## **Ultrarelativistic nuclear collisions: Direction of spectator flow**

Sergei A. Voloshin and Takafumi Niida

*Wayne State University, 666 West Hancock, Detroit, Michigan 48201, USA* (Received 22 April 2016; published 17 August 2016)

In high-energy heavy-ion collisions, the directed flow of particles is conventionally measured with respect to that of the projectile spectators, which is defined as positive  $x$  direction, but it is not known if the spectators deflect in the outward or inward directions—outward or toward the center line of the collision. In this Communication we discuss how the measurements of the directed flow at midrapidity, especially in asymmetric collision such as  $Cu + Au$ , can be used to answer this question. We show that the existing data strongly favor the case that the spectators, in the ultrarelativistic collisions, on average deflect outward.

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In an ultrarelativistic nuclear collision, only part of all nucleons from the colliding nuclei experience a truly inelastic collision. Some of nucleons, called spectators, stay mostly intact (or might experience a transition to an excited state). Nevertheless, those nucleons do experience a nonzero momentum transfer and deflect from the original nucleus trajectory. The direction of such projectile nucleon ("spectator") deflection is conventionally taken as a positive  $x$  direction in the description of any anisotropic particle production (anisotropic flow [\[1\]](#page-2-0)). At the same time, while this direction has been measured experimentally at very low collision energies, nothing is known about which direction the spectators really deflect at high energies—toward the center of the collision or outward. Note that this question is not of purely academic interest: It is intimately related to understanding of the nucleon wave function in the nucleus, as well as momentum distribution of the nucleons confined in a nucleus [\[2\]](#page-2-0). It is also important for the interpretation of the anisotropic flow measurements. In particular, the knowledge of the spectator flow is requited for determination of the direction of the magnetic field created in the collision as well as the system orbital momentum. The latter, for example, is needed for the measurements of the so-called global polarization [\[3–5\]](#page-2-0).

Direct determination of the spectator nucleons' deflection direction was performed at the energies  $E/A \sim 100 \,\text{MeV}$  by measuring the polarization of emitted photons [\[6\]](#page-2-0). It was observed (see also Refs.  $[7,8]$  $[7,8]$ ) that around this energy the direction of the deflection direction changes from inward (due to attractive potential at lower energies) to outward at higher energies. No similar measurements were performed at higher collision energies. Theoretically, this question is also not well understood. As has been shown in Ref. [\[2\]](#page-2-0), the direction of the spectator deflection is likely dependent on the nucleon transverse momentum. These calculations show that at relatively large transverse momentum (more than  $\sim$ 200 MeV) the nucleons are likely deflected inward, while at low transverse momentum they might deflect outward. One reason for the latter might be the Coulomb interaction (repulsion) of the spectator protons.

In this article we show how the study of the charge particle directed flow at midrapidity measured relative to the spectator deflection direction (directed flow) can help to answer the question of which direction the spectators are deflected on average. We do not distinguish between low- and high- $p<sub>T</sub>$ spectators in this study, though in principle this question can be studied experimentally.

The main idea of our approach is based on the observation that in the case of asymmetric initial density distribution in the system, the high(er) transverse momentum particles on average are flowing or emitted in the direction of the largest density gradient, while the lower  $p_T$  particles flow in the opposite direction  $[9,10]$ . If the mean transverse momentum of all particles is zero (e.g., at midrapidity region in symmetric collisions) then the average, integrated over all transverse momenta, directed flow is in the same direction as that of low- $p_T$  particles.

Then the strategy in establishing the direction of the spectator flow becomes straight-forward. First, one has to measure the directed flow of particles at midrapidity with respect to the spectator deflection. Comparing that to the initial density gradients calculated relative to the position of spectators, one can determine the direction of spectator flow. The direction of the highest density gradient in the system has to be determined with the help of a model, but this appears to be a very robust procedure, as this direction depends mostly on the distribution of the matter inside the nucleus. As we argue below, there is no real model dependence or ambiguity here. In asymmetric collisions, such as  $Cu + Au$ , the direction of the density gradient can be established unambiguously on average, over all events. In symmetric collisions, e.g., Au+Au at RHIC or  $Pb + Pb$  at LHC, one has to account for the fluctuation nature of the density distribution and look for the density gradients relative to the position of the spectators.

To quantify the anisotropic flow we use a standard Fourier decomposition of the azimuthal particle distribution with respect to the *n*th harmonic symmetry planes  $[11,12]$ :

$$
E\frac{d^3N}{d^3p} = \frac{1}{2\pi} \frac{d^2N}{p_T dp_T dy} \bigg( 1 + \sum_{n=1}^{\infty} 2v_n \cos[n(\phi - \Psi_n)] \bigg), \quad (1)
$$

where  $v_n$  is the *n*th harmonic flow coefficient and  $\Psi_n$  is the nth harmonic symmetry plane determined by the initial geometry of the system (as given by the participant nucleon distribution, see below). According to model calculations (see Ref. [\[13\]](#page-3-0) and references therein) the event-by-event fluctuations in anisotropic flow closely follow the fluctuations



FIG. 1. Schematic view of the collision. Arrows indicate the direction of the spectator flow, outward from the center line.

in the corresponding eccentricities of the initial density distribution. Following Ref. [\[10\]](#page-3-0), for the latter we use the definition, for  $n \geqslant 2$ :

$$
\varepsilon_{n,x} = \varepsilon_n \cos(n\Psi_n) = -\langle r^n \cos(n\phi) \rangle / \langle r^n \rangle, \tag{2}
$$

$$
\varepsilon_{n,y} = \varepsilon_n \sin(n\Psi_n) = -\langle r^n \sin(n\phi) \rangle / \langle r^n \rangle, \tag{3}
$$

and for  $n = 1$  (most important for this study)

$$
\varepsilon_{13,x} = \varepsilon_{13} \cos(\Psi_{13}) = -\langle r^3 \cos(\phi) \rangle / \langle r^3 \rangle, \tag{4}
$$

$$
\varepsilon_{13,y} = \varepsilon_{13} \sin(\Psi_{13}) = -\langle r^3 \sin(\phi) \rangle / \langle r^3 \rangle, \tag{5}
$$

where  $\varepsilon_n = \sqrt{\varepsilon_{n,x}^2 + \varepsilon_{n,y}^2}$  is the so-called *participant* eccentricity  $[14]$ ; for the  $n = 1$  case we extend the subscript notation to 13 to emphasize the fact that in this definition the third power of r is used as a weight instead of the first power. In our Monte Carlo model, in calculations of the average quantities in eccentricity definitions we weight with the number of participating nucleons (those undergoing inelastic collision). For the nucleon distribution in the nuclei we use the Woods-Saxon density distribution with standard parameters (for the exact values, see Ref.  $[15]$ ; the inelastic nucleon-nucleon cross section is taken to be 42 mb for calculations of at  $\sqrt{s_{_{NN}}}$  = 200 GeV (Cu + Au collisions discussed below) and 64 mb for  $\sqrt{s_{_{NN}}}$  = 2.76 TeV (Pb + Pb collisions). In our model calculations we chose the positive  $x$  direction to point along the impact parameter vector, and assume that the spectators deflect in the outward direction (target spectators flow in the impact parameter vector direction, as indicated in Fig. 1), and then check if this agrees with the experimental observations.

There exist several measurements of directed flow at midrapidity relative to the spectator nucleons in Au+Au and Cu+Cu collisions at RHIC. Unfortunately, all those measurements reported only rapidity odd component of the directed flow, that is not suitable for our discussion, as in symmetric collision this component is exactly zero at midrapidity. Rapidity even component, not zero at midrapidity even in symmetric collisions due to fluctuations in initial density distribution, has been measured only in  $Pb + Pb$ collisions at LHC by ALICE Collaboration [\[16\]](#page-3-0). We will analyze these measurements below first, and then discuss less ambiguous directed flow measurements in asymmetric Cu + Au collisions at  $\sqrt{s_{_{NN}}}$  = 200 GeV by PHENIX [\[17\]](#page-3-0) and STAR [\[18\]](#page-3-0) Collaborations.



FIG. 2.  $\langle \cos(\Psi_{13}) \rangle$  as function of the difference in number of target and projectile nucleon participants in  $Pb + Pb$  collision in the impact parameter range  $2 < b < 3$  fm.

In symmetric nuclear collisions, such as  $Pb + Pb$ , the directed flow at midrapidity due to density fluctuations, if measured relative to the projectile spectator flow, can be nonzero only due to decorrelation in the flow directions of target and projectile spectators (and corresponding geometry) or fluctuations in the relative reaction plane resolutions due to fluctuations in the number of the spectators. We test the latter by calculating the directed flow at midrapidity,  $cos(\Psi_{13})$ , as a function of the difference in the number of projectile and target participants. An example of such calculations for the impact parameter range  $2 < b < 3$  fm is shown in Fig. 2. From that plot it follows that in the case of the smaller number of projectile participants  $\langle \cos(\Psi_{13}) \rangle > 0$  and the average directed flow would be negative. The smaller number of participants corresponds to the larger number of spectators that have to lead to better event plane resolution and thus dominate the measurements. Having in mind that the measurements [\[16\]](#page-3-0) indicate *negative* rapidity even component of the directed flow, one has to conclude that the flow of spectators must be outward (as assumed in the model). This reasoning one can check with direct measurement of flow as a function of the difference in number of spectators (e.g., as measured by zero-degree calorimeters). Unfortunately at present there are no such results published.

The effect of the projectile and target spectator flow direction decorrelation, and the correlations of the corresponding directions with the direction of the density gradient at midrapidity, can be studied as follows. Let us assume that the direction of the spectator flow is along the line between the center of the nucleus and the so-called center of gravity of the projectile spectators in the transverse plane. We denote the corresponding angle  $\Psi_{sp}$ . We calculate the correlation of that angle with  $\Psi_{13}$ , indicative of the direction of the (participant) density gradients that determined directed flow at midrapidity. The results of these calculations for  $Pb + Pb$  collision are shown in Fig. [3](#page-2-0) by open red markers. One can clearly see positive correlations, which again would lead to a conclusion that on average the flow at midrapidity should be negative (recall that on average the directed flow is in the opposite direction to  $\Psi_{13}$  $\Psi_{13}$  $\Psi_{13}$ ). Blue open points in Fig. 3 show the results

<span id="page-2-0"></span>

FIG. 3.  $\langle \cos(\Psi_{13} - \Psi_{sp}) \rangle$  and  $\langle \cos(\Psi_{13}) \rangle$  as function of the impact parameter for Pb + Pb collisions at  $\sqrt{s_{NN}} = 2.76 \text{ TeV}$  (open markers) and  $Cu + Au$  collisions at 200 GeV (filled markers). In  $Cu + Au$ collisions the Au nucleus is defined as the projectile;  $\Psi_{sp}$  is calculated using Au spectators.

for  $\langle \cos(\Psi_{13}) \rangle$  and are consistent with zero, as expected for symmetric nuclear collisions.

While the discussion above about directed flow at midrapidity in symmetric collisions is based on rather subtle details of the treatment and modeling of the fluctuations in the initial density distributions, in the asymmetric collisions, such as  $Cu + Au$ , the direction of the density gradient practically is insensitive to the fluctuations. In this case, the line of arguments and the conclusion become totally unambiguous. In the calculations discussed below we treat Au nucleus as the projectile, and Au spectators are used in calculations of the angle  $\Psi_{sp}$ .

Figure 4 presents the nucleon participant distributing in  $Cu + Au$  collisions in the impact parameter range  $2 < b <$ 3 fm. The distribution looks rather symmetric, but a more detailed study indicates that the density gradient is larger in the positive x direction. This is clearly seen in Fig.  $3$  (filled blue points). The effect of the density fluctuations and the corresponding correlations between the density gradients and the position of spectators (shown by red points) is rather insignificant in this case unless one considers very central collisions. In peripheral collision we observe that the red points are slightly below the blue points, which can be explained by the decorrelations of the direction of spectator flow relative to the reaction plane determined by the impact parameter.

- [1] S. A. Voloshin, A. M. Poskanzer, and R. Snellings, Collective phenomena in non-central nuclear collisions, in *Landolt-Boernstein*, in *Relativistic Heavy Ion Physics*, Vol. I/23 (Springer-Verlag, Berlin, Heidelberg, 2010), pp. 5–54.
- [2] M. Alvioli and M. Strikman, Beam fragmentation in heavy ion collisions with realistically correlated nuclear configurations, [Phys. Rev. C](http://dx.doi.org/10.1103/PhysRevC.83.044905) **[83](http://dx.doi.org/10.1103/PhysRevC.83.044905)**, [044905](http://dx.doi.org/10.1103/PhysRevC.83.044905) [\(2011\)](http://dx.doi.org/10.1103/PhysRevC.83.044905).
- [3] Z.-T. Liang and X.-N. Wang, Globally Polarized Quark-Gluon Plasma in Non-Central A + A Collisions, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.94.102301) **[94](http://dx.doi.org/10.1103/PhysRevLett.94.102301)**, [102301](http://dx.doi.org/10.1103/PhysRevLett.94.102301) [\(2005\)](http://dx.doi.org/10.1103/PhysRevLett.94.102301); Erratum: Globally Polarized Quark-Gluon Plasma in Non-Central A + A Collisions, **[96](http://dx.doi.org/10.1103/PhysRevLett.96.039901)**, [039901\(E\)](http://dx.doi.org/10.1103/PhysRevLett.96.039901) [\(2006\)](http://dx.doi.org/10.1103/PhysRevLett.96.039901).





FIG. 4. Participant distribution in  $Cu + Au$  collisions in the impact parameter range  $2 < b < 3$  fm. Positive x direction is toward the Au nucleus.

The measurements of directed flow at midrapidity in  $Cu + Au$  collisions  $[17,18]$  show that charged particles on average flow in the opposite direction to that of the projectile spectators. Thus, once again, we are to conclude that on average the spectators flow outward from the collision center. We note that the experimental values of the mean  $v_1$  in  $Cu + Au$  collisions is about an order of magnitude larger than the values of even  $v_1$  in Pb + Pb collisions (while the magnitude of the odd  $v_1$  component at LHC is only about 3 times smaller than that at top RHIC energies), which is consistent with much stronger values of  $\langle \cos(\Psi_{13} - \Psi_{sp}) \rangle$  in Cu + Au collisions compared to  $Pb + Pb$  collisions as shown in Fig. 3.

In summary, we have analyzed the recent directed flow measurements at midrapidity in  $Pb + Pb$  collisions at LHC and  $Cu + Au$  collisions at RHIC in order to determine the direction of flow of the spectator nucleons. We conclude that all the measurements strongly support the picture of spectators flowing outward from the collision center line.

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- [4] S. A. Voloshin, Polarized secondary particles in unpolarized high energy hadron-hadron collisions?, [arXiv:nucl-th/0410089](http://arxiv.org/abs/arXiv:nucl-th/0410089) (unpublished).
- [5] B. I. Abelev *et al.* (STAR Collaboration), Global polarization measurement in Au+Au collisions, [Phys. Rev. C](http://dx.doi.org/10.1103/PhysRevC.76.024915) **[76](http://dx.doi.org/10.1103/PhysRevC.76.024915)**, [024915](http://dx.doi.org/10.1103/PhysRevC.76.024915) [\(2007\)](http://dx.doi.org/10.1103/PhysRevC.76.024915).
- [6] R. C. Lemmon *et al.*, Direct Observation of the Inversion of Flow, [Phys. Lett. B](http://dx.doi.org/10.1016/S0370-2693(98)01545-7) **[446](http://dx.doi.org/10.1016/S0370-2693(98)01545-7)**, [197](http://dx.doi.org/10.1016/S0370-2693(98)01545-7) [\(1999\)](http://dx.doi.org/10.1016/S0370-2693(98)01545-7).
- [7] D. Krofcheck *et al.*, Disappearance of Flow in Heavy-Ion Collisions, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.63.2028) **[63](http://dx.doi.org/10.1103/PhysRevLett.63.2028)**, [2028](http://dx.doi.org/10.1103/PhysRevLett.63.2028) [\(1989\)](http://dx.doi.org/10.1103/PhysRevLett.63.2028).

- <span id="page-3-0"></span>[8] C. A. Ogilvie et al., Disappearance of flow and its relevance to nuclear matter physics, [Phys. Rev. C](http://dx.doi.org/10.1103/PhysRevC.42.R10) **[42](http://dx.doi.org/10.1103/PhysRevC.42.R10)**, [R10](http://dx.doi.org/10.1103/PhysRevC.42.R10) [\(1990\)](http://dx.doi.org/10.1103/PhysRevC.42.R10).
- [9] U. W. Heinz and P. F. Kolb, Rapidity dependent momentum anisotropy at RHIC, [J. Phys. G](http://dx.doi.org/10.1088/0954-3899/30/8/096) **[30](http://dx.doi.org/10.1088/0954-3899/30/8/096)**, [S1229](http://dx.doi.org/10.1088/0954-3899/30/8/096) [\(2004\)](http://dx.doi.org/10.1088/0954-3899/30/8/096).
- [10] D. Teaney and L. Yan, Triangularity and dipole asymmetry in heavy ion collisions, [Phys. Rev. C](http://dx.doi.org/10.1103/PhysRevC.83.064904) **[83](http://dx.doi.org/10.1103/PhysRevC.83.064904)**, [064904](http://dx.doi.org/10.1103/PhysRevC.83.064904) [\(2011\)](http://dx.doi.org/10.1103/PhysRevC.83.064904).
- [11] S. Voloshin and Y. Zhang, Flow study in relativistic nuclear collisions by Fourier expansion of azimuthal particle distributions, [Z. Phys. C](http://dx.doi.org/10.1007/s002880050141) **[70](http://dx.doi.org/10.1007/s002880050141)**, [665](http://dx.doi.org/10.1007/s002880050141) [\(1996\)](http://dx.doi.org/10.1007/s002880050141).
- [12] B. Alver and G. Roland, Collision-geometry fluctuations and triangular flow in heavy-ion collisions, [Phys. Rev. C](http://dx.doi.org/10.1103/PhysRevC.81.054905) **[81](http://dx.doi.org/10.1103/PhysRevC.81.054905)**, [054905](http://dx.doi.org/10.1103/PhysRevC.81.054905) [\(2010\)](http://dx.doi.org/10.1103/PhysRevC.81.054905); Erratum: Collision-geometry fluctuations and triangular flow in heavy-ion collisions, **[82](http://dx.doi.org/10.1103/PhysRevC.82.039903)**, [039903\(E\)](http://dx.doi.org/10.1103/PhysRevC.82.039903) [\(2010\)](http://dx.doi.org/10.1103/PhysRevC.82.039903).
- [13] J. Noronha-Hostler, L. Yan, F. G. Gardim, and J. Y. Ollitrault, Linear and cubic response to the initial eccentricity in heavy-ion collisions, [Phys. Rev. C](http://dx.doi.org/10.1103/PhysRevC.93.014909) **[93](http://dx.doi.org/10.1103/PhysRevC.93.014909)**, [014909](http://dx.doi.org/10.1103/PhysRevC.93.014909) [\(2016\)](http://dx.doi.org/10.1103/PhysRevC.93.014909).
- [14] B. Alver *et al.* (PHOBOS Collaboration), System Size, Energy, Pseudorapidity, and Centrality Dependence of Elliptic Flow, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.98.242302) **[98](http://dx.doi.org/10.1103/PhysRevLett.98.242302)**, [242302](http://dx.doi.org/10.1103/PhysRevLett.98.242302) [\(2007\)](http://dx.doi.org/10.1103/PhysRevLett.98.242302).
- [15] S. A. Voloshin, Testing the Chiral Magnetic Effect with Central U+U Collisions, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.105.172301) **[105](http://dx.doi.org/10.1103/PhysRevLett.105.172301)**, [172301](http://dx.doi.org/10.1103/PhysRevLett.105.172301) [\(2010\)](http://dx.doi.org/10.1103/PhysRevLett.105.172301).
- [16] B. Abelev et al. (ALICE Collaboration), Directed Flow of Charged Particles at Midrapidity Relative to the Spectator Plane in Pb-Pb Collisions at  $\sqrt{s_{NN}} = 2.76$  TeV, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.111.232302) [111](http://dx.doi.org/10.1103/PhysRevLett.111.232302), [232302](http://dx.doi.org/10.1103/PhysRevLett.111.232302) [\(2013\)](http://dx.doi.org/10.1103/PhysRevLett.111.232302).
- [17] A. Adare *et al.* (PHENIX Collaboration), Measurements of directed, elliptic, and triangular flow in  $Cu + Au$  collisions at  $\sqrt{s_{NN}}$  = 200 GeV, [arXiv:1509.07784.](http://arxiv.org/abs/arXiv:1509.07784)<br>T. Niida *et al.* (STAR Collaboration),
- [18] T. Niida *et al.* (STAR Collaboration), Chargedependent anisotropic flow in  $Cu + Au$  collisions, [arXiv:1601.01017.](http://arxiv.org/abs/arXiv:1601.01017)