Transverse isospin response function of asymmetric nuclear matter from a local isospin density functional

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The time dependent local isospin density approximation (TDLIDA) has been extended to the study of the transverse isospin response function in nuclear matter with an arbitrary neutron-proton asymmetry parameter ξ. The energy density functional has been chosen in order to fit existing accurate quantum Monte Carlo calculations with a density dependent potential. The evolution of the response with ξ in the $\Delta T_z = \pm 1$ channels is quite different. While the strength of the $\Delta T_z = +1$ channel disappears rather quickly by increasing the asymmetry, the $\Delta T_z = -1$ channel develops a stronger and stronger collective mode that in the regime typical of neutron star matter at β equilibrium almost completely exhausts the excitation spectrum of the system. The neutrino mean free paths obtained from the TDLIDA responses are strongly dependent on ξ and on the presence of collective modes, leading to a sizable difference with respect to the prediction of the Fermi gas model.

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I. INTRODUCTION

Weak interaction processes are a key ingredient in a number of phenomena both in terrestrial and astrophysical environments. In particular, in recent time there has been an increasing interest in neutrino physics, not only related to detection mechanisms, but also in connection to supernova explosions and the cooling of neutron stars. In general, the energy range of neutrinos of astroparticle interest is 0.1–50 MeV, implying a direct connection with the excitation modes of nuclei and nuclear matter [1]. The Weinberg-Salam model [2] allows for deriving an expression of the neutrino cross section in terms of the matrix elements of the weak current operators [3–6], and of the corresponding response functions. The involved excitation operators describe fluctuations in number, spin, and isospin density. An accurate computation of the linear response to these operators is fundamental to correctly analyze the interaction of neutrinos with dense matter.

One of the open issues in this context is understanding how the collective modes that are commonly observed in nuclei (such as the giant dipole resonance) translate into collective modes in the infinite matter, and how the energy and the strength of such modes evolve with the density and with the composition of the system [7,8]. Collective excitations tend to suppress, for instance, the mean free path of neutrinos, and could therefore play an important role in the cooling process of neutron stars [5,6]. A thorough analysis of the response function in nuclei and nuclear matter in the framework of mean field theories was performed in Refs. [9–11]. In this case particular attention was devoted to the study of the contribution of tensor terms in the Skyrme-like functionals used. The result is that many quantities related to the response function (the occurrence and position of collective modes, mechanical instability in nuclear matter, and the neutrino mean free path) are particularly sensitive to the details of the interaction.

In general, we can conclude that all these problems can be correctly addressed only if quantum correlations due to the nucleon-nucleon interactions are taken into account.

The effect of the correlations induced by the strong interaction on the response function has been widely studied by Cowell and Pandharipande [5,12] both for cold and hot neutron matter. In that case the response function is computed within a fully microscopic scheme. The correlations induced in the wave function by the interaction, described by a realistic Hamiltonian, are determined by the correlated basis function (CBF) method. An effective interaction is then built, and the response function is computed by diagonalizing the matrix elements of the excitation operators on the space of particle-hole states (correlated Tamm-Dancoff approximation (CTD), and further extensions[13,14]). Calculations have been carried out both at zero and finite temperature. This scheme has been recently further extended with the inclusion of three-body force effects by Lovato *et al.* [6], who studied the density and spin density response in pure neutron matter.

The extension of these calculations to the case of nuclear matter with an arbitrary value of the asymmetry parameter $\xi = \frac{(N-Z)}{A}$ is still difficult, and therefore it is important to develop a scheme that can include as much as possible the information coming from available fully microscopic results and can be extended to arbitrary isospin polarizations. In a previous work [15], we introduced a time dependent local isospin density approximation (TDLIDA) that allowed us to compute the longitudinal response function related to the excitation operator

$$F^z = \sum_k f(\mathbf{r}_k)\tau^z_k = \sum_k \exp(i\mathbf{q}\cdot\mathbf{r}_k)\tau^z_k \equiv \tau^z_f$$

where $\mathbf{q}$ is the transferred momentum and $\mathbf{r}_k$ are the coordinates of the k-th particle with $\Delta T_z = 0$, where $T_z = \sum_i \tau^z_i$ and τ^z is the third component of the isospin operator ($\tau^z|n\rangle = |n\rangle$, $\tau^z|p\rangle = -|p\rangle$, and n,p stands for neutrons and protons,

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respectively), $\langle \sum_i \tau_i^z \rangle = N - Z$ with N and Z neutron and proton numbers, respectively, and we have assumed that the isospin polarization is along the z direction.

The connection with *ab initio* calculations was made by using a density functional fitted on the equation of state (EoS) of pure neutron matter (PNM) and symmetric nuclear matter (SNM) computed by Auxiliary Field Diffusion Monte Carlo (AFDMC) method with a Hamiltonian including a density dependent term [16], and that gives results in good agreement with the pressure constraints coming from heavy ion collision experiments and the prediction on the mass of neutron stars.

In this paper we extend the calculations to the transverse isospin channel ($\Delta T_z = \pm 1$), providing information useful to investigate the contributions from the Fermi operator $F^{\pm} = \sum_k f(\mathbf{r_k})\tau_k^{\pm}$. Calculations are provided in a range of densities $0.5\rho_0 < \rho < 2\rho_0$, where ρ_0 is the nuclear saturation density $\rho_0 = 0.16$ fm^{-3} and for a few values of the asymmetry $0 < \xi < 1$ analyzing the occurrence and the strength of collective modes. The computed response function very accurately fulfills the m_0 sum rule, i.e. the static structure factor, and therefore the prediction on the strength, necessary to understand the evolution of the collective mode with the asymmetry parameter. Besides looking at values of ξ that are typical of matter at β equilibrium in the neutron star core, we also studied a situation that is close to the typical asymmetry of neutron rich nuclei near the drip line, which is interesting both for understanding the behavior of neutrino scattering in a neutron star crust, and in principle some phenomenology of exotic beams.

The paper is organized as follows. In Sec. II the transverse isospin density in the TDLIDA scheme is derived. In Sec. III the response function for a few values of nucleon density and asymmetry are presented and discussed. Section IV is devoted to conclusions.

II. TRANSVERSE ISOSPIN RESPONSE IN THE TDLIDA APPROXIMATION

For this study we work within the framework of Density Functional Theory (DFT) [17], and in particular of the local-isospin density approximation (LIDA), where the single-particle states are obtained as solutions of the LIDA equations:

$$\left[-\tfrac{1}{2}\nabla_{\mathbf{r}}^2 + \tfrac{1}{2}\omega_C \tau_z + v(\mathbf{r}) + w(\mathbf{r})\tau_z\right]\varphi_i^{\tau}(\mathbf{r}) = \varepsilon_{i,\tau}\, \varphi_i^{\tau}(\mathbf{r}). \tag{1}$$

In Eq. (1) i indicates the set of quantum numbers, excluding isospin, characterizing the single-particle wave functions. The term $\frac{1}{2}\omega_C \tau_z$ is an effective isovector potential responsible for the isospin polarization of the nuclear matter. In neutron star matter it is fixed by charge neutrality and β equilibrium, in nuclei by the proton Coulomb potential. The parameter ω_C can be related to the equilibrium asymmetry ξ [$\xi = m/\rho = (N-Z)/A$] by imposing that the variation of the LIDA energy with respect to ξ be zero. One gets

$$\int d\mathbf{r}(\rho_n - \rho_p) = N - Z = \omega_C \frac{\frac{3A}{4\epsilon_F}}{1 + \frac{3\rho}{2\epsilon_F}\frac{\partial w}{\partial m}}, \tag{2}$$

where $\epsilon_F = k_F^2/2m$ is the Fermi energy, with the Fermi momentum k_F given by $k_F = (\frac{3\pi^2}{2}\rho)^{1/3}$ and the neutron and proton momenta given by $k_F^n = k_F(1+\xi)^{1/3}$ and $k_F^p = k_F(1-\xi)^{1/3}$, respectively.

The nuclear neutron (proton) density is given by

$$\rho_\tau = \sum_i |\varphi_i^{\tau}(\mathbf{r})|^2,$$

with $\tau = n(p)$, and

$$v(\mathbf{r}) = \frac{\partial \rho \epsilon_V[\rho(\mathbf{r}),m]}{\partial \rho(\mathbf{r})}, \quad w(\mathbf{r}) = \frac{\partial \rho \epsilon_V[\rho(\mathbf{r}),m]}{\partial m(\mathbf{r})}, \tag{3}$$

and the total density ρ and isospin polarization m are given by $\rho = \rho_n + \rho_p$ and $m = \rho_n - \rho_p$. In Eq. (3) the interaction-correlation energy $\epsilon_V(\rho,m)$ was derived by an AFDMC calculation of the EoS of SNM and PNM [16] and is given by

$$\epsilon_V(\rho,m) = \epsilon_0(\rho) + \xi^2[\epsilon_1(\rho) - \epsilon_0(\rho)], \tag{4}$$

where

$$\epsilon_q(\rho) = \epsilon_q^0 + a_q(\rho - \rho_0) + b_q(\rho-\rho_0)^2 + c_q(\rho-\rho_0)^3 e^{\gamma_q(\rho-\rho_0)} \tag{5}$$

is the interaction correlation energy of SNM ($\xi = 0, q = 0$), and PNM ($\xi = 1, q = 1$). The parameters in Eq. (5) are: $\epsilon_0^0 = -38.10$ MeV, $a_0 = -92.10$ MeV/fm^3, $b_0 = 630.1$ MeV fm^6, $c_0 = -1717.2$ MeV fm^9, $\gamma_0 = 2.360$ fm^3 for $\epsilon_0(\rho)$; $\epsilon_1^0 = -19.80$ MeV, $a_1 = -21.0$ MeV fm^3, $b_1 = 533.0$ MeV fm^6, $c_1 = -1327.7$ MeV fm^9, $\gamma_1 = -2.201$ fm^3 for $\epsilon_1(\rho)$, respectively. These values very well reproduce the AFDMC results of Ref. [16] in the density range $\rho_0/2 < \rho < 2\rho_0$.

In the present work we will use the above formalism to study the transverse TDLIDA response, which is related to the neutron β decay and electron capture on proton weak processes:

$$p + e^- \to n + \nu, \quad \nu + n \to p + e^-. \tag{6}$$

One of the excitation operators involved in the description of these reactions ($\Delta T_z = \pm 1$) is the Fermi operator:

$$F^{\pm} = \sum_k f(\mathbf{r_k})\tau_k^{\pm} \equiv \tau_f^{\pm},$$

where the isospin-flip operators $\frac{1}{2}(\tau_x \pm i\tau_y)$ are defined by

$$\tau_+|\uparrow\rangle = \tau_-|\downarrow\rangle = 0,$$
$$\tau_+|\downarrow\rangle = |\uparrow\rangle,$$
$$\tau_-|\uparrow\rangle = |\downarrow\rangle.$$

The method followed here to derive the transverse response function was developed by Rajagopal [18] and was applied to quantum dots by Lipparini *et al.* [19,20].

In the $\Delta T_z = \pm 1$ channel, for an isospin polarization m different from zero in the ground state, the LIDA equations can be rewritten as

$$\left[-\tfrac{1}{2}\nabla_{\mathbf{r}}^2 + \tfrac{1}{2}\omega_C \tau_z + v(\mathbf{r}) + \mathcal{W}\mathbf{m}\cdot\boldsymbol{\tau}\right]\varphi_i^{\tau}(\mathbf{r}) = \varepsilon_{i,\tau}\varphi_i^{\tau}(\mathbf{r}), \tag{7}$$

where we have introduced the *isospin polarization vector* $\mathbf{m}$ and assumed that the interaction-correlation energy only depends on ρ and $|\mathbf{m}|$, i.e., $\epsilon_V = \epsilon_V[\rho,|\mathbf{m}|]$ so that the isospin-dependent interaction-correlation potential w in Eq. (3) can be written now as

$$\mathcal{W}\mathbf{m} = w[\rho,|\mathbf{m}|]\,\mathbf{m}/|\mathbf{m}|,$$

with

$$w[\rho,|\mathbf{m}|] = \partial\epsilon_V[\rho,|\mathbf{m}|]/\partial|\mathbf{m}|.$$

and $\mathcal{W}[\rho,|\mathbf{m}|] \equiv w[\rho,|\mathbf{m}|]/|\mathbf{m}|$. Defining the spherical components $\pm$ of the vectors $\mathbf{m}$ and $\boldsymbol{\tau}$, we have made the substitution

$$w\tau_z \rightarrow \mathcal{W}[\rho,|\mathbf{m}|]\,[m_z\tau_z + 2(m_+\tau_- + m_-\tau_+)]. \tag{8}$$

In the static case, the inclusion of the densities m_+ and m_- makes no difference since they vanish identically. The situation is different when the system interacts with a time-dependent field that couples to the nucleon isospin through

$$\mathbf{F}\cdot\boldsymbol{\tau} = F_z\tau_z + 2(F_+\tau_- + F_-\tau_+).$$

If the time dependence is harmonic, the interaction Hamiltonian causing transverse isospin excitations may be written as

$$H_{\rm int} \sim \tau_f^- e^{-\imath\omega t} + \tau_f^+ e^{\imath\omega t}. \tag{9}$$

$H_{\rm int}$ causes nonvanishing variations in the densities m_+ and m_- which, in turn, generate induced potentials through first-order perturbation variations in the mean field of (7). This equation shows that the induced interaction is $2\,\mathcal{W}(\rho,m)\,\delta(\mathbf{r}_1 - \mathbf{r}_2)\,\tau_-$ for m_+, and $2\,\mathcal{W}(\rho,m)\,\delta(\mathbf{r}_1 - \mathbf{r}_2)\,\tau_+$ for m_-, computed for the ground-state values of ρ and m. The transverse linear response function is defined by

$$\chi_t(F,\omega) = \langle(\tau_f^-)^\dagger\rangle = \int d\mathbf{r} f^*(\mathbf{r})\delta m^+(\mathbf{r},\omega), \tag{10}$$

with

$$\delta m^+(\mathbf{r},\omega) = \int d\mathbf{r}'\alpha^\pm(\mathbf{r},\mathbf{r}',\omega) f(\mathbf{r}'), \tag{11}$$

and $\alpha^\pm(\mathbf{r},\mathbf{r}',\omega)$ is the correlation function, solution of the Dyson type integral equation

$$\alpha^\pm(\mathbf{r},\mathbf{r}',\omega) = \alpha_0^\pm(\mathbf{r},\mathbf{r}',\omega) + 2\int d\mathbf{r}_1 d\mathbf{r}_2 \alpha_0^\pm(\mathbf{r},\mathbf{r}',\omega)\mathcal{W}(\rho,m) \times \delta(\mathbf{r}_1 - \mathbf{r}_2)\alpha^\pm(\mathbf{r}_2,\mathbf{r}',\omega), \tag{12}$$

where

$$\alpha_0^\pm(\mathbf{r},\mathbf{r}';\omega) = \sum_{mi}\sum_{\tau\tau'}\left[\frac{\phi^*_{i\tau'}(\mathbf{r}) f^*(\mathbf{r})\tau^+\phi_{m\tau}(\mathbf{r})\phi^*_{m\tau}(\mathbf{r}') f(\mathbf{r}')\tau^-\phi_{i\tau}(\mathbf{r}')}{\omega - \epsilon_m + \epsilon_i} - \frac{\phi^*_{m\tau'}(\mathbf{r}) f^*(\mathbf{r})\tau^+\phi_{i\tau}(\mathbf{r})\phi^*_{i\tau}(\mathbf{r}') f(\mathbf{r}')\tau^-\phi_{m\tau}(\mathbf{r}')}{\omega + \epsilon_m - \epsilon_i}\right]. \tag{13}$$

is the free transverse correlation function, built with the solutions of the LIDA equations (1).

In the homogeneous case where $f(\mathbf{r})$ is given by $f(\mathbf{r}) = \exp(i\mathbf{q}\cdot\mathbf{r})$, one gets for the transverse linear response

$$\chi_t(q,\omega) = \sum_n\left(\frac{|\langle n|\sum_k \exp(i\mathbf{q}\cdot\mathbf{r}_k)\tau_k^-|0\rangle|^2}{\omega - \omega_{n0}} - \frac{|\langle n|\sum_k \exp(-i\mathbf{q}\cdot\mathbf{r}_k)\tau_k^+|0\rangle|^2}{\omega + \omega_{n0}}\right), \tag{14}$$

and the TDLIDA solution of Eqs. (7)–(13) (V is the volume):

$$\chi_t(q,\omega) = \frac{\chi_t^0(q,\omega)}{1 - \frac{2}{V}\mathcal{W}(\rho,m)\chi_t^0(q,\omega)}, \tag{15}$$

where $\chi_t^0(q,\omega)$ is the free transverse linear response. In the $qv_F \ll \epsilon_F$ limit, where $v_F = k_F/m$ is the Fermi velocity, it is given by

$$\frac{\chi_t^0(q,\omega)}{V} = -\frac{3}{4}\frac{\rho}{\epsilon_F}\left(1 + \frac{\omega}{2qv_F}\ln\frac{\omega - \omega_a - qv_F}{\omega - \omega_a + qv_F}\right), \tag{16}$$

where

$$\omega_a = \frac{\omega_C}{\left(1 + \frac{3\rho\mathcal{W}(\rho,m)}{2\epsilon_F}\right)} = \frac{2}{3}\frac{k_F^2}{m}\xi,$$

with $\xi = (N - Z)/A$ and the last step has been obtained by using relation (2).

Equation (15) is the main result of the present work. It provides the correct description of the response function in the macroscopic regime characterized by the conditions qv_f, ω, and $\omega_c \ll \epsilon_F$, where the TDLIDA is supposed to correctly describe the system response. In the absence of an isovector component of the Coulomb potential ($\omega_C = \omega_a = 0$) the isospin polarization is $m = 0$ ($N = Z$) in the ground state, and consequently $\mathcal{W}(\rho,m=0) = \frac{\partial w}{\partial m}|_{m=0}$. In this case the transverse response coincides with the longitudinal isovector response previously studied [15]. However, when $N \neq Z$ the two responses are different and new solutions emerge.

Also, notice that in the limit $qv_F \ll \omega_C$, χ_t approaches the expression

$$\chi_t(q,\omega) = -\frac{3}{4}\frac{A}{\epsilon_F}\frac{\omega_a}{(\omega - \omega_C)}, \tag{17}$$

showing that the TDLIDA transverse pole is correctly given by ω_C as stated by the Larmor theorem, and not by the renormalized frequency ω_a, which gives the poles of the free response function $\chi_t^0(q,\omega)$ in the same low q limit.

From Eq. (15), by taking the imaginary part of χ_t, it is possible to calculate the excitations strengths $S^\pm(q,\omega) = \sum_n |\langle n|\tau_f^\pm|0\rangle|^2\delta(\omega - \omega_{no})$ corresponding to the $\Delta T_z = \pm 1$

channels, respectively, through the relation

$$S^-(q,\omega) - S^+(q,-\omega) = -\frac{1}{\pi}\mathrm{Im}(\chi_t). \quad (18)$$

It is also interesting to investigate the properties of $\chi_t(q,\omega)$ in terms of the moments [21,22]:

$$m_k^{\pm} = \int_o^{\infty} d\omega\omega^k[S^-(q,\omega) \pm S^+(q,\omega)]$$
$$= \sum_n \omega_{n0}^k(|\langle n|\tau_f^-|0\rangle|^2 \pm |\langle n|\tau_f^+|0\rangle|^2). \quad (19)$$

From Eq. (14) one obtains

$$\lim_{\omega\to\infty} \chi_t(q,\omega) = \frac{1}{\omega}\left(m_0^- + \frac{1}{\omega}m_1^+ + \cdots\right), \quad (20)$$

with

$$m_0^- = \sum_n(|\langle n|\tau_f^-|0\rangle|^2 - |\langle n|\tau_f^+|0\rangle|^2)$$
$$= \langle 0|[\tau_f^+,\tau_f^-]|0\rangle, \quad (21)$$

and

$$m_1^+ = \sum_n \omega_{n0}(|\langle n|\tau_f^-|0\rangle|^2 + |\langle n|\tau_f^+|0\rangle|^2)$$
$$= \langle 0|[\tau_f^+,[H,\tau_f^-]]|0\rangle. \quad (22)$$

Explicit evaluation of the commutator of Eq. (21) shows that the sum rule m_0^- is model independent and given by

$$m_0^- = N - Z. \quad (23)$$

This sum rule is exactly satisfied by the TDLIDA scheme as it can be explicitly shown by taking the $\omega \to \infty$ limit of Eq. (15) and recalling that $\xi = \frac{3\omega_a}{2\epsilon_F}$.

Different from m_0^-, the m_1^+ sum rule is model dependent, involving an explicit assumption for the Hamiltonian. For the LIDA energy functional used in this work the result of the evaluation of the first energy momentum is

$$m_1^+ = (N+Z)\frac{q^2}{2m} + (N-Z)\omega_C, \quad (24)$$

where the first term originates from the kinetic energy, and the second one from the isovector component of the Coulomb potential. The interaction-correlation energy term does not give any contribution since it is completely local. This point is rather delicate. For realistic nonlocal interactions more contributions to the energy-weighted m_1^+ sum rule would be included that are not contained in the TDLIDA scheme. This reflects a failure of the TDLIDA model in reproducing quantities in the high-ω region where many-particle-many-hole excitations not described by the TDLIDA may be important. An important effect of many-particle-many-hole excitations is to give a width to the collective states. This width which can be of some few MeV is completely absent in the TDLIDA scheme.

III. NUMERICAL RESULTS

A. Response and excitation strengths

Introducing the adimensional quantities $s = \frac{\omega}{qv_F}$ and $z = \frac{3q}{2k_F\xi}$, Eqs. (16) and (15) can be recast in the following way:

$$\frac{\chi_t^0(q,\omega)}{V\nu} \equiv \frac{\chi_t^0(s,z)}{V\nu} = \Omega_{\pm}(s,z), \quad (25)$$

with

$$\nu = mk_F/\pi^2,$$
$$\Omega_{\pm}(s,z) = -\left(1 + \frac{s}{2}\ln\frac{s-1-1/z}{s+1-1/z}\right),$$

and

$$\frac{\chi_t(q,\omega)}{V\nu} \equiv \frac{\chi_t(s,z)}{V\nu} = \frac{\Omega_{\pm}(s,z)}{1-2\nu\mathcal{W}(\rho,m)\Omega_{\pm}(s,z)}. \quad (26)$$

The TDLIDA excited states are given by the poles of the response function (26), which are solutions of the equation

$$\Omega_{\pm}(s,z) = \frac{1}{2\nu\mathcal{W}(\rho,m)}. \quad (27)$$

Positive values of s describe the excited states in the $\Delta T_z = -1$ channel, while for negative values of s the excited states in the $\Delta T_z = +1$ channel are obtained. Depending on the strength of the interaction and on the value of z these solutions are essentially of two types: (i) real solutions producing a discrete peak in the total strength with no attenuation (collective modes) and (ii) solutions with some imaginary component, and corresponding modes which decay by exciting single quasi-particle quasi-hole pairs. The quasi-particle quasi-hole strengths $S(s,z)/(V\nu)$ have been calculated by direct evaluation of the imaginary part of Eq. (26).

The strengths of the collective states in the two excitation channels $\Delta T_z = \pm 1$ are obtained by expanding the adimensional expression for the response in Eq. (26) around the pole at $\bar{s}$. Naming $N(s)$ and $D(s)$ the numerator and the denominator of Eq. (26), respectively, one gets $S(s)/(V\nu) = [N(s)/\frac{\partial D}{\partial s}]\delta(s-\bar{s})$.

The fraction of the m_0^-/A sum rule exhausted by the collective states is easily calculated to be

$$\frac{3q}{2k_F}\left(\frac{N(\bar{s})}{\frac{\partial D}{\partial s}\big|_{s=\bar{s}}}\right).$$

The integral

$$\frac{3}{2}\frac{q}{k_F}\int_0^{\infty}\frac{[S^-(s,z) - S^+(s,z)]}{V\nu}ds\ ,$$

when taking into account both the contributions of collective and quasi-particle quasi-hole excitations, is equal to ξ. In all calculations we have also checked that the particle-hole and collective contributions to the strength completely exhaust also the m_1^+ sum rule.

The interaction parameter $G = 2\nu\mathcal{W}(\rho,m)$ entering Eqs. (26) and (27) is shown in Fig. 1 as a function of of the nuclear density ρ. Due to the specific choice of a quadratic dependence on m of the parametrization of Eq. (4), this strength does not depend on m. From the figure one sees that G is practically constant also with respect to ρ in the range of density (from $\rho_0/2$ to 3ρ) over which the LIDA functional reliably reproduces quantum Monte Carlo (QMC) results. For this reason in the following calculations we only present results

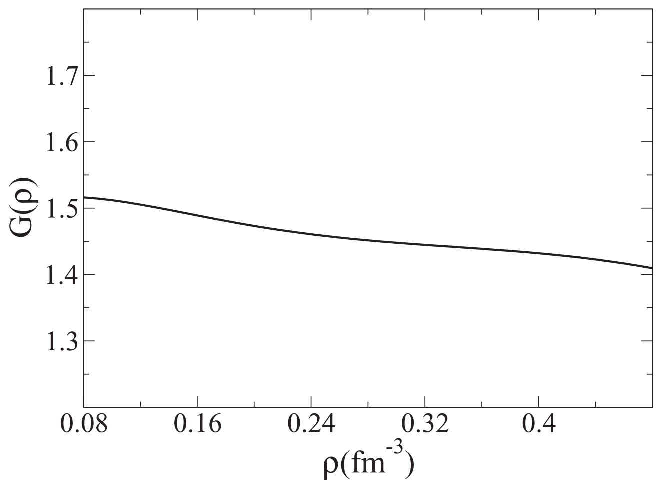

FIG. 1. Adimensional interaction parameter $G = 2\nu\mathcal{W}(\rho,m)$ as a function of the total nucleon density ρ.

in which the interaction strength has been fixed at the $\rho = \rho_0$ value.

The transverse response of Eq. (26) is plotted in Figs. 2–5 in two relevant cases: (a) $z \leqslant 1$ ($\xi \simeq 1$), and (b) $z \geqslant 1$ (small ξ). The first case is typical of neutron stars at β equilibrium, where the proton fraction does not exceed 10%. The second case is instead typical in neutron rich nuclei that can be found in exotic radioactive beams with masses below the drip line. The response function shows a continuum of single particle states (depicted as gray bands in the figures), and one or two poles at positive and negative values of s. Such poles correspond to collective states of the system. The presence of a collective mode in cold symmetric matter was already predicted by the CTD calculations in the $\Delta T_z = -1$ channel. However, for large z *both* the $\Delta T_z = \pm 1$ channels are present and when z approaches ∞ ($\xi = 0$) the two poles in the transverse response coincide with each other at the same value of $|s|$ which in turn would be the same position of the pole of the

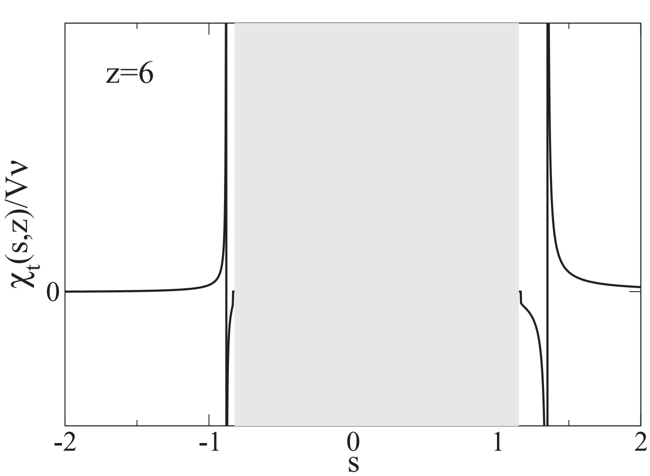

FIG. 2. Transverse response in units of $V\nu$ for $z = 3q/(2k_F\xi) = 6$. The gray area corresponds to the continuum of quasi-particle quasi-hole excitations. The poles at real values of s correspond to the collective states in the system.

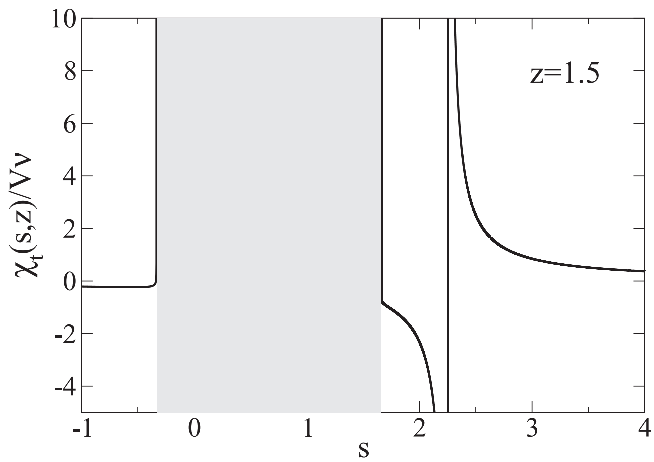

FIG. 3. Same as Fig. 2 for $z = 1.5$.

longitudinal response function (the $\Delta T_z = 0$ channel [15]). It should be mentioned that an explicit treatment of tensor forces should bring to a slight difference between the longitudinal and transverse responses also in the case of symmetric matter, as discussed by Cowell and Pandharipande [5,12].

By decreasing the value of z, the energy of the collective state in the $\Delta T_z = -1$ increases and moves further away from the continuum of single particle states. Also in this case (closer to the limit of pure neutron matter), the results essentially confirm the behavior predicted by CTD calculations [6]. On the other hand the energy of the collective state in the $\Delta T_z = +1$ channel quickly approaches the continuum of single particle states until it disappears. For even smaller values of z, Pauli blocking in the $\Delta T_z = +1$ channel becomes total.

A better intuition about the character of the excitation is given by the analysis of the excitation strengths. In Figs. 6–9 the strength is plotted for the same values of z considered above as a function of $|s|$. As we pointed out earlier in the paper, the TDLIDA response function allows for a precise estimate of the strength of the continuum and of the collective modes, and the strengths rigorously satisfy the number sum rule. In correspondence with the previous analysis of the response function, at small z the spectrum is clearly dominated by a

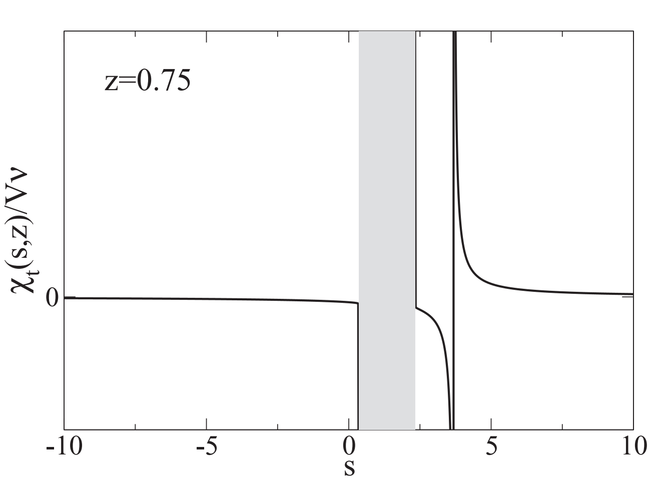

FIG. 4. Same as Fig. 2 for $z = 0.75$.

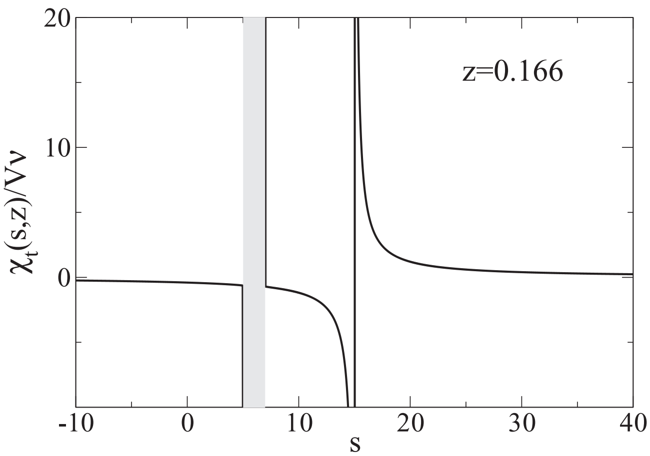

FIG. 5. Same in Fig. 2 for $z = 0.166$.

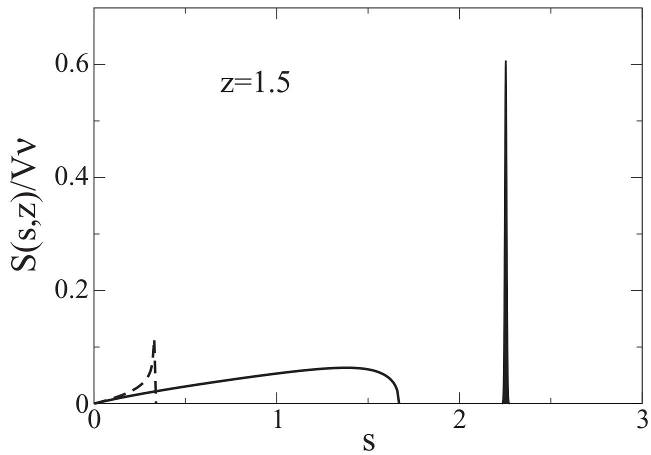

FIG. 7. Same as in Fig. 6 for $z = 1.5$. Notice that the collective state in the $\Delta T_z = +1$ channel has disappeared, and all the residual strength is absorbed by quasi-particle quasi-hole excitations.

strong collective state at high energy in the channel $\Delta T_z = -1$ (neutrino absorption by neutrons), while in the $\Delta T_z = +1$ channel (electron capture by protons) Pauli Blocking is total, and no strength is present. This means that within this model, in the core of neutron stars the only active channel in the Fermi response is neutrino capture, that excites a strongly collective isospin wave

Increasing the value of z, there is a decreasing of the strength of the collective mode, and at the same time a gain of single particle strength in the $\Delta T_z = +1$ channel. For larger values, the $\Delta T_z = +1$ collective peaks appears. In any case, in this regime the fraction of m_0^- sum rule exhausted by the collective states in both channel strongly decreases.

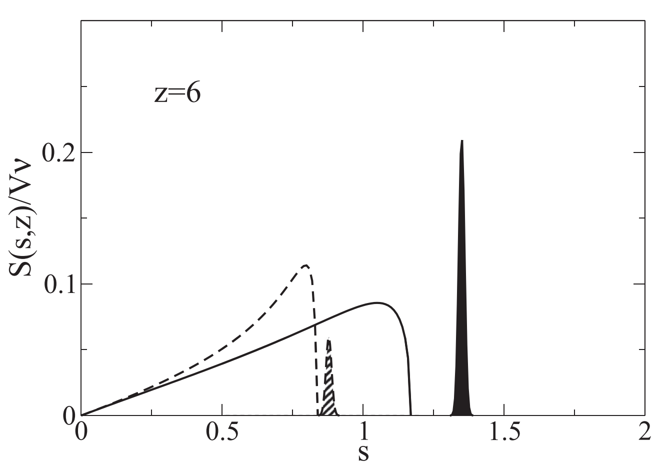

FIG. 6. Excitation strengths in units of $V\nu$ for $z = 3q/(2k_F\xi) = 6$ as a function of the absolute value of the adimensional parameter s. The full and dashed lines indicate the quasi-particle quasi-hole and collective strengths in the $\Delta T_z = -1$ and $\Delta T_z = +1$ channels (which would correspond to negative values of s), respectively. Note that a fictitious width has been given to the collective state in order to correctly reproduce in the figure the collective contribution to the non-energy-weighted sum rule.

B. Neutrino mean free path

The contribution of the isospin transverse channel to the determination of the neutrino mean free path (NMFP) can be computed by integrating the transverse excitation strength $S(q,\omega)$ to first obtain the total neutrino cross section σ [4,5]:

$$\sigma = \frac{G_F^2}{2}\frac{1}{E}\int dq \int d\omega (E-\omega) q \times \left(1 + \frac{E^2 + (E-\omega)^2 - q^2}{2E(E-\omega)}\right) S(q,\omega), \tag{28}$$

where E is the incident neutrino energy (which can be related to the temperature), and $G_F = 1.166 \times 10^{-5}$ GeV^{-2}. Integration must be performed on a region of q and ω compatible with the kinematics of the scattering process, as discussed in Ref. [4]. We will assume neutrinos to be ultrarelativistic and nondegenerate. The NMFP λ can be derived from σ from the relation $\lambda = 1/\sigma\rho$.

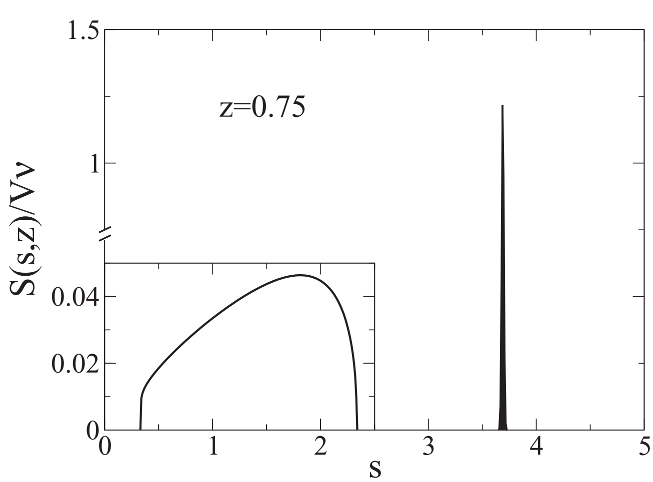

FIG. 8. Same as in Fig. 6 for $z = 0.75$. By now, Pauli blocking of the $\Delta T_z = +1$ channel is total.

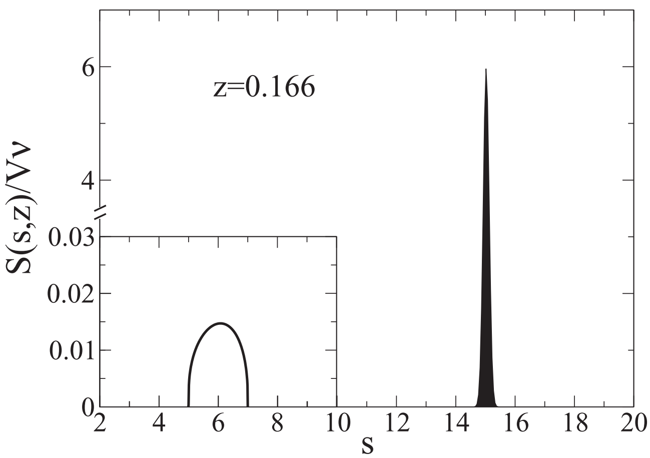

FIG. 9. Same as in Fig. 6 for $z = 0.166$. Notice once again that the Pauli blocking of the $\Delta T_z = +1$ channel is total, and that the strength in the $\Delta T_z = -1$ channel is almost completely exhausted by the collective state.

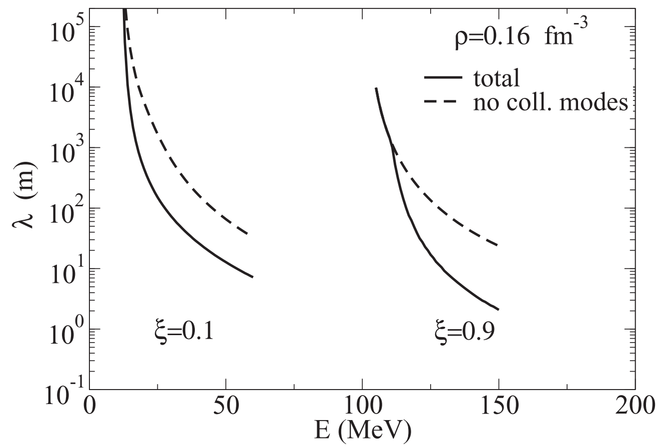

FIG. 11. NMFP computed including (solid lines) and excluding (dashed lines) the contributions from collective modes to the total strength.

The NMFP for nuclear matter at saturation density $\rho = 0.16\ \mathrm{fm}^{-3}$ is displayed in Fig. 10 as a function of the incident energy E, and for different values of the asymmetry parameter ξ. We consider two different asymmetry regimes. The region where $\xi \leqslant 0.2$ is typical of neutron rich nuclei, whereas values of $\xi > 0.8$ are expected in neutron star matter at β equilibrium. It can be noticed how the incident energy threshold below which matter becomes transparent to neutrinos increases by increasing ξ. For small values of the asymmetry parameter the transverse channel is still active in depleting the NMFP at energies typical of supernova explosions. At large values of the asymmetry parameter, instead, the isospin transverse channel would influence the NMFP only at neutrino incident energies that are not typical of the stellar physics context. We should point out that the expression for the free response function we have used in our TDLDA approach is valid only for low values of energy and momentum transfer. This approximation might definitely affect the results for large asymmetry, but without qualitatively changing them.

In Fig. 11 we have plotted the values of λ computed including and excluding the contribution from the collective modes for two representative values of the asymmetry parameter, $\xi = 0.1$ and $\xi = 0.9$. As it can be seen, in the small ξ regime, the mean free path is affected by the presence of collective excitations at all values of the incident energy. At larger asymmetries, collective modes would enter into play only beyond a given energy threshold. In general, the presence of collective modes tends to suppress the NMFP by a substantial factor, which becomes more than one order of magnitude at energies of 20 MeV or less.

In Figs. 12 and 13 we plot the normalized NMFP $\bar{\lambda} = \lambda/\lambda_{FG}$, where λ_{FG} is the NMFP computed for a noninteracting matter. This quantity is shown at different

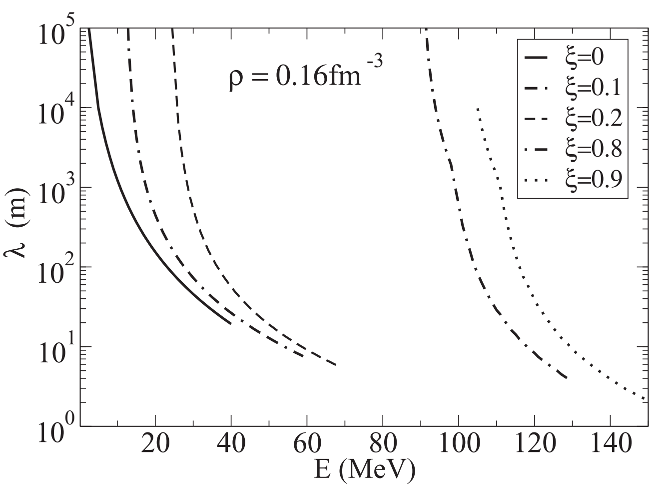

FIG. 10. Neutrino mean free path for an interacting, asymmetric nuclear matter at saturation density computed considering only the contribution of the transverse isospin channel. The values of the asymmetry parameter are displayed in the figure.

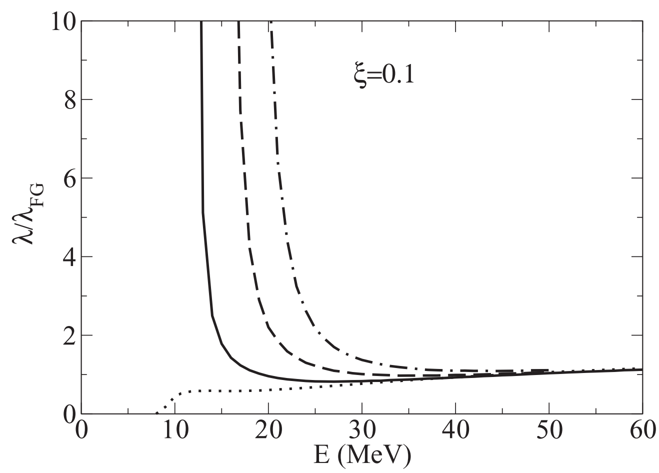

FIG. 12. Ratio of the NMFP computed for an interacting system and for a Fermi gas for different values of the density. The value of the asymmetry parameter is $\xi = 0.1$. The densities considered are 0.5 ρ_0 (dotted line), ρ_0 (solid line), 1.5 ρ_0 (dashed line), and 2 ρ_0 (dotted-dashed line).

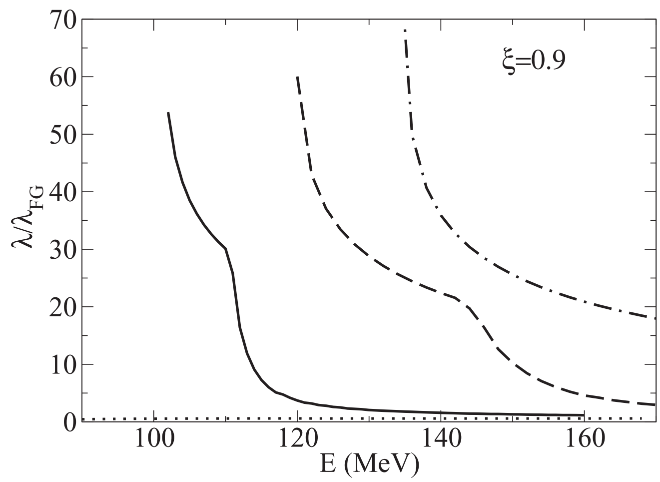

FIG. 13. Same as Fig. 12, but for a value of the asymmetry parameter $\xi = 0.9$.

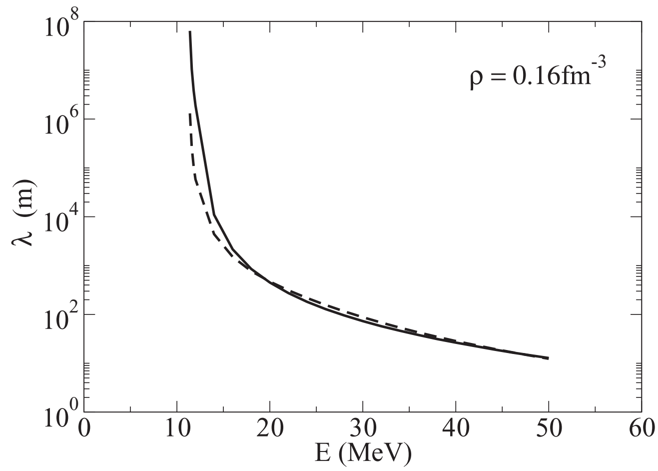

FIG. 15. Same as Fig. 14, but at saturation density ρ_0.

values of the density ($0.5\rho_0, \rho_0, 1.5\rho_0$, and $2\rho_0$), and once again for two sample values of the asymmetry parameter ($\xi = 0.1$ and $\xi = 0.9$). In the low asymmetry regime, there are two interesting facts to notice. First, at the lowest density considered, $\bar{\lambda}$ does not diverge at low incident energy, but it does rather go to zero, indicating that in this region the interaction would strongly suppress the NMFP. The second interesting fact is that $\bar{\lambda}$ tends to be completely independent of the density for large values of the incident energy. At larger values of ξ one can still see the strong suppression of the NMFP at low densities, while for larger densities the results at high incident energy are more strongly density dependent. In this regime the contribution of the collective modes is very evident.

A possible explanation of the strong NMFP suppression at sub-saturation density might come from the fact that in this region the contribution from the single particle spectrum is still comparable to that of the Fermi gas. This has the consequence

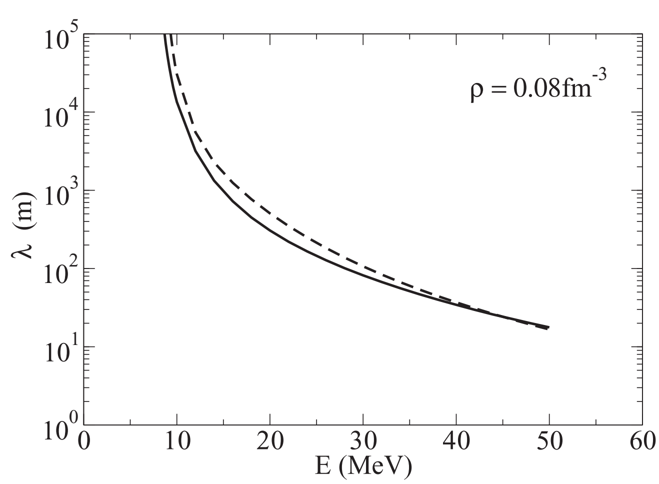

FIG. 14. NMFP at density $0.5\rho_0$ computed in a Fermi gas (dashed line) and in an interacting asymmetric nuclear matter (full line). The asymmetry parameter of the system is $\xi = 0.1$.

that, as soon as matter becomes opaque to neutrinos, the combined effect of the single particle excitations and of the collective mode is large enough to enhance the neutrino cross section with respect to the Fermi gas prediction. This is well illustrated in Fig. 14, where we directly compare λ to λ_{FG}. At saturation density, instead, the situation is reversed (see Fig. 15). The contribution of the single particle excitations is reduced, and it is not completely replaced by the contribution of the collective mode. As a result, the cross section is suppressed, and the NMFP is enhanced at low energies. Notice that at incident energies around 20 MeV the two curves cross, and the interaction once again gives a slight suppression of the NMFP up to energies of about 45 MeV.

In general, our calculations show that the influence of the isospin transverse channel on the NMFP is relevant in a regime of low asymmetry and low density. In particular, for energies below 10 MeV these processes are essentially not active, and this would limit the importance of this mechanism to the case of supernova explosions. The nuclear interaction has the effect of further enhancing the NMFP with respect to the noninteracting medium. Collective modes, instead, play an important role in depleting the NMFP, in particular at low densities.

IV. CONCLUSIONS

We have extended the time dependent local isospin approximation to the case of the Fermi response in homogeneous nuclear matter with arbitrary asymmetry. The energy density functional used reproduces quantum Monte Carlo results obtained with a density dependent potential in the range $0.5\rho_0 < \rho < 2\rho_0$. A first interesting result is a substantial independence of the effective interaction appearing in the TDLIDA response function on the density, at least in the range considered. The computed response functions show a systematic behavior with the asymmetry parameter. In particular, for situations closer to SNM, typical of nuclei, there is a contribution from both $\Delta T_z = \pm 1$ channels. However, the collective mode associated to the $\Delta T_z = +1$ channel disappears very quickly, and also the quasi-particle quasi-hole strength is rapidly suppressed. When approaching the PNM limit, in a regime of asymmetry typical of matter at β equilibrium, the Pauli blocking of the $\Delta T_z = +1$ channel is total, and the collective mode in the $\Delta T_z = -1$ channel gains strength until it exhausts almost completely the m_0 sum rule. We have also computed and discussed the contribution of the isospin transverse channel to the determination of the neutrino mean free path, pointing out the role of the interaction in the different regimes of density and asymmetry. An increase in the density and in the asymmetry strongly depletes the contribution of this channel in the range of incident energies of interest for stellar processes.

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