Coexistence and $B(E2)$ values in ^{72}Ge

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An earlier coexistence model of Ge nuclei is applied to $E2$ strengths connecting low-lying 0^+ and 2^+ states in ^{72}Ge. New data have smaller uncertainties and, for the first time, a value for the transition strength from the third 2^+ state to the second 0^+ state. This $B(E2)$ for the third 2^+ state clearly indicates that it is the one that should be included in the mixing, rather than the second 2^+ state. My results confirm that the 0^+ states are maximally mixed, the 2^+ states are weakly mixed, and the $E2$ matrix element involving the lower 0^+ basis state is significantly larger than the one involving the second 0^+ basis state.

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I. INTRODUCTION

Many years ago, Carchidi and co-workers developed a coexistence model [1–3] that allowed self-consistent treatment of nuclear data for a chain of isotopes. Initially intended for use in analyzing results of (t,p) and (p,t) experiments [1,4–6], the model was later extended to treatments of α transfer [7,8], proton occupancies [9], and $E2$ transition strengths [4,10]. Of special interest for the present purposes is the topic of $B(E2)$'s connecting 0^+ and 2^+ states in ^{72}Ge. These strengths were analyzed long ago [10] in the coexistence model just referred to. Two recent developments have prompted a return to this problem: (i) New values of $E2$ transition strengths have appeared [11] with somewhat improved uncertainties, and for the first time, a value for the transition strength from the third 2^+ state to the second 0^+ state; (ii) the understanding of the structures of the low-lying 2^+ states in ^{72}Ge has changed over the years [10,11].

In a two-state model of 0^+ states in $^{68-78}$Ge, analysis [1] of $2n$ transfer data produced a set of basis states that had several simple properties: (i) The energy difference between the basis states turned out to be a linear function of mass number A; (ii) the mixing matrix element between the basis states was nearly constant over the entire isotopic chain; (iii) the lower basis state in ^{68,70}Ge became the upper basis state in ^{74,76}Ge with a crossover at ^{72}Ge. Calculations with these wave functions reproduced 0^+ (t,p) and (p,t) cross-sectional ratios in all Ge nuclei [1].

These wave functions were later confronted with $E2$ transitions in ^{70,72}Ge where all four matrix elements connecting the first two 0^+ and 2^+ states were known [12–14]. Calculations [10] were able to reproduce the $E2$ strengths, to provide the amount of 2^+ mixing, and to determine the underlying $E2$ strengths connecting basis states—while still maintaining agreement with $2n$ transfer. In ^{72}Ge, we demonstrated that the second 2^+ state did not participate in the mixing and the upper 0^+ basis state did not have an appreciable $E2$ connection to any 2^+ state.

II. ANALYSIS AND RESULTS

In the earlier analysis [10], the first and second 2^+ states of ^{72}Ge were the focus. However, the second 2^+ state has very small strength for both 0^+ states, contrary to the expectation in a two-state model. The first two 0^+ states have long been thought to be maximally mixed ([1] and references therein]) with approximately equal admixtures of two basis states. The earlier $E2$ analysis concluded that the second 0^+ basis state was not connected to either of the first two 2^+ states. With the measurement of the $B(E2)$ connecting the third 2^+ state to the second 0^+ state [11], the reason is obvious—it is the third 2^+ state that should be included in the mixing. The experimental values of $E2$ transition matrix elements that are relevant to the present analysis are listed in Table I [11]. It can be noted that the $E2$'s have the expected property: the sum of $B(E2)$'s from the ground state (g.s.) to the first and third 2^+ states is approximately equal to that from the excited 0^+ state—$0.211(4)e^2\,\mathrm{b}^2$ vs $0.200(11)e^2\,\mathrm{b}^2$ as expected for nearly maximal mixing of the 0^+ states. Thus, in the analysis presented here, I consider two-state mixing between the first and the third 2^+ states. The second 2^+ state is apparently the band head of a $K=2$ band [11]. I denote 0^+ basis states as φ_g and φ_e, 2^+ basis states as ψ_g and ψ_e where the subscripts g and e refer to ground and excited bands, respectively.

I write

$$\text{g.s.} = a\varphi_g + b\varphi_e, \qquad 0_2^+ = b\varphi_g - a\varphi_e;$$
$$2_1^+ = A\psi_g + B\psi_e, \qquad 2_3^+ = B\psi_g - A\psi_e.$$

I define $M_g = \langle\varphi_g|E2|\psi_g\rangle$, $M_e = \langle\varphi_e|E2|\psi_e\rangle$, and I assume there is no strength connecting g to e. Then, we have four unknowns, viz., two mixing amplitudes, two $E2$ matrix elements, and four experimental numbers to fit with labels 0–3 in Table I. In the notation of Table I, the experimental and

TABLE I. Relevant $E2$ matrix elements in ^{72}Ge [11].

Label	Initial	Final	$M(E2)$ (e b)
0	0_1	2_1	0.457(4)
1	2_1	0_2	$0.35^{+0.01}_{-0.02}$
	0_1	2_2	0.020(1)
	0_2	2_2	0.0144(6)
2	0_1	2_3	0.044(1)
3	0_2	2_3	$0.279^{+0.002}_{-0.004}$

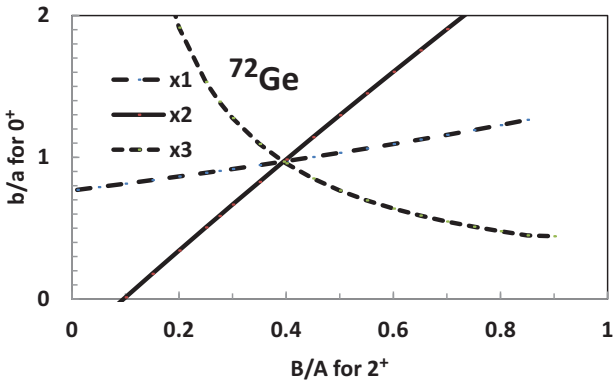

FIG. 1. The ratio of mixing amplitudes in the two lowest 0^+ states in ^{72}Ge is plotted vs the amplitude ratio for the first and third 2^+ states for agreement with $E2$ matrix elements labeled 0–3 in Table I. (See the text.)

basis-state M's are related by the expressions [4],

$$M_0M_3 - M_1M_2 = M_gM_e \text{ and } \Sigma M_i^2 = M_g^2 + M_e^2.$$

These constraint equations are similar to other sum-rule equations, but these are not defined in terms of nuclear shape.

Taking ratios, with the matrix element labeled 0 as the denominator in each case, produces three equations in three unknown ratios: $x = b/a$, $y = B/A$, and $R = M_e/M_g$. With the constraints above, inspection demonstrates solutions near $R = 0.3$. The x values from the equations labeled 1–3 are plotted vs y in Fig. 1 for R near this value. We note that the three are equal for y near 0.40. Results of the best fit are listed in Table II. Amplitudes derived from these best-fit ratios are listed in Table III. Values of b and B are compared there with those from Ref. [11]. Agreement is excellent.

Changing the sign of the matrix element labeled 2 produces another slightly different fit, whose parameters are also listed in Tables II and III. We note that both fits have the properties that the 0^+ states are approximately maximally mixed, the 2^+ states are weakly mixed, and the value of M_e is significantly smaller than M_g. These conclusions all agree with the earlier analysis [10].

TABLE II. Results of best fit to four $M(E2)$'s labeled 0–3 in Table I.

Parameter	$x = b/a$	$y = B/A$	$R = M_e/M_g$
Value fit 1	0.970(5)	0.395(5)	0.297(1)
Value fit 2	0.96(1)	0.283(3)	0.404(2)

TABLE III. Derived quantities from best-fit parameters.

Quantity	b	B	M_g (e b)	M_e (e b)
Value fit 1	0.696(2)	0.367(4)	0.615	0.182
Value fit 2	0.693(4)	0.272(3)	0.594	0.240
Reference [11]	0.694(4)	0.360		

III. DISCUSSION AND SUMMARY

The present coexistence model has the advantage that no information about the properties of the basis states is needed as input—this information emerges from the fit. In this case, the resulting structural information consists of the extracted values of M_g and M_e. There is no predetermined assumption of shape coexistence, spherical-deformed mixing, or axial-triaxial mixing. In the present case, the smallness of the resulting M_e suggests that the upper 0^+ basis state is nearly spherical with no strong $E2$ to any 2^+ state—again as concluded earlier [10]. Reference [13] later stated "We, therefore, conclude that there is no rotational band based on the 0_2^+. This strongly suggests that the 0_2^+ state has a spherical shape and can be treated as an "intruder" state." Reference [11] also found nearly maximal mixing for the 0^+ states. They concluded that their analysis "provided compelling evidence for the coexistence of two triaxially-deformed configurations associated with the 0_1^+ and 0_2^+ states." I do not disagree with this conclusion, but my results do not require it. It is good enough that the second basis states are almost spherical.

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