Quark-Pauli effects in three octet-baryons

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(Received 23 June 2016; published 13 September 2016)

To sustain a neutron star with about two times the solar mass, multibaryons including hyperons are expected to produce repulsive effects in the interior of its high-baryon-density region. To examine possible quark-Pauli repulsion among the baryons, we solve the eigenvalue problem of the quark antisymmetrizer for three octet-baryons that are described by most compact spatial configurations. We find that the Pauli blocking effect is weak in the Λnn system, while it is strong in the $\Sigma^- nn$ system. The appearance of the Σ^- hyperon is suppressed in the neutron star interior but no quark-Pauli repulsion effectively works for the Λ hyperon.

DOI: 10.1103/PhysRevC.94.035803

I. INTRODUCTION

Recently, the properties of multibaryons including hyperons (Y's) have attracted much attention in the study of neutron stars. Though the neutron star is primarily composed of neutrons (n's), the presence of a Y appears to be energetically unavoidable [1]. If n's become superabundant in the interior of the neutron star and the neutron Fermi energy greatly increases, the n becomes unstable against decaying into the Λ hyperon via a weak interaction. On top of that, if the electron (e^-) chemical potential grows with an increase in the baryon density in the neutron star, the Σ^- hyperon may be formed through the weak interaction, $n + e^- \to \Sigma^- + \nu$. Furthermore, it is suggested that Σ^- may appear at a lower density earlier than Λ in spite of the fact that Σ^- is more massive than Λ [2]. It is also suggested that the Ξ^- hyperon may appear at a relatively low density depending on the strength of Ξ^- attraction in the interior of the neutron star [3,4].

The appearance of Y's in the neutron star, however, leads to a softening of the equation of state [5]. Because of this softening the maximum mass of the neutron star predicted by solving the equation of state with Y-nucleon (N) and YY interactions used in hypernuclear physics turns out to be incompatible with the recent observation [6,7] that finds the neutron star with about double the solar mass. A resolution of this problem calls for a mechanism that could provide additional repulsion to make the equation of state stiffer [8]. In search of a candidate mechanism for the additional repulsion, various proposals have been made, e.g., vector-meson exchange in baryon-baryon interactions [9,10], a repulsive Λnn three-body force [11], a universal three-baryon repulsion [12,13], and cold quark matter rather than hadronic matter [14]. See also Refs. [15,16] for other cases.

Both of N and Y are members of octet-baryons (B_8's). Describing them as three-quark clusters, we investigate the quark-Pauli effect in three-B_8 systems because it could be responsible for the needed additional repulsion. The quark-Pauli effect becomes most apparent when the three baryons strongly overlap. Any three-B_8 channel that is (almost) Pauli forbidden provides a short-range three-body repulsion that is independent of the baryon-baryon interactions.

The quark-Pauli effect in the two-B_8 system has already been studied [17,18]. The effect often leads to important repulsion regardless of the detail of the baryon-baryon interaction [19]. For example, the repulsive Σ single-particle potential in nuclei [20] is considered to originate from the strong Pauli repulsion in the $\Sigma N(I=\frac{3}{2})\,{}^3S_1$ state [21]. The most recent Nijmegen ESC08 potential incorporates this quark effect phenomenologically because it is difficult to achieve strongly repulsive short-range interactions in this channel [22]. There are some earlier studies on the quark-Pauli effect in three- and more-baryon systems. See, for example, Refs. [23–27]. These are mostly for multibaryon systems composed of N's and a single hyperon such as N^n and ΛN^n.

The plan of this paper is as follows. We construct antisymmetric three-B_8 states in Sec. II with a particular emphasis on the most compact spatial quark configurations. We discuss in Sec. III the quark-Pauli effect by solving the eigenvalue problem of the antisymmetrizer of 9 quarks. Conclusions are drawn in Sec. IV.

II. THREE OCTET-BARYON STATES

The octet baryons (B_8) with spin $S=\frac{1}{2}$ include N,Λ,Σ, and Ξ, all belonging to a member of the flavor SU(3) symmetry $(\lambda\mu)=(11)$. We use the Elliott notation for the SU(3) group [28]. The B_8's are classified by the SU(2)×U(1) subgroup label, $a=YI$, the hypercharge Y and the isospin I: $N(YI=1\frac{1}{2})$, $\Lambda(00)$, $\Sigma(01)$, $\Xi(-1\,\frac{1}{2})$. Assuming that the B_8 is a three-quark cluster, we describe its orbital part $\phi^{(\mathrm{orb})}(123)$ by the $(0s)^3$ harmonic-oscillator wave function with a common size parameter. Since $\phi^{(\mathrm{orb})}(123)$ is totally symmetric and the B_8 color wave function $C(123)$ is totally antisymmetric, its spin-flavor part represented by $W^{[3]}(123)$ must be totally symmetric, as indicated by [3] symmetry. By specifying the z components of the spin and the isospin by S_z and I_z, respectively, a full quark-model description of B_8 reads [29]

$$\psi_{(11)aS_zI_z}(123)=\phi^{(\mathrm{orb})}(123)W^{[3]}_{aS_zI_z}(123)C(123). \qquad (1)$$

More explicitly, $W^{[3]}_{aS_zI_z}(123)$ is given by

$$W^{[3]}_{aS_zI_z}(123) = \frac{1}{\sqrt{2}} \sum_{S'(\lambda'\mu')=0(01),1(20)} \big[\big[w_{\frac{1}{2}}(1)w_{\frac{1}{2}}(2)\big]_{S'} w_{\frac{1}{2}}(3)\big]_{\frac{1}{2}S_z} \times [[F_{(10)}(1)F_{(10)}(2)]_{(\lambda'\mu')}F_{(10)}(3)]_{(11)aI_z}, \quad (2)$$

where $w_{\frac{1}{2}}$ and $F_{(10)}$ are the spin and flavor functions of the single quark. The square bracket [ ] is used to stand for spin and/or flavor SU(3) couplings.

Equation (1) gives the normalized B_8 wave function that satisfies the required symmetry at the quark level. By combining two B_8 wave functions, it is possible to express the spin-isospin coupled basis in terms of a combination of the spin-flavor coupled basis [18,30]. Physically allowed two-baryon states have to satisfy the generalized Pauli principle that demands the total wave function to be antisymmetric under the exchange of quarks. We extend this to a special three-B_8 state in which all nine quarks occupy the same $0s$ harmonic-oscillator function. The orbital configuration of that state is most compact and such a three-B_8 state is expected to be most strongly influenced by the quark-Pauli principle.

To construct the fully antisymmetric nine-quark states, we first start from the three-B_8 configuration that is antisymmetric under the exchange of baryons. The $(0s)^9$ configuration is apparently symmetric under the exchange of baryons. The color part is also totally symmetric with respect to the exchange of baryons. Therefore the spin-flavor part of the three-B_8 state must be antisymmetric under the exchange of baryons. To construct such three-B_8 spin-flavor states, we combine the spin-isospin coupled two-B_8 state, $[W^{[3]}_{a_1}(123)W^{[3]}_{a_2}(456)]_{S'a'}$, with the third B_8 as follows:

$$\big[\big[W^{[3]}_{a_1}(123)W^{[3]}_{a_2}(456)\big]_{S'a'} W^{[3]}_{a_3}(789)\big]_{SaS_zI_z}. \quad (3)$$

Here S is the total spin that couples S' with $\frac{1}{2}$ and a includes the isospin coupling of I' and I_3. The hypercharge is trivially given as $Y' = Y_1 + Y_2$ and $Y = Y' + Y_3$. The z components of S and I, S_z and I_z, are abbreviated in what follows throughout this section. Since in this section we focus on constructing such three-B_8 spin-flavor functions that are antisymmetric under the baryon exchange, we suppress the quark labels and simplify $W^{[3]}_a(123)$ by $B_a(1)$ and express Eq. (3) as

$$\big[\big[B_{a_1}(1)B_{a_2}(2)\big]_{S'a'} B_{a_3}(3)\big]_{Sa}. \quad (4)$$

The fully antisymmetrized spin-flavor function constructed from Eq. (4) is

$$\Psi^{(\mathrm{SF})}_{Sa}(a_1a_2a_3,S'a') = \mathcal{N}\big\{\big[\big[B_{a_1}(1)B_{a_2}(2)\big]_{S'a'}B_{a_3}(3)\big]_{Sa} - \big[\big[B_{a_1}(2)B_{a_2}(1)\big]_{S'a'}B_{a_3}(3)\big]_{Sa} + \big[\big[B_{a_1}(2)B_{a_2}(3)\big]_{S'a'}B_{a_3}(1)\big]_{Sa} - \big[\big[B_{a_1}(3)B_{a_2}(2)\big]_{S'a'}B_{a_3}(1)\big]_{Sa} + \big[\big[B_{a_1}(3)B_{a_2}(1)\big]_{S'a'}B_{a_3}(2)\big]_{Sa} - \big[\big[B_{a_1}(1)B_{a_2}(3)\big]_{S'a'}B_{a_3}(2)\big]_{Sa}\big\}, \quad (5)$$

which is characterized by S,a and S',a' as well as a_1,a_2,a_3. Here $\mathcal{N}$ is a normalization constant.

With use of the angular-momentum recoupling or Racah coefficients U in unitary form, function (5) can be expressed as

$$\begin{aligned}\Psi^{(\mathrm{SF})}_{Sa}(a_1a_2a_3,S'a') = \mathcal{N}\Bigg\{&[[B_{a_1}(1)B_{a_2}(2)]_{S'a'}B_{a_3}(3)]_{Sa} + (-1)^{S'+I_1+I_2-I'}[[B_{a_2}(1)B_{a_1}(2)]_{S'a'}B_{a_3}(3)]_{Sa}\\
&+ \sum_{S_{12}I_{12}}\Bigg[(-1)^{1/2+S_{12}-S+I_1+I_{12}-I}U\left(\frac{1}{2}\frac{1}{2}S\frac{1}{2};S'S_{12}\right)U(I_1I_2II_3;I'I_{12})[[B_{a_2}(1)B_{a_3}(2)]_{S_{12}a_{12}}B_{a_1}(3)]_{Sa}\\
&+(-1)^{1/2-S+I_1+I_2+I_3-I}U\left(\frac{1}{2}\frac{1}{2}S\frac{1}{2};S_{12}S'\right)U(I_3I_2II_1;I_{12}I')[[B_{a_3}(1)B_{a_2}(2)]_{S_{12}a_{12}}B_{a_1}(3)]_{Sa}\\
&+(-1)^{1/2+S'-S+I'+I_3-I}U\left(\frac{1}{2}\frac{1}{2}S\frac{1}{2};S_{12}S'\right)U(I_3I_1II_2;I_{12}I')[[B_{a_3}(1)B_{a_1}(2)]_{S_{12}a_{12}}B_{a_2}(3)]_{Sa}\\
&+(-1)^{1/2-S'+S_{12}-S+I_1+2I_2+I_{12}-I'-I}U\left(\frac{1}{2}\frac{1}{2}S\frac{1}{2};S'S_{12}\right)U(I_2I_1II_3;I'I_{12})[[B_{a_1}(1)B_{a_3}(2)]_{S_{12}a_{12}}B_{a_2}(3)]_{Sa}\Bigg]\Bigg\},\end{aligned} \quad (6)$$

where the order of the particle labels is always arranged to 1, 2, and 3, while that of the baryon species, $a_1a_2a_3$, is changed appropriately. In what follows, we often abbreviate $[[B_{a_1}(1)B_{a_2}(2)]_{S'a'}B_{a_3}(3)]_{Sa}$ as $[[B_{a_1}B_{a_2}]_{S'a'}B_{a_3}]_{Sa}$.

It should be noted that for a given set of $a_1a_2a_3$, functions (6) generated using all possible values of $S'a'$ provide a full set of antisymmetric functions but they are not always independent. Also the order of $a_1a_2a_3$ is not important. For example, two independent functions, $\Psi^{(\mathrm{SF})}_{\frac{1}{2}1\frac{1}{2}}(\Sigma\Lambda N,01)$ and $\Psi^{(\mathrm{SF})}_{\frac{1}{2}1\frac{1}{2}}(\Sigma\Lambda N,11)$, are related to $\Psi^{(\mathrm{SF})}_{\frac{1}{2}1\frac{1}{2}}(\Lambda N\Sigma,0\frac{1}{2})$ and $\Psi^{(\mathrm{SF})}_{\frac{1}{2}1\frac{1}{2}}(\Lambda N\Sigma,1\frac{1}{2})$ as

$$\begin{aligned}\Psi^{(\mathrm{SF})}_{\frac{1}{2}1\frac{1}{2}}(\Sigma\Lambda N,01) &= \frac{1}{2}\Psi^{(\mathrm{SF})}_{\frac{1}{2}1\frac{1}{2}}\left(\Lambda N\Sigma,0\frac{1}{2}\right) + \frac{\sqrt{3}}{2}\Psi^{(\mathrm{SF})}_{\frac{1}{2}1\frac{1}{2}}\left(\Lambda N\Sigma,1\frac{1}{2}\right),\\
\Psi^{(\mathrm{SF})}_{\frac{1}{2}1\frac{1}{2}}(\Sigma\Lambda N,11) &= -\frac{\sqrt{3}}{2}\Psi^{(\mathrm{SF})}_{\frac{1}{2}1\frac{1}{2}}\left(\Lambda N\Sigma,0\frac{1}{2}\right) + \frac{1}{2}\Psi^{(\mathrm{SF})}_{\frac{1}{2}1\frac{1}{2}}\left(\Lambda N\Sigma,1\frac{1}{2}\right).\end{aligned} \quad (7)$$

In the case of ΣNN with $S=\frac{1}{2}, I=1$, there are four possible $S'a'$ values but only two independent, antisymmetric functions can be constructed. By introducing a label v to enumerate the antisymmetric, orthogonal functions, they read

$$\Psi^{(\mathrm{SF})}_{\frac{1}{2}21}(\Sigma NN, v=1) = \frac{1}{\sqrt{3}}[[NN]_{021}\Sigma]_{\frac{1}{2}21} + \frac{1}{3}\big[[\Sigma N]_{01\frac{1}{2}}N\big]_{\frac{1}{2}21} + \frac{1}{\sqrt{18}}\big[[\Sigma N]_{01\frac{3}{2}}N\big]_{\frac{1}{2}21} - \frac{1}{\sqrt{3}}\big[[\Sigma N]_{11\frac{1}{2}}N\big]_{\frac{1}{2}21} - \frac{1}{\sqrt{6}}\big[[\Sigma N]_{11\frac{3}{2}}N\big]_{\frac{1}{2}21},$$

$$\Psi^{(\mathrm{SF})}_{\frac{1}{2}21}(\Sigma NN, v=2) = \frac{1}{\sqrt{3}}[[NN]_{120}\Sigma]_{\frac{1}{2}21} + \frac{1}{\sqrt{6}}\big[[\Sigma N]_{01\frac{1}{2}}N\big]_{\frac{1}{2}21} - \frac{1}{\sqrt{3}}\big[[\Sigma N]_{01\frac{3}{2}}N\big]_{\frac{1}{2}21} + \frac{1}{\sqrt{18}}\big[[\Sigma N]_{11\frac{1}{2}}N\big]_{\frac{1}{2}21} - \frac{1}{3}\big[[\Sigma N]_{11\frac{3}{2}}N\big]_{\frac{1}{2}21}. \quad (8)$$

In the case where only one antisymmetric function is possible, the label v is suppressed. All of the totally antisymmetric spin-flavor functions of three-B_8 systems are tabulated for both $S=\frac{1}{2}$ and $\frac{3}{2}$ in Appendix A of the Supplemental Material [31]. The spin part of the three-B_8 state with $S=\frac{3}{2}$ is totally symmetric under the baryon exchange, so that its flavor part is totally antisymmetric.

The flavor SU(3) symmetry was used to good advantage in studying the B_8B_8 interaction [29]. This is based on the assumption that the underlying Hamiltonian for the octet-baryon system is approximately SU(3) scalar. Similarly we expect that the flavor SU(3) symmetry plays an important role in the three-B_8 systems. To exploit this possibility, we represent all three-B_8 states obtained in the spin-isospin coupled basis in the flavor SU(3) basis. This is done in exactly the same way as the two-baryon case [18,30] with use of reduced SU(3) Wigner coefficients [32]:

$$\big[B_{a_1}(1)B_{a_2}(2)\big]_{S_{12}a_{12}} = \sum_{(\lambda_{12}\mu_{12})\rho_{12}} \langle (11)a_1\,(11)a_2||(\lambda_{12}\mu_{12})a_{12}\rangle_{\rho_{12}} \times [B_{(11)}(1)B_{(11)}(2)]_{S_{12}(\lambda_{12}\mu_{12})\rho_{12}a_{12}}, \quad (9)$$

where $(\lambda_{12}\mu_{12})$ takes (22), (11), (00) for $S_{12}=0$ and (30), (03), (11) for $S_{12}=1$, respectively. The label ρ_{12} distinguishes possible multiple occurrences of $(\lambda_{12}\mu_{12})$. Two representations appear for $(\lambda_{12}\mu_{12})=(11)$, and $\rho_{12}=1$ stands for an antisymmetric coupling, while $\rho_{12}=2$ a symmetric coupling. Further application of the SU(3) coupling with $B_{a_3}(3)$ makes it possible to express the three-B_8 spin-isospin coupled state as follows:

$$\big[\big[B_{a_1}(1)B_{a_2}(2)\big]_{S_{12}a_{12}}B_{a_3}(3)\big]_{Sa} = \sum_{(\lambda_{12}\mu_{12})\rho_{12}(\lambda\mu)\rho} \langle (11)a_1\,(11)a_2||(\lambda_{12}\mu_{12})a_{12}\rangle_{\rho_{12}} \times \langle (\lambda_{12}\mu_{12})a_{12}\,(11)a_3||(\lambda\mu)a\rangle_{\rho} \times |S_{12}(\lambda_{12}\mu_{12})\rho_{12}; S(\lambda\mu)\rho a\rangle \quad (10)$$

with

$$|S_{12}(\lambda_{12}\mu_{12})\rho_{12}; S(\lambda\mu)\rho a\rangle = \big[[B_{(11)}(1)B_{(11)}(2)]_{S_{12}(\lambda_{12}\mu_{12})\rho_{12}}B_{(11)}(3)\big]_{S(\lambda\mu)\rho a}. \quad (11)$$

For the sake of convenience, short-hand notation for $|S_{12}(\lambda_{12}\mu_{12})\rho_{12}; S(\lambda\mu)\rho a\rangle$ is introduced as shown in Table I.

Applying Eq. (10) in Eq. (6) and following the construction of the function $\Psi^{(\mathrm{SF})}_{Sa}(a_1a_2a_3,v)$ defines the totally antisymmetric spin-flavor three-B_8 state in the flavor SU(3) basis as follows:

$$\Psi^{(\mathrm{SF})}_{Sa}(a_1a_2a_3,v) = \sum_{S_{12}(\lambda_{12}\mu_{12})\rho_{12}(\lambda\mu)\rho} G(a_1a_2a_3,v,S_{12}(\lambda_{12}\mu_{12})\rho_{12},(\lambda\mu)\rho;Sa) \times |S_{12}(\lambda_{12}\mu_{12})\rho_{12}; S(\lambda\mu)\rho a\rangle. \quad (12)$$

Tables II and III tabulate the coefficients G for some interesting three-B_8 systems including (i) NNN, (ii) YNN and YYN that couple each other, (iii) high-isospin systems that may be important in the neutron-star interior, and (iv) those systems that lead to almost Pauli-forbidden states. Other three-B_8 systems are tabulated in Appendix B of the Supplemental Material [31].

As Table II shows for the $S=\frac{1}{2}$ case, a group of NNN ($I=\frac{1}{2}$), $\Xi\Sigma\Sigma$ ($I=\frac{5}{2}$), and $\Xi\Xi\Sigma$ ($I=2$) states belongs to $|14\rangle_1-|14\rangle_2$, and likewise a group of ΣNN ($I=2$), $\Sigma\Sigma N$ ($I=\frac{5}{2}$), and $\Xi\Xi\Xi$ ($I=\frac{1}{2}$) belongs to $|41\rangle_1-|41\rangle_2$. In the $S=\frac{3}{2}$ sector, Table III shows that ΣNN ($I=1$), $\Xi\Sigma N$ ($I=2$), and

TABLE I. Short-hand notation $|\lambda\mu\rangle_n$ for the antisymmetric spin-flavor functions in the flavor SU(3) basis, $|S_{12}(\lambda_{12}\mu_{12})\rho_{12}; S(\lambda\mu)\rho a\rangle$, where the multiplicity label ρ is explicitly written for the SU(3) couplings of $(11)\times(11)\to(11)$ and $(22)\times(11)\to(22)$, but it is suppressed in multiplicity-free cases. The total spin S and the label a are abbreviated in the short-hand notation.

$\|41\rangle_1$	$\|0(22); S(41)a\rangle$	$\|11\rangle_1$	$\|0(22); S(11)a\rangle$
$\|41\rangle_2$	$\|1(30); S(41)a\rangle$	$\|11\rangle_2$	$\|1(30); S(11)a\rangle$
$\|30\rangle_1$	$\|0(22); S(30)a\rangle$	$\|11\rangle_3$	$\|1(03); S(11)a\rangle$
$\|30\rangle_2$	$\|1(30); S(30)a\rangle$	$\|11\rangle_4$	$\|1(11)1; S(11)1a\rangle$
$\|30\rangle_3$	$\|1(11)1; S(30)a\rangle$	$\|11\rangle_5$	$\|1(11)1; S(11)2a\rangle$
$\|30\rangle_4$	$\|0(11)2; S(30)a\rangle$	$\|11\rangle_6$	$\|0(11)2; S(11)1a\rangle$
$\|22\rangle_1$	$\|0(22); S(22)1a\rangle$	$\|11\rangle_7$	$\|0(11)2; S(11)2a\rangle$
$\|22\rangle_2$	$\|0(22); S(22)2a\rangle$	$\|11\rangle_8$	$\|0(00); S(11)a\rangle$
$\|22\rangle_3$	$\|1(30); S(22)a\rangle$	$\|03\rangle_1$	$\|0(22); S(03)a\rangle$
$\|22\rangle_4$	$\|1(03); S(22)a\rangle$	$\|03\rangle_2$	$\|1(03); S(03)a\rangle$
$\|22\rangle_5$	$\|1(11)1; S(22)a\rangle$	$\|03\rangle_3$	$\|1(11)1; S(03)a\rangle$
$\|22\rangle_6$	$\|0(11)2; S(22)a\rangle$	$\|03\rangle_4$	$\|0(11)2; S(03)a\rangle$
$\|14\rangle_1$	$\|0(22); S(14)a\rangle$	$\|00\rangle_1$	$\|0(11)1; S(00)a\rangle$
$\|14\rangle_2$	$\|1(03); S(14)a\rangle$	$\|00\rangle_2$	$\|0(11)2; S(00)a\rangle$

TABLE II. Coefficients G in Eq. (12) for some three-B_8 systems with $S = 1/2$. The label v distinguishes the multiple occurrence of the orthogonal, antisymmetric states for a given SYI.

$Y\,I$	$3\,\frac{1}{2}$	$2\,0$		$2\,1$			$2\,2$	$1\,\frac{1}{2}$				
	NNN	ΛNN	ΣNN	ΛNN	ΣNN $v=1$	ΣNN $v=2$	ΣNN	ΞNN $v=1$	ΞNN $v=2$	$\Lambda\Lambda N$	$\Sigma\Sigma N$ $v=1$	$\Sigma\Sigma N$ $v=2$
$\vert 41\rangle_1$							$-\frac{1}{\sqrt{2}}$					
$\vert 41\rangle_2$							$\frac{1}{\sqrt{2}}$					
$\vert 30\rangle_1$												
$\vert 30\rangle_2$												
$\vert 30\rangle_3$												
$\vert 30\rangle_4$												
$\vert 22\rangle_1$				$\frac{3}{4\sqrt{2}}$	$\frac{\sqrt{3}}{4}$	$-\frac{1}{4\sqrt{2}}$		$\frac{\sqrt{15}}{8}$	$-\frac{1}{8}\sqrt{\frac{3}{5}}$	$-\frac{9}{8\sqrt{10}}$	$\frac{1}{8}\sqrt{\frac{3}{10}}$	$\frac{1}{8\sqrt{5}}$
$\vert 22\rangle_2$				$\frac{1}{4}\sqrt{\frac{7}{30}}$	$-\frac{1}{4}\sqrt{\frac{7}{5}}$	$-\frac{1}{4}\sqrt{\frac{21}{10}}$		$-\frac{\sqrt{7}}{40}$	$-\frac{3\sqrt{7}}{40}$	$-\frac{1}{40}\sqrt{\frac{21}{2}}$	$-\frac{1}{8}\sqrt{\frac{7}{2}}$	$\frac{\sqrt{21}}{40}$
$\vert 22\rangle_3$					$-\frac{1}{\sqrt{6}}$	$-\frac{1}{3}$		$-\frac{1}{\sqrt{30}}$	$-\frac{1}{\sqrt{30}}$		$-\frac{1}{\sqrt{15}}$	$\frac{1}{3\sqrt{10}}$
$\vert 22\rangle_4$				$\frac{1}{4}$	$-\frac{1}{2\sqrt{6}}$	$-\frac{5}{12}$		$\frac{1}{4\sqrt{30}}$	$-\frac{1}{4}\sqrt{\frac{5}{6}}$	$-\frac{3}{8\sqrt{5}}$	$-\frac{7}{8\sqrt{15}}$	$\frac{1}{12}\sqrt{\frac{5}{2}}$
$\vert 22\rangle_5$				$-\frac{1}{2}$	$-\frac{1}{\sqrt{6}}$	$\frac{1}{6}$		$-\frac{1}{2}\sqrt{\frac{5}{6}}$	$\frac{1}{2\sqrt{30}}$	$\frac{3}{4\sqrt{5}}$	$-\frac{1}{4\sqrt{15}}$	$-\frac{1}{6\sqrt{10}}$
$\vert 22\rangle_6$				$-\frac{1}{2\sqrt{15}}$	$\frac{1}{\sqrt{10}}$	$\frac{1}{2}\sqrt{\frac{3}{5}}$		$\frac{1}{10\sqrt{2}}$	$\frac{3}{10\sqrt{2}}$	$\frac{\sqrt{3}}{20}$	$\frac{1}{4}$	$-\frac{1}{10}\sqrt{\frac{3}{2}}$
$\vert 14\rangle_1$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{2}\sqrt{\frac{3}{2}}$	$\frac{1}{2\sqrt{2}}$	$-\frac{\sqrt{3}}{4}$	$\frac{1}{2\sqrt{2}}$	$-\frac{\sqrt{3}}{4}$		$-\frac{1}{4\sqrt{6}}$	$-\frac{1}{4}\sqrt{\frac{3}{2}}$	$\frac{3}{8}$	$-\frac{1}{8\sqrt{3}}$	$\frac{1}{4\sqrt{2}}$
$\vert 14\rangle_2$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{2}\sqrt{\frac{3}{2}}$	$-\frac{1}{2\sqrt{2}}$	$\frac{\sqrt{3}}{4}$	$-\frac{1}{2\sqrt{2}}$	$\frac{\sqrt{3}}{4}$		$\frac{1}{4\sqrt{6}}$	$\frac{1}{4}\sqrt{\frac{3}{2}}$	$-\frac{3}{8}$	$\frac{1}{8\sqrt{3}}$	$-\frac{1}{4\sqrt{2}}$
$\vert 11\rangle_1$								$-\frac{11}{20\sqrt{3}}$	$-\frac{\sqrt{3}}{20}$	$-\frac{3}{10\sqrt{2}}$	$\frac{1}{2\sqrt{6}}$	$-\frac{1}{5}$
$\vert 11\rangle_2$								$-\frac{1}{2\sqrt{30}}$	$-\frac{1}{2\sqrt{30}}$		$\frac{2}{\sqrt{15}}$	$\frac{1}{\sqrt{10}}$
$\vert 11\rangle_3$								$-\frac{1}{2\sqrt{30}}$	$-\frac{1}{2}\sqrt{\frac{5}{6}}$	$-\frac{1}{2\sqrt{5}}$	$\frac{1}{2\sqrt{15}}$	
$\vert 11\rangle_4$								$\frac{1}{\sqrt{6}}$		$\frac{1}{4}$	$-\frac{1}{4\sqrt{3}}$	$\frac{1}{2\sqrt{2}}$
$\vert 11\rangle_5$									$\frac{1}{\sqrt{30}}$	$\frac{1}{4\sqrt{5}}$	$\frac{1}{4}\sqrt{\frac{3}{5}}$	$\frac{1}{2\sqrt{10}}$
$\vert 11\rangle_6$									$-\frac{1}{\sqrt{10}}$	$-\frac{1}{4}\sqrt{\frac{3}{5}}$	$-\frac{3}{4\sqrt{5}}$	$-\frac{1}{2}\sqrt{\frac{3}{10}}$
$\vert 11\rangle_7$								$\frac{1}{\sqrt{50}}$	$-\frac{\sqrt{2}}{5}$	$\frac{1}{20\sqrt{3}}$	$\frac{1}{4}$	$\frac{3}{10}\sqrt{\frac{3}{2}}$
$\vert 11\rangle_8$								$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2\sqrt{6}}$	$-\frac{1}{2\sqrt{2}}$	
$\vert 03\rangle_1$		$-\frac{1}{2}\sqrt{\frac{3}{10}}$	$-\frac{3}{2\sqrt{10}}$					$\frac{1}{4}\sqrt{\frac{3}{5}}$	$-\frac{1}{4}\sqrt{\frac{3}{5}}$	$\frac{3}{4\sqrt{10}}$	$\frac{1}{4}\sqrt{\frac{3}{10}}$	$-\frac{3}{4\sqrt{5}}$
$\vert 03\rangle_2$		$\frac{1}{2\sqrt{6}}$	$\frac{1}{2\sqrt{2}}$					$-\frac{1}{4\sqrt{3}}$	$\frac{1}{4\sqrt{3}}$	$-\frac{1}{4\sqrt{2}}$	$-\frac{1}{4\sqrt{6}}$	$\frac{1}{4}$
$\vert 03\rangle_3$		$\frac{1}{2\sqrt{3}}$	$\frac{1}{2}$					$-\frac{1}{2\sqrt{6}}$	$\frac{1}{2\sqrt{6}}$	$-\frac{1}{4}$	$-\frac{1}{4\sqrt{3}}$	$\frac{1}{2\sqrt{2}}$
$\vert 03\rangle_4$		$-\frac{1}{2\sqrt{5}}$	$-\frac{1}{2}\sqrt{\frac{3}{5}}$					$\frac{1}{2\sqrt{10}}$	$-\frac{1}{2\sqrt{10}}$	$\frac{1}{4}\sqrt{\frac{3}{5}}$	$\frac{1}{4\sqrt{5}}$	$-\frac{1}{2}\sqrt{\frac{3}{10}}$

$Y\,I$	$1\,\frac{1}{2}$		$1\,\frac{3}{2}$					$1\,\frac{5}{2}$	$-1\,\frac{5}{2}$	$-2\,0$		$-2\,2$	$-3\,\frac{1}{2}$
	$\Sigma\Lambda N$ $v=1$	$\Sigma\Lambda N$ $v=2$	ΞNN	$\Sigma\Sigma N$ $v=1$	$\Sigma\Sigma N$ $v=2$	$\Sigma\Lambda N$ $v=1$	$\Sigma\Lambda N$ $v=2$	$\Sigma\Sigma N$	$\Xi\Sigma\Sigma$	$\Xi\Xi\Lambda$	$\Xi\Xi\Sigma$	$\Xi\Xi\Sigma$	$\Xi\Xi\Xi$
$\vert 41\rangle_1$			$-\frac{1}{4}\sqrt{\frac{5}{3}}$	$-\frac{1}{8\sqrt{3}}$	$-\frac{\sqrt{5}}{8}$	$\frac{\sqrt{5}}{8}$	$-\frac{\sqrt{15}}{8}$	$\frac{1}{\sqrt{2}}$		$-\frac{1}{2}\sqrt{\frac{3}{2}}$	$-\frac{1}{2\sqrt{2}}$		$\frac{1}{\sqrt{2}}$
$\vert 41\rangle_2$			$\frac{1}{4}\sqrt{\frac{5}{3}}$	$\frac{1}{8\sqrt{3}}$	$\frac{\sqrt{5}}{8}$	$-\frac{\sqrt{5}}{8}$	$\frac{\sqrt{15}}{8}$	$-\frac{1}{\sqrt{2}}$		$\frac{1}{2}\sqrt{\frac{3}{2}}$	$\frac{1}{2\sqrt{2}}$		$-\frac{1}{\sqrt{2}}$
$\vert 30\rangle_1$			$-\frac{1}{2}\sqrt{\frac{3}{10}}$	$\frac{1}{4}\sqrt{\frac{3}{2}}$	$\frac{3}{4\sqrt{10}}$	$-\frac{3}{4\sqrt{10}}$	$-\frac{1}{4}\sqrt{\frac{3}{10}}$			$-\frac{1}{2}\sqrt{\frac{3}{10}}$	$\frac{3}{2\sqrt{10}}$		
$\vert 30\rangle_2$			$\frac{1}{2\sqrt{6}}$	$-\frac{1}{4}\sqrt{\frac{5}{6}}$	$-\frac{1}{4\sqrt{2}}$	$\frac{1}{4\sqrt{2}}$	$\frac{1}{4\sqrt{6}}$			$\frac{1}{2\sqrt{6}}$	$-\frac{1}{2\sqrt{2}}$		
$\vert 30\rangle_3$			$-\frac{1}{2\sqrt{3}}$	$\frac{1}{4}\sqrt{\frac{5}{3}}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4\sqrt{3}}$			$-\frac{1}{2\sqrt{3}}$	$\frac{1}{2}$		
$\vert 30\rangle_4$			$-\frac{1}{2\sqrt{5}}$	$\frac{1}{4}$	$\frac{1}{4}\sqrt{\frac{3}{5}}$	$-\frac{1}{4}\sqrt{\frac{3}{5}}$	$-\frac{1}{4\sqrt{5}}$			$-\frac{1}{2\sqrt{5}}$	$\frac{1}{2}\sqrt{\frac{3}{5}}$		

TABLE II. (*Continued.*)

$Y\ I$	$1\ \frac{1}{2}$		$1\ \frac{3}{2}$					$1\ \frac{5}{2}$	$-1\ \frac{5}{2}$	$-2\ 0$		$-2\ 2$	$-3\ \frac{1}{2}$
	$\Sigma\Lambda N$	$\Sigma\Lambda N$	ΞNN	$\Sigma\Sigma N$	$\Sigma\Sigma N$	$\Sigma\Lambda N$	$\Sigma\Lambda N$	$\Sigma\Sigma N$	$\Xi\Sigma\Sigma$	$\Xi\Xi\Lambda$	$\Xi\Xi\Sigma$	$\Xi\Xi\Sigma$	$\Xi\Xi\Xi$
	$v=1$	$v=2$		$v=1$	$v=2$	$v=1$	$v=2$						
$\|22\rangle_1$	$\frac{3}{4\sqrt{5}}$	$-\frac{1}{8}\sqrt{\frac{3}{5}}$	$\frac{1}{4}\sqrt{\frac{3}{2}}$	$\frac{1}{8}\sqrt{\frac{15}{2}}$	$-\frac{5}{8\sqrt{2}}$	$-\frac{3}{8\sqrt{2}}$	$-\frac{1}{8}\sqrt{\frac{3}{2}}$						
$\|22\rangle_2$	$\frac{1}{20}\sqrt{\frac{7}{3}}$	$\frac{\sqrt{7}}{8}$	$\frac{1}{4}\sqrt{\frac{7}{10}}$	$\frac{1}{8}\sqrt{\frac{7}{2}}$	$-\frac{1}{8}\sqrt{\frac{21}{10}}$	$\frac{1}{8}\sqrt{\frac{35}{6}}$	$-\frac{1}{8}\sqrt{\frac{7}{10}}$						
$\|22\rangle_3$		$\sqrt{\frac{2}{15}}$	$\frac{1}{4\sqrt{3}}$	$\frac{1}{8}\sqrt{\frac{5}{3}}$	$\frac{7}{24}$	$\frac{3}{8}$	$-\frac{1}{8\sqrt{3}}$						
$\|22\rangle_4$	$\frac{1}{2\sqrt{10}}$	$\frac{7}{4\sqrt{30}}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{4}\sqrt{\frac{5}{3}}$	$\frac{1}{12}$	$\frac{1}{4}$	$-\frac{1}{4\sqrt{3}}$						
$\|22\rangle_5$	$-\frac{1}{\sqrt{10}}$	$\frac{1}{2\sqrt{30}}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{4}\sqrt{\frac{5}{3}}$	$\frac{5}{12}$	$\frac{1}{4}$	$\frac{1}{4\sqrt{3}}$						
$\|22\rangle_6$	$-\frac{1}{5\sqrt{6}}$	$-\frac{1}{2\sqrt{2}}$	$-\frac{1}{2\sqrt{5}}$	$-\frac{1}{4}$	$-\frac{1}{4}\sqrt{\frac{3}{5}}$	$-\frac{1}{4}\sqrt{\frac{5}{3}}$	$\frac{1}{4\sqrt{5}}$						
$\|14\rangle_1$	$\frac{1}{2\sqrt{2}}$	$-\frac{1}{4}\sqrt{\frac{3}{2}}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{4}\sqrt{\frac{5}{3}}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{\sqrt{3}}{4}$		$-\frac{1}{\sqrt{2}}$			$\frac{1}{\sqrt{2}}$	
$\|14\rangle_2$	$-\frac{1}{2\sqrt{2}}$	$\frac{1}{4}\sqrt{\frac{3}{2}}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{4}\sqrt{\frac{5}{3}}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{\sqrt{3}}{4}$		$\frac{1}{\sqrt{2}}$			$-\frac{1}{\sqrt{2}}$	
$\|11\rangle_1$	$\frac{1}{5}$												
$\|11\rangle_2$		$\frac{1}{\sqrt{30}}$											
$\|11\rangle_3$	$-\frac{1}{\sqrt{10}}$	$-\frac{1}{\sqrt{30}}$											
$\|11\rangle_4$	$-\frac{1}{2\sqrt{2}}$												
$\|11\rangle_5$	$\frac{1}{2\sqrt{10}}$	$\frac{1}{\sqrt{30}}$											
$\|11\rangle_6$	$-\frac{1}{2}\sqrt{\frac{3}{10}}$	$-\frac{1}{\sqrt{10}}$											
$\|11\rangle_7$	$-\frac{3}{10}\sqrt{\frac{3}{2}}$												
$\|11\rangle_8$													
$\|03\rangle_1$		$\frac{1}{4}\sqrt{\frac{3}{5}}$											
$\|03\rangle_2$		$-\frac{1}{4\sqrt{3}}$											
$\|03\rangle_3$		$-\frac{1}{2\sqrt{6}}$											
$\|03\rangle_4$		$\frac{1}{2\sqrt{10}}$											

TABLE III. Same as Table II, but for $S = 3/2$.

$Y\ I$	$2\ 0$	$2\ 1$	$1\ \frac{1}{2}$			$1\ \frac{3}{2}$		$0\ 2$	$-1\ \frac{1}{2}$			$-2\ 0$	$-2\ 1$
	ΛNN	ΣNN	ΞNN	$\Sigma\Sigma N$	$\Sigma\Lambda N$	$\Sigma\Sigma N$	$\Sigma\Lambda N$	$\Xi\Sigma N$	$\Xi\Xi N$	$\Xi\Sigma\Sigma$	$\Xi\Sigma\Lambda$	$\Xi\Xi\Lambda$	$\Xi\Xi\Sigma$
$\|30\rangle_2$						$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$		$\frac{1}{\sqrt{3}}$		$\frac{1}{\sqrt{3}}$	$\sqrt{\frac{2}{3}}$	
$\|30\rangle_3$						$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$		$\frac{1}{\sqrt{6}}$		$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{3}}$	
$\|22\rangle_3$		$\frac{2}{3}$	$\sqrt{\frac{2}{15}}$	$\frac{2}{3}\sqrt{\frac{2}{5}}$	$\sqrt{\frac{2}{15}}$	$-\frac{1}{3}$	$\frac{1}{\sqrt{3}}$	$-\frac{2}{3}$	$\sqrt{\frac{2}{15}}$	$-\frac{2}{3}\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{2}{15}}$		$\frac{2}{3}$
$\|22\rangle_4$		$-\frac{2}{3}$	$-\sqrt{\frac{2}{15}}$	$-\frac{2}{3}\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{2}{15}}$	$\frac{1}{3}$	$-\frac{1}{\sqrt{3}}$	$\frac{2}{3}$	$-\sqrt{\frac{2}{15}}$	$\frac{2}{3}\sqrt{\frac{2}{5}}$	$\sqrt{\frac{2}{15}}$		$-\frac{2}{3}$
$\|22\rangle_5$		$-\frac{1}{3}$	$-\frac{1}{\sqrt{30}}$	$-\frac{1}{3}\sqrt{\frac{2}{5}}$	$-\frac{1}{\sqrt{30}}$	$\frac{1}{6}$	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$	$-\frac{1}{\sqrt{30}}$	$\frac{1}{3}\sqrt{\frac{2}{5}}$	$\frac{1}{\sqrt{30}}$		$-\frac{1}{3}$
$\|11\rangle_2$			$\frac{1}{\sqrt{30}}$	$-\frac{1}{\sqrt{10}}$	$\frac{1}{\sqrt{30}}$				$\frac{1}{\sqrt{30}}$	$\frac{1}{\sqrt{10}}$	$-\frac{1}{\sqrt{30}}$		
$\|11\rangle_3$			$-\frac{1}{\sqrt{30}}$	$\frac{1}{\sqrt{10}}$	$-\frac{1}{\sqrt{30}}$				$-\frac{1}{\sqrt{30}}$	$-\frac{1}{\sqrt{10}}$	$\frac{1}{\sqrt{30}}$		
$\|11\rangle_5$			$-\sqrt{\frac{2}{15}}$	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{2}{15}}$				$-\sqrt{\frac{2}{15}}$	$-\sqrt{\frac{2}{5}}$	$\sqrt{\frac{2}{15}}$		
$\|03\rangle_2$	$\sqrt{\frac{2}{3}}$		$\frac{1}{\sqrt{3}}$		$-\frac{1}{\sqrt{3}}$								
$\|03\rangle_3$	$-\frac{1}{\sqrt{3}}$		$-\frac{1}{\sqrt{6}}$		$\frac{1}{\sqrt{6}}$								

$\Xi\Xi\Sigma$ ($I=1$) systems are all expressed by $2|22\rangle_3 - 2|22\rangle_4 - |22\rangle_5$.

III. QUARK EXCHANGES OF THREE OCTET-BARYON STATES

The nine-quark three-B_8 wave function with SaS_zI_z that is antisymmetric under the baryon exchange is given by

$$\Psi_{SaS_zI_z}(B_1B_2B_3,v) = \Psi^{(\mathrm{orb})}(B_1B_2B_3)\Psi^{(\mathrm{SF})}_{SaS_zI_z}(a_1a_2a_3,v)\Psi^{(\mathrm{color})}(B_1B_2B_3), \tag{13}$$

where $\Psi^{(\mathrm{orb})}(B_1B_2B_3)$ denotes the orbital part with the $(0s)^9$ configuration, and $\Psi^{(\mathrm{color})}(B_1B_2B_3)$ is the color wave function $C(123)C(456)C(789)$. The spin-flavor part $\Psi^{(\mathrm{SF})}_{SaS_zI_z}(a_1a_2a_3,v)$ is given by Eq. (12) with B_{a_i} being replaced by the corresponding three-quark wave function $W^{[3]}_{a_i}$. In order to examine the quark-Pauli effect, we have to solve the eigenvalue problem of the antisymmetrizer $\mathcal{A}$ that makes the nine-quark wave function totally antisymmetric under the exchange of quarks among the baryons. Those eigenfunctions that correspond to vanishing eigenvalues or considerably small eigenvalues are called Pauli forbidden or almost Pauli forbidden. Three-B_8 baryons cannot occupy such forbidden configurations, namely, they exhibit quark-Pauli repulsion.

It is useful to note that $\mathcal{A}$ is scalar with respect to the total spin and flavor SU(3) label, that is, it has no matrix elements between different SU(3) labels though it mixes the content of three-baryon species. This is the reason why we express the basis (13) in the flavor SU(3) representation.

Since we have three baryons each of which consists of three quarks and is antisymmetrized, $\mathcal{A}$ reduces to 55 terms of five basic types [26]:

$$\mathcal{A} = [1 - 9(P_{36}+P_{69}+P_{93}) + 27(P_{396}+P_{369}) + 54(P_{36}P_{59}+P_{69}P_{83}+P_{93}P_{26})]\left(\sum_{\mathcal{P}=1}^{6}(-1)^{\pi(\mathcal{P})}\mathcal{P}\right) - 216P_{26}P_{59}P_{83}, \tag{14}$$

where P_{ij} exchanges quarks i and j and acts on the full orbital, color, spin, and flavor degrees of freedom. The six $\mathcal{P}$ include those quark exchanges that are equivalent to baryon exchanges. Of the five basic types of terms in $\mathcal{A}$, the first is the direct term, and the second, $(P_{36}+P_{39}+P_{69})(\sum_{\mathcal{P}=1}^{6}(-1)^{\pi(\mathcal{P})}\mathcal{P})$, involves only the exchange of baryons and one quark pair. Terms in $\mathcal{A}$ of the third to fifth categories involve the exchange of quark pairs between different baryon pairs.

Our three-B_8 wave function (13) is already antisymmetrized with respect to the baryon exchanges, so $\mathcal{A}$ can be effectively replaced by $\mathcal{A}' = \frac{1}{6}\mathcal{A}$. The matrix elements of $\mathcal{A}'$ with the basis functions (13) are obtained by combining those of the orbital, spin-isospin, and color parts. The spin-isospin matrix elements are evaluated by making use of the decomposition (12) in the flavor SU(3) basis. Apparently $\mathcal{A}'$ conserves S as well as the subgroup labels I and Y of the flavor SU(3) group. The orbital matrix element is unity, independent of the quark exchange because of the fully symmetric $(0s)^9$ configuration. The color matrix element is also simple: the fifth term in $\mathcal{A}'$, $P_{26}P_{59}P_{83}$, has no color matrix element, whereas those of the other terms are $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{9}$, for P_{36}, P_{396}, $P_{36}P_{59}$, respectively [23,26]. The full matrix elements including the orbital, spin-flavor, and color parts between the basis functions,

$$\langle S_{12}(\lambda_{12}\mu_{12})\rho_{12}; S(\lambda\mu)\rho a|\mathcal{A}'|S'_{12}(\lambda'_{12}\mu'_{12})\rho'_{12}; S(\lambda\mu)\rho a\rangle, \tag{15}$$

are calculated by the use of SU(3) 6-$(\lambda\mu)$ and 9-$(\lambda\mu)$ coefficients [33] and are tabulated in Appendix C of the Supplemental Material [31]. Assuming the eigenfunction of $\mathcal{A}'$ to be

$$\sum_{B_1B_2B_3v} C_{Sa}(B_1B_2B_3,v)\Psi_{SaS_zI_z}(B_1B_2B_3,v), \tag{16}$$

we solve the eigenvalue problem

$$\sum_{B'_1B'_2B'_3v'}\left\langle\Psi_{SaS_zI_z}(B_1B_2B_3,v)\middle|\mathcal{A}'\middle|\Psi_{SaS_zI_z}(B'_1B'_2B'_3,v')\right\rangle \times C_{Sa}(B'_1B'_2B'_3,v') = \mu_{Sa}C_{Sa}(B_1B_2B_3,v). \tag{17}$$

The eigenvalues μ_{Sa} are given in Tables IV and V. The eigenfunctions are available from the authors of this paper upon request.

As seen in Tables IV and V, some three-B_8 systems do not couple with other systems. They are $NNN(I=\frac{1}{2})$, $\Sigma NN(2)$, $\Sigma\Sigma N(\frac{5}{2})$, $\Xi\Sigma\Sigma(\frac{5}{2})$, $\Xi\Xi\Sigma(2)$, $\Xi\Xi\Xi(\frac{1}{2})$ for $S=\frac{1}{2}$ and $\Lambda NN(0)$, $\Sigma NN(1)$, $\Xi\Sigma N(2)$, $\Xi\Xi\Lambda(0)$, $\Xi\Xi\Sigma(1)$ for $S=\frac{3}{2}$, respectively. Among these, $\Sigma NN(2)$, $\Sigma\Sigma N(\frac{5}{2})$, $\Xi\Xi\Xi(\frac{1}{2})$, and $\Sigma NN(1)$, $\Xi\Sigma N(2)$, $\Xi\Xi\Lambda(0)$, $\Xi\Xi\Sigma(1)$ are all considered almost Pauli-forbidden states because the corresponding μ_{Sa} values are fairly small. The strong quark-Pauli repulsion in the $S=\frac{1}{2}$ systems, $\Sigma NN(2)$, $\Sigma\Sigma N(\frac{5}{2})$, and $\Xi\Xi\Xi(\frac{1}{2})$, results from the fact that they consist of the $|41\rangle_1$ and $|41\rangle_2$ components (see Table II) whose matrix elements of $\mathcal{A}'$

$$\begin{pmatrix} \frac{2}{81} & -\frac{2}{81} \\ -\frac{2}{81} & \frac{2}{81} \end{pmatrix} \tag{18}$$

are small. The reason that the above-mentioned systems with $S=\frac{3}{2}$ become almost Pauli forbidden is similar to the $S=\frac{1}{2}$ case. As seen in Table III, $\Sigma NN(1)$, $\Xi\Sigma N(2)$, and $\Xi\Xi\Sigma(1)$ consist of $|22\rangle_n$ components and $\Xi\Xi\Lambda(0)$ consists of $|30\rangle_n$ components. The matrix elements of those components are relatively small as given in Appendix C of the Supplemental Material [31].

In the coupled-channel three-B_8 case, we find many completely Pauli-forbidden states Φ^{FS}_{Sa}. Examples of Φ^{FS}_{Sa} that are uniquely determined are

$$\Phi^{\mathrm{FS}}_{\frac{1}{2}20} = \frac{1}{2}\Psi_{\frac{1}{2}20}(\Lambda NN) + \frac{\sqrt{3}}{2}\Psi_{\frac{1}{2}20}(\Sigma NN),$$

$$\Phi^{\mathrm{FS}}_{\frac{1}{2}21} = -\frac{1}{4}\Psi_{\frac{1}{2}21}(\Lambda NN) + \sqrt{\frac{3}{8}}\Psi_{\frac{1}{2}21}(\Sigma NN,v=1) + \frac{3}{4}\Psi_{\frac{1}{2}21}(\Sigma NN,v=2),$$

TABLE IV. Eigenvalues μ_{Sa} in Eq. (17), given in increasing order, for three-B_8 systems with $S = 1/2$. The expectation value of $\mathcal{A}'$ calculated for each $B_8B_8B_8$ system is given in the $\langle\mathcal{A}'\rangle$ column.

$Y\ I$	$B_8B_8B_8$	$\langle\mathcal{A}'\rangle$	μ_{Sa}	$Y\ I$	$B_8B_8B_8$	$\langle\mathcal{A}'\rangle$	μ_{Sa}	$Y\ I$	$B_8B_8B_8$	$\langle\mathcal{A}'\rangle$	μ_{Sa}
$3\ \frac{1}{2}$	NNN	$\frac{100}{81}$	$\frac{100}{81}$	$0\ 0$	$\Xi\Lambda N\ v=1$	$\frac{5}{6}$	0	$-1\ \frac{1}{2}$	$\Xi\Xi N\ v=1$	$\frac{17}{81}$	0
$2\ 0$	ΛNN	$\frac{25}{27}$	0		$\Xi\Lambda N\ v=2$	$\frac{55}{54}$	0		$\Xi\Xi N\ v=2$	$\frac{73}{81}$	0
	ΣNN	$\frac{25}{81}$	$\frac{100}{81}$		$\Xi\Sigma N\ v=1$	$\frac{5}{54}$	0		$\Xi\Lambda\Lambda$	$\frac{1}{2}$	0
$2\ 1$	ΛNN	$\frac{25}{27}$	0		$\Xi\Sigma N\ v=2$	$\frac{55}{486}$	$\frac{200}{243}$		$\Xi\Sigma\Sigma\ v=1$	$\frac{83}{162}$	0
	$\Sigma NN\ v=1$	$\frac{50}{81}$	$\frac{200}{243}$		$\Sigma\Sigma\Lambda$	$\frac{10}{27}$	$\frac{130}{81}$		$\Xi\Sigma\Sigma\ v=2$	$\frac{17}{243}$	$\frac{4}{81}$
	$\Sigma NN\ v=2$	$\frac{125}{243}$	$\frac{100}{81}$	$0\ 1$	$\Xi\Lambda N\ v=1$	$\frac{33}{54}$	0		$\Xi\Sigma\Lambda\ v=1$	$\frac{1}{36}$	$\frac{200}{243}$
$2\ 2$	ΣNN	$\frac{4}{81}$	$\frac{4}{81}$		$\Xi\Lambda N\ v=2$	$\frac{17}{162}$	0		$\Xi\Sigma\Lambda\ v=2$	$\frac{83}{324}$	$\frac{130}{81}$
$1\ \frac{1}{2}$	$\Xi NN\ v=1$	$\frac{25}{27}$	0		$\Xi\Sigma N\ v=1$	$\frac{11}{162}$	0	$-1\ \frac{3}{2}$	$\Xi\Xi N$	$\frac{34}{81}$	0
	$\Xi NN\ v=2$	$\frac{35}{81}$	0		$\Xi\Sigma N\ v=2$	$\frac{17}{27}$	0		$\Xi\Sigma\Sigma\ v=1$	$\frac{35}{162}$	0
	$\Lambda\Lambda N$	$\frac{5}{6}$	0		$\Xi\Sigma N\ v=3$	$\frac{673}{1458}$	0		$\Xi\Sigma\Sigma\ v=2$	$\frac{253}{486}$	$\frac{4}{81}$
	$\Sigma\Sigma N\ v=1$	$\frac{85}{162}$	0		$\Xi\Sigma N\ v=4$	$\frac{295}{729}$	$\frac{4}{81}$		$\Xi\Sigma\Lambda\ v=1$	$\frac{1}{3}$	$\frac{200}{243}$
	$\Sigma\Sigma N\ v=2$	$\frac{35}{243}$	$\frac{200}{243}$		$\Sigma\Lambda\Lambda$	$\frac{4}{9}$	$\frac{200}{243}$		$\Xi\Sigma\Lambda\ v=2$	$\frac{50}{81}$	$\frac{100}{81}$
	$\Sigma\Lambda N\ v=1$	$\frac{5}{9}$	$\frac{100}{81}$		$\Sigma\Sigma\Lambda$	$\frac{23}{81}$	$\frac{100}{81}$	$-1\ \frac{5}{2}$	$\Xi\Sigma\Sigma$	$\frac{100}{81}$	$\frac{100}{81}$
	$\Sigma\Lambda N\ v=2$	$\frac{20}{81}$	$\frac{130}{81}$		$\Sigma\Sigma\Sigma$	$\frac{19}{27}$	$\frac{130}{81}$	$-2\ 0$	$\Xi\Xi\Lambda$	$\frac{1}{27}$	0
$1\ \frac{3}{2}$	ΞNN	$\frac{10}{27}$	0	$0\ 2$	$\Xi\Sigma N\ v=1$	$\frac{1}{3}$	0		$\Xi\Xi\Sigma$	$\frac{1}{81}$	$\frac{4}{81}$
	$\Sigma\Sigma N\ v=1$	$\frac{73}{162}$	0		$\Xi\Sigma N\ v=2$	$\frac{125}{243}$	$\frac{4}{81}$	$-2\ 1$	$\Xi\Xi\Lambda$	$\frac{13}{27}$	0
	$\Sigma\Sigma N\ v=2$	$\frac{235}{486}$	$\frac{4}{81}$		$\Sigma\Sigma\Lambda$	$\frac{13}{27}$	$\frac{200}{243}$		$\Xi\Xi\Sigma\ v=1$	$\frac{26}{81}$	$\frac{4}{81}$
	$\Sigma\Lambda N\ v=1$	$\frac{5}{18}$	$\frac{200}{243}$		$\Sigma\Sigma\Sigma$	$\frac{7}{9}$	$\frac{100}{81}$		$\Xi\Xi\Sigma\ v=2$	$\frac{17}{243}$	$\frac{200}{243}$
	$\Sigma\Lambda N\ v=2$	$\frac{85}{162}$	$\frac{100}{81}$					$-2\ 2$	$\Xi\Xi\Sigma$	$\frac{100}{81}$	$\frac{100}{81}$
$1\ \frac{5}{2}$	$\Sigma\Sigma N$	$\frac{4}{81}$	$\frac{4}{81}$					$-3\ \frac{1}{2}$	$\Xi\Xi\Xi$	$\frac{4}{81}$	$\frac{4}{81}$

TABLE V. Same as Table IV, but for $S = 3/2$.

Y	I	$B_8B_8B_8$	$\langle\mathcal{A}'\rangle$	μ_{Sa}
2	0	ΛNN	$\frac{25}{27}$	$\frac{25}{27}$
2	1	ΣNN	$\frac{35}{243}$	$\frac{35}{243}$
1	$\frac{1}{2}$	ΞNN	$\frac{50}{81}$	$\frac{35}{243}$
		$\Sigma\Sigma N$	$\frac{95}{243}$	$\frac{5}{9}$
		$\Sigma\Lambda N$	$\frac{50}{81}$	$\frac{25}{27}$
1	$\frac{3}{2}$	$\Sigma\Sigma N$	$\frac{31}{486}$	$\frac{1}{27}$
		$\Sigma\Lambda N$	$\frac{19}{162}$	$\frac{35}{243}$
0	0	$\Xi\Lambda N$	$\frac{20}{27}$	$\frac{35}{243}$
		$\Xi\Sigma N$	$\frac{140}{243}$	$\frac{5}{9}$
		$\Sigma\Sigma\Sigma$	$\frac{55}{81}$	$\frac{35}{27}$
0	1	$\Xi\Lambda N$	$\frac{34}{81}$	$\frac{1}{27}$
		$\Xi\Sigma N\ v=1$	$\frac{134}{729}$	$\frac{35}{243}$
		$\Xi\Sigma N\ v=2$	$\frac{565}{729}$	$\frac{5}{9}$
		$\Sigma\Sigma\Lambda$	$\frac{23}{81}$	$\frac{25}{27}$
0	2	$\Xi\Sigma N$	$\frac{35}{243}$	$\frac{35}{243}$
-1	$\frac{1}{2}$	$\Xi\Xi N$	$\frac{14}{81}$	$\frac{1}{27}$
		$\Xi\Sigma\Sigma$	$\frac{95}{243}$	$\frac{35}{243}$
		$\Xi\Sigma\Lambda$	$\frac{14}{81}$	$\frac{5}{9}$
-1	$\frac{3}{2}$	$\Xi\Sigma\Sigma$	$\frac{355}{486}$	$\frac{35}{243}$
		$\Xi\Sigma\Lambda$	$\frac{55}{162}$	$\frac{25}{27}$
-2	0	$\Xi\Xi\Lambda$	$\frac{1}{27}$	$\frac{1}{27}$
-2	1	$\Xi\Xi\Sigma$	$\frac{35}{243}$	$\frac{35}{243}$

$$\Phi^{\mathrm{FS}}_{\frac{1}{2}02} = \frac{\sqrt{3}}{4}\Psi_{\frac{1}{2}02}(\Xi\Sigma N, v=1) + \frac{3}{4}\Psi_{\frac{1}{2}02}(\Xi\Sigma N, v=2) - \frac{1}{2}\Psi_{\frac{1}{2}02}(\Sigma\Sigma\Lambda),$$

$$\Phi^{\mathrm{FS}}_{\frac{1}{2}-20} = \frac{1}{2}\Psi_{\frac{1}{2}-20}(\Xi\Xi\Lambda) - \frac{\sqrt{3}}{2}\Psi_{\frac{1}{2}-20}(\Xi\Xi\Sigma),$$

$$\Phi^{\mathrm{FS}}_{\frac{1}{2}-21} = \frac{1}{4}\Psi_{\frac{1}{2}-21}(\Xi\Xi\Lambda) + \sqrt{\frac{3}{8}}\Psi_{\frac{1}{2}-21} \times (\Xi\Xi\Sigma, v=1) + \frac{3}{4}\Psi_{\frac{1}{2}-21}(\Xi\Xi\Sigma, v=2). \quad (19)$$

The existence of these Pauli-forbidden states indicates that they are not allowed to take the $(0s)^9$ configuration in, e.g., few-body calculations consisting of N's and Y's. Other completely Pauli-forbidden states have degeneracy. In the coupled-channel three-B_8 systems, we also find several almost-Pauli-forbidden states, for example, $\Xi\Xi\Lambda + \Xi\Xi\Sigma$ with $SI = \frac{1}{2}0$ and $\Sigma\Lambda N + \Sigma\Sigma N$ with $SI = \frac{3}{2}\frac{3}{2}$.

Whether or not the ΛNN system receives a strong quark-Pauli repulsion is particularly interesting for the neutron star problem as noted in the Introduction. The ΛNN system takes $I = 0,1$ in $S = \frac{1}{2}$ and $I = 0$ in $S = \frac{3}{2}$. In the latter case the value of μ_{Sa} is large, indicating that the ΛNN system with $SI = \frac{3}{2}0$ is completely allowed. In the former case we have one completely Pauli-forbidden state in both $I = 0,1$ states. The probability of finding the ΛNN state in those Pauli-forbidden states is, however, rather small as shown in Eq. (19), so the ΛNN system receives a minor quark-Pauli repulsion. Thus the quark-Pauli repulsion is unlikely to suppress the emergence of the Λ hyperon that is a candidate for the first hyperon in increasing baryon density.

In contrast to Λ, the Σ hyperon is involved in producing several almost-Pauli-forbidden ΣNN states. A specific case is $\Sigma NN(I=2)$ including the $\Sigma^- nn$ state. The quark-Pauli effect prevents Σ^- from appearing with increasing baryon density in the neutron star.

The H dibaryon is conjectured to be a B_8B_8 bound or resonant state with $S=0, Y=-1, I=0$. Its flavor symmetry is SU(3) scalar, that is, $(\lambda_{12}\mu_{12})=(00)$ in Eq. (9). No clear experimental confirmation of the H dibaryon has been made yet. A question arises if the quark-Pauli effect demolishes the most compact three-B_8 configuration consisting of HB_8 if the H is assumed to have a main configuration of $(0s)^6$. The HB_8 state should have $(\lambda\mu)=(11)$ and $S=\frac{1}{2}$. We identify the HB_8 state as $|11\rangle_8$ in Table I. The expectation value of $\mathcal{A}'$ with this state is $\frac{5}{12}$ as seen in the table of Appendix C in the Supplemental Material [31]. This implies a somewhat repulsive quark-Pauli effect, which is qualitatively consistent with the corresponding result of Ref. [34].

IV. CONCLUSION

The hyperons appear to be present in the interior of the neutron star with increasing baryon density. Since their appearance generally leads to the softening of its equation of state, some repulsive mechanism to suppress the role of the hyperons is called for in order to be consistent with the observation that the mass of the neutron star can be twice as heavy as the solar mass. At present the information on two- and three-baryon forces including the hyperons is very much limited, and it is hard to draw clear conclusions on the required repulsion. In this work we have considered the role of the quark-Pauli blocking effect on the three octet-baryons.

Assuming a common orbital wave function for all the octet-baryons, we have considered the most compact spatial configuration in which the three particles are located on top of each other, as a possible quark-Pauli effect becomes largest. We have first constructed all possible states with the total spin $S=\frac{1}{2}$ and $\frac{3}{2}$ that are antisymmetric in the simultaneous exchange of spin and flavor degrees of freedom of the baryons. The flavor SU(3) symmetry of the three particles is exploited to classify the constructed states. The quark-Pauli effect is then quantified by calculating the eigenvalues of the nine-quark antisymmetrizer in those three-baryon states.

Several systems have been found to have vanishing or small eigenvalues that lead to the strong quark-Pauli repulsion. In the $S=\frac{1}{2}$ case, they are $\Sigma NN(I=2)$, $\Sigma\Sigma N(\frac{5}{2})$, $\Xi\Xi\Xi(\frac{1}{2})$, $\Xi\Xi\Lambda(0)$, and $\Xi\Xi\Sigma(0)$, where I is the total isospin of the three baryons. In the $S=\frac{3}{2}$ case, they are $\Sigma NN(1)$, $\Xi\Sigma N(2)$, $\Xi\Xi\Lambda(0)$, $\Xi\Xi\Sigma(1)$, $\Sigma\Lambda N(\frac{3}{2})$, and $\Sigma\Sigma N(\frac{3}{2})$. The Λ and Σ hyperons behave differently with respect to the quark-Pauli repulsion. The ΛNN system receives minor quark-Pauli effects and are allowed to be present in the interior of the neutron star unless the ΛNN three-body force is strongly repulsive, whereas the $\Sigma NN(I=2)$ system regardless of $S=\frac{1}{2}$ or $\frac{3}{2}$, including, e.g., $\Sigma^- nn$ is almost Pauli forbidden.

It will be interesting to study three octet-baryon forces using a quark-model Hamiltonian. The spin and flavor SU(3) symmetry developed here should be useful to the extent to which the underlying Hamiltonian is SU(3) scalar. Work along this direction is in progress.

ACKNOWLEDGMENT

This work was supported in part by JSPS KAKENHI Grants No. 24654071, No. 24540261, and No. 15K05072.

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