Hadronization time of heavy quarks in nuclear matter

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(Received 20 January 2016; revised manuscript received 8 June 2016; published 8 September 2016)

We study the hadronization time of heavy quark in nuclear matter by using the coalescence model and the spatial diffusion constant of a heavy quark from lattice quantum chromodynamic calculations, assuming that the main interaction of a heavy quark at the critical temperature is hadronization. It is found that the hadronization time of a heavy quark is about 3 fm/c for $2\pi T_c D_s = 6$, if a heavy quark is combined with the nearest light antiquark in coordinate space without any correlation between the momentum of a heavy quark and that of a light antiquark which forms a heavy meson. However, the hadronization time reduces to 0.6–1.2 fm/c for charm and 0.4–0.9 fm/c for bottom, depending on the heavy meson radius, in the presence of momentum correlation. Considering the interspace between quarks and antiquarks at the critical temperature, it seems that the hadronization of a heavy quark does not happen instantaneously but gradually for a considerable time, if started from the thermal distribution of quarks and antiquarks.

DOI: 10.1103/PhysRevC.94.034901

I. INTRODUCTION

Relativistic heavy-ion collisions are practically the only way to create extremely hot dense nuclear matter in laboratories. The BNL Relativistic Heavy Ion Collider (RHIC) and the CERN Large Hadron Collider (LHC), respectively, accelerate heavy nuclei and make collisions up to the energies of 200 GeV and 2.76 TeV. Such collisions produce a strong elliptic flow in semicentral collisions and induce a significant energy loss of high p_T particles, which indicate the formation of extremely dense and strongly interacting nuclear matter, so-called strongly interacting quark-gluon plasma (sQGP).

Searching for the properties of the hot dense nuclear matter is very interesting and also challenging. Heavy flavor is one of the promising probes for the properties. It has a couple of advantages over other probes. Firstly, it might have the information about the early stage of nuclear matter, because it is produced early in relativistic heavy-ion collisions. Secondly, different from the light quark, its production is well described in perturbative quantum chromodynamics (pQCD) [1].

Experimental data from RHIC and LHC show a large suppression of the nuclear modification factor and strong elliptic flow for heavy flavors [2–7]. This indicates that heavy flavors also strongly interact with the nuclear matter produced in relativistic heavy-ion collisions. There have been numerous theoretical studies to explain and describe the experimental data of heavy flavors [8–21]. Most of them take the following steps: First of all, heavy quark pairs are produced through nucleon-nucleon binary collisions. Produced heavy quarks and heavy antiquarks then interact with partonic matter in the QGP phase. At the critical temperature for the phase transition, heavy quarks and heavy antiquarks are hadronized into heavy mesons. Finally, the heavy mesons interact with other hadrons until they freeze-out.

The interactions of a heavy flavor with nuclear matter have been extensively studied in the QGP phase as well as in the hadron gas phase. In the dynamical quasiparticle model (DQPM), the heavy quark interacts with the off-shell partons whose spectral functions are determined from a fit to lattice equation-of-state (EoS) [22]. It shows that the spatial diffusion constant of the heavy quark decreases as temperature approaches the critical temperature [23]. This results are in good agreement with the recent results from lattice quantum chromodynamics (lQCD) [24]. On the other hand, the spatial diffusion constant of a heavy meson in hadron gas has been calculated by using an effective Lagrangian, and it decreases with increasing temperature [25]. Interestingly the diffusion constant of the heavy quark in QGP meets that of the heavy meson in hadron gas around the critical temperature (T_c). In other words, the diffusion constant is smoothly connected and has the minimum value around T_c. Since the spatial diffusion constant is defined as the squared displacement of a particle per unit time, the small diffusion constant at the critical temperature implies the strong interaction of a heavy quark with nuclear matter in phase transition.

Quark coalescence is one of the most popular models to describe the hadronization of partons in nuclear matter [26,27]. In this model, a pair of quark and antiquark forms a meson, and three quarks and three antiquarks, respectively, form a baryon and an antibaryon. In this process, the heavy quark gains momentum from a coalescence partner or coalescence partners. Since the spatial diffusion constant is related to the momentum transferred to a heavy quark per unit time, if the hadronization time of a heavy quark is given, the diffusion constant can be calculated.

In this study, we calculate the hadronization time of a heavy quark in nuclear matter by using the spatial diffusion constant of a heavy quark from lQCD calculations and the momentum transfer to heavy quark in the coalescence model.

This paper is organized as follows. We describe in Sec. II the spatial diffusion constant of a heavy quark, and in Sec. III the coalescence model. Combining them, our results are presented in Sec. IV, and the summary is given in Sec. V.

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II. DIFFUSION CONSTANT

The spatial diffusion constant, D_s, is defined as the squared distance per unit time which a particle travels in matter:

$$\langle x_i(t)x_j(t)\rangle = 2D_s t\delta_{ij}, \quad (1)$$

where $\langle\cdots\rangle$ is the ensemble average and the particle is located at $\mathbf{x}=0$ at $t=0$. Using the relation in the nonrelativistic limit

$$x_i(t) = \int_0^t dt' \frac{p_i(t')}{M}, \quad (2)$$

where M is the heavy quark mass, we have

$$6D_s t = \langle \mathbf{x}(t)\cdot\mathbf{x}(t)\rangle = \frac{1}{M^2}\int_0^t dt_1 \int_0^t dt_2 \langle \mathbf{p}(t_1)\cdot\mathbf{p}(t_2)\rangle. \quad (3)$$

On the other hand, the momentum as a function of time is given by random kicks in matter:

$$p_i(t) = \int_{-\infty}^t dt' e^{\eta_D(t'-t)}\xi_i(t'), \quad (4)$$

where η_D is a momentum drag coefficient and ξ_i is the random force which has the correlation

$$\langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t'), \quad (5)$$

where

$$\kappa = \frac{1}{3}\frac{d\langle(\Delta p)^2\rangle}{dt}. \quad (6)$$

Substituting Eq. (4) into Eq. (3) and assuming $t \gg \eta_D^{-1}$, we have

$$D_s = \frac{\kappa}{2\eta_D^2 M^2}. \quad (7)$$

Using the relation

$$3MT = \langle \mathbf{p}(t)\cdot\mathbf{p}(t)\rangle = \frac{3\kappa}{2\eta_D}, \quad (8)$$

where T is the temperature of matter, the spatial diffusion constant is reexpressed as

$$D_s = \frac{2T^2}{\kappa}. \quad (9)$$

From Eqs. (6) and (9), we finally have

$$\frac{d\langle(\Delta p)^2\rangle}{dt} = \frac{6T^2}{D_s}. \quad (10)$$

III. HEAVY QUARK COALESCENCE

A heavy quark is produced by pairs through a hard collision, and hadronized into a heavy meson or baryon. The hadronization in vacuum is well described by using a fragmentation function, where a heavy quark emits soft gluons to be hadronized. On the other hand, a heavy quark in nuclear matter is mostly hadronized by the coalescence with a neighboring parton [13,14,16–21]. Therefore, we focus in this study on the coalescence of a heavy quark.

The squared transition amplitude for two-particle coalescence is given by

$$|M|^2 = |\langle \mathbf{P}|\mathbf{p_1p_2}\rangle|^2 = \int d^3\mathbf{x_1}d^3\mathbf{x_2}d^3\mathbf{x}'_1 d^3\mathbf{x}'_2 \times \langle \mathbf{P}|\mathbf{x_1x_2}\rangle\langle \mathbf{x_1x_2}|\mathbf{p_1p_2}\rangle\langle \mathbf{p_1p_2}|\mathbf{x}'_1\mathbf{x}'_2\rangle\langle \mathbf{x}'_1\mathbf{x}'_2|\mathbf{P}\rangle, \quad (11)$$

where two particles with momenta $\mathbf{p_1}$ and $\mathbf{p_2}$ form one particle with the momentum $\mathbf{P}$. Since instant transition is assumed, there is no time difference between initial and final states in Eq. (11).

Defining new variables,

$$\mathbf{R_1} = \frac{\mathbf{x_1}+\mathbf{x}'_1}{2}, \quad \mathbf{R_2} = \frac{\mathbf{x_2}+\mathbf{x}'_2}{2}, \quad \mathbf{r_1} = \mathbf{x_1}-\mathbf{x}'_1, \quad \mathbf{r_2} = \mathbf{x_2}-\mathbf{x}'_2, \quad (12)$$

the scalar products in Eq. (11) are, respectively, expressed as

$$\begin{aligned}&\langle \mathbf{P}|\mathbf{x_1x_2}\rangle\langle \mathbf{x}'_1\mathbf{x}'_2|\mathbf{P}\rangle \\ &= \frac{1}{V}e^{-i\mathbf{P}\cdot\frac{\mathbf{x_1}+\mathbf{x_2}}{2}}\psi(\mathbf{x_1}-\mathbf{x_2})e^{i\mathbf{P}\cdot\frac{\mathbf{x}'_1+\mathbf{x}'_2}{2}}\psi^*(\mathbf{x}'_1-\mathbf{x}'_2) \\ &= \frac{1}{V}e^{-i\mathbf{P}\cdot\frac{\mathbf{r_1}+\mathbf{r_2}}{2}}\psi\left(\mathbf{R_1}+\mathbf{R_2}+\frac{\mathbf{r_1}}{2}+\frac{\mathbf{r_2}}{2}\right) \\ &\quad\times\psi^*\left(\mathbf{R_1}-\mathbf{R_2}+\frac{\mathbf{r_1}}{2}-\frac{\mathbf{r_2}}{2}\right),\end{aligned} \quad (13)$$

where $\psi(\mathbf{p})$ is the wave function of two particles and V the volume, and

$$\langle \mathbf{x_1x_2}|\mathbf{p_1p_2}\rangle\langle \mathbf{p_1p_2}|\mathbf{x}'_1\mathbf{x}'_2\rangle = \frac{1}{V^2}e^{i\mathbf{p_1}\cdot(\mathbf{x_1}-\mathbf{x}'_1)}e^{i\mathbf{p_2}\cdot(\mathbf{x_2}-\mathbf{x}'_2)} = \frac{1}{V^2}e^{i(\mathbf{p_1}\cdot\mathbf{r_1}+\mathbf{p_2}\cdot\mathbf{r_2})}. \quad (14)$$

Introducing new variables again,

$$\mathbf{R} = \frac{\mathbf{R_1}+\mathbf{R_2}}{2}, \quad \mathbf{r} = \frac{\mathbf{r_1}+\mathbf{r_2}}{2}, \quad \mathbf{R}' = \mathbf{R_1}-\mathbf{R_2}, \quad \mathbf{r}' = \mathbf{r_1}-\mathbf{r_2}, \quad (15)$$

the squared transition amplitude is simplified into

$$|M|^2 = \frac{(2\pi)^3}{V^2}\delta^3(\mathbf{p_1}+\mathbf{p_2}-\mathbf{P})\int \mathbf{d^3R'}\Phi(\mathbf{R}',\mathbf{k}), \quad (16)$$

where $\mathbf{k} = (\mathbf{p_1}-\mathbf{p_2})/\mathbf{2}$ and $\Phi(\mathbf{R}',\mathbf{k})$ is the Wigner function,

$$\Phi(\mathbf{R}',\mathbf{k}) = \int \mathbf{d^3r'e^{ik\cdot r'}}\psi\left(\mathbf{R}'+\frac{\mathbf{r}'}{\mathbf{2}}\right)\psi^*\left(\mathbf{R}'-\frac{\mathbf{r}'}{\mathbf{2}}\right). \quad (17)$$

By using Eq. (16), the particle yield from coalescence is given by

$$\begin{aligned}N &= V^3\int \frac{d^3\mathbf{P}}{(2\pi)^3}\frac{d^3\mathbf{p_1}}{(2\pi)^3}\frac{d^3\mathbf{p_2}}{(2\pi)^3}f_1(\mathbf{p_1})f_2(\mathbf{p_2})|M|^2 \\ &= V\int d^3\mathbf{R}'\frac{d^3\mathbf{p_1}}{(2\pi)^3}\frac{d^3\mathbf{p_2}}{(2\pi)^3}f_1(\mathbf{p_1})f_2(\mathbf{p_2})\Phi(\mathbf{R}',\mathbf{k}),\end{aligned} \quad (18)$$

and the differential density by

$$\frac{d(N/V)}{d^3\mathbf{P}} = \frac{1}{(2\pi)^6}\int d^3\mathbf{R}'d^3\mathbf{k} f_1(\mathbf{p_1})f_2(\mathbf{p_2})\Phi(\mathbf{R}',\mathbf{k}), \quad (19)$$

where $f_i(\mathbf{p}_i)$ is the distribution function of particle i. Equation (19) clearly shows that the coalescence probability is nothing but the Wigner function which depends on distances between two particles in coordinate and momentum spaces.

Using the wave function from the simple harmonic oscillator (SHO),

$$\psi(r) = \left(\frac{mk}{\pi^2}\right)^{3/8} e^{-\frac{1}{2}\sqrt{mk}r^2}, \quad (20)$$

where m and k are, respectively, the particle mass and spring constant, and $\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$, we have the Wigner function

$$\Phi(\mathbf{r},\mathbf{p}) = 8\exp\left[-\frac{r^2}{\sigma^2} - \sigma^2 p^2\right], \quad (21)$$

where $\sigma = 1/\sqrt{mk}$ and $\mathbf{p} = (\mathbf{p_1} - \mathbf{p_2})/2$.

In the case of the heavy meson which is composed of partons with asymmetric masses, the mass m in Eq. (20) is substituted by the reduced mass, $\mu = m_1m_2/(m_1+m_2)$, and σ and $\mathbf{p}$ in Eq. (21), respectively, by $\sigma = 1/\sqrt{\mu k}$ and $\mathbf{p} = (m_2\mathbf{p_1} - m_1\mathbf{p_2})/(m_1+m_2)$.

Defining the mean-squared radius of a meson as the average of the squared distance of a quark and that of an antiquark from their center of mass [28], it is expressed as

$$\begin{aligned}\langle r_M^2\rangle &= \frac{1}{2}\langle(\mathbf{R}-\mathbf{r_1})^2 + (\mathbf{R}-\mathbf{r_2})^2\rangle \\ &= \frac{1}{2}\frac{m_1^2+m_2^2}{(m_1+m_2)^2}\langle r^2\rangle = \frac{3}{4}\frac{m_1^2+m_2^2}{(m_1+m_2)^2}\sigma^2, \quad (22)\end{aligned}$$

where $\mathbf{R} = (m_2\mathbf{r_1} + m_1\mathbf{r_2})/(m_1+m_2)$. We note that the coefficient in Eq. (22) is different from the one in Ref. [20] due to the different definitions of $\mathbf{r}$ and $\mathbf{p}$.

IV. RESULTS

Figure 1 shows the spatial diffusion constant of charm as a function of temperature. Below the critical temperature (T_c) the spatial diffusion constants of the D meson are calculated by using an effective Lagrangian [25], while those of the charm quark above the critical temperature are calculated by using the dynamical quasiparticle model (DQPM), which reproduce the results from the lattice calculations [24].

The DQPM describes QCD properties in terms of resumed single-particle Greens functions. The degrees of freedom of the QGP are interpreted as being strongly interacting massive effective quasiparticles with broad spectral functions whose pole position and width are directly related to the real and imaginary parts of the related self-energy [29]. The entropy density from the dynamical quasiparticles has been fitted to lattice QCD calculations, which allows to fix entire parameters in the DQPM.

The spatial diffusion coefficients D_s are expressed in two different ways [15]: It can be obtained from $\eta_D = A/p_Q$, where A and p_Q are, respectively, the drag coefficient and heavy quark momentum [23],

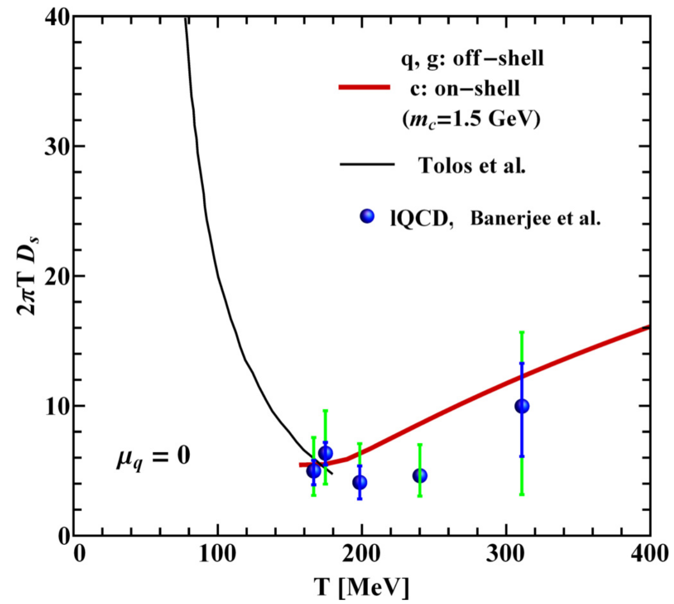

FIG. 1. The spatial diffusion constant of charm as a function of temperature. The black solid line below $T = 180$ MeV is the hadronic diffusion coefficients [25], and the red solid line above $T_c \approx 160$ MeV partonic ones [23]. The lattice QCD calculations are from Ref. [24].

$$D_s = \lim_{p_Q\to 0} T/(M_Q\eta_D), \quad (23)$$

or from the diffusion coefficient, $\kappa = \frac{1}{3}d\langle(\boldsymbol{p}_Q - \boldsymbol{p}'_Q)^2\rangle/dt$ [25],

$$D_s = \lim_{p_Q\to 0}\frac{\kappa}{2M_Q^2\eta_D^2}. \quad (24)$$

Both definitions agree with each other, if the Einstein relation is valid. Since in the case of the DQPM the deviation from the Einstein relation for small momenta p_Q is of the order 10–15%, Eq. (24) is used in Fig. 1.

η_D and κ are calculated using the partonic scattering processes. The transport coefficient $\mathcal{X}$ is defined by [23]

$$\begin{aligned}\frac{d\langle\mathcal{X}\rangle}{dt} &= \sum_{q,g}\frac{1}{(2\pi)^5 2E_Q}\int\frac{d^3q}{2E_q}f(\boldsymbol{q})\int\frac{d^3q'}{2E_{q'}}\int\frac{d^3p'_Q}{2E'_Q} \\ &\quad\times\delta^{(4)}(P_{\rm in} - P_{\rm fin})\,\mathcal{X}\,\frac{1}{g_Q g_p}|\mathcal{M}_{2,2}|^2, \quad (25)\end{aligned}$$

where p'_Q (E'_Q) is the final momentum (energy) of a heavy quark with the initial energy E_Q,q (E_q) and q' ($E_{q'}$) the initial and final momenta (energies) of colliding parton whose thermal distribution is given by $f(\boldsymbol{q})$, $|\mathcal{M}_{2,2}|^2$ the transition matrix-element squared for $2\to 2$ scattering, and g_Q and g_p the degeneracy factors of a heavy quark and colliding parton. We note that $g_Q = 6$, and $g_p = 16$ for gluons and 6 for light quarks. We neglect in Eq. (25) the Pauli blocking and Bose enhancement factors, $1 \pm f(\boldsymbol{p}')$, in the final states, since in our case the occupation numbers $f(\boldsymbol{p}')$ are rather small in the temperature range of interest due to the rather massive degrees of freedom with pole masses larger than twice the temperature. Employing $\mathcal{X} = E - E'$ and $\mathcal{X} = \boldsymbol{p}_Q - \boldsymbol{p}'_Q$, we can calculate, respectively, the energy loss, $d\langle E\rangle/dt$, and the drag coefficient, $d\langle \boldsymbol{p}_Q\rangle/dt = A(p_Q,T)$.

Figure 1 shows that the diffusion constant of the D meson is smoothly connected with that of the charm quark around the critical temperature and it has the minimum value there. Since the spatial diffusion constant is defined as the squared displacement of a particle per unit time, a small diffusion constant implies a strong interaction with matter. In other words, a charm strongly interacts with matter near the critical temperature. It is clearly shown as the strong coupling which increases rapidly near the critical temperature in the DQPM [30]. There is a simple reason for the large strong coupling near the critical temperature: All partons must be hadronized without exception. The calculations of the diffusion constant of the D meson and those of a charm quark in Fig. 1 do not have a hadronization process. However, the momentum transfer to charm per unit time, which is related to the diffusion constant of the D meson in the hadronic side and that of the charm quark in the partonic side, must be smoothly connected by hadronization, considering the phase transition is a crossover in baryon-free nuclear matter. Therefore, though the hadronization at the critical temperature is a completely different process from the (quasi)elastic scattering near the critical temperature, the hadronization time and the diffusion time can be related to each other through the momentum transfer to charm per unit time, as will be shown below.

In order to simplify the situation, we prepare a box of which temperature is slightly above T_c. Then the temperature suddenly drops slightly below T_c. In this case most interactions will be hadronization. In fact, the temperature of nuclear matter produced in relativistic heavy-ion collisions drops slowly near T_c, because it takes time to change phase from QGP to hadron gas. In this sense our study provides the minimum time for a heavy quark to be hadronized.

In coalescence the momentum transfer to heavy quark is nothing but the momentum of absorbed antiquark. Then the average of momentum transfer squared to heavy quark is given by

$$\langle(\Delta p_{\rm coal.})^2\rangle = \frac{\int d^3\mathbf{k}\int d^3\mathbf{q}\, q^2 f_{\bar q}(q) f_Q(k)\phi(\mathbf{k},\mathbf{q},r_M)}{\int d^3\mathbf{k}\int d^3\mathbf{q}\, f_{\bar q}(q) f_Q(k)\phi(\mathbf{k},\mathbf{q},r_M)}, \quad (26)$$

where $f_Q(k)$ and $f_{\bar q}(q)$ are, respectively, the Fermi-Dirac distribution functions of the heavy quark and of light antiquark at T_c, and $\phi(\mathbf{k},\mathbf{p},r_M)$ the momentum part of the coalescence probability in Eq. (21): $\phi(\mathbf{k},\mathbf{q},r_M) \sim e^{-\sigma^2 p^2}$ with $\mathbf{p} = (m_{\bar q}\mathbf{k} - m_Q\mathbf{q})/(m_{\bar q} + m_Q)$ in the center-of-mass frame of $\mathbf{k}$ and $\mathbf{q}$, and $m_{\bar q}$ and m_Q being, respectively, the masses of the light antiquark and heavy quark. In other words, we assume the homogenous distribution of particles in coordinate space.

Now we apply Eq. (10) into the hadronization process. Since the left-hand side of Eq. (10) is contributed from all kinds of interactions, we separate it into the contribution from coalescence and that from others, for example, from elastic scattering:

$$\frac{d\langle(\Delta p)^2\rangle}{dt} = \frac{d\langle|\Delta\mathbf{p}_{\rm coal.} + \Delta\mathbf{p}_{\rm others}|^2\rangle}{dt} = \frac{d\langle(\Delta p_{\rm coal.})^2\rangle}{dt} + \frac{d\langle(\Delta p_{\rm others})^2\rangle}{dt}, \quad (27)$$

where $\langle\Delta\mathbf{p}_{\rm coal.}\cdot\Delta\mathbf{p}_{\rm others}\rangle = 0$, because there is no correlation between $\Delta\mathbf{p}_{\rm coal.}$ and $\Delta\mathbf{p}_{\rm others}$. Substituting Eq. (27) into Eq. (10), the diffusion constant near the critical temperature is expressed by

$$D_s = 6T_c^2 \frac{t}{\langle(\Delta p_{\rm coal.})^2 + (\Delta p_{\rm others})^2\rangle}, \quad (28)$$

where t is the hadronization time and Δp the momentum transfer during the hadronization. Equation (28) clearly shows that coalescence cannot happen instantaneously ($t \approx 0$), unless the diffusion constant vanishes at T_c. From the equation we can obtain the minimum time required for heavy quark coalescence, $t_{\rm min}$:

$$t = \frac{D_s\langle(\Delta p_{\rm coal.})^2 + (\Delta p_{\rm others})^2\rangle}{6T_c^2} \geqslant \frac{D_s\langle(\Delta p_{\rm coal.})^2}{6T_c^2} \equiv t_{\rm min}. \quad (29)$$

Since hadronization would be the most dominant process in the nuclear matter near T_c, $t \approx t_{\rm min}$ would not be a bad approximation.

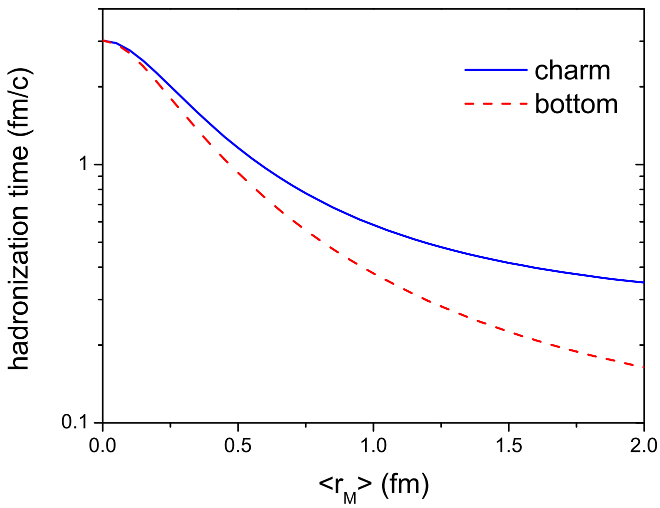

FIG. 2. Hadronization times of charm (solid) and bottom (dashed) quarks as functions of heavy meson radius for $2\pi T_c D_s = 6$ [24].

Figure 2 shows the hadronization times of charm and bottom quarks as functions of heavy-meson radius for $2\pi T_c D_s = 6$ [24]. Heavy quark mass m_Q is taken to be 1.5 GeV for charm and 4.5 GeV for bottom, and $m_{\bar q}$ and T_c are, respectively, 0.3 GeV and 160 MeV.

The hadronization time is about 3 fm/c for the vanishing radius of a heavy meson. From Eqs. (21) and (22), the vanishing radius implies that a heavy quark does coalesce with the nearest light antiquark in coordinate space. In this case, there is no correlation between the momentum of the heavy quark and that of the light antiquark, and the momentum transfer due to coalescence is largest. Since the momentum transfer per unit time is fixed by D_s from the lattice calculations, the hadronization time must be longest from Eq. (29).

As the coalescence radius increases, a small relative momentum between the heavy quark and light antiquark is favored for coalescence, and the momentum transfer due to the absorption of the antiquark becomes small. It reduces the hadronization time of the heavy quark as shown in Fig. 2. We can also see that the hadronization time of the bottom quark is smaller than that of the charm quark as the coalescence radius is large. The large coalescence radius means that only the heavy quark and light antiquark which almost comove can be combined in coalescence. Since the thermal motion of the charm quark is larger than that of the bottom quark at the critical temperature, the momentum of the light antiquark which is combined with the charm quark is larger and the momentum transfer to charm quark is also larger.

Assuming that the radius of the heavy meson is 0.5–1.0 fm, the hadronization time of the charm quark is 0.6–1.2 fm$/c$ and that of the bottom quark 0.4–0.9 fm$/c$. They are of reasonable time scale and support the results on the spatial diffusion constant from lattice calculations. Since we neglect the elastic scattering which might give additional momentum transfer to the heavy quark, our estimate on the hadronization time of the heavy quark is a lower limit.

The number density of thequark and antiquark at the critical temperature is about 1 fm^{-3}, and the interspace between them 1 fm. Considering the typical size of hadrons, it is highly likely that a heavy quark is combined with the nearest antiquark in coordinate space for hadronization. In this case, the hadronization time of the heavy quark is considerably long and it can be interpreted as follows: Above the critical temperature, quarks and antiquarks have thermal motion, which is random and does not have any correlation between the momentum of the quark and that of the antiquark. As energy density decreases, the quark and antiquark begin to cluster, and the relative momentum between quark and antiquark becomes small. In this environment, the heavy quark needs a shorter time for hadronization as discussed and shown in Fig. 2. This interpretation suggests that the hadronization of the heavy quark does not happen instantaneously, rather requires a considerable time, if started from the thermal distribution of quarks and antiquarks.

V. SUMMARY

Heavy flavor is one of the promising probes for the properties of extremely hot dense nuclear matter created in relativistic heavy-ion collisions. Since it is massive, heavy flavor is produced mainly through initial nucleon-nucleon binary collisions and exists in the very early stages of relativistic heavy-ion collisions. After production, the heavy quark interacts with partons in the QGP phase. The interactions change the energy-momentum of heavy quark, and it is shown as a highly suppressed nuclear modification factor at large transverse momentum and large elliptic flow in semicentral heavy-ion collisions.

From lattice QCD calculations and the DQPM, the spatial diffusion constant of heavy quark decreases with decreasing temperature in the QGP phase. On the other hand, the spatial diffusion constant of the heavy meson from effective Lagrangian calculations increases with increasing temperature in the hadron gas phase. Both diffusion constants meet each other around the critical temperature for the phase transition and have minimum value there. Since the spatial diffusion constant is defined as the squared displacement per unit time, a small diffusion constant near critical temperature implies that strong interactions happen in phase transition. It is reasonable in respect that without exception all partons should be hadronized in the phase transition. In other words, the small diffusion constant at critical temperature is mostly attributed to hadronization.

The coalescence model has widely been used in describing the hadronization of partons. In this model, a heavy quark is hadronized into a heavy meson by absorbing a light antiquark nearby in coordinate and momentum spaces. The absorption transfers the momentum of the antiquark to the heavy quark. If the radius of the heavy meson is small, the heavy quark favors the antiquark near coordinate space as its coalescence partner. It allows the coalescence with the antiquark whose momentum is rather far from that of the heavy quark. In this case, the momentum transfer to heavy quark due to hadronization is large. On the contrary, the momentum transfer is small for the large radius of the heavy meson.

Since the spatial diffusion constant is proportional to the squared momentum transfer per unit time, if the momentum transfer and diffusion constant are given, the time for the momentum transfer can be calculated. We have calculated the hadronization time of the heavy quark by using the spatial diffusion constant from lattice QCD and DQPM calculations, and the momentum transfer from the coalescence model. If the radius of the heavy meson is extremely small, in other words, a heavy quark does coalesce with any nearest antiquark in coordinate space, the hadronization time is as long as 3 fm$/c$ for $2\pi T_c D_s = 6$. Assuming that the heavy meson radius is 0.5–1.0 fm, the hadronization time is 0.6–1.2 fm$/c$ for charm and 0.4–0.9 fm$/c$ for bottom, where the small (large) radius corresponds to the long (short) hadronization time. The longer hadronization time for the smaller radius of the heavy meson is not so intuitive. It can be understood that the small radius in coordinate space allows large momentum transfer to the heavy quark for hadronization in the coalescence model.

In principle, there could be additional interactions such as elastic scattering other than hadronization near the critical temperature. Considering that, our estimate on the hadronization time of the heavy quark is a lower limit.

Finally, the consideration of interspace between the quark and antiquark at the critical temperature favors the coalescence of the heavy quark with the nearest antiquark in coordinate space, and it suggests the gradual progress of heavy quark hadronization for a couple of fermi of time.

ACKNOWLEDGMENTS

This work was supported by DFG under contract no. BR 4000/3-1 and by the LOEWE center “HIC for FAIR”. The computational resources have been provided by the LOEWE-CSC.

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