Analysis of inelastic pion scattering from the low-lying 2^+ states in ^{14}C

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I have analyzed data for inelastic π^+ and π^- scatterings to the two lowest 2^+ states of ^{14}C to determine proton and neutron matrix elements. In a two-state model, I have then derived one-body transition amplitudes for p shell and $(sd)^2$ transitions. Data are found to be consistent with equal mixing of the 2^+ states but with slightly nonstandard ratios of effective charges. Using effective charges in common usage, mixing is found to be only slightly different from equal—55% of the $(sd)^2$ component in the lower 2^+ state.

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I. INTRODUCTION

For many years, the conventional picture of the first two 2^+ states of ^{14}C has been that they are approximately equal admixtures of two basis states—a pure p-shell configuration and a configuration having two neutrons in the sd shell. Inelastic pion scattering is quite sensitive to proton and neutron excitations separately so that combining data for both π^+ and π^- scatterings usually allows both the proton and the neutron transition matrix elements to be determined. Such an experiment was reported by Hayes *et al.* [1] with a very thorough analysis. Those data contained two quite unexpected results: (1) For the 7.01-MeV state, π^+ and π^- cross sections were virtually identical; (2) for the 8.32-MeV state, both cross sections were very small with that for π^- nearly vanishing. Cross sections at the peak of the angular distributions (about 35°) are listed in Table I. Some of the calculations were able to reproduce the results for the lower state, but all calculations for the upper state were unsatisfactory. Here, I have revisited this problem.

II. ANALYSIS

At kinetic energies near 165 MeV, distorted-wave impulse-approximation calculations have demonstrated that $E2$ angular distributions are well described by the relations,

$$\sigma^+(T_\pi,\theta) = K^+(T_\pi,\theta)(2.7M_p + M_n)^2 \quad \text{and}$$
$$\sigma^-(T_\pi,\theta) = K^-(T_\pi,\theta)(M_p + 2.7M_n)^2,$$

where M_p and M_n are proton and neutron transition matrix elements normalized such that $B(E2; J_i \to J_f) = M_p^2/(2J_i+1)$. Ignoring slight differences in K^+ and K^-, and defining $M' = K^{1/2}M$, we have

$$\sigma^+ = (2.7M'_p + M'_n)^2 \quad \text{and} \quad \sigma^- = (M'_p + 2.7M'_n)^2,$$

where the cross sections are understood to be taken at the maximum of the angular distributions. The units of the M's are thus (mb/sr)$^{1/2}$. As stated by Ref. [1], the near equality of π^+ and π^- cross sections for the 7.01-MeV state requires that $M'_p = M'_n$ for that state. For the 8.32-MeV state, proton and neutron contributions interfere destructively. In what follows, I have taken $M'_n(8.32)$ to be positive. The values of both matrix elements will depend on what I use for the π^- cross section for this state. It is so small that I first perform the analysis with this cross section set to zero. I return to this point later. Results are listed in Table II. They clearly indicate that both p-shell and sd-shell transitions are important as noted in Ref. [1].

Even with the admixtures in the 2^+ states, the $(sd)^2$ components would not contribute if the ^{14}C ground state (g.s.) had no $(sd)^2$ components. Such an admixture is known to exist, i.e.,

$$^{14}\mathrm{C}\,(\mathrm{g.s.}) = A\ {}^{14}\mathrm{C}_{1p} + \varepsilon\ {}^{12}\mathrm{C}_{1p} \times (sd)^2{}_0\,.$$

From an analysis of the ^{12}C(t,p) reaction to the g.s. and excited 0^+ state, a value of $\varepsilon^2 = 0.12(3)$ was derived [2]. The Hayes analysis [1] had a smaller value—$\varepsilon^2 \sim 0.08$. Others have had similar estimates [3]. Elsewhere, I have suggested another method to measure this impurity [4]. If we write

$$^{14}\mathrm{C}\,(7.01) = \alpha\ {}^{14}\mathrm{C}_{1p}(2^+) + \beta\ {}^{12}\mathrm{C}_{1p}\,(\mathrm{g.s.}) \times (sd)^2{}_2\,,$$

and

$$^{14}\mathrm{C}\,(8.32) = -\beta\ {}^{14}\mathrm{C}_{1p}(2^+) + \alpha\ {}^{12}\mathrm{C}_{1p}\,(\mathrm{g.s.}) \times (sd)^2{}_2\,,$$

it is simple to see how both p-shell and $(sd)^2$ components will contribute. The 2^+ shell-model wave functions of Ref. [1] might make it appear that three-state mixing is present. But, calculations with those wave functions did not produce agreement with the pion data. The fact that the third 2^+ state is mostly of $(sd)^2$ character does not automatically mean that it should be included in the mixing. The shell model has already included its mixing with the $(sd)^2$ component in the lower two states—a component to which it is orthogonal. Furthermore, the shell-model 0^+ wave functions might also appear to suggest three-state mixing, whereas two-state mixing works well there. The evidence is that the third 0^+ state does not mix with the lower two. The nonparticipation of the next (third) 0^+ state in ^{14}C can be seen clearly [5] in the fact that it [the second $(sd)^2$ 0^+ state] behaves nearly identically to the second 0^+ state in ^{16}C, which has no p-shell state. In my model, the third 2^+ state [the second $(sd)^2$ one] is orthogonal to the $(sd)^2$ configuration that is present in the first two 2^+ states. Finally, there is no evidence in the pion data for a third 2^+ state with measurable strength.

First, I investigate whether the pion data are consistent with values $\alpha^2 = \beta^2 = 0.50$.

The matrix elements are conventionally written as

$$M_p = e_pA_p + e_nA_n, \qquad M_n = e_nA_p + e_pA_n,$$

TABLE I. Cross sections for π^+ and π^- inelastic scattering to the first two 2^+ states of ^{14}C [1].

E_x(MeV)	Cross section (mb/sr)[a]	
	π^+	π^-
7.01	1.5	1.5
8.32	0.18	Very small

[a]At the peak of the angular distributions (about 35°).

where e_p and e_n are effective charges for protons and neutrons, respectively (not necessarily the same for p and sd shells). The A's are bare one-body transition amplitudes from a structure calculation. Here, I use the same expressions for relating M' to A'. For the A's, the p-shell transition will involve only protons (the neutron p shell is filled for the first component above.) For the $(sd)^2$ transition, only neutrons will contribute (there are no sd-shell protons.) With the wave functions above, we have $A'_p = A\alpha\tilde{\mathbf{A}}'_p$ for the 7.01-MeV state and $A'_p = -A\beta\tilde{\mathbf{A}}'_p$ for 8.32, where $\tilde{\mathbf{A}}'_p$ is the microscopic transition amplitude for the pure p-shell $0^+ \to 2^+$ transition. For A'_n, the results are $A'_n(7.01) = \varepsilon\beta\tilde{\mathbf{A}}'_n$ and $A'_n(8.32) = \varepsilon\alpha\tilde{\mathbf{A}}'_n$ for the two states. Here, $\tilde{\mathbf{A}}'_n$ is the transition amplitude for a pure $(sd)^2$ transition from the lowest 0^+ to the lowest 2^+ state. Of course, even with A'_p involving only the p shell and A'_n involving only the sd shell, both will still contribute to the M's.

The approach now is to determine whether the data are consistent with this model with $\alpha^2 = \beta^2 = 0.50$. For this purpose, we start with the values of M' in Table II and evaluate the $\tilde{\mathbf{A}}$'s. It will turn out that two results emerge for each $\tilde{\mathbf{A}}$—one multiplied by e_p, the other multiplied by e_n. Making them agree need not coincide with the ratio of effective charges in common usage. Results of this procedure are listed in Table III. Recall that we are still using an equal mixture in the two 2^+ states and $\sigma^- = 0$ for the 8.32-MeV state. For the p shell, one set of effective charges has $e_p = 1.20e$ and $e_n = 0.43e$ [6]. For the $(sd)^2$ space, $e_p = 1.5e$ and $e_n = 0.5e$ have met with success [7]. It can be noted from the table that neither set of values gives the proper e_p/e_n ratio. So, which condition should we relax—the equal 2^+ mixing or the zero π^- cross section for the upper state? It turns out that a value of $\sigma^-(8.32) = 0.010$ mb/sr (easily consistent with the data [1]) produces results that agree with the p-shell effective charges but not with those for the sd shell.

Alternatively, we can relax the assumption of equal mixing, input the affective charges from the beginning, and investigate whether a reasonable set of mixing amplitudes emerges. The answer is yes. In fact, with effective charges given, the data allow a determination of the mixing and of the pure amplitudes $\tilde{\mathbf{A}}$ for p and sd and provide a prediction for the π^- cross section for the 8.32-MeV state. We have four equations in four unknowns: A'_p, A'_n, α, and $\sigma^-(8.32)$. Combining the equations in pairs and eliminating A'_p produces two independent equations for A'_n in terms of α and σ^-. Equating them provides a relationship between α and σ^-. Repeating the procedure, but eliminating A'_n gives another (independent) equation relating α and σ^-. Solving these two equations yields $\alpha^2 = 0.45$ and $\sigma^- = 0.016$ mb/sr. It might be surprising that the lower state has more $(sd)^2$ intensity, but the predicted energy [5] of the $(sd)^2$ 2^+ state is nearly 1 MeV below that for the p-shell 2^+ [8]. The final results are listed in Table IV.

If I use the current effective charges and derived values of $\tilde{\mathbf{A}}'_p$ and $\tilde{\mathbf{A}}'_n$ to fit the ratio of observed $B(E2)$ values for the 2^+ states {1.8(3) for 7.01 and 0.39(15) for 8.32 (both in Weisskopf units) ([9] as quoted in Ref. [1])}, the result for β/α is 1.26(29), to be compared with the ratio in Table IV of 1.1. Thus, the $B(E2)$ values are in agreement with the current pion analysis, but they do not set a meaningful constraint on the fit.

On the other hand, if I compute the expected $B(E2)$ ratio from the values of M' in Table II (recall that these were derived assuming equal mixing), the result is $B(E2;8.32)/B(E2;7.01) = 0.297$, compared to the experimental ratio of 0.219(85). Clearly, a better value for the 8.32-MeV state is desirable for a meaningful test of the $B(E2)$'s.

Of course, a two-state mixing model is always an approximation. The question is: Does it contain most of the important physics? In the present case, this simple model has been shown

TABLE II. Proton and neutron matrix elements [in (mb/sr)$^{1/2}$] for the first two 2^+ states of ^{14}C.

E_x(MeV)	Matrix element	
	M'_p	M'_n
7.01	0.33	0.33
8.32[a]	−0.18	0.068

[a]Assuming the π^- cross section is zero.

TABLE III. Pure single-shell amplitudes [(mb/sr)$^{1/2}$] assuming equal mixing and $\sigma^-(8.32) = 0$.

Quantity	Value	
	p shell	$(sd)^2$
$e_p\tilde{\mathbf{A}}'_p$	0.39	
$e_n\tilde{\mathbf{A}}'_p$	0.20	
$e_p\tilde{\mathbf{A}}'_n$		0.80
$e_n\tilde{\mathbf{A}}'_n$		0.30

TABLE IV. Final results with $e_p = 1.20e$ and $e_n = 0.43e$ in the p shell [6]; $e_p = 1.5e$ and $e_n = 0.50e$ in the sd shell [7].

Quantity	Value
$e\tilde{\mathbf{A}}'_p$ p shell	0.33(mb/sr)$^{1/2}$
$e\tilde{\mathbf{A}}'_n$ sd shell	0.62(mb/sr)$^{1/2}$
α^2	0.45
β^2	0.55
$\sigma^-(8.32)$	0.016 mb/sr
M'_p (8.32)	−0.20(mb/sr)$^{1/2}$
M'_n(8.32)	0.12(mb/sr)$^{1/2}$

to agree with the pion data and to agree with the $B(E2)$'s, even though the latter are not precise enough for a stringent test. It might be interesting to investigate the larger problem of the 2^+ states in ^{14}O and the 2^+, $T = 1$ states in ^{14}N. Energies, $B(E2)$'s, and proton decay widths should all be fruitful areas of interest. The 2^+ splitting is 1.31 MeV in ^{14}C and 1.178(14) MeV in ^{14}O. The nearly equal spacing of the first two 2^+ states in ^{14}C and ^{14}O might appear as a challenge for calculations, but in one simple procedure of computing mirror energy differences (see, for example, Ref. [10]), wave-function admixtures are assumed to be equal in mirror nuclei so that equal spacing is consistent with approximately equal mixing in both nuclei.

III. CONCLUSIONS

I have analyzed data [1] for inelastic π^+ and π^- scatterings to the first two 2^+ states of ^{14}C using a two-state mixing model for them and for the g.s. of ^{14}C. The outstanding features of the data are large and virtually equal π^+ and π^- cross sections for the 7.01-MeV state but small cross sections for the 8.32-MeV state with the π^- yield nearly vanishing. Assuming zero for the latter (and then letting the model predict it), I have extracted proton and neutron matrix elements. These were then used to derive the underlying microscopic one-body transition amplitudes, first assuming equal mixing of the 2^+ states and then relaxing this condition. I used a previously determined two-component wave function for ^{14}C(g.s.). The data are found to be consistent with equal 2^+ mixing, but extracted ratios of effective charges e_p/e_n are somewhat different from those in common usage. Allowing the 2^+ mixing to be determined from the data and using established effective charges, agreement is obtained—with nearly equal mixing, namely, 55% of the $(sd)^2$ component in the lower 2^+ state. One-body transition amplitudes are obtained for the pure 0^+ to 2^+ p-shell transition and for the lowest $(sd)^2$ 0^+ to 2^+. The model prediction for the π^- cross section for the 8.32-MeV state is 0.016 mb/sr, consistent with the very small observed yield.

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