PHYSICAL REVIEW C **94**, 024339 (2016)

Single-particle $s_{1/2}$ and $d_{5/2}$ states in ^{15}N and ^{15}O

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(Received 22 June 2016; published 25 August 2016)

For states in ^{15}N and ^{15}O that have large single-particle strengths for $2s_{1/2}$ and $1d_{5/2}$, I have computed energy differences for mirror states and widths for unbound states in ^{15}O. I consider both $T = 1$ and $T = 0$ cores. Calculated and experimental energies agree well. Results indicate that actual $\ell = 0$ spectroscopic factors for two $3/2^+$ states are significantly smaller than those recently reported.

DOI: 10.1103/PhysRevC.94.024339

I. INTRODUCTION

In ^{13}C, the $2s_{1/2}$ and $1d_{5/2}$ single-particle (sp) strength is primarily contained in the $1/2^+$ and $5/2^+$ states at excitation energies of 3.09 and 3.85 MeV, respectively [1]. In ^{14}C, coupling these sp states to the $1/2^-$ ground state (g.s.) of ^{13}C produces doublets with $J^\pi = 0^-$, 1^- ($s_{1/2}$) and 2^-, 3^- ($d_{5/2}$). These states are well known [1], and their properties agree well with this description [2]. Properties of their analogs in ^{14}N and of their mirrors in ^{14}O generally agree with expectations of this simple model, but some discrepancies remain [3]. Here, I address these sp states in ^{15}N and ^{15}O. Some mirror assignments and the dependence of Coulomb energies on the configuration have been discussed long ago [4]. Shell-model wave functions for $1/2^+$, $3/2^+$, and $5/2^+$ states have been published [5]. I have been aided by a recent thorough examination of the reaction ^{14}N(d,p) [6] and by an earlier study of the ^{14}N(^{3}He, d) reaction [7]. Low-lying $T = 1/2$ states in ^{15}N can have sp parentages from both $T = 0$ and $T = 1$ in $A = 14$. Mertin *et al.* [6] did not mention the possibility of a $T = 1$ core. So, I address them first. But first, I address a problem with the absolute scales in the two transfer experiments.

II. ABSOLUTE SPECTROSCOPIC FACTORS

As mentioned in Ref. [6], the spectroscopic factors from the recent ^{14}N(d,p) experiment are systematically larger than those from the reaction ^{14}N(^{3}He, d) to the mirror states in ^{15}O [7]. Of course, it is expected that spectroscopic factors for mirror states should be equal—especially if S is large. The comparison is plotted in Fig. 1. Best-fit linear dependences provide $S(^{15}\mathrm{O})/S(^{15}\mathrm{N}) = 0.65$ and 0.63 for $\ell = 0$ and 2, respectively. Of course, this deviation of the ratio from unity does not identify which of the absolute scales is incorrect. In this connection, I note that, for states with large S, the (d,p) values are also significantly larger than those from a shell-model calculation [6]. The average ratio is $S_{\mathrm{th}}/S_{\mathrm{exp}} = 0.68$ for strong s and d transfers. This is remarkably close to the ratio of S_p to S_n mentioned above. This near equality of ratios may be a coincidence, but it might signify that the absolute cross-sectional scale of Mertin *et al.* [6] is too large by a factor of about 1.5.

III. CALCULATIONS AND RESULTS

A. $T = 1$ cores

Because of isospin Clebsch-Gordan (CG) coefficients, the $T = 1/2$ states of ^{15}N with a $T = 1$ core have the structures (2/3) ^{14}C$+p$ and (1/3) ^{14}N$(T = 1) + n$. Mirror states in ^{15}O are (1/3) ^{14}N$(T = 1) + p$ and (2/3) ^{14}O$+n$. The lowest $1/2^+$ and $5/2^+$ states of ^{15}N at $E_x = 5.299$ and 5.270 MeV, respectively [1] have been shown to have large spectroscopic factors in the reaction ^{14}C(^{3}He, d) [8], and hence, they are predominantly of this structure. Their spectroscopic factors in ^{14}N(d,p) [6] are significantly smaller as expected. The aim here is to compute the energies of the mirrors of these states in ^{15}O.

I have used a Woods-Saxon potential well with parameters r_0, a, $r_{0c} = 1.26$, 0.60, 1.40 fm. For ^{14}C$+p$ states in ^{15}N, I adjust the potential depth to reproduce the ^{14}C$+p$ separation energy, and I then use that same potential depth to compute the ^{14}O$+n$ separation energy in ^{15}O. For ^{14}N$(T = 1) + n$ states in ^{15}N, I adjust the potential depth to reproduce the ^{14}N$(T = 1) + n$ separation energy in ^{15}N, and I then use that same potential depth to compute the ^{14}N$(T = 1) + p$ separation energy in ^{15}O. I then weight the two sets of results with the appropriate CGs to obtain energies in ^{15}O. Results for these two states are listed in Table I. The two S's for (^{3}He, d) are the "renormalized" ones from Ref. [8] where the authors reduced their initial results to (incorrectly) make the p-shell sum of $S = 1$. These authors label their quantities S, but they appear to be C^2S—the shell-model p-shell results certainly are. Neither of the values in Table I is near unity, but the same reaction on a ^{12}C target [8] gave only $S = 0.23$ for the $1/2^+$ state of ^{13}N, which has long been known as a $2s_{1/2}$ sp state [1]. Also listed in Table I are ^{15}O energies computed assuming the core for these two states is the $T = 0$ g.s. of ^{14}N. I then give the energy obtained by weighting with the $T = 1$ and $T = 0$ spectroscopic factors. Agreement between calculated and experimental energies in ^{15}O is good but not as good as usually encountered in such calculations. Neglected components in the wave functions have intensities

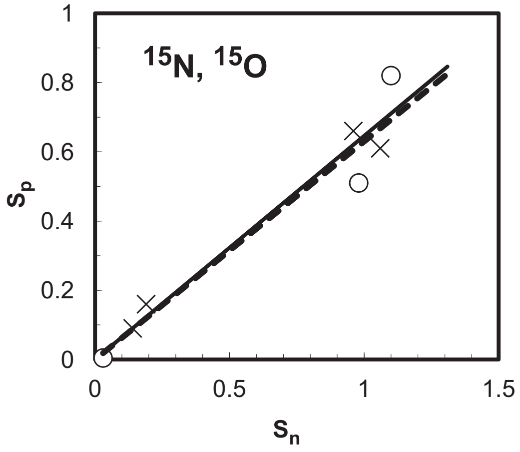

FIG. 1. Proton spectroscopic factors for $\ell = 0$ (circles) and $\ell = 2$ (crosses) from the reaction ^{14}N(^{3}He, d) ^{15}O [7] are plotted vs neutron S's from ^{14}N(d,p) ^{15}N [6] for mirror pairs. The straight lines correspond to $S_p/S_n = 0.65$ (solid lines, $\ell = 0$) and 0.63 (dashed lines, $\ell = 2$).

of 4%–6% [5] and could easily be large enough to account for the differences.

B. $T = 0$ cores

I turn now to states of ^{15}N that have a ^{14}N(g.s.) core. The $s_{1/2}$ coupling gives rise to states with $J^\pi = 1/2^+$ and $3/2^+$, whereas the $d_{5/2}$ coupling produces three states having $J^\pi = 3/2^+, 5/2^+$, and $7/2^+$. By far the majority of the $\ell = 0$ strength [6] lies in the $3/2^+$ and $1/2^+$ states at $E_x = 7.300$ and 8.312 MeV, respectively. Note that the lowest $1/2^+$ state at 5.27 MeV has less than 3% of the $s_{1/2}$ strength. For $5/2^+$ and $7/2^+$, most of the $d_{5/2}$ strength is concentrated in single states—at 7.155 and 7.567 MeV, respectively. These four states are listed in Table II. However, the $3/2^+$ strength is split among at least three states at $E_x = 7.300$, 8.571, and 10.066 MeV. This splitting is demonstrated in Table III. The $d_{5/2}S$-weighted centroid of these energies is 9.324 MeV, and the sum of the $d_{5/2}S$'s is 0.97.

To obtain the s and d centroids in ^{15}N, I weight the energies by a factor $(2J + 1)$ for s and d separately. For d, I use the $3/2^+$ centroid mentioned above. For the other J states, no further weighting is used because most of the strength is concentrated in a single state for each J. The results are $E(2s_{1/2}) = 7.637$ MeV, $E(1d_{5/2}) = 7.820$. [I have assumed all the $\ell = 2$ strengths in these states correspond to $d_{5/2}$ because the $d_{3/2}$ strength will lie much higher. This is borne out in the shell-model calculations.] Thus, the s and d sp energies are approximately degenerate as concluded by Ref. [6]. My energies are lower than theirs because I have considered only the lowest strongest states and they correctly included weaker states at higher excitation. However, my energies seem more appropriate for comparing with energies of the lowest $1/2^+$ and $5/2^+$ states in ^{17}O and ^{13}C. To use the centroids of Ref. [6] in such a comparison would require the inclusion of weaker high-lying states in ^{17}O and ^{13}C.

For the four states with the largest spectroscopic factors, the computed energies in ^{15}O agree with the experimental ones to within 30–60 keV. It is encouraging that these computed energies are all lower than the experimental ones because any small neglected component in the wave functions is likely to have little or no excitation-energy shift from ^{15}N to ^{15}O.

The three $3/2^+$ states that share the $d_{5/2}$ strength deserve special attention. As mentioned above, the $s_{1/2}$ strength of the 7.3-MeV state is as expected. However, as we will see now, the $\ell = 0$ strengths of the other two $3/2^+$ states are probably in error. The 8.57-MeV state is reported to have $S(\ell = 0) = 0.07(2)$ [6]. However, the mirror state at 8.284 MeV in ^{15}O has a proton width of only 3.6(7) keV [1]. The $\ell = 0$ sp width for an ^{15}O state at this excitation energy is 440 keV so that $S = \Gamma_{\text{exp}}/\Gamma_{\text{sp}}$ for $\ell = 0$ turns out to be at most 0.008(2)—about an order of magnitude smaller than reported for the ^{15}N state. If I use the reported $\ell = 2$ spectroscopic factor of 0.13(2) and the $\ell = 2$ sp width of 6.6 keV, the $\ell = 2$ component of the experimental width is 0.86(13) keV, leaving 2.7(7) keV for the $\ell = 0$ width—resulting in $S(\ell = 0) = 0.006(2)$. I note that the shell-model $S(\ell = 0)$ is reported as 0.00 [6]. I also note that the $\ell = 0 + 2$ curve given for this state provides a bad fit to its angular distribution.

The $3/2^+$ state at 10.066 MeV in ^{15}N has $S(\ell = 0) = 0.32(4)$, $S(\ell = 2) = 0.65(2)$ [6], but the experimental width of the mirror at 9.484 MeV in ^{15}O is ~200 keV [1]. Here the sp widths are 110 and about 2100 keV for d and s, respectively.

TABLE I. Properties of single-particle states in ^{15}N and ^{15}O with predominately $T = 1$ cores.

^{15}N					^{15}O	
E_x (MeV)[a]	J^{π} [a]	ℓ	S		E_x (MeV)	
			^{14}C(^{3}He, d)[b]	^{14}N(d,p)[c]	Expt.[a]	Calc.[d]
5.299	$1/2^+$	0	0.25	0.03(2)	5.183	5.388,4.724,5.317
5.270	$5/2^+$	2	0.45	0.14(1)	5.241	5.311,4.946,5.224

[a] Reference [1].
[b] Reference [8]. They also have $S = 0.23$ for the $1/2^+$ state of ^{13}N.
[c] Reference [6].
[d] The first number is for the $T = 1$ core; the second is for the $T = 0$ core; the third is the average of the first two, weighted by spectroscopic factors.

TABLE II. Properties of positive-parity mirror states in ^{15}N and ^{15}O with large S.

E_x (^{15}N) (MeV)[a]	J^{π} [a]	ℓ	S[b]	E_x (^{15}O) (MeV)			S_{th} [b]		
				Calc.[c]	Mixed[c]	Expt.[a,b]	s	$d_{3/2}$	$d_{5/2}$
7.155	$5/2^+$	2	1.06(2)	6.829	6.829	6.859		0.05	0.65
7.301	$3/2^+$	0	0.98(3)	6.686	6.733	6.793	0.72		
7.301	$3/2^+$	2	0.19(3)	6.974				0.01	0.07
7.567	$7/2^+$	2	0.96(2)	7.240	7.240	7.276			0.73
8.313	$1/2^+$	0	1.10(5)	7.458	7.493	7.557	0.65		
8.313	$1/2^+$	2	0.10(4)	7.870				0.00	

[a] Reference [1].
[b] Reference [6].
[c] Present paper.

Thus, the reported d-wave S implies a width of about 72(2) keV, leaving ∼130 keV for the $\ell = 0$ width so that the s-wave S would be about 0.06. I note that the shell-model value is quoted as 0.04. In fact, the published angular distribution for this state appears to be consistent with a pure $\ell = 2$ shape.

We thus see that the actual $\ell = 0$ spectroscopic factors for the second and third $3/2^+$ states are considerably smaller than the published ones. This remains the case even if the assumption of equal spectroscopic factors for mirror states is not exact but only approximate.

In a manner similar to that for the other states with $T = 0$ cores, I have computed the ^{15}O energies for the second and third $3/2^+$ states. In Table III, the results labeled Calc1 are those for the pure $\ell = 0$ and 2 configurations. For the Calc2 results, I have combined these pure energies with the appropriate S's discussed above and then assumed the remainder of the wave function has no shift from ^{15}N to ^{15}O. This is certainly not exact, but it is the best I can do for these two states. Agreement is reasonable, although the Calc2 results are only marginally better than those for pure $\ell = 2$.

IV. SUMMARY

I have examined single-particle $s_{1/2}$ and $d_{5/2}$ states in ^{15}N that have $T = 1$ and $T = 0$ cores and their mirrors in ^{15}O. The lowest excited states at 5.270 and 5.299 MeV appear to contain most of the $s_{1/2}$ and $d_{5/2}$ strengths expected for states having $T = 1$ cores. For states with $T = 0$ cores, four of the five expected states have their strengths concentrated in one state of the appropriate J^{π}. Computed energies for their mirrors in ^{15}O are in good agreement with experimental energies. The exception is that for $J^{\pi} = 3/2^+$ coming from the $d_{5/2}$ configuration. Its strength appears to be split among three states, even though about 65% of it is in the third $3/2^+$ state at 10.066 MeV. I demonstrate that the $\ell = 0$ spectroscopic factors reported for the second and third $3/2^+$ states are much larger than the actual ones—if mirror symmetry is assumed. Computed energies for these two states are reasonable but not as good as for the other four states.

I have pointed out that the absolute cross-sectional scale of Mertin *et al.* [6] is probably too large by a factor of about 1.5.

TABLE III. Splitting of $d_{5/2}$ strength among $3/2^+$ states in ^{15}N.

E_x (^{15}N) (MeV)[a]	ℓ	$S(d,p)$[b]	E_x (^{15}O) (MeV)			Γ_{exp} (keV)[a]	Γ_{sp} (keV)[c]	S_{Γ}[a]	S_{th}[b]		
			Calc1[c]	Calc2[c]	Expt.[a]				s	$d_{3/2}$	$d_{5/2}$
7.301	0	0.98(3)	6.686	6.733	6.793			Bound	0.72		
7.301	2	0.19(3)	6.974							0.01	0.07
8.571	0	0.07(2)	7.600	8.440	8.284	3.6(7)	440	<0.0081(16)	0.00		
8.571	2	0.13(2)	8.085		8.284		6.6			0.13	0.20
10.066	0	0.32(4)	8.424	9.425	9.484	∼200	2100	<0.1, <0.06	0.04		
10.066	2	0.65(2)	9.333		9.484		110			0.03	0.55

[a] Reference [1].
[b] Reference [6].
[c] Present paper.

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