11]. It is quite natural that the coupling to the closed channels is less important at higher incident energy. One can clearly see this for both angular distribution [Fig. 2(a)] and breakup energy distribution [Fig. 2(b)]. In fact, the adopted $k_{\rm max}$ (0.9 fm^{-1}) for this reaction that gives convergence is very close to the threshold of the open channels, 1.05 fm^{-1}. It is thus quite trivial that the two lines agree with each other in Figs. 2(a) and 2(b). In any case, checking the convergence with respect to $k_{\rm max}$ is necessary.

Figure 3 is the result for ^{10}Be $(d,pn)^{10}$Be at $E_d = 21$ MeV. The role of the closed channels and the agreement between the converged CDCC and FAGS results are the same as in Fig. 1, although the role of the odd partial waves is appreciable in this reaction.

We reinvestigated deuteron elastic breakup reactions on ^{12}C and ^{10}Be at low incident energies, in which significant difference in the cross sections between the CDCC method and FAGS theory was reported [11]. We checked carefully the convergence of CDCC, with respect to the maximum p-n breakup momentum $k_{\rm max}$ in particular. The crucial importance of the closed channels was shown, and the converged CDCC results agree well with the FAGS results shown in Ref. [11]. At higher energy, the closed channels turned out to be less important, as expected.

In conclusion, we demonstrated the applicability of the CDCC method to elastic breakup reactions ^{10}Be $(d,pn)^{10}$Be at $E_d = 21$ MeV and ^{12}C $(d,pn)^{12}$C at $E_d = 12$ MeV by confirming the convergence of the CDCC model space with respect to $k_{\rm max}$. As a next step, a more systematic investigation on the role of closed channels, in transfer reactions in particular, will be important.

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