

Search for dilute excited states in ^{16}O

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The root mean square radii of ^{16}O in the short-lived 0^+ excited states were experimentally deduced for the first time from the analyses of $\alpha + ^{16}\text{O}$ diffraction scattering. Differential cross sections of the elastic and inelastic $\alpha + ^{16}\text{O}$ and $^{16}\text{O} + ^{16}\text{O}$ scattering in the incident energy range from a few MeV/nucleon up to 100 MeV/nucleon were analyzed by the modified diffraction model. No significant radius enhancement in any state in comparison with the ground state was observed. This concerns, in particular, the 15.1-MeV 0_6^+ state of ^{16}O , located in the vicinity of the four- α -particle complete dissociation threshold, for which we did not confirm the “gigantic” size predicted by the α -particle condensation model. This result does not support the idea that ^{16}O in the 0_6^+ state has a dilute structure and can be considered as an analog of the famous 7.65-MeV 0_2^+ Hoyle state of ^{12}C .

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As long ago as 1959, Baz predicted [1] that nuclear states located near the nucleon or cluster emission thresholds should possess enhanced radii. In 1968, Ikeda and co-workers visualized the possible cluster formation close to decay thresholds for $N = Z$ nuclei in the so-called Ikeda diagrams [2]. Practically all cluster models (see, e.g., the review articles [3,4] and the references therein) predict enhanced size of ^{12}C in the famous Hoyle state (7.65-MeV 0_2^+), crucial for stellar nucleosynthesis. A comparison of its radius as predicted by different models is given in Ref. [5].

Until recently there were no direct experimental methods of measuring the radii of short-lived unstable nuclear states. Nevertheless, some indirect experimental data (e.g., the electron inelastic scattering form factors [6,7]) pointed to the enlarged radius of the Hoyle state. We have developed two methods: the modified diffraction method (MDM) [8–10] and a method based on the inelastic nuclear rainbow scattering (INRS) [11], which allowed us to experimentally deduce the radii of the excited short-lived states from the analysis of inelastic scattering data. The INRS method was elaborated as a generalization of the proposal presented by Ohkubo and Hirabayashi in Ref. [12] for refractive effects and Airy structure in inelastic rainbow scattering [13,14]. The enhanced radius of ^{12}C in the Hoyle state was confirmed in Refs. [10,11] and found to be $R_{\text{rms}} = 2.89 \pm 0.04$ fm, which is about 25% larger than the radius of ^{12}C in its ground state ($R_{\text{rms}} = 2.34$ fm). This value is in the best agreement with the AMD calculations [15] [$R_{\text{rms}}(0_2^+) = 2.90$ fm] and the no-core symplectic model (NCSpM) calculations [$R_{\text{rms}}(0_2^+) = 2.93$ fm] [16]. These results showed that ^{12}C in the Hoyle state really has a fairly dilute structure. Dilute excited states with enlarged radii were also determined by the MDM in the ^{12}C neighboring nuclei: ^{11}B [17] and ^{13}C [18]. This concerns the 8.86-MeV $1/2_2^-$ state of ^{13}C ($R_{\text{rms}} = 2.91 \pm 0.12$ fm) and the 8.56-MeV $3/2^-$ state of ^{11}B ($R_{\text{rms}} = 2.99 \pm 0.18$ fm), which were predicted in Ref. [19] to be analogs of the Hoyle state, so that an extra nucleon or hole added to the 3α -cluster configuration does not destroy its dilute structure and original α -cluster features.

A question naturally arises: do analogs of the Hoyle state exist in ^{16}O ? Really, the excited state of ^{16}O located 570 keV above the 4α -particle complete dissociation threshold (14.44 MeV) and 285 keV above the $\alpha + ^{12}\text{C}^*(0_2^+)$ threshold (14.81 MeV), namely, the 15.1-MeV 0_6^+ state, which was discovered by Marvin and Singh [20] as early as 1972, now is considered as the most probable candidate to be an analog of the Hoyle state. Suggestions about the structure of the 15.1-MeV 0_6^+ state were proposed in the framework of the α -particle Bose-Einstein condensation model (α BEC) [21], which predicted the appearance of nuclear states with unusual dilute α -cluster structures resembling a gas of almost noninteracting α particles. Funaki *et al.* realized [22,23] four-body orthogonality condition model (OCM) calculations of the ^{16}O energy spectrum using the α -particle condensation wave functions and found that the 0_6^+ state has the “gigantic” rms radius $R_{\text{rms}} = 5.6$ fm comparable to the radius of the uranium nucleus. For the 13.6-MeV 0_4^+ state first found by Wakasa *et al.* [24], the calculations [22,23] also showed surprising enhancement of radius. However, no experimental information about the size of ^{16}O in these states was obtained until now.

In our work, inspired by the appearance of recent experimental data on $\alpha + ^{16}\text{O}$ inelastic scattering at 386 MeV [25], we present results of determining the radii of ^{16}O in the 0_4^+ , 0_5^+ , and 0_6^+ states located near and above the 4α -particle dissociation threshold deduced from the analysis of these data by means of the MDM.

The MDM was widely used for the direct determination of the radii of light nuclei in the excited states (see, e.g., [8–11,17,18,26–28]). The model uses differential cross sections of elastic and inelastic scattering of light projectiles (deuterons ^3He , ^6Li , etc.) with an incident energy of several tens of MeV on stable and radioactive targets, which exhibit a well-developed oscillatory structure at small angles (the Fraunhofer-type angular distributions). It is suggested that the scattering has a diffraction nature, i.e., the minima and maxima of the angular distributions correspond to the extremes of the cylindrical Bessel functions squared depending on the argument qR_{dif} . Here q is the linear transferred momentum

and R_{dif} is a parameter (diffraction radius) determined from the minima and maxima positions of the angular distributions. The diffraction pattern should be confirmed (in controversial cases) either by demonstrating the agreement with characteristic diffraction systematics or by more elaborated methods of the analysis, such as the distorted-wave Born approximation (DWBA) or coupled channels (CC) calculations.

The main assumption of the MDM is that the rms radius of a nucleus in the excited state, R_{rms}^* , can be determined as an increment to its rms radius in the ground state, R_{rms}^0 . This increment is equal to the difference of the diffraction radii for the excited state, $R_{\text{dif}}(\text{in})$, and the ground state, $R_{\text{dif}}(\text{el})$:

$$R_{\text{rms}}^* = R_{\text{rms}}^0 + [R_{\text{dif}}(\text{in}) - R_{\text{dif}}(\text{el})]. \quad (1)$$

Evidently, the diffraction radius depends not only on the “real” radius of a nucleus in a particular state, but also on the collision dynamics and some structural peculiarities as well. However, one may expect that these factors in elastic and inelastic scattering are mainly similar and cancel each other. There are many experimental findings speaking in favor of the MDM. Among them it is an excellent fulfillment of the Blair’s phase rules [30] in a lot of cases when the radii of the ground and excited states are known to be equal. Another strong argument that supports Eq. (1) is the observed independence of the second term in Eq. (1) on the incident energy in a wide range and for different projectiles [10]. The most important evidence of a validity of the MDM is the equality of results obtained by the three independent methods: MDM, INRS, and ANC (asymptotic normalization coefficients), by determining the radius of ^{13}C in the 3.09-MeV $1/2^+$ state possessing a neutron halo. The rms radius of ^{13}C in this state was found to be 1.2 times larger than the radius of ^{13}C in its ground state [31].

A comparison of the rms radii obtained by the MDM with the DWBA or CC cross-section calculations is not effective, because the diffraction scattering is mostly determined by absorption, which usually screens the nuclear interior, while the calculated inelastic cross sections are more sensitive to the extension of the transition density than to the radius of the excited state (see, e.g., Ref. [29]).

An illustration of the MDM application to the ^{12}C excited states is given in Fig. 1. The latter shows the diffraction radii pertaining to the first three states of ^{12}C deduced from the $\alpha + ^{12}\text{C}$ elastic and inelastic scattering data. The diffraction radii for the ground and the first excited (2^+ , 4.44 MeV) states of ^{12}C are practically the same, in excellent agreement with the expectations. The diffraction radius for the 7.65-MeV Hoyle state is approximately 0.55 fm larger. A difference between both lines remains almost constant in the energy per nucleon range from $E/A \approx 15$ to 60 MeV. A similar result was obtained with other projectiles (^3He , ^6Li , ^{12}C) scattered from ^{12}C .

However, at higher incident energies, the difference $[R_{\text{dif}}(\text{in}) - R_{\text{dif}}(\text{el})]$ increases. At $E/A = 100$ MeV, the diffraction radius for the 4.44-MeV 2^+ state of ^{12}C becomes about 0.4 fm greater than that for the ground state. A similar change is observed for the 7.65-MeV 0_2^+ state as well. Probably, it is an effect of nuclear dynamics indicating the limitation of the diffraction approach at energies $E/A \gtrsim 70$ MeV. However, *a*

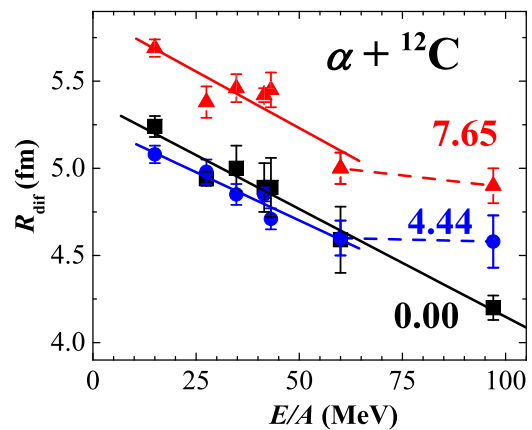


FIG. 1. Energy dependence of the diffraction radii extracted from the $\alpha + ^{12}\text{C}$ elastic and inelastic scattering populating the 4.44-MeV 2^+ and 7.65-MeV 0_2^+ states (adopted from Ref. [10]). The straight lines show linear approximations of the data at $E/A \lesssim 60$ MeV (solid lines) and $E/A \gtrsim 60$ MeV (dashed lines).

fortiori it did not influence the results obtained in Ref. [10] where the diffraction data were analyzed at smaller energies.

Now we fulfill the MDM analysis of the $\alpha + ^{16}\text{O}$ scattering, which allows us for the first time to determine the radii of ^{16}O in the excited states. We analyze the differential cross sections of the $\alpha + ^{16}\text{O}$ elastic and inelastic scattering populated excited states: 6.13-MeV 3^- [25,32–34], 6.92-MeV 2^+ [25,32–35], 7.12-MeV 1^- [32,33], 11.52-MeV 2^+ [25,33–35], and 12.44-MeV 1^- [33] at different energies. The extracted diffraction radii are presented in Fig. 2. For comparison, we also analyzed the available $^{16}\text{O} + ^{16}\text{O}$ data [10,40] related to the elastic and inelastic (leading to the 6.13-MeV 3^- and 6.92-MeV 2^+ states)

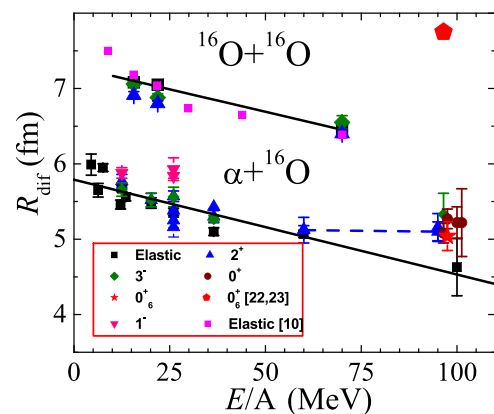


FIG. 2. Energy dependence of the diffraction radii extracted by the MDM analysis of $\alpha + ^{16}\text{O}$ elastic and inelastic scattering data from Refs. [24,25,32–39]. The solid lines represent linear approximations of the elastic scattering data. The diffraction radii for the elastic scattering, 1^- , 2^+ , 3^- , and 0^+ states are denoted by filled squares, down triangles, triangles, diamonds, and circles, correspondingly. The extracted diffraction radius for the 15.1-MeV 0_6^+ state is marked by a filled star, while a prediction from Refs. [22,23] is pointed out by a pentagon. The radii for some states measured at about 100 MeV/nucleon are slightly shifted for convenience of observation.

scattering. In both cases the energy dependence of the elastic diffraction radii is well approximated by a straight line similar to that found for ^{12}C in Fig. 1. At $E/A = 15\text{--}70$ MeV, the diffraction radii $R_{\text{dif}}(\text{in})$ extracted from the inelastic scattering data (indicated by various symbols in Fig. 2) are generally concentrated along the same line with average deviations from this line that do not exceed 0.3 fm and carry on a statistical character. An exception to this behavior is found for a group at $E/A \approx 100$ MeV.

Let us underline some important results following from the analysis presented in Fig. 2. First, an application of the MDM is quite adequate for the inelastic $\alpha + ^{16}\text{O}$ and $^{16}\text{O} + ^{16}\text{O}$ scattering at $E/A \leq 70$ MeV. Similarity of the results obtained with such different projectiles as ^4He and ^{16}O gives another strong argument in favor of applicability of the MDM.

Second, the rms radii of ^{16}O in the studied states with $J^\pi = 1^-, 2^+$, and 3^- do not differ from the rms radius of ^{16}O in the ground state, in accordance with Eq. (1). This is especially true for the radius of the 6.92-MeV 2^+ state located only 240 keV below the α -particle emission threshold, for which one might expect an enhancement of a radius. Moreover, the obtained results provide indirect evidence that all the members of the rotational band based on the 6.05-MeV 0_2^+ state [they correspond to the $^{12}\text{C}(\text{g.s.}) + \alpha$ configuration] have radii similar to that of the ground state.

Third, the deviation of the diffraction radius for the 7.12-MeV 1^- state from the straight line presented in Fig. 2 slightly exceeds the average value. The position of this state is only 45 keV below the α -particle emission threshold, and the observed diffraction radius enhancement might really reflect an increase of the rms radius of ^{16}O in this state.

Fourth, the diffraction radii corresponding to the excitation of the 2^+ states ($E_x = 6.92$ and 11.52 MeV) and the 3^- state ($E_x = 6.13$ MeV) measured nearly the incident energy per nucleon of 100 MeV lie about 0.5 fm above the elastic scattering line. This result reveals once again an increase of the difference $[R_{\text{dif}}(\text{in}) - R_{\text{dif}}(\text{el})]$ at such high energies just in the same way as it is observed for the $\alpha + ^{12}\text{C}$ inelastic scattering and demonstrates a common (dynamic) origin of the effect. Evidently the diffraction radii obtained directly from the angular distributions at high energies should be corrected. Taking into account the results shown in Figs. 1 and 2, we estimate an increase of $R_{\text{dif}}(\text{in})$ at high energies as

$$\Delta R_{\text{dif}}(\text{in}) = 0.5 \pm 0.2 \text{ fm}. \quad (2)$$

Now let us estimate the rms radii of the 0^+ states measured up to now only at high energy. Figure 3 shows the differential cross sections of the elastic (at three incident energies: 400 [24], 80.7, and 50.0 MeV [32]) and inelastic $\alpha + ^{16}\text{O}$ scattering as a function of the transferred linear momentum $q = 2k \sin(\theta_{\text{c.m.}}/2)$ (where k is the wave number of the projectile) populating the excited 13.6-MeV 0_4^+ state at $E_\alpha = 400$ MeV [24], and the 12.0-MeV 0_3^+ , 14.0-MeV 0_5^+ , and 15.1-MeV 0_6^+ states at $E_{\text{lab}} = 386$ MeV [25].

The MDM analysis of angular distributions presented in Fig. 3 leads to the following conclusions. The shapes of the inelastic scattering differential cross sections leading to the excitation of the 12.0-MeV 0_3^+ , 14.0-MeV 0_5^+ , and 15.1-MeV 0_6^+ states are quite similar, and a few closely locating minima

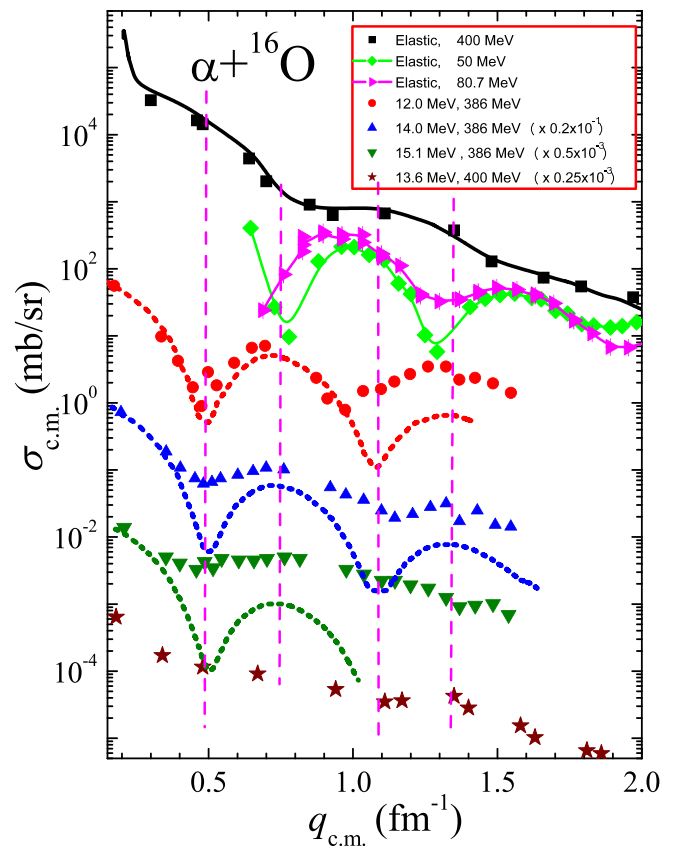


FIG. 3. Cross sections of the elastic and inelastic $\alpha + ^{16}\text{O}$ scattering plotted vs momentum transfer q . Three upper graphs present the elastic scattering data at $E_\alpha = 400, 80.7,$ and 50 MeV (adopted from Refs. [24,32]). The solid curves in the three upper graphs show the optical model and spline fits. The next three graphs present the inelastic scattering cross sections at $E_\alpha = 386$ MeV (adopted from Ref. [25]) with an excitation of the 12.0-MeV 0_3^+ , 14.0-MeV 0_5^+ , and 15.1-MeV 0_6^+ states. The dotted curves show the $L = 0$ cross-section components in the multipole decomposition fitted the data. The lower graph presents the inelastic scattering data at $E_\alpha = 400$ MeV (adopted from Refs. [24]) populating the 13.6-MeV 0_4^+ state. The vertical lines are drawn through the minima and maxima of the same order.

and maxima can be identified. The angular distribution related to the 13.6-MeV 0_4^+ state is almost unstructured except for a weakly defined maximum near $q = 1.34 \text{ fm}^{-1}$. The elastic scattering cross section also does not demonstrate a developed structure and exhibits three weak extremes at $q = 0.84, 1.11,$ and 1.50 fm^{-1} . Taking into account the low accuracy of determining the positions of the latter extremes one may conclude that the cross section of the elastic scattering and those of three inelastic transitions corresponding to the 12.0, 14.0, and 15.1 MeV states are out of phase, as they should be for the 0^+ states. This means that qualitatively the diffraction radii R_{dif} of the all four states are similar.

The mentioned inelastic scattering cross sections to the $0_3^+, 0_5^+,$ and 0_6^+ states correspond to the finite intervals of excitation energies. Therefore they can contain some contributions from a formation of states with different spin

TABLE I. Predicted and experimental rms radii (in fm) of ^{16}O in the 0^+ states.

State	E_x (MeV)	R_{rms} AMD [41]	R_{rms} OCM [42]	R_{rms} [43]	R_{rms} OCM [44]	$R_{\text{dif}}^{\text{a}}$	$R_{\text{rms}}^{\text{b}}$
0_1^+	0.00	2.9	2.5	2.47	2.7	4.63 ± 0.38	2.70
0_2^+	6.05	3.5	2.9	3.03	3.0	$5.16 \pm 0.18^{\text{c}}$	$2.73 \pm 0.47^{\text{c}}$
0_3^+	12.05	3.9	2.8		3.1	5.27 ± 0.13	2.84 ± 0.45
0_4^+	13.60	3.5			4.0	5.22 ± 0.45	2.79 ± 0.62
0_5^+	14.01	3.8			3.1	5.08 ± 0.13	2.65 ± 0.45
0_6^+	15.10				5.6	5.04 ± 0.19	2.61 ± 0.47

^aThe inelastic diffraction radii are shown without corrections of Eq. (2).

^bThe rms radii are shown with corrections Eq. (2) of the diffraction radii.

^cRadius is taken equal to that of the 6.92-MeV state (see text).

and parities. A multipole decomposition analysis carried out in Ref. [25] has singled out the cross-section components with different transferred angular momenta L . We show in Fig. 3 the $L = 0$ components, whereas the other ones are presented in Fig. 3 of Ref. [25]. Figure 3 shows that the positions of the first two extremes (the first minimum and second maximum) practically coincide with those of the $L = 0$ components. Besides, the second maximum of the angular distributions to the 12.0- and 14.0-MeV states are almost completely exhausted by the $L = 0$ component. These findings clearly point to the similarity of their diffraction radii, which were evaluated as $R_{\text{dif}}(0_3^+, 0_5^+, 0_6^+) = 5.27 \pm 0.13, 5.08 \pm 0.13,$ and 5.04 ± 0.19 fm, correspondingly.

The diffraction radius of the ground state obtained from the position of the three extremes of the elastic scattering angular distribution was determined to be $R_{\text{dif}} = 4.63 \pm 0.38$ fm. This value coincides with the diffraction radii of the $0_3^+, 0_5^+$, and 0_6^+ states within the error bars, but also does not contradict to the found enhancement of the inelastic scattering diffraction radii given by expression (2).

As for the 13.6-MeV 0_4^+ state, its diffraction radius can be only roughly estimated from the single weak maximum at $q = 1.34 \text{ fm}^{-1}$. The obtained value $R_{\text{dif}} = 5.22 \pm 0.45$ fm is in line with the diffraction radii for other 0^+ states under consideration and leads to the rms radius almost equal to the radius of the ground state (we have to note a large error related to an uncertainty of the minima and maxima determination from these data). This result contradicts the claim made in Ref. [24] that this state has a large rms radius consistent with its α -particle condensation structure. The rms radii determined by applying Eq. (1) and the corrections Eq. (2) are shown in Table I in comparison with some theoretical predictions.

The results presented in Table I definitely show that ^{16}O in the $0_3^+, 0_5^+$, and 0_6^+ states, and probably the 0_2^+ and 0_4^+ states, are very similar. This follows from almost equal values of their diffraction radii presented in the last but one column of Table I. Consequently, the 15.1-MeV 0_6^+ level cannot represent a gigantic state with dimensions comparable with those of the uranium nucleus predicted in Refs. [22,23]. If its rms radius were equal to 5.6 fm, it would then have, according to Eq. (1), the diffraction radius equal to $R_{\text{dif}} = 7.6$ fm. This point, as

shown in Fig. 2, is located much higher than the experimental value. Moreover, the first minimum of the corresponding angular distribution would be at the angle $\theta_{\text{c.m.}} = 2^\circ$ (see the lowest panel of Fig. 3), which definitely is not the case.

In connection with these results, we note that the calculations in the α BEC model have predicted the existence of similar gigantic states in ^{12}C (the 2_2^+ state [45]) and ^{11}B (the 12.6-MeV state [46]). No such large radius enhancement was observed in these nuclei [17,26,47].

It is reasonable to suggest that the rms radius of the 6.92-MeV 2^+ state is similar to that of the 6.05-MeV 0_2^+ state, because both states belong to the same rotational band. Thus we have evidence that the members of the well-known positive-parity α -cluster rotational band in ^{16}O have rms radii similar to that of the ground 0_1^+ state (2.7 fm).

To conclude, the differential cross sections of the inelastic $\alpha + ^{16}\text{O}$ scattering in the energy interval from a few tens MeV up to 400 MeV were analyzed. We determined directly the rms radii of ^{16}O in a number of states with excitation energies up to 15.1 MeV applying the MDM. No significant radius enhancement in any states, with the possible exception of the 7.12-MeV 1^- and 13.6-MeV 0_4^+ states (in the latter case, some ambiguity is connected with a large error bar), was observed. This result concerns, first, the 0^+ states located in the vicinity of 4α -particle dissociation threshold. In particular, we did not confirm the existence of a dilute state with super-large radius associated with the 15.1-MeV 0_6^+ state, which was predicted by the α BEC model. The rms radius of ^{16}O in this state was found similar to the radius of ^{16}O in the ground state. From this point of view, the 0_6^+ state cannot be considered as an analog of the Hoyle state in ^{12}C .

Finally we hope that the results reported in this paper will lead to more understanding of the problem of nuclear radii in the excited states and will stimulate further experimental and theoretical studies of the properties of light nuclei.

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