Multipolarity of the $2^- \rightarrow 1^-$, ground-state transition in ^{210}Bi via multivariable angular correlation analysis

N. Cieplicka-Oryńczak,[1,2] B. Szpak,[2] S. Leoni,[1,3] B. Fornal,[2] D. Bazzacco,[4] A. Blanc,[5] G. Bocchi,[1,3] S. Bottoni,[1,3] G. de France,[6] M. Jentschel,[5] U. Köster,[5] P. Mutti,[5] G. Simpson,[7] T. Soldner,[5] C. Ur,[8] and W. Urban[9]

[1]*Istituto Nazionale di Fisica Nucleare Sezione di Milano, Via Celoria 16, 20133 Milano, Italy*
[2]*Institute of Nuclear Physics, Polish Academy of Sciences, PL-31342 Kraków, Poland*
[3]*Università degli Studi di Milano, Via Celoria 16, 20133 Milano, Italy*
[4]*Dipartimento di Fisica e Astronomia dell'Università and Istituto Nazionale di Fisica Nucleare Sezione di Padova, I-35131 Padova, Italy*
[5]*Institut Laue-Langevin, 71 Avenue des Martyrs, 38042 Grenoble Cedex 9, France*
[6]*GANIL, Boulevard Becquerel, BP 55027, 14076 CAEN Cedex 05, France*
[7]*LPSC, Université Joseph Fourier Grenoble 1, Centre National de la Recherche Scientifique/IN2P3, Institut National Polytechnique de Grenoble, F-38026 Grenoble Cedex, France*
[8]*Istituto Nazionale di Fisica Nucleare Sezione di Padova, Via F. Marzolo 8, 35131 Padova, Italy*
[9]*Faculty of Physics, University of Warsaw, ul. Pasteura 5, PL-02-093 Warszawa, Poland*

(Received 23 March 2016; revised manuscript received 23 May 2016; published 14 July 2016)

The multipolarity of the main transition leading to the ground state in ^{210}Bi was investigated using the angular correlations of γ rays. The analyzed γ-coincidence data were obtained from the ^{209}Bi$(n,\gamma)^{210}$Bi experiment performed at Institut Laue-Langevin Grenoble at the PF1B cold-neutron facility. The EXILL (EXOGAM at the ILL) multidetector array, consisting of 16 high-purity germanium detectors, was used to detect γ transitions. The mixing ratio of the 320-keV γ ray was defined by minimizing a multivariable χ^2_Σ function constructed from the coefficients of angular correlation functions for seven pairs of strong transitions in ^{210}Bi. As a result, the almost pure $M1$ multipolarity of the 320-keV γ ray was obtained, with an $E2$ admixture of less than 0.6% only (95% confidence limit). Based on this multipolarity the neutron-capture cross section leading to the ground state in ^{210}Bi, that decays in turn to radiotoxic ^{210}Po, was determined to be within the limits 21.3(9) and 21.5(9) mb. This result is important for nuclear reactor applications.

DOI: 10.1103/PhysRevC.94.014311

I. INTRODUCTION

The ^{210}Bi nucleus has one valence proton and one valence neutron with respect to the doubly magic ^{208}Pb core. This makes it an excellent test case for studying nuclear structure properties around the closed shells. Moreover, due to the fact that Pb-Bi can be used as a coolant material in fast reactor systems or as a spallation neutron-production target in accelerator driven systems, the studies of the ^{209}Bi$(n,\gamma)^{210}$Bi reaction are very important for applications as well.

The ground state of ^{210}Bi was proven to have spin parity $J^\pi = 1^-$ and to belong to the fully identified multiplet $\pi h_{9/2}\nu g_{9/2}$ [1], while the member of this multiplet with maximum spin, $J^\pi = 9^-$, lies at 271 keV and is the second excited state in this nuclide. Due to the large spin difference, the 9^- excitation is a long-lived, α-decaying isomer with $T_{1/2} = 3.04 \times 10^6$ yr. ^{210}Bi can be populated by the neutron-capture process in a state at 4605 keV which then decays by γ-ray cascades to the 1^- ground state or to the long-lived isomeric 9^- excitation. The cross section for production of these two states is of primary interest for assessing the amount of long-term waste production when bismuth is used in the coolant of nuclear reactors. While the radioactive decay chain starting from the ground state (with the half-life of 5.013 d), which includes the ^{210}Po nucleus with $T_{1/2} = 138$ d, contributes to short-term radiotoxicity, the decay of the 9^- isomer with the half-life of 3.04×10^6 yr with emission of α particle is a source of long-term radiotoxicity. This issue was investigated earlier, and the value of the neutron-capture cross section leading to the isomeric state was established as 17.7(7) mb [2]. In contrast, the neutron-capture cross section for the ground-state population is more difficult to obtain, because it relies on the precise knowledge of the conversion coefficient of the main ground-state transition, $2^- \rightarrow 1^-$, with energy of 320 keV. This conversion coefficient depends strongly on the $M1/E2$ multipolarity mixing of the 320-keV line, which so far has not been measured with sufficient precision. According to Ref. [2], when one assumes the following mixing values—pure $M1$, 50% $M1$ + 50% $E2$, or pure $E2$—the final neutron-capture cross section to the ground state results in 21.5(9), 19.3(8), and 17.2(7) mb, respectively [2].

In the past, based on theoretical considerations it was suggested that this 320-keV transition could be of almost pure $M1$ character. This, however, has not been confirmed experimentally and, in the present work, we made the effort of measuring the multipolarity mixing for the 320-keV line with high accuracy. To this end, we employed the γ angular correlation technique using the EXILL (EXOGAM at the ILL) multidetector Ge array localized at Institut Laue-Langevin (ILL) in Grenoble (France). The analysis was based on the minimization of the multivariable χ^2_Σ function constructed from the experimental angular correlation coefficients of seven pairs of strong transitions in ^{210}Bi. In Sec. II we discuss the experimental setup, the basics of theory of γ angular correlations and the experimental data analysis. Section III is devoted to the construction of the multivariable χ^2_Σ function and its minimization. Conclusions are given in Sec. IV.

II. EXPERIMENTAL METHODS

A. Experimental setup

^{210}Bi was populated in a cold-neutron capture reaction performed at the Institut Laue-Langevin (Grenoble) using the PF1B cold-neutron facility [3]. After collimation to a halo-free pencil beam, the capture flux on target was 10^8 neutrons/(s $\times$ cm^2). The target, made of ^{209}Bi pieces (total weight of 3 g), was placed in the center of the EXILL array. This detection system consisted of 16 high-purity germanium (HPGe) detectors: eight EXOGAM (GAMma-ray from EXOtic nuclei spectrometer) clovers [4], six GASP (GAmma-ray SPectrometer) detectors [5] with their bismuth germanium oxide (BGO) anti-Compton shields, as well as two clovers from the ILL LOHENGRIN instrument. The eight detectors of EXOGAM were arranged into one ring in octagonal geometry around the target at every 45° in a plane perpendicular with respect to the beam. Digital electronics was used to collect and process the signals from the detectors and the data were stored triggerless. The events contained information about energy and time of the registered γ rays (with a time stamp every 10 ns) as well as the identification number of the specific detector that fired [6]. A detailed description of the experimental setup is presented in Ref. [7]. Based on the data collected in this experiment, the decay scheme of ^{210}Bi from the capture states was established and the angular correlations of the strongest γ rays were used to assign spins and parities of several populated levels. The latter analysis was performed making use of three $\gamma\gamma$-coincidence matrices constructed for the angles 0, 45, and 90° between the EXOGAM detectors [8]. In the following we describe the angular correlation analysis, focusing on the 320-keV transition in ^{210}Bi.

B. Angular correlations—theory

The angular correlations of γ rays allow us to study the anisotropy in the emission of γ rays with respect to the nuclear spin direction. The angular correlation function is usually expressed as the sum of Legendre polynomials $P_m(\cos\theta)$ parametrized by the coefficients A_2 and A_4 [9,10]:

$$W(\theta) = A_0[1 + A_2P_2(\cos\theta) + A_4P_4(\cos\theta)], \qquad (1)$$

where A_0 is the normalization coefficient, while A_m ($m = 2,4$) are the products of two coefficients $A_m(1)$ and $A_m(2)$ depending on the character of two considered transitions:

$$A_m = q_m A_m(1)A_m(2), \quad \text{with} \quad m = 2,4 \qquad (2)$$

while q_m are attenuation coefficients associated with the finite solid angle of the detectors (geometry and size of the crystal, distance from the radiation source). The q_m coefficients may be calculated from theory or extracted from experimental data. From the coefficients A_2 and A_4 one may deduce the multipole orders of the transitions, the spins of involved nuclear states, and the mixing ratios. If a γ ray is a mixture of two multipoles, then the equation for the $A_m(k)$ coefficients (where $k = 1,2$ indicates the first and second transition in the investigated pair) includes also the mixing ratio parameter δ:

$$A_m(k) = \frac{1}{1+\delta_k^2}\big[F_m(j_kL_kL_kj) + 2\delta_kF_m(j_kL_kL'_kj) + \delta_k^2F_m(j_kL'_kL'_kj)\big]. \qquad (3)$$

The F_m functions are tabulated [11]. The ratio of the intensities I_{L_k} and $I_{L'_k}$ of L pole to L' pole radiation defines the mixing ratio:

$$\delta_k^2 \equiv \frac{I_{L'_k}}{I_{L_k}}. \qquad (4)$$

The analysis performed in cases of having nonstretched transitions requires the minimization of the χ^2 cost function:

$$\chi^2 = \left(\frac{A_2 - A_2^{\text{theor}}(\delta_k)}{\Delta A_2}\right)^2 + \left(\frac{A_4 - A_4^{\text{theor}}(\delta_k)}{\Delta A_4}\right)^2. \qquad (5)$$

In the previous expression, the A_m^{theor} coefficients are calculated under a particular hypothesis for the spins and multipolarities as a function of δ mixing ratio. The A_m coefficients are the experimental results obtained by fitting the data with the function given in Eq. (1). The minimum of the χ^2 function defines the most probable value of the δ_k parameter.

The analysis presented in this work was based on double coincidence data sorted into three matrices corresponding to average angles between EXOGAM detectors: 180° (corresponding to 0° in the following analysis), 45° (and 135°), and 90°. The clovers of EXOGAM were used as single detectors, including add-back between individual Ge crystals. The normalization factors for the number of counts in a given peak of a particular matrix were then applied, depending on the number of detector pairs available in each of the three groups (4, 16, and 8 for the angles 0, 45, and 90°, respectively). Moreover, the attenuation coefficients q_m were extracted by using cascades from ^{60}Co and ^{152}Eu sources with well-defined anisotropies [$q_2 = 0.86(2)$ and $q_4 = 0.60(3)$] [7].

C. Angular correlations—experimental results

The complete information about the order of multipole and possible mixing ratio of a given γ ray can be obtained directly from angular correlations only if the second transition from the investigated pair is pure or its mixing ratio is known. The type of multipole (electric or magnetic) and, as a result, the change of parity cannot, however, be defined—for this, one must use a complementary method or deduce it from the decay pattern. In ^{210}Bi, the knowledge of the transitions multipolarities is rather scarce and it is not possible to find a transition with firm multipolarity assignment in coincidence with the 320-keV line. As the first step in determining the multipolarity mixing in the 320-keV transition, one can consider the 674–320-keV cascade, depopulating the level at 993 keV. This is the strongest cascade leading to the ground state in ^{210}Bi (Fig. 1). In Fig. 2 we show the angular correlation function for those two γ rays. From the fit to the experimental points (red curve), we obtained the following values of the coefficients: $A_2 = 0.03(1)$ and $A_4 = -0.01(2)$. They are

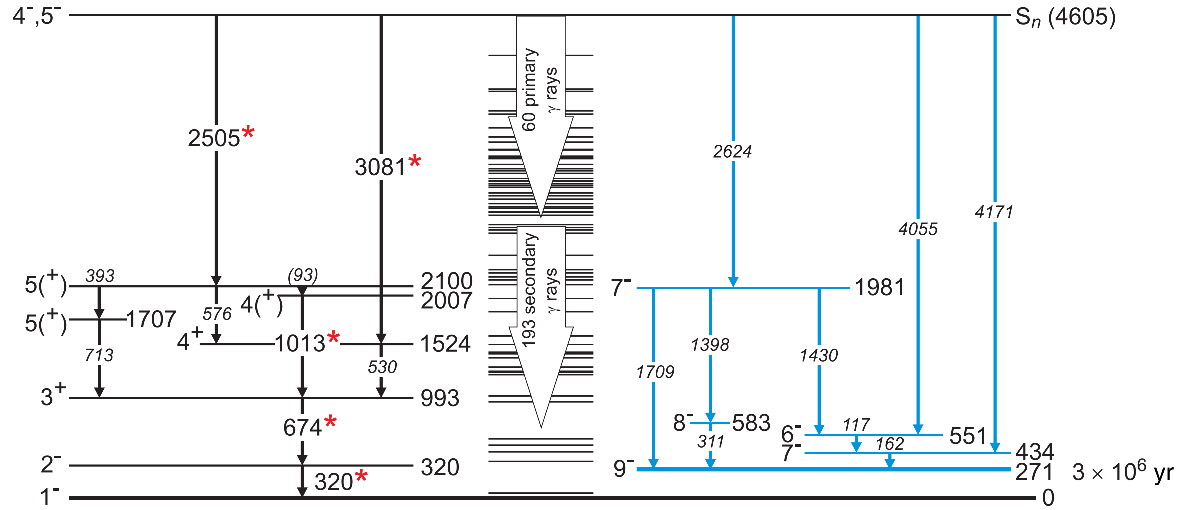


FIG. 1. Partial level scheme of ^{210}Bi produced in cold-neutron capture reaction. The left side shows the strongest decay cascades leading to the ground state, with transitions used in the analysis marked by stars. The right side, marked in blue, represents the partial decay scheme leading from the capture state at neutron binding energy S_n to the long-lived isomer at energy 271 keV. The arrows in the middle schematically represent the remaining part of the γ decay flux from the capture states. The full level scheme of ^{210}Bi from the present data is reported in Ref. [8].

indeed very close to the ones calculated for the pure $E1$-$M1$ cascade, that is, 0.05 and 0.00 for A_2 and A_4, respectively (the corresponding curve is marked in yellow in Fig. 2). In fact, the 320-keV line is expected to be rather a pure $M1$ [12], as it was mentioned before, while the 674-keV line ($3^+ \to 2^-$) should be rather of pure $E1$ character, because the mixing with $M2$ would result in a lifetime of the order of ns, not observed in our data. The small discrepancy observed in Fig. 2 between the experimental fit and the theoretical curve for pure $E1$-$M1$ multipolarity suggests that in one (or both) of the transitions the admixture of higher order of the multipole may exist.

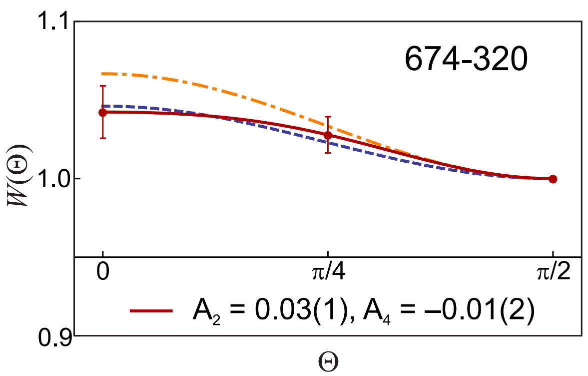


FIG. 2. Angular correlation function for the 320–674-keV pair of transitions. The red solid line is the function fitted to the experimental points, with the A_2 and A_4 values given in the legend [see Eq. (1)]. The yellow dash-dotted line is the theoretical curve for pure $E1 - M1$ cascade. The blue dashed line is the theoretical curve calculated with A_2 and A_4 from Eq. (3) and mixing ratios $\delta_{320} = 0.05$ and $\delta_{674} = 0.02$, extracted from the multivariable minimization described in Sec. III. Due to the applied normalization [$W(90^\circ)$ was chosen to be 1] the error of the third point is included in the errors of the $W(0^\circ)$ and $W(45^\circ)$ points.

III. MINIMIZATION OF A MULTIVARIABLE COST FUNCTION

To determine with higher precision the possible admixture of $E2$ multipolarity in the $2^- \to 1^-$, 320-keV transition, we employed an advanced statistical method based on the minimization of a multivariable cost function, denoted as χ^2_Σ, as described in the following. We want to note that applying the cost function analysis is very common in mathematical optimization and statistics, while this is a novel approach in analysis of γ-ray angular correlation data.

When both multipolarities of γ rays in a given pair are not known, a minimum of three transitions, coincident with each other, is requested in order to define the mixing ratios with angular correlation technique. In the case of the 320-keV transition we have found three very intense γ rays, being in coincidence with the 674–320-keV cascade (i.e., the 1013-, 2505-, and 3081-keV lines). This gives five transitions, marked by stars in Fig. 1, which can be combined into the seven pairs of coincident transitions reported in Table I. By applying the angular correlations formalism to these seven pairs of γ rays, we obtained seven independent angular correlation functions, from which the fitted A_{n2} and A_{n4} coefficients (Fig. 3) may be taken to construct the χ^2_n functions with the formalism given by Eq. (5) (here n indicates a given pair of γ rays).

We note that not all pairs consist of consecutive γ rays; nevertheless, the presence of intermediate transitions is not going to affect the final result. The possible disalignment caused by the presence of connecting transitions, especially in the pairs 2505-320, 2505-674, and 3081-320 keV, was checked by increasing the attenuation coefficients by up to 35% and this was found not to increase the final value. Moreover, we note that in the following analysis the primary γ rays 2505 and 3081 keV are assumed to be $5^- \to 5^{(+)}$ and $5^- \to 4^{(+)}$ transitions, respectively, as assigned in Ref. [8]. However,

TABLE I. The measured angular-correlations coefficients A_2 and A_4, assigned multipolarities, and theoretical A_2^{theor} and A_4^{theor} values, calculated by using the mixing ratios extracted from the multivariable minimization procedure, are presented for each pair of $E_{\gamma 1}$-$E_{\gamma 2}$ transitions considered in the analysis.

No.	$E_{\gamma 1}$-$E_{\gamma 2}$	Multipolarity	A_2	A_4	A_2^{theor}	A_4^{theor}	δ_{n1}	δ_{n2}
1	674-320	$E1(+M2)$ — $M1(+E2)$	0.03(1)	−0.01(2)	0.03	0.00	0.02(3)	0.05(3)
2	1013-320	$(M1+E2)$ — $M1(+E2)$	0.10(5)	0.01(11)	0.10	0.00	−0.12(10)	0.05(3)
3	1013-674	$(M1+E2)$ — $E1(+M2)$	0.10(2)	0.01(4)	0.10	0.00	−0.12(10)	0.02(3)
4	2505-320	$E1(+M2)$ — $M1(+E2)$	−0.14(2)	−0.03(3)	−0.14	0.00	0.02(13)	0.05(3)
5	2505-674	$E1(+M2)$ — $E1(+M2)$	−0.13(2)	−0.02(5)	−0.13	0.00	0.02(13)	0.02(3)
6	3081-320	$E1(+M2)$ — $M1(+E2)$	0.07(2)	0.03(4)	0.07	0.00	−0.04(5)	0.05(3)
7	3081-674	$E1(+M2)$ — $E1(+M2)$	0.07(2)	−0.01(5)	0.07	0.00	−0.04(5)	0.02(3)

since at least two states having spin-parity values 4^- or 5^- can contribute to the capture level populated in the ^{209}Bi$(n,\gamma)^{210}$Bi reaction (see Fig. 1), we checked that a possible decay from a 4^- capture state has no impact on the values of mixing ratios of the 320-, 674-, and 1013-keV transitions.

As no value of mixing ratio is known for either pair of γ rays, the χ^2 function from Eq. (5) (denoted as χ_n^2, where n numbers the pairs of γ rays) in each case depends on two parameters, δ_{n1} and δ_{n2}:

$$\chi_n^2 = \left(\frac{A_{n2} - A_{n2}^{\text{theor}}(\delta_{n1},\delta_{n2})}{\Delta A_{n2}}\right)^2 + \left(\frac{A_{n4} - A_{n4}^{\text{theor}}(\delta_{n1},\delta_{n2})}{\Delta A_{n4}}\right)^2. \tag{6}$$

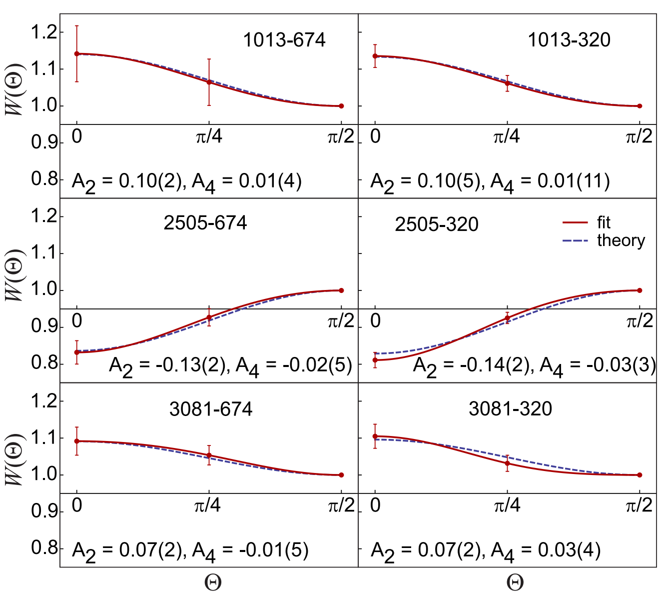


FIG. 3. Angular correlations functions for ^{210}Bi for the considered pair of lines. The red solid lines are the functions fitted to the experimental points, with the A_2 and A_4 coefficients given in the legend [see Eq. (1)]. The blue dashed lines are the theoretical curves calculated with A_2 and A_4 from Eq. (3) and mixing ratios extracted from the multivariable minimization described in Sec. III (see Table I). In the case of the 2505-keV line, the δ_{2505} value for solution II was taken, which provides very similar results to solution III. Due to the applied normalization [$W(90^\circ)$ was chosen to be 1] the error of the third point is included in the errors of the $W(0^\circ)$ and $W(45^\circ)$ points.

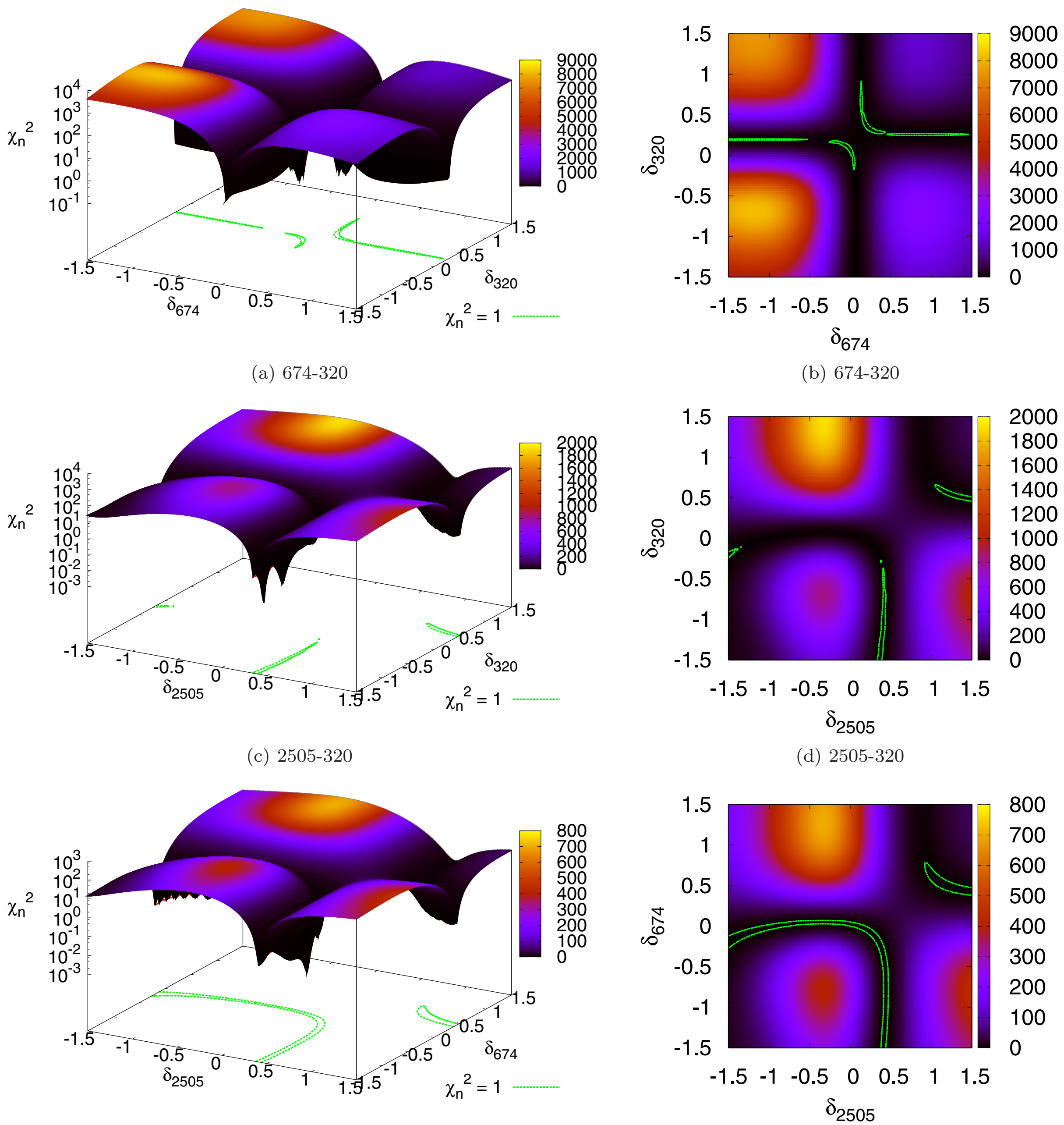
Theoretical expression of the coefficients $A_{n2}^{\text{theor}}(\delta_{n1},\delta_{n2})$ and $A_{n4}^{\text{theor}}(\delta_{n1},\delta_{n2})$ are calculated from Eq. (3). We note that not all δ_{n1} and δ_{n2} values are independent, since only five γ rays are used to construct the seven pairs. In particular, $\delta_{11} = \delta_{32} = \delta_{52} = \delta_{72}$, $\delta_{12} = \delta_{22} = \delta_{42} = \delta_{62}$, $\delta_{21} = \delta_{31}$, $\delta_{41} = \delta_{51}$, and $\delta_{61} = \delta_{71}$, as one can see from Table I. This results in only five independent values of mixing ratios to be determined, which may be denoted later as δ_{320}, δ_{674}, δ_{1013}, δ_{2505}, and δ_{3081}, where the index refers to the energy of the transition.

Examples of the single χ_n^2 functions ($n = 1,4,5$) are reported in Fig. 4 for the pairs 674-320, 2505-320, and 2505-674 keV. As one can see, the χ_n^2 function does not have any well-defined minimum, so many $(\delta_{n1},\delta_{n2}) \neq 0$ are possible in each case. Therefore, in order to define the δ_{320} mixing ratio, the following multivariable analysis was performed. The cost function χ_Σ^2 was constructed, starting from the seven χ_n^2 functions defined by Eq. (6), in the form

$$\chi_\Sigma^2 = \sum_n \chi_n^2, \tag{7}$$

where n indicates the pair of γ rays ($n = 1,\ldots,7$). This function was then normalized by the factor $1/\nu$, where ν is the number of degrees of freedom, that is the difference between the number of experimental data points (equal to 14 in our case: seven pairs of A_{n2} and A_{n4} coefficients) and the number of adjusted parameters (equal to 5, i.e., the mixing ratios to be determined).

Equation (7) forms a nonlinear least-square problem. We solve it by minimizing the cost function χ_Σ^2, using the Downhill Simplex algorithm (also known as the Nelder-Mead method) [13]. This is a popular, general purpose optimization algorithm commonly applied to nonlinear optimization problems. To be sure that we obtain a global minimum, the optimization is repeated 5000 times using random starting points in the five-dimensional space $(\delta_1,\delta_2,\delta_3,\delta_4,\delta_5)$, with the initial condition $|\delta_i| < 1$. This number of repetitions is sufficiently large to avoid local minima and eliminate the nonstationary points, to which the Simplex algorithm may converge. Such points are seen in Fig. 5: they connect the flat minima and represent the solutions which did not converge to the minimum in a number of allowed iterations. In the calculation, only solutions with $|\delta_i| < 10$ were considered, since higher than 10 and lower than -10 values of δ_i would be rather nonphysical. As shown in Fig. 5, the minimization algorithm finds the three lowest minima, that is three sets of mixing ratios with



(a) 674-320 (b) 674-320

(c) 2505-320 (d) 2505-320

(e) 2505-674 (f) 2505-674

FIG. 4. (a), (c), (e) χ_n^2 function constructed according to Eq. (6) for the pairs of transitions indicated in the legend ($n = 1,4,5$ as indicated in Table I). (b), (d), (f) 2D projection of the χ_n^2 function shown on the left panels, respectively.

very similar values of $\chi_\Sigma^2 \approx 0.5$. In the following, only these three minima will be considered since we note that the next minimum corresponds to χ_Σ^2 more than twice larger (equal to 1.124). The results of the minimization are presented in Table II, where the δ mixing ratios are given for the five transitions considered in this multivariable procedure. The errors of the estimated mixing ratios are calculated using the covariance matrix obtained from the linearization of the cost function [14]. To demonstrate the quality of the minimization procedure we show in Fig. 6 the projections of the χ_Σ^2 cost function for each combination of the mixing ratios. We note that the multivariable minimization procedure always provides a well-defined local minimum in contrast to the independent minimization of the χ_n^2 functions, defined by Eq. (6) (see examples in Fig. 4).

As shown in Table II, it is found that the mixing ratios for the transitions 674, 1013, and 3081 keV are independent of the choice of the minimum, pointing to the multipolarity assignments given in Table I. Concerning the 2505-keV primary γ-ray transition, two solutions are found for δ_{2505}, namely, 0.02(6) and $-0.79(9)$. Although none of them can be excluded, this ambiguity is not affecting the results for the 320-keV main line of interest. In this case there are two possible values of mixing ratio: $-3.04(13)$ and 0.05(2). We

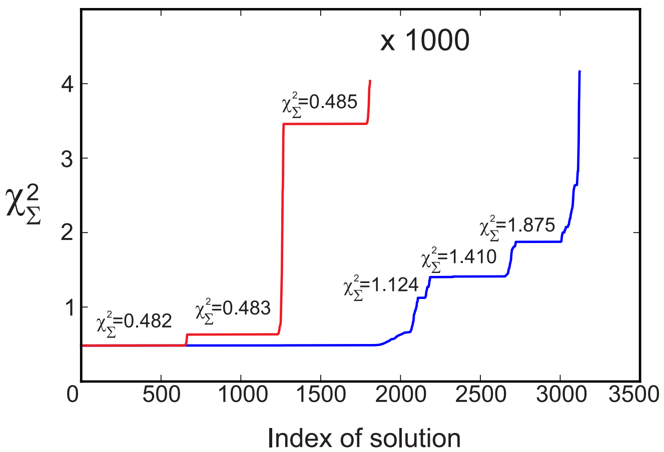


FIG. 5. The subset of results of the χ^2_Σ cost function minimization (approx. 3100 from 5000 iterations), corresponding to the condition $|\delta_i| < 10$ (blue curve). The x axis numbers the solutions, while the value of the χ^2_Σ cost function is given on the y axis (note that the solutions are ordered ascending with the χ^2_Σ value). The red curve is the zoom (multiplied by 1000 and normalized to the first minimum of the original curve) showing the three solutions characterized by the lowest values of χ^2_Σ (see Table II).

note that the $\delta_{320} = -3.04$ value would imply a significant (90%) admixture of $E2$ multipolarity. This would have a consequence on the lifetime of the 320-keV state deexcited by this line. By checking the half-lives $T_{1/2}$ measured in the neighboring nuclei, one notices that typical $T_{1/2}$ values for $E2$ would be much longer than the one measured in this case $T_{1/2} = 7.5(14)$ ps [15]. Moreover, the reduced transition probability $B(E2)$ for the 320-keV transition was calculated assuming the lifetime known previously from Ref. [15] and the mixing ratios given by our analysis. The $B(E2)$ value obtained for the $\delta_{320} = -3.04$ solution is unrealistically large ($18 \times 10^3\ e^2\mathrm{fm}^4$) and very far from the shell-model prediction, i.e., $B(E2) = 0.17\ e^2\mathrm{fm}^4$. This is an additional indication that this solution is highly unlikely. Therefore, we take into consideration only the value $\delta_{320} = 0.05(2)$ [giving $B(E2) = 41\ e^2\mathrm{fm}^4$]. The precise value for the standard deviation is 0.013 and the calculated δ_{320} range, with a 95% confidence limit, was found to be equal to 0.024–0.076. This range is very close to the interval expected for one-dimensional random Gaussian variables ($0.05 \pm 1.96\sigma$), implying that the correlations between parameters have no effect on the estimate of the confidence limits.

As noted previously in this section, an attenuation of γ-γ correlation may occur when intermediate transitions are present in the cascade. We have verified that the χ^2_Σ minimization procedure assuming such attenuation always leads to a value of the mixing parameter δ for the 320-keV transition lower than the value quoted above. We should therefore consider the value of 0.076 as an upper limit for δ_{320}, which gives an upper limit of 0.6% [calculated from Eq. (4)] for the $E2$ admixture. We note that the $B(E2)$ value calculated from the shell model is within this limit.

The theoretical angular correlation functions for the 320–674-keV cascade, as well as for the rest of pairs considered in the analysis, are shown by blue lines in Fig. 2, confirming the very good quality of the fit.

TABLE II. Results of the χ^2_Σ cost function minimization corresponding to the three lowest minima with $|\delta_i| < 10$.

No.	δ_{320}	δ_{674}	δ_{1013}	δ_{2505}	δ_{3081}	χ^2_Σ
I	−3.04(13)	0.02(1)	−0.12(3)	0.03(5)	−0.04(2)	0.482
II	0.05(2)	0.02(1)	−0.12(3)	−0.79(9)	−0.04(2)	0.483
III	0.05(2)	0.02(1)	−0.12(3)	0.02(6)	−0.04(2)	0.485

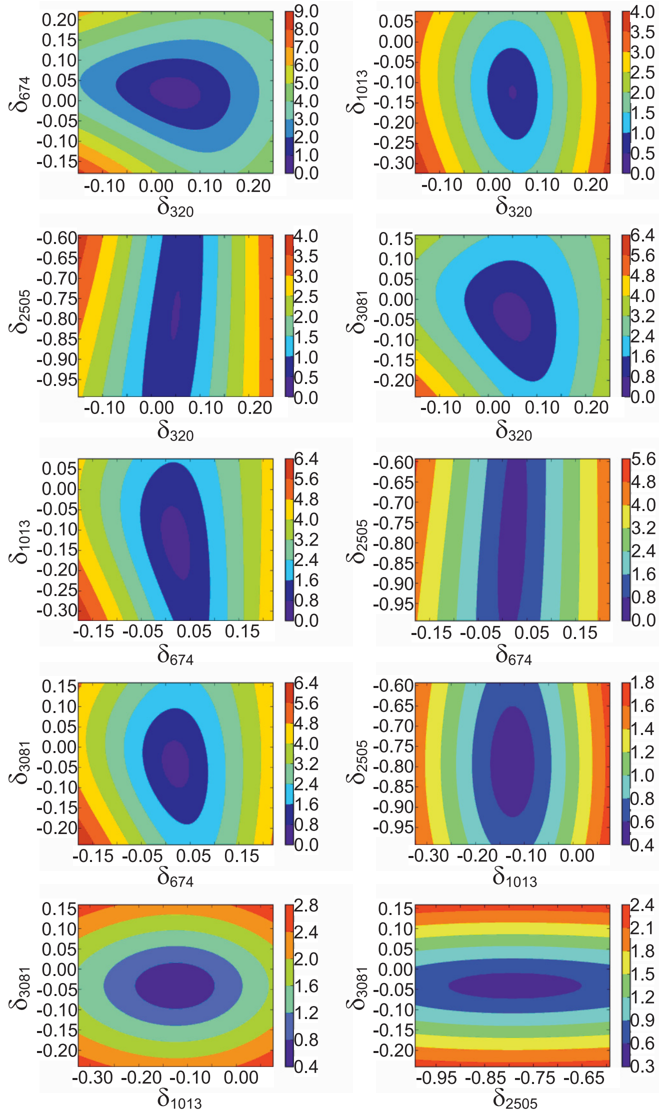


FIG. 6. 2D projections of the χ^2_Σ cost function around the II minimum (see Table II). The χ^2_Σ cost function is projected on the plane defined by the corresponding δ_i, δ_j mixing ratios.

IV. CONCLUSIONS AND SUMMARY

The present paper aimed at establishing the $M1/E2$ multipolarity mixing of the $2^- \rightarrow 1^-$, 320-keV line in ^{210}Bi.

The analysis of the data was based on a minimization of a multivariable χ^2_Σ cost function constructed from the coefficients of angular correlation functions for seven pairs of strong transitions. It is concluded that the mixing ratio of the 320-keV transition is very small [$\delta_{320} = 0.05(2)$] and corresponds to an upper limit of 0.6% (95% confidence limit) admixture of $E2$, pointing to an almost pure $M1$ character.

The present result may be further employed for an accurate evaluation of the neutron capture cross section to the ground state, σ_{GS}, in ^{210}Bi. With the clearly established upper limit of 0.6% for $E2$ admixture in the 320-keV transition, one can follow the analysis described in Ref. [12] in order to obtain the cross-section value from the formula

$$\sigma_{gs} = \frac{\sum_i I_i}{I_{4055}} \sigma_{4055}, \tag{8}$$

where $\sum_i I_i$ is the sum of the intensities of the γ rays leading to the ground state, reported by Borella *et al.*, relative to the partial capture cross section σ_{4055} for the very intense 4055-keV line in ^{210}Bi [12]. In this estimate the intensity of the 517-keV γ ray, not observed by Borella *et al.*, was taken from [16]. Moreover, the correction for the conversion coefficient of the 320-keV transition has been made: it was assumed to range from the one corresponding to pure $M1$ transition to a value calculated for 0.6% $E2$ admixture. Based on these intensities one can recalculate the total cross section for the ^{210}Bi ground-state population as being within the limits 21.3(9) and 21.5(9) mb. In these calculations the 21.5-mb value was obtained assuming pure $M1$ character of the 320-keV transition, while the 21.3-mb value results from allowing the 0.6% upper limit of $E2$ admixture.

The ^{210}Bi ground state decays to radiotoxic ^{210}Po that dominates the short-term radiotoxicity of nuclear reactors or accelerator driven systems with Pb-Bi coolant. Our measurement of the pure $M1$ character of the 320-keV transition allowed us to reduce the relative uncertainty on the ^{209}Bi$(n,\gamma)^{210}$Bi ground-state cross-section from 25 to 0.9%. The newly established cross-section limits may now serve for accurate projections of the ^{210}Po inventory in nuclear reactors and accelerator driven systems using Pb-Bi coolant.

ACKNOWLEDGMENTS

The authors thank the technical services of the Institut Laue-Langevin, LPSC, and GANIL for supporting the EXILL campaign. The EXOGAM collaboration and the Istituto Nazionale di Fisica Nucleare Legnaro are acknowledged for the loan of Ge detectors. This work is supported by the Polish National Science Centre under Contracts No. DEC-2011/01/N/ST2/04612, No. 2014/14/M/ST2/00738, and No. 2013/08/M/ST2/00257.

[1] J. R. Erskine, W. W. Buechner, and H. A. Enge, Phys. Rev. **128**, 720 (1962).

[2] A. Borella, P. Schillebeeckx, G. Molnar, T. Belgya, Zs. Revay, L. Szentmiklési, E. Berthoumieux, F. Gunsing, A. Letourneau, and F. Marie, AIP Conf. Proc. **769**, 648 (2005).

[3] H. Abele *et al.*, Nucl. Instr. Meth. A **562**, 407 (2006).

[4] J. Simpson, F. Azaiez, G. de France, J. Fouan, J. Gerl, R. Julin, W. Korten, P. J. Nolan, B. M. Nyako, G. Sletten, P. M. Walker, and the EXOGAM Collaboration, Acta Phys. Hung. N.S., Heavy Ion Physics **11**, 159 (2000).

[5] http://gasp.lnl.infn.it/.

[6] P. Mutti, A. Blanc, G. de France, M. Jentschel, U. Köster, E. Ruiz Martinez, G. Simpson, T. Soldner, C. A. Ur, and W. Urban, in *2013 Third International Conference on Advancements in Nuclear Instrumentation Measurement Methods and their Applications (ANIMMA)* (IEEE, Marseille, 2013), p. 1.

[7] M. Jentschel *et al.* (unpublished).

[8] N. Cieplicka-Oryńczak *et al.*, Phys. Rev. C **93**, 054302 (2016).

[9] H. Morinaga and T. Yamazaki, *In-Beam Gamma-Ray Spectroscopy* (North-Holland, Amsterdam, 1976).

[10] A. J. Ferguson, *Angular Correlation Methods in Gamma-Ray Spectroscopy* (North-Holland, Amsterdam, 1965).

[11] K. S. Krane, R. M. Steffen, and R. M. Wheeler, Nucl. Data Tables **11**, 351 (1973).

[12] A. Borella, T. Belgya, S. Kopecky, F. Gunsing, M. Moxon, M. Rejmund, P. Schillebeeckx, and L. Szentmiklési, Nucl. Phys. A **850**, 1 (2011).

[13] J. A. Nelder and R. Mead, Comput. J. **7**, 308 (1965).

[14] S. Brandt, *Data Analysis—Statistical and Computational Methods for Scientists and Engineers* (Springer, New York, 1998).

[15] D. J. Donahue, O. Häusser, R. L. Hershberger, R. Lutter, and F. Riess, Phys. Rev. C **12**, 1547 (1975).

[16] R. K. Sheline, R. L. Ponting, A. K. Jain, J. Kvasil, B. bu Nianga, and L. Nkwambiaya, Czech. J. Phys. B **39**, 22 (1989).