Theoretical study of the $\alpha + d \rightarrow {}^6\mathrm{Li} + \gamma$ astrophysical capture process in a three-body model

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The astrophysical capture process $\alpha + d \rightarrow {}^6\mathrm{Li}$ is studied in a three-body model. The initial state is factorized into the deuteron bound state and the $(\alpha + d)$-scattering state. The final nucleus ${}^6\mathrm{Li}(1^+)$ is described as a three-body bound state $\alpha + n + p$ in the hyperspherical Lagrange-mesh method. The contribution of the $E1$-transition operator from the initial isosinglet states to the isotriplet components of the final state is estimated to be negligible. An estimation of the forbidden $E1$ transition to the isosinglet components of the final state is comparable with the corresponding results of the two-body model. However, the contribution of the $E2$-transition operator is found to be much smaller than the corresponding estimations of the two-body model. The three-body model perfectly matches the new experimental data of the LUNA Collaboration with the spectroscopic factor of 2.586 estimated from the bound-state wave functions of ${}^6\mathrm{Li}$ and a deuteron.

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I. INTRODUCTION

In the Big Bang nucleosynthesis (BBN) model of the Universe estimations of the primordial abundance of the light ${}^2\mathrm{H}$, ${}^3\mathrm{He}$, and ${}^4\mathrm{He}$ nuclei are in very good agreement with astrophysical observations [1]. However, the situation is very different for the primordial abundance of the ${}^6\mathrm{Li}$ and ${}^7\mathrm{Li}$ nuclei [2–6]. Recent observations of ${}^6\mathrm{Li}$ in metal-poor stars [3] suggest a large production of this isotope. The data for the ${}^6\mathrm{Li}/{}^7\mathrm{Li}$ ratio of about 0.05 is almost three orders of magnitude larger than estimations from the BBN model [7]. Understanding of this phenomenon is one of the open problems in nuclear astrophysics.

In BBN the light ${}^6\mathrm{Li}$ nucleus is produced mainly through the radiative capture process,

$$\alpha + d \rightarrow {}^6\mathrm{Li} + \gamma, \tag{1}$$

at low energies within the range of $50 \leqslant E_{\mathrm{cm}} \leqslant 400$ keV [7]. This process was experimentally studied in detail at energies around the 3^+ resonance of $E_{\mathrm{cm}} = 0.711$ MeV and above [8,9]. Until recently the direct measurement of the cross section of the process at low energies was not possible due to serious experimental difficulties [10,11]. In Ref. [11] breakup of the ${}^6\mathrm{Li}$ nucleus in the field of heavy-ion ${}^{208}\mathrm{Pb}$ was studied with the aim to extract data on the cross section of the inverse process at astrophysical energies in laboratory conditions. However, dominance of the nuclear breakup over the Coulomb-induced process did not allow implementing this idea. The LUNA Collaboration has recently reported new data at two astrophysical energies of $E = 94$ and $E = 134$ keV [12]. The results turn out to be much lower than the old data from Ref. [10]. Recently in Ref. [13] a way to improve the accuracy of the direct experiment has been proposed based on the photon angular distribution calculated in the potential model. The results provide the best kinematic conditions for the measurement of the ${}^2\mathrm{H}(\alpha,\gamma){}^6\mathrm{Li}$ reaction.

From the theoretical side, different two-body and three-body potential models [14–21] and *ab initio* approaches [22] have been developed. These studies have demonstrated that the main contribution to the process at energies around and beyond the 3^+ resonance comes from the $E2$ transition. However, at low astrophysical energies the situation is different. Here the dominant contribution comes from the $E1$-transition operator. The most realistic two-body model of Ref. [19] is based on the well-known asymptotic form of the two-body $\alpha + d$ bound-state wave function at low energies and a complicated potential derived from the original Woods-Saxon potential via the integrodifferential transformation at higher energies. Recently these results have been reproduced with a much simpler $\alpha - d$ potential of the Gaussian form describing both bound-state [asymptotic normalization coefficient (ANC), binding energy] and scattering-state (phase shifts in the S, P, and D waves) properties [21] of the $\alpha + d$ system.

On the other hand, in the two-body models the $E1$ transition is forbidden by the isospin-selection rule since both initial and final states are isospin singlet. To overcome this problem, an appropriate correction to the $E1$-transition operator was introduced to take into account the difference between mass of the α particle and twice the deuteron mass. Without this correction the $E1$ transition does not contribute to the S factor of the process. However, this drawback has been common for all the models developed so far.

There is another possible development for the estimation of the $E1$- and $E2$-transition matrix elements for the ${}^4\mathrm{He}(d,\gamma){}^6\mathrm{Li}$ capture process. In realistic three-body models the $E1$ transition is allowed from the initial $T_i = 0$ states to the $T_f = 1$ components of the final ${}^6\mathrm{Li}(1^+)$ bound state of the $\alpha + n + p$ system. Indeed, the ground state of the ${}^6\mathrm{Li}$ nucleus contains a small isospin-triplet component. The norm square of this component of the three-body wave function in hyperspherical coordinates [23,24] is about 1.13×10^{-5}.

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However, it still can make some additional contribution to the process.

The aim of the present study is to estimate the $E1$- and E2-transition contributions to the S factor of the aforementioned process in a three-body model. The initial three-body wave function is factorized into the deuteron bound state and the $(\alpha + d)$-scattering wave functions. The final ^{6}Li(1^+) state is described as an $\alpha + p + n$ three-body bound system. The hyperspherical wave function on the Lagrange-mesh basis available for the ^{6}Li(1^+) bound state [23,24] will be used.

In Sec. II we describe the model, in Sec. III we discuss obtained numerical results, and finally, in the last section we make conclusions.

II. THEORETICAL MODEL

A. Cross sections of the radiation capture process

The cross sections of the radiative capture process read

$$\sigma_E(\lambda) = \sum_{J_i T_i \pi_i} \sum_{J_f T_f \pi_f} \sum_{\Omega\lambda} \frac{(2J_f+1)}{[I_1][I_2]} \frac{32\pi^2(\lambda+1)}{\hbar\lambda([\lambda]!!)^2} k_\gamma^{2\lambda+1} C_S^2 \times \sum_{l_\omega I_\omega} \frac{1}{k_\omega^2 v_\omega} \left|\left\langle \Psi^{J_f T_f \pi_f} \middle\| M_\lambda^\Omega \middle\| \Psi^{J_i T_i \pi_i}_{l_\omega I_\omega} \right\rangle\right|^2, \tag{2}$$

where $\Omega = E$ or M (electric or magnetic transition), ω denotes the entrance channel, k_ω, v_ω, and I_ω are the wave number, velocity of the $\alpha - d$ relative motion, and the spin of the entrance channel, respectively, J_f, T_f, π_f, and (J_i,T_i,π_i) are the spin, isospin, and parity of the final (initial) state, I_1,I_2 are channel spins, and $k_\gamma = E_\gamma/\hbar c$ is the wave number of the photon corresponding to the energy $E_\gamma = E_{\rm th} + E$ with the threshold energy of $E_{\rm th} = 1.474$ MeV. The wave functions $\Psi^{J_i T_i \pi_i}_{l_\omega I_\omega}$ and $\Psi^{J_f T_f \pi_f}$ present the initial and final states, respectively. They are given in a common form for both two-body and three-body models. The reduced matrix elements are evaluated between the initial and the final states. The constant C_S^2 is the spectroscopic factor [25]. We also use shorthand notations $[I] = 2I + 1$ and $[\lambda]!! = (2\lambda + 1)!!$.

The electric-transition operator in the Jacobi coordinates can be written as [23]

$$\begin{aligned} M^E_{\lambda\mu}(\vec{x},\vec{y}) &= e\left[\hat{Z}_{12}\left(\frac{-A_3}{A}\right)^\lambda + \hat{Z}_3\left(\frac{A_{12}}{A}\right)^\lambda\right] M^E_{\lambda\mu}(\vec{y}) \\ &+ e\left[\hat{Z}_1\left(\frac{-A_2}{A_{12}}\right)^\lambda + \hat{Z}_2\left(\frac{A_1}{A_{12}}\right)^\lambda\right] M^E_{\lambda\mu}(\vec{x}) \\ &+ e\sum_{k>0}^{\lambda-1} \alpha_{\lambda k}\left(\frac{-A_3}{A}\right)^k \left[\hat{Z}_1\left(\frac{-A_2}{A_{12}}\right)^{\lambda-k} \right. \\ &\left. + \hat{Z}_2\left(\frac{A_1}{A_{12}}\right)^{\lambda-k}\right] \left\{M^E_k(\vec{y}) \otimes M^E_{\lambda-k}(\vec{x})\right\}_{\lambda\mu}, \end{aligned} \tag{3}$$

with

$$M^E_{\lambda\mu}(\vec{x}) = \left(\frac{x}{\sqrt{\mu_{12}}}\right)^\lambda \quad Y_{\lambda\mu}(\hat{x}) \equiv r^\lambda Y_{\lambda\mu}(\hat{r}), \tag{4}$$

$$M^E_{\lambda\mu}(\vec{y}) = \left(\frac{y}{\sqrt{\mu_{12}}}\right)^\lambda \quad Y_{\lambda\mu}(\hat{y}) \equiv R^\lambda Y_{\lambda\mu}(\hat{R}), \tag{5}$$

and

$$\alpha_{\lambda k} = \left(\frac{4\pi[\lambda]!}{[k]![\lambda-k]!}\right)^{1/2}, \tag{6}$$

where $\frac{1}{\mu_{12}} = \frac{1}{A_1} + \frac{1}{A_2}$ and $\frac{1}{\mu_{(12)3}} = \frac{1}{A_{12}} + \frac{1}{A_3}$ are the reduced masses. The Jacobi coordinates $\boldsymbol{x}$ (between the proton and the neutron), $\boldsymbol{y}$ (between the $p + n$ and the α particle), and relative $\boldsymbol{r},\boldsymbol{R}$ coordinates are related as

$$\boldsymbol{x} = \sqrt{\mu_{12}}\boldsymbol{r}, \quad \boldsymbol{y} = \sqrt{\mu_{(12)3}}\boldsymbol{R}. \tag{7}$$

B. Wave functions

In the present three-body model the initial state is factorized as

$$\begin{aligned} \Psi_i^{J'M',T'0}(\vec{x},\vec{y}) &= \frac{u^d_{l'}(r)}{r}\frac{u_{L'}(R)}{R} \\ &\times \left\{Y_{L'}(\hat{y}) \otimes \{Y_{l'}(\hat{x}) \otimes \chi_{s'}(1,2)\}_{j'}\right\}_{J'M'} \\ &\times \zeta^{T',0}_{1/2,1/2}(1,2), \end{aligned} \tag{8}$$

where s' and L' are the spin and orbital angular momenta of the entrance channel, respectively, and l' is the orbital angular momentum of the deuteron. Although in the present study we restrict ourselves to the S-wave component of the deuteron, and hence the quantum numbers $s' = 1$ and $l' = 0$ are fixed, we aim to derive the analytical expressions of the matrix elements for a general case of arbitrary s' and l'. In addition, $u^d_{l'}(r)$ is the radial wave function of the deuteron, and $u_{L'}(R)$ is the scattering wave function of the $\alpha - d$ pair. The latter asymptotically behaves as

$$u_{L'}(R) \underset{R\to\infty}{\to} F_{L'}(k_\omega R)\cos\,\delta_{L'}(E) + G_{L'}(k_\omega R)\sin\,\delta_{L'}(E), \tag{9}$$

where $F_{L'}$ and $G_{L'}$ are Coulomb functions and $\delta_{L'}(E)$ is the phase shift in the L' wave at energy E. The parity of the state is defined from the intrinsic parities of the α particle and deuteron, which are positive, and the orbital momentum L'.

The spin and isospin wave functions of the two nucleons as a bound state of the deuteron read, respectively,

$$\chi_{s'm'}(1,2) = \{\chi_{1/2}(1) \otimes \chi_{1/2}(2)\}_{s'm'}, \tag{10}$$

and

$$\zeta^{T',0}_{1/2,1/2}(1,2) = \{\zeta_{1/2}(1) \otimes \zeta_{1/2}(2)\}_{T',0}. \tag{11}$$

The antisymmetry condition requires $S' + T' + l'$ to be odd. Since for the deuteron $l' = 0$ and $S' = 1$, the initial three-body system is in the isosinglet state $T' = 0$. The final three-body wave function of the ^{6}Li$(1^+,0)$ ground state in the hyperspherical basis reads as

$$\Psi_f^{JM,T0}(\vec{x},\vec{y}) = \frac{1}{\rho^{5/2}}\sum_{\gamma,k}\chi_{\gamma k}(\rho)\left\{\mathcal{Y}^L_{l_x l_y}(\hat{x},\hat{y}) \otimes \chi^S(\vec{\xi})\right\}_{JM} \times \Phi_k^{l_x l_y}(\alpha)\zeta^{T,0}_{1/2,1/2}(1,2), \tag{12}$$

where ρ (hyperradius) and α (hyperangle) are defined as

$$\rho^2 = x^2 + y^2, \quad \alpha = \arctan(y/x). \tag{13}$$

Hyperangle α varies between 0 and $\pi/2$. The hyperspherical harmonics are defined as [23,24]

$$\Phi_k^{l_x l_y}(\alpha) = N_k^{l_x l_y}(\cos\,\alpha)^{l_x}(\sin\,\alpha)^{l_y} P_n^{l_y+1/2,l_x+1/2}(\cos\,2\alpha), \tag{14}$$

where $P_n^{l_y+1/2,l_x+1/2}(\cos\,2\alpha)$ are the Jacobi polynomials and $N_k^{l_x l_y}$ is the normalization factor (see Ref. [23] for details).

The astrophysical S factor of the process is expressed in terms of the cross section as [26]

$$S(E) = E\sigma_E(\lambda)\exp(2\pi\eta), \tag{15}$$

where η is the Coulomb parameter.

C. Isospin transition-matrix elements

We rewrite the charge operators of the proton and neutron in Eq. (3) with the help of the isospin operators as

$$\hat{Z}_1 = \tfrac{1}{2} + \hat{m}_{t1}, \quad \hat{Z}_2 = \tfrac{1}{2} + \hat{m}_{t2}. \tag{16}$$

Then the matrix element of the isospin operator,

$$\hat{T}_y = \left[\left(\frac{1}{2} + \hat{m}_{t1}\right) + \left(\frac{1}{2} + \hat{m}_{t2}\right)\right]\left(-\frac{A_3}{A}\right)^{\lambda} + Z_3\left(\frac{A_{12}}{A}\right)^{\lambda}, \tag{17}$$

of the first term in the Eq. (3) between the initial and the final three-body isospin wave functions reads as

$$\langle \zeta_{1/2,1/2}^{T,0}|\hat{T}_y|\zeta_{1/2,1/2}^{T',0}\rangle = \left[\left(-\frac{A_3}{A}\right)^{\lambda} + Z_3\left(\frac{A_{12}}{A}\right)^{\lambda}\right]\delta_{T,T'}. \tag{18}$$

The matrix element of the second isospin operator,

$$\hat{T}_x = \left(\frac{1}{2} + \hat{m}_{t1}\right)\left(-\frac{A_2}{A_{12}}\right)^{\lambda} + \left(\frac{1}{2} + \hat{m}_{t2}\right)\left(\frac{A_1}{A_{12}}\right)^{\lambda} \tag{19}$$

can be evaluated using the angular momentum algebra,

$$\begin{aligned}\langle \zeta_{1/2,1/2}^{T,0}|\hat{T}_x|\zeta_{1/2,1/2}^{T',0}\rangle = {}& \frac{1}{2}\left[\left(-\frac{A_2}{A_{12}}\right)^{\lambda} + \left(\frac{A_1}{A_{12}}\right)^{\lambda}\right]\delta_{T,T'} \\ &+ \frac{1}{2}\left[\left(-\frac{A_2}{A_{12}}\right)^{\lambda} - \left(\frac{A_1}{A_{12}}\right)^{\lambda}\right] \\ &\times (\delta_{T,0}\delta_{T',1} + \delta_{T,1}\delta_{T',0}).\end{aligned} \tag{20}$$

The isospin operator in the last term of Eq. (3) is evaluated in the same way as the second term.

From last equation one can note that the $E1$ transition is allowed from the isospin-singlet states to the isospin-triplet components of the final ^{6}Li(1^+) three-body bound state. The spin-angular parts of the matrix elements for the $E1$- and $E2$-transition operators in the three-body model are given in the Appendix.

III. NUMERICAL RESULTS

A. Details of the calculations

The radial wave function $u_{l'}^d(r)$ of the deuteron is the solution of the bound-state Schrödinger equation with the central Minnesota potential V_{NN} [27,28] with $\hbar^2/2m_N = 20.7343$ MeV fm^2. The Schrödinger equation is solved using a highly accurate Lagrange-Laguerre-mesh method [29]. It yields $E_d = -2.202$ MeV for the deuteron ground-state energy with the number of mesh points $N = 40$ and a scaling parameter $h_d = 0.40$.

The scattering wave function $u_L(E,R)$ of the $\alpha - d$ relative motion is calculated as a solution of the Schrödinger equation using the Numerov method with an appropriate potential subject to the boundary condition Eq. (9). In the present study we use the well-known deep potential of Dubovichenko and Dzhazairov-Kakhramanov [30] with a small modification in the S wave [21]: $V_d^{(S)}(R) = -92.44\exp(-0.25R^2)$ MeV. The potential parameters in the $^3P_0, {}^3P_1, {}^3P_2$ and $^3D_1, {}^3D_2, {}^3D_3$ partial waves are the same as in Ref. [30]. The potential contains additional states in the S and P waves forbidden by the Pauli principle. The above modification allows to better describe the phase shifts in the S wave and, most importantly, reproduce the empirical value $C_{\alpha d} = 2.31$ fm$^{-1/2}$ of the ANC of the ^{6}Li(1^+) ground state derived from $\alpha - d$ elastic-scattering data [31].

In order to check the sensitivity of the $E1$- and $E2$-transition-matrix elements on the short-range part of the $\alpha - d$ wave function, we also test the $\alpha - d$ potential V_d^S obtained from the initial V_d potential in the S and P waves by a supersymmetric (SUSY) transformation [32]. The resulting potential gives the same phase shifts and the same ground-state energy as the initial potential. However, the forbidden state is removed, and the role of the Pauli principle is simulated by a short-range core.

The final ^{6}Li(1^+) ground-state wave function was calculated using the hyperspherical Lagrange-mesh method [23,24,33] with the same Minnesota NN potential. For the $\alpha - N$ nuclear interaction the potential of Voronchev *et al.* [34] was employed, which contains a deep Pauli forbidden state in the S wave. The potential was slightly renormalized by a scaling factor of 1.008 to reproduce the experimental binding energy $E_b = 3.70$ MeV. The Coulomb $\alpha - p$ interaction is parametrized as $V_C(r) = 2e^2\,\mathrm{erf}(r/R_C)$ with a radius of $R_C = 1.2$ fm. The Pauli forbidden states in the three-body configuration space are eliminated with the help of the orthogonalizing pseudopotential method [35,36].

The hypermomentum expansion includes terms up to $K_{\max} = 20$, which ensures a good convergence of the energy. The matter rms radius of the ground state (with 1.4 fm for the radius of the α particle) was found as $\sqrt{\langle r\rangle^2} = 2.25$ fm, a value slightly lower than the experimental data (2.32 ± 0.03 fm [37]). The ground state is essentially $S = 1$ (96%). As noted above, the three-body wave function also includes a small isotriplet component $l_x = l_y = S = T = 1$ with the norm

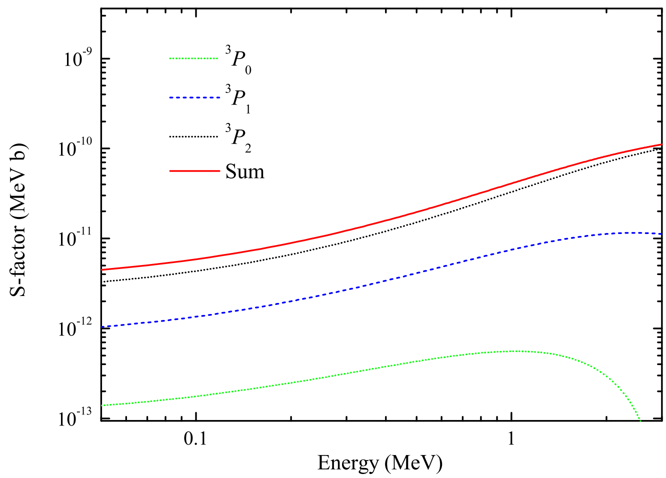

FIG. 1. Contribution of the $E1$-transition operator from the initial isosinglet state to the isotriplet component of the final state for the astrophysical S factor of the capture process $\alpha + d \to {}^6\mathrm{Li} + \gamma$.

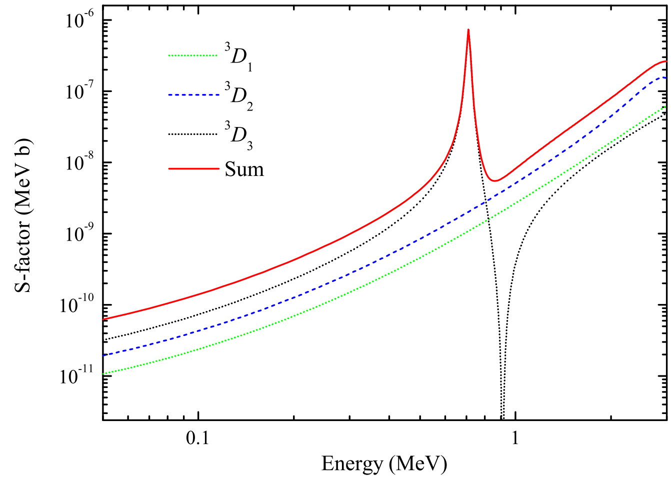

FIG. 3. Contribution of the $E2$-transition operator to the astrophysical S factor of the capture process $\alpha + d \to {}^6\mathrm{Li} + \gamma$.

square 1.13 $\times 10^{-5}$ which can give a contribution to the $E1$-transition-matrix elements.

B. Estimation of the astrophysical S factor

First we estimate the allowed $E1$-transition contribution to the capture process ${}^4\mathrm{He}(d,\gamma)\,{}^6\mathrm{Li}$ in the three-body model when the isospin changes. Here contributions come from the initial ${}^3P_0, {}^3P_1, {}^3P_2$ partial waves and the $l_x = l_y = S = T = 1$ components of the final state. In Fig. 1 we show the corresponding estimation for the astrophysical S factor. As can be seen from the picture the contribution is rather small, which means that the small isotriplet component of the ${}^6\mathrm{Li}(1^+)$ ground state does not make a significant contribution to the capture process. Figure 2 shows the estimated contribution of the $E1$-transition operator to the

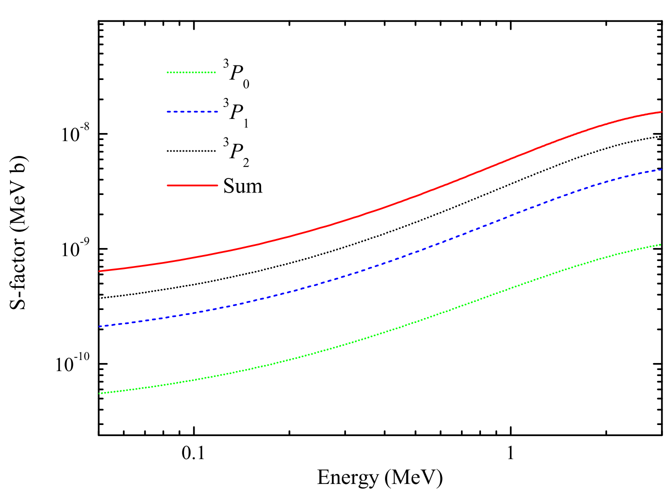

FIG. 2. Contribution of the $E1$-transition operator from the initial isosinglet state to the isotriplet and isosinglet components of the final state for the astrophysical S factor of the capture process $\alpha + d \to {}^6\mathrm{Li} + \gamma$.

astrophysical S factor including the correction to the mass numbers $A_n = 1.008\,664\,915\,97$, $A_n = 1.007\,276\,466\,77$, and $A_3 = 4.001\,506\,179\,127$ a.u. This yields an additional contribution to the S factor, larger than isospin-transition terms in Fig. 1 approximately by two orders of magnitude.

In Fig. 3 the contribution of the $E2$-transition operator to the astrophysical S factor is demonstrated for different initial partial waves ${}^3D_1, {}^3D_2$, and ${}^3D_3$. As can be seen from the figure the estimations are essentially less than the corresponding numbers for the two-body model [21]. The magnitude of underestimation is larger at low astrophysical energies.

Additionally, unlike the two-body model, in the three-body model there is a contribution of the initial ${}^3S_1$ state to the $E2$-transition-matrix elements. However, our numerical study shows this contribution to be very small. For the energy range from 0.1 to 1.0 MeV the S-wave contribution to the astrophysical S factor increases from 1.0×10^{-12} to 2.02×10^{-12} MeV b. This is why we do not show the S-wave contribution in Fig. 3.

We also have tested the SUSY-transformed $\alpha - d$ potentials V_d^S. It turns out that this transformation increases the S-wave contribution to the S factor by about 12%–13% in the energy range from 0.1 to 1.0 MeV. But the total S-wave contribution is still negligible. The SUSY transformation of the P-wave potentials yields a very small increase in the S factor by 0.52%–0.60% in the aforementioned energy range. The situation is different from the β- and $M1$-transition processes [24,33,38] where the main contribution comes from the S-wave $(\alpha - d)$-scattering state, hence a sensitivity of the transition probability to the short-range behavior of the wave function was essential.

Figure 4 demonstrates the convergence of the evaluated S factor in the three-body model for different choices of the number of integration points $N = 300,500,700$ with a fixed step of $h = 0.05$ fm. As one can see, the convergent results are obtained with $N = 500$ mesh points. In Fig. 5 we compare the $E1$- and $E2$-transition components. At low energies the $E1$ transition dominates, and at higher energies the $E2$ component is stronger.

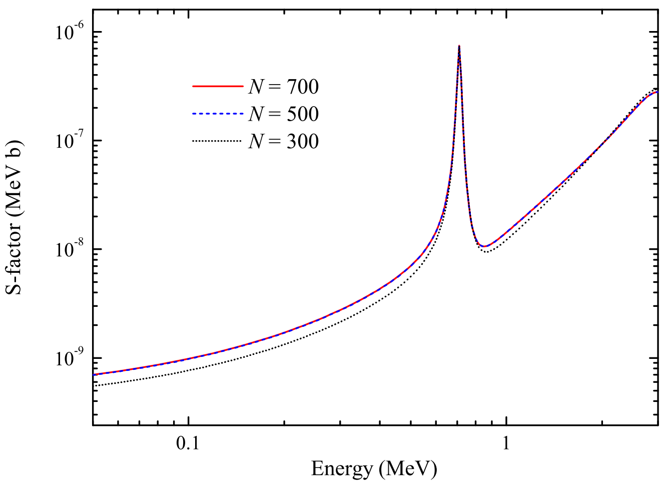

FIG. 4. Convergence of the astrophysical S factor for the capture process $\alpha + d \rightarrow {}^6\mathrm{Li} + \gamma$ with respect to the number of integration points with the fixed step of $h = 0.05$ fm.

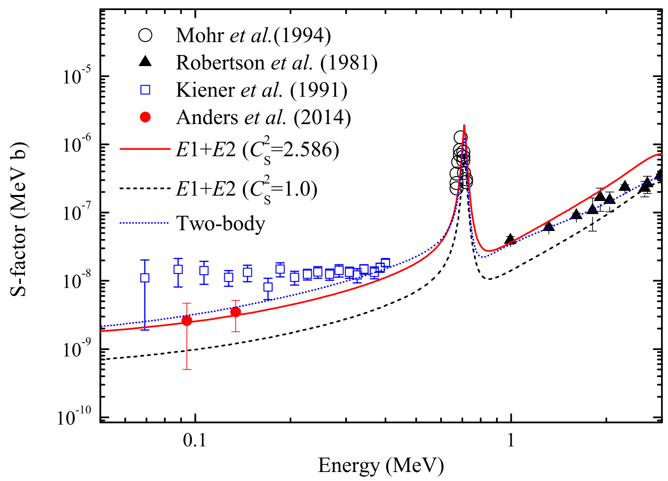

FIG. 6. Comparison of the theoretical estimations obtained in the two- and three-body models for the astrophysical S factor of the capture process $\alpha + d \rightarrow {}^6\mathrm{Li} + \gamma$ with available experimental data.

Finally, in Fig. 6 we compare the obtained theoretical results with the estimations of the two-body model [21] and experimental data from Refs. [8–10,12]. One can see from the figure that the results of the two-body and three-body models differ essentially for the spectroscopic factor of $C_S^2 = 1$. At the resonance energy they differ by a factor of 0.565, which is consistent with the square of the overlap integral $I = 0.748$ of the three-body bound-state wave function with the deuteron and the two-body $\alpha - d$ bound-state wave functions.

We have estimated the integral $P_{\alpha d} = \int |\Psi(\vec{R})|^2 d\vec{R}$ with $\Psi(\vec{R}) = \langle \Psi_3(\vec{r},\vec{R}) | \psi_d(\vec{r}) \rangle$ and found its value to be 0.3867. That yields for the spectroscopic factor an estimation of $C_S^2 = 1/P_{\alpha d} = 2.586$. As was shown in Fig. 6 with this value of the spectroscopic factor the three-body model perfectly describes the new experimental data of the LUNA Collaboration better than the two-body models. Any value of the spectroscopic factor from the interval between 1.50 and 4.25 is able to describe these data within the error bar.

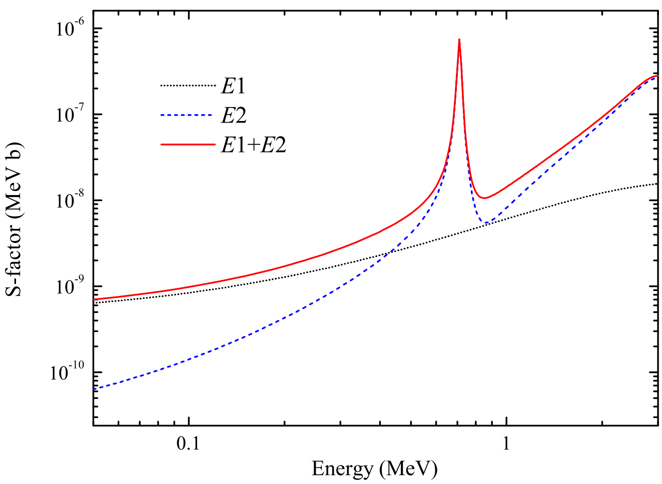

FIG. 5. Comparison of the contributions of the $E1$- and $E2$-transition operators to the astrophysical S factor of the capture process $\alpha + d \rightarrow {}^6\mathrm{Li} + \gamma$.

IV. CONCLUSIONS

The astrophysical capture process $\alpha + d \rightarrow {}^6\mathrm{Li} + \gamma$ has been studied in the three-body model. The contribution of the $E1$-transition operator has been estimated from the initial isosinglet states to the isotriplet components of the final ${}^6\mathrm{Li}(1^+)$ bound state. It is shown that this contribution is small. The most important contribution of the $E1$ transition comes due to the mass difference of the proton and neutron with the violation of the isospin selection rule. The situation is close to the two-body model where the $E1$ transition, forbidden by the isospin selection rule, is only possible due to the mass difference of the α particle and twice the deuteron mass. The three-body model perfectly matches the new experimental data of the LUNA Collaboration with the spectroscopic factor of 2.586 derived from the overlap integral of the ^{6}Li and deuteron bound-state wave functions.

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APPENDIX: SPIN-ANGULAR MATRIX ELEMENTS OF THE $E\lambda$-TRANSITION OPERATOR IN THE THREE-BODY MODEL

The spin-angular matrix elements of the $E\lambda$ transition are given as

$$\langle \psi_f^{JM} | M^E_{\lambda\mu}(\vec{x},\vec{y}) | \psi_i^{J'M'} \rangle = \left\langle \frac{1}{\rho^{5/2}} \sum_{\gamma,k} \chi_{\gamma k}(\rho) \{ Y^L_{l_x l_y}(\hat{x},\hat{y}) \otimes \chi^S(\vec{\xi}) \}_{JM} \Phi_k^{l_x l_y}(\alpha) \middle| M^E_{\lambda\mu}(\vec{x},\vec{y}) \right. \\ \times \left| \frac{u^{pn}_{l'}(r)}{r} \frac{u_{L'}(R)}{R} \{ Y_{L'}(\hat{y}) \otimes \{ Y_{l'}(\hat{x}) \otimes \chi_{s'}(1,2) \}_{j'} \}_{J'M'} \right\rangle, \tag{A1}$$

where

$$M^E_{\lambda\mu}(\vec{x},\vec{y}) = A_x M^E_{\lambda\mu}(\vec{x}) + A_y M^E_{\lambda\mu}(\vec{y}) + \sum_{k>0}^{\lambda-1} A^{(k)}_{xy} \{ M^E_{\lambda-k}(\vec{x}) \otimes M^E_k(\vec{y}) \}_{\lambda\mu}, \tag{A2}$$

and

$$\langle \{ Y^L_{l_x l_y}(\hat{x},\hat{y}) \otimes \chi^S(1,2) \}_{JM} | A^{(k)}_{xy} \{ M^E_{\lambda-k}(\vec{x}) \otimes M^E_k(\vec{y}) \}_{\lambda\mu} | \{ Y_{L'}(\hat{y}) \otimes \{ Y_{l'}(\hat{x}) \otimes \chi_{s'}(1,2) \}_{j'} \}_{J'M'} \rangle \\ = \frac{A^{(k)}_{xy}}{4\pi} \left(\frac{x}{\sqrt{\mu_{12}}} \right)^{\lambda-k} \left(\frac{y}{\sqrt{\mu_{(12)3}}} \right)^k \delta_{ss'} [\sigma][\tau]([k][\lambda-k][\lambda][l'][j'][L'][L][J'])^{1/2} \sum_{\sigma\tau} (-1)^{2J+2M+l_x+l_y+L-\tau+L'-l'-2\sigma} \\ \times C^{l_x 0}_{\lambda-k 0 l' 0} C^{l_y 0}_{k 0 L' 0} \begin{Bmatrix} l_y & k & L' \\ l_x & \lambda-k & l' \\ L & \lambda & \tau \end{Bmatrix} \begin{Bmatrix} S & L & J \\ l' & \tau & L' \\ j' & \lambda & \sigma \end{Bmatrix} \begin{Bmatrix} \sigma & j' & \lambda \\ J' & J & L' \end{Bmatrix} C^{JM}_{J'M'\lambda\mu}. \tag{A3}$$

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