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Masses of ^{17,18,19,20}Mg

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A previous simple parametrization of mirror energy differences in pairs of nuclei consisting of a p-shell core plus two sd-shell nucleons is applied to a series of mirrors that contain sd-shell nucleons in the core. Results for ^{19,20}Mg agree with experiment and with a potential model. Predictions are made for $2p$ separation energies of ^{17,18}Mg.

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I. INTRODUCTION

Recently, I discovered a simple parametrization of mirror energy differences of nuclei whose structures are predominantly a p-shell core plus two sd-shell nucleons [1]. For this purpose, I defined the mirror energy difference (MED) as MED $= S_{2n}$ (neutron-excess nucleus) $-S_{2p}$ (proton-excess mirror) and then fitted to the expression $\mathrm{MED}(A,Z) = \{f[S_{2n}, P(s^2)]\}Z_</A^{1/3}$. Here, S_{2n} and S_{2p} are separation energies [2], A is the mass number, $Z_<$ is the proton number of the core of the core $+2p$ nucleus, and $P(s^2)$ is the occupation probability of the $2s_{1/2}$ orbital—assumed to be equal in the two members of a mirror pair. The function $f = C + aS_{2n} - bP(s^2)$ produced agreement with experimental values for five pairs of nuclei (^{18}O, ^{17}N, ^{16}C, ^{14}C, and ^{12}Be and their mirrors) with a root-mean-square deviation of 4 keV (better than the experimental uncertainties in some cases). Applying the fit parameters from the other nuclei to ^{13}B /^{13}O, I was able to deduce $P(s^2) = 0.21$ for that pair—in reasonable agreement with other estimates [3,4]. As far as I know, this expression is not derivable in any first-principles approach, but its simplicity demands further scrutiny. My aim here is to test this simple parametrization for nuclei for which the cores already have some $2s_{1/2}$ occupancy and then to use it to predict the mass of ^{17}Mg.

I then estimated the ground-state (g.s.) mass of the unbound nucleus ^{15}Ne using the known mass of ^{15}B. Because S_{2n}'s for ^{15}B and ^{12}Be are very similar, the ^{15}Ne prediction did not involve an extrapolation and is thus likely to be reasonably robust. The s^2 parentage is not well known in ^{15}B, but two estimates [5,6] are that it is large. I gave S_{2p} predictions for ^{15}Ne for $P(s^2) = 0.66(10)$. If this quantity is ever determined, it is a simple matter to revise the prediction. The result was $S_{2p}(^{15}\mathrm{Ne}) = -2.68(24)$ MeV for $P(s^2) = 0.66(10)$. With the mass excess of 23.115(10) MeV [2] for ^{13}O, this value of S_{2p} corresponded to a mass excess of (^{15}Ne) = 40.37(24) MeV. The dependence of predicted S_{2p} on $P(s^2)$ is plotted in Fig. 1. In a very recent experiment [7], the ^{15}Ne ground state was found to be unbound by 2.522(66) MeV. Those authors stated that this value corresponded to $P(s^2) = 0.63(5)$. But they must have misread something because Fig. 1 illustrates that agreement occurs for $P(s^2) = 0.73(3)$.

A small caveat is in order here. Approximately the same group of experimenters reported a mass excess for ^{12}O of 31.914(24) MeV [8], considerably different from the value of 32.048(18) MeV from the 2003 mass evaluation [9]. The new and old values differ by 134(30) keV—a 4.5σ difference. Two unpublished results from the ^{12}C(π^+,π^-) reaction [10,11] are 32.036(24) and 32.016(22) MeV. I know of nothing wrong with the measurement of Jager *et al.* [8], but I think it needs to be repeated. If they have an undiscovered systematic error that results in lower mass excesses, the value for ^{15}Ne might need to be revised upward—and hence toward smaller $P(s^2)$.

II. CALCULATIONS AND RESULTS

My purpose here is to attempt to extend the simple parametrization to nuclei that also have sd-shell nucleons in the core. I look first at the Mg isotopes with $A = 17-20$. Long ago, in a potential model assuming mirror symmetry, we calculated the mass excess of ^{20}Mg and missed it by only calc − exp $= -21(27)$keV [12]. This simple model uses the same spectroscopic factors for $^{20}\mathrm{Mg} \rightarrow {}^{19}\mathrm{Na}$ as for $^{20}\mathrm{O} \rightarrow {}^{19}\mathrm{O}$. In a new calculation using all the ^{19}O core states for which the spectroscopic factor is larger than 0.02, the prediction for ^{20}Mg(g.s.) was $S_{2p} = 2.341$ MeV [13]. The new mass evaluation [2] lists $S_{2p} = 2.337(27)$. The new calculation had an s^2 occupancy of $P(s^2) = 0.17$.

A spectacular success of our simple potential model was the prediction of the mass of ^{19}Mg. We predicted $E_{2p} = 0.87(7)$ MeV [14]. A later experiment [15] found $E_{2p} = 0.75(5)$ MeV, just at the 1σ limit of the combined uncertainties. For that calculation, we needed to compute energies of several states in the core nucleus ^{18}Na because they were not known experimentally. Later, results appeared from an experiment [16] to measure energies in ^{18}Na, and we used these to recalculate the g.s. energy of ^{19}Mg(g.s.) [17]. Using the experimental ^{18}Na energies and a slightly different geometry for the potential well ($r_0 = 1.26$, $a = 0.60$, $r_{0c} = 1.40$ fm rather than $r_0 = r_{0c} = 1.25$, $a = 0.65$ fm), our prediction was 0.76(7) MeV. [These geometrical parameters have long been used for the bound (and unbound) state potentials in the analysis of proton transfer reactions.] We recalculated the energy of ^{19}Mg(g.s.) for a number of different inputs: potential set 1 vs set 2, S from the shell model vs S from the shell model + weak coupling, and calculated energies in ^{18}Na vs the new [16] experimental ones. All predictions were in the range of 0.76–0.87 MeV, so we felt our calculation was robust. Of course, we preferred the one that used experimental

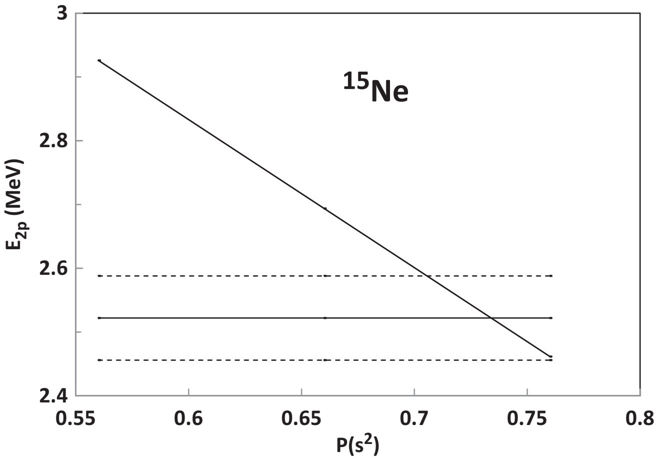

FIG. 1. For ^{15}Ne, the sloping line is the predicted $2p$ separation energy vs the s^2 occupancy $P(s^2)$. The horizontal lines represent the recent experimental value [7].

core energies and shell-model spectroscopic factors. In that calculation, the s^2 occupancy was $P(s^2) = 0.245$.

Table I lists the ^{20}O /^{20}Mg and ^{19}N /^{19}Mg cases discussed above and the predictions of the parametrization of Ref. [1]. Spectacular agreement can be noted. The aim now is to predict the separation energy of ^{18}Mg. The $2n$ separation energy of its mirror ^{18}C is known [2], but information concerning $P(s^2)$ for that nucleus is sketchy. One method that has been used to estimate s^2 occupancy involves matter radii. Because computed matter radii depend on the identity of the valence orbital(s), if the matter radius is well known, the occupancies can be estimated (Ref. [18], and references therein). Unfortunately, a relatively small uncertainty in R_m gives rise to a somewhat large uncertainty in the occupancies. [On the other hand, if the occupancies are even approximately known, the matter radius can be computed reliably.] For ^{18}C, the only reported matter radius with a small uncertainty is $R_m = 2.82(4)$fm [19]. For small separation energies, matter radii for configurations s^2 and d^2 differ considerably, but for the ^{18}C value of $S_{2n} = 4.92$ MeV, they are not very different: 3.01 fm for s^2 and 2.77 fm for d^2 [20]. Requiring a fit to the experimental value produces an s^2 occupancy of $P(s^2) = 0.21(17)$—not very precise, but I have used it in what follows. Two theoretical values are 0.32 from a shell-model calculation [21] and 0.26 from a Hartree-Fock-Bogoliubov approximation [22]. If a better value of $P(s^2)$ becomes available, a new prediction is trivial.

TABLE I. Separation energies (MeV) and s^2 parentages for selected Mg nuclei and their mirrors.

Nucl.	S_{2n}[a]	$P(s^2)$[b]	Mirror	S_{2p} (expt.)[a]	S_{2p}(calc)	
					Present	Potential model
^{20}O	11.564(1)	0.17	^{20}Mg	2.337(27)	2.270	2.341[c]
^{19}N	8.157(22)	0.245	^{19}Mg	−0.75(5)	−0.744	−0.76(7)[d]
^{18}C	4.92(3)	0.208	^{18}Mg		−3.84	−3.87(10)[e]
		0.375			−3.48	
		0.042			−4.18	
^{17}B	1.33(17)	0.51	^{17}Mg		−6.51	
		0.59			−6.49	
		0.43			−6.53	

[a] Reference [2].
[b] See the text.
[c] Reference [13].
[d] Reference [17].
[e] Reference [23].

TABLE II. Comparison of predictions for ^{18}Mg and ^{15}Ne.

Nucleus	S_{2p}(MeV)		
	Reference [27]	Present	Expt.
^{18}Mg	−4.233(34)	−3.84(35)[a],−3.87(10)[b]	
^{15}Ne	−3.532(23)	−2.64(24)	−2.522(66)[c]

[a] Present paper.
[b] Reference [23].
[c] Reference [7].

With this range of $P(s^2)$ values, the predicted separation energies are as listed in the table: $S_{2p} = -3.84(36)$ MeV, still a reasonably narrow range. Earlier, we used a potential model, together with spectroscopic factors from a combination of weak coupling and a shell-model calculation, to compute the mass of the ground state of ^{18}Mg, considered as a mirror of ^{18}C. The result was $E_{2p} = 3.87(10)$ MeV [23]—not very different from the present result. I encourage an experiment to measure this quantity.

There must be some fundamental reason why this simple parametrization produces results that are nearly identical to results of a potential model, but I do not know what it is. The question clearly deserves further thought.

I turn now to ^{17}Mg, whose mirror ^{17}B is bound by 1.33(17) MeV to ^{15}B $+2n$. Several values of $P(s^2)$ are available for ^{17}B [18,24–26], and their weighted average is 0.51(8) [18]. Predictions of S_{2p} for ^{17}Mg are listed in the table. The uncertainty in $P(s^2)$ causes only a small uncertainty in S_{2p}, but, of course, the uncertainty in S_{2n} produces an uncertainty of about 170 keV. This nucleus may be very difficult to populate, but it would be interesting to try.

A paper concerning improved Kelson-Garvey mass relations for proton-rich nuclei [27] contains calculations for ^{18}Mg and ^{15}Ne discussed above. Their results for these two nuclei are compared with mine in Table II.

III. CONCLUSIONS

To summarize, a simple parametrization of mirror energy differences of nuclei whose structures are predominantly a p-shell core plus two sd-shell nucleons also appears to work well even if the core contains some sd-shell nucleons. For ^{19,20}Mg, the simple parametrization agrees, both with experimental values and with results of a potential model. I then used the model to predict $2p$ separation energies for ^{17,18}Mg. The ^{18}Mg results agree with the potential-model calculations. I urge an attempt to produce these two nuclei and measure their separation energies. Some explanation for why the simple parametrization produces the same results as a potential-model calculation would be very welcome.

[1] H. T. Fortune, Phys. Lett. B **718**, 1342 (2013).
[2] M. Wang, G. Audi, A. H. Wapstra, F. G. Kondev, M. MacCormick, X. Xu, and B. Pfeiffer, Chin. Phys. C **36**, 1603 (2012).
[3] H. T. Fortune and R. Sherr, Phys. Rev. C **68**, 024301 (2003).
[4] N. Aoi *et al.*, Phys. Rev. C **66**, 014301 (2002).
[5] E. Sauvan *et al.*, Phys. Rev. C **69**, 044603 (2004).
[6] M. Labiche *et al.*, Phys. Rev. Lett. **86**, 600 (2001).
[7] F. Wamers *et al.*, Phys. Rev. Lett. **112**, 132502 (2014).
[8] M. F. Jager *et al.*, Phys. Rev. C **86**, 011304(R) (2012).
[9] G. Audi, A. H. Wapstra, and C. Thibault, Nucl. Phys. A **729**, 337 (2003).
[10] R. A. Ivie, Master's thesis, University of Pennsylvania, 1992.
[11] H. T. Fortune and R. Sherr, J. Phys. G: Nucl. Part. Phys. **40**, 055102 (2013).
[12] R. Sherr, H. T. Fortune, and B. A. Brown, Eur. Phys. J. A **5**, 371 (1999).
[13] H. T. Fortune, R. Sherr, and B. A. Brown, Phys. Rev. C **85**, 054304 (2012).
[14] H. T. Fortune and R. Sherr, Phys. Rev. C **76**, 014313 (2007).
[15] I. Mukha *et al.*, Phys. Rev. Lett. **99**, 182501 (2007).
[16] M. Assie *et al.*, Phys. Lett. B **712**, 198 (2012).
[17] H. T. Fortune and R. Sherr, Phys. Rev. C **85**, 051302 (2012).
[18] H. T. Fortune and R. Sherr, Eur. Phys. J. A **48**, 103 (2012).
[19] A. Ozawa *et al.*, Nucl. Phys. **A691**, 599 (2001).
[20] H. T. Fortune and R. Sherr, Eur. Phys. J. A **47**, 154 (2011).
[21] Y. Kondo *et al.*, Phys. Rev. C **79**, 014602 (2009).
[22] K. Hagino, N. Takahashi, and H. Sagawa, Phys. Rev. C **77**, 054317 (2008).
[23] H. T. Fortune and R. Sherr, Phys. Rev. C **87**, 044315 (2013).
[24] Y. Yamaguchi *et al.*, Phys. Rev. C **70**, 054320 (2004).
[25] T. Suzuki *et al.*, Nucl. Phys. A **658**, 313 (1999).
[26] T. Suzuki *et al.*, Phys. Rev. Lett. **89**, 012501 (2002).
[27] J. Tian, N. Wang, C. Li, and J. Li, Phys. Rev. C **87**, 014313 (2013).