Decay width of $d^*(2380) \to NN\pi\pi$ processes

Yubing Dong,[1,2] Fei Huang,[3] Pengnian Shen,[1,2,4] and Zongye Zhang[1,2]
[1]*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*
[2]*Theoretical Physics Center for Science Facilities (TPCSF), CAS, Beijing 100049, China*
[3]*School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 101408, China*
[4]*College of Physics and Technology, Guangxi Normal University, Guilin 541004, China*
(Received 1 April 2016; revised manuscript received 15 June 2016; published 22 July 2016)

The decay widths of four-body double-pion decays $d^* \to pn\pi^0\pi^0$, $d^* \to pn\pi^+\pi^-$, and isoscalar parts of $d^* \to pp\pi^0\pi^-$ and $d^* \to nn\pi^+\pi^0$ are explicitly calculated with the help of the d^* wave function obtained in a chiral SU(3) quark model calculation. The effect of the dynamical structure on d^*'s width is analyzed both in the single $\Delta\Delta$ channel and the $\Delta\Delta + CC$ coupled-channel approximations. It is found that in the latter case, the obtained partial decay widths of $d^* \to pn\pi^0\pi^0$, $d^* \to pn\pi^+\pi^-$, and those of d^* to the isoscalar parts of $pp\pi^0\pi^-$ and $nn\pi^+\pi^0$ are about 9.6 MeV, 20.6 MeV, 3.5 MeV and 3.5 MeV, respectively. As a consequence, the total width is about 71.9 MeV. These widths are consistent with our previous estimation by using cross section data and observed width. Apparently, the resultant mass and width in the $\Delta\Delta + CC$ coupled-channel calculation again support our assertion that the d^* resonance is a six-quark dominated exotic state.

DOI: 10.1103/PhysRevC.94.014003

In recent years, CELSIUS/WASA and WASA@COSY Collaborations clearly observed a resonancelike structure in double pionic fusion channels $pn \to d\pi^0\pi^0$ and $pn \to d\pi^+\pi^-$ when they studied the ABC effect and in dealing with the neutron-proton scattering data with newly measured analyzing power A_y. This possible resonance has a mass of about 2380 MeV and a width of about 70 MeV [1–3]. Because the observed resonance cannot simply be explained by either the intermediate Roper excitation or the t-channel $\Delta\Delta$ process, they proposed a d^* hypothesis, in which its quantum number, mass, and width are $I(J^P) = 0(3^+)$, $M \approx 2370$ MeV, and $\Gamma \approx 70$ MeV [1,4] (in their recent paper [4], the averaged mass and width are $M \approx 2375$ MeV and $\Gamma \approx 75$ MeV, respectively). Because its baryon number is 2, it would be regarded as a dibaryon, and could be either "an exotic compact particle or a hadronic molecule" [5]. Moreover, according to the experimental data, the mass of d^* is about 80 MeV smaller than the $\Delta\Delta$ threshold and about 70 MeV larger than the $\Delta\pi N$ threshold, so the threshold (or cusp) effect is expected to be not so important as that in the XYZ study, and therefore, the internal structure of d^* would be essentially significant.

The existence of such a nontrivial six-quark configuration with $I(J^P) = 0(3^+)$ (called d^* lately) has caused a great attention of theoreticians, and it, in fact, has intensively been studied since Dyson's estimation [6–11]. It should especially be mentioned that one of those calculations reported in 1999 predicted a binding energy of about 40–80 MeV by taking into account a $\Delta\Delta$ channel and a hidden-color channel (denoted by CC hereafter) simultaneously, and pointed out the importance of the contribution from the CC channel [10]. That predicted binding energy is quite close to the recent observation, unfortunately, the width of the state was not studied.

Since COSY reported their discovery, there are mainly three types of explanation. Based on the SU(2) quark model, Ref. [12] proposed a $\Delta\Delta$ resonance structure and performed a multichannel scattering calculation. They obtained a binding energy of about 71 MeV (namely $M_{d^*} = 2393$ MeV) and a width of about 150 MeV which is apparently much larger than the observation. On the other hand, Ref. [13] studied a three-body system of $\Delta N\pi$ and found a resonance pole with a mass of 2363 ± 20 MeV and a width of 65 ± 17 MeV. However, one argued that an additional factor of 2/3 should not be included in the width estimation [14]. An important viewpoint, claimed by Bashkanov, Brodsky, and Clement [15] in 2013, is that a dominant hidden-color structure (or six-quark configuration) of d^* is necessary for understanding its narrow width. Soon after, following our previous prediction [10], Huang and his collaborators made an explicit dynamical calculation by using a chiral SU(3) quark model [16–18] in the framework of the resonating group method (RGM), and showed that the d^* state has a mass of 2380–2414 MeV, which agrees with COSY's observation, and does have a "CC" configuration of about 66%–68% in its wave function [19]. Based on the obtained wave functions of d^* and deuteron, Dong and his collaborators calculated the partial decay widths of the "Golden" decay channel $d^* \to d + 2\pi^0(\pi^+\pi^-)$ [20]. They showed that inclusion of the CC configuration inside d^* would make its width greatly suppressed and the resultant widths for both $d^* \to d\pi^0\pi^0$ and $d^* \to d\pi^+\pi^-$ are consistent with the data. Further using observed cross sections, they gave an estimate of the total width of about 69 MeV, which is fairly close to the observation [20]. All these outcomes imply that d^* is probably a six-quark dominated exotic state.

However, a blemish in our previous calculation is that the four-body $\pi\pi$ decays $d^* \to pn\pi^0\pi^0$, $d^* \to pn\pi^+\pi^-$, and isoscalar parts of $d^* \to pp\pi^0\pi^-$ and $d^* \to nn\pi^+\pi^0$ were not explicitly calculated [20]. A naive conjecture from the $d^* \to \Delta\Delta \to np\pi\pi$ process showed a very large value [15], which does not fit the observed data [4]. Because the only difference between these four-body decays and the corresponding three-body decays is that the produced proton and neutron are free particles rather than a weakly bound state of deuteron, it is our purpose to check if one can achieve reasonable partial widths for these four-body decays by using the obtained d^* wave function in our previous calculation.

The phenomenological effective Hamiltonian for the pseudoscalar interaction among quark, pion, and quark in the nonrelativistic approximation reads

$$\mathcal{H} = g_{qq\pi}\vec{\sigma}\cdot\vec{k}_\pi \tau\cdot\phi \times \frac{1}{(2\pi)^{3/2}\sqrt{2\omega_\pi}}, \tag{1}$$

where $g_{qq\pi}$ is the coupling constant, ϕ stands for the π meson field, ω_π and $\vec{k}_\pi$ are the energy and three-momentum of the π meson, respectively, and $\sigma(\tau)$ represents the spin (isospin) operator of a single quark. The wave functions of the nucleon and $\Delta(1232)$ resonance in the conventional constituent quark model can be found in [20]. The decay width for $\Delta \to \pi N$ reads

$$\Gamma_{\Delta\to\pi N} = \frac{4}{3\pi}k_\pi^3(g_{qq\pi}I_o)^2\frac{\omega_N}{M_\Delta}, \tag{2}$$

where $\omega_{\pi,N} = \sqrt{M_{\pi,N}^2 + \vec{k}_\pi^2}$ are the energies of the pion and nucleon, respectively, $k_\pi \sim 0.229$ GeV, and I_o denotes the spatial overlap integral of the internal wave functions of the nucleon and the Δ resonance. In terms of the measured decay width of the $\Delta \to \pi N$ process, for example, $\Gamma_{\Delta^+\to\pi^0 p} \sim 117$ MeV, we extract the coupling constant $g_{qq\pi}$ by calculating $\Gamma_{\Delta\to N\pi} = \langle\Delta|\sum_{i=1}^3 \mathcal{H}|N\rangle$ [21] (the details can be found in Ref. [20]). Defining $G = g_{qq\pi}I_o$, the obtained G value is about 5.41 GeV^{-1}.

As mentioned in Refs. [19,20], our model wave function is obtained by dynamically solving the bound-state RGM equation of the six-quark system in the framework of the extended chiral SU(3) quark model. Further projecting the wave function in the quark level onto the two-cluster wave function in the baryon level, namely hadronize to the physical state, we end up with a wave function of d^* as

$$\begin{aligned}\Psi_{d^*} = [&\phi_\Delta(\vec{\xi}_1,\vec{\xi}_2)\phi_\Delta(\vec{\xi}_4,\vec{\xi}_5)\chi_{\Delta\Delta}(\vec{R})\zeta_{\Delta\Delta}\\ &+\phi_C(\vec{\xi}_1,\vec{\xi}_2)\phi_C(\vec{\xi}_4,\vec{\xi}_5)\chi_{CC}(\vec{R}))\zeta_{CC}]_{(SI)=(30)},\end{aligned} \tag{3}$$

where ϕ_Δ, and ϕ_C denote the internal wave functions of Δ and C (color-octet configuration) in the coordinate space, $\chi_{\Delta\Delta}$ and χ_{CC} represent the channel wave functions between Δs and Cs (in the single $\Delta\Delta$ channel case, the CC component is absent), and $\zeta_{\Delta\Delta}$ and ζ_{CC} stand for the spin-isospin wave functions in the $\Delta\Delta$ and CC channels, respectively [19]. It should be especially mentioned that in such a wave function, these two channel wave functions are orthogonal to each other, and the totally antisymmetric effect is implicitly included in the channel wave functions through the above-mentioned two steps [19].

In terms of the obtained wave function of d^*, we are able to calculate the four-body decay width of $d^* \to pn\pi^0\pi^0$,

$$\begin{aligned}\Gamma_{d^*\to pn\pi^0\pi^0} = &\frac{1}{2!2!}\int d^3k_1d^3k_2d^3p_1(2\pi)\delta(\Delta E)\\ &\times \mid\overline{\mathcal{M}(k_1,k_2;p_1)}\mid^2,\end{aligned} \tag{4}$$

where $\mid\overline{\mathcal{M}(k_1,k_2;p_1)}\mid^2$ stands for the squared transition matrix element with a sum over the final four-body state and an average of the polarizations of the initial state d^*, the factor of $2!\times 2!$ is from the identical particles of $\pi^0\pi^0$ and pn, respectively, and $\delta(\Delta E)$ denotes the energy conservation with $\Delta E = M_{d^*} - \omega_\pi(k_1) - \omega_\pi(k_2) - E_N(p_1) - E_N(-p_1 - k_1 - k_2)$; ω_π and E_N represent the energies of the outgoing pion and nucleon, respectively.

Based on the on-shell factorized form of the Bethe-Salpeter equation (BSE), for a specific partial amplitude, the transition matrix $\mathcal{M}(k_1,k_2;p_1)$ in Eq. (4), where the final state interaction (FSI) between proton and neutron is taking into account, can formally be written as [22–25]

$$\mathcal{M}(k_1,k_2;p_1) = \mathcal{M}^{\text{bare}}(k_1,k_2;p_1)\times\mathcal{I}, \tag{5}$$

where $\mathcal{M}^{\text{bare}}(k_1,k_2;p_1)$ represents the transition matrix without FSI, and $\mathcal{I}$ denotes the enhancement factor caused by FSI. In the S-partial wave approximation, namely in the lower energy region, $\mathcal{I}$ can be expressed as

$$\mathcal{J}^{-1}(k) = \frac{k+i\alpha}{k-i\kappa}, \tag{6}$$

with $\mathcal{J}(k)$ being the Jost function and k the relative three-momentum between the proton and neutron. In our calculation, we only consider the $^3S_1$ np FSI and ignore FSI in other channels. The values of two parameters for the $^3S_1$ np channel are $\alpha = 178.7$ MeV, and $\kappa = (2\mu\epsilon_d)^{1/2} = 45.7$ MeV with μ being the reduced mass of the np system and ϵ_d being the binding energy of the deuteron. Then, we assume that the Jost function can approximately be linked to the np $^3S_1$ phase shift by a Watson-type enhancement factor [26],

$$\begin{aligned}\mathcal{J}^{-1}(k) &= C(k^2)\frac{\sin\delta e^{i\delta}}{k},\\ C(k^2) &= \frac{k^2+\alpha^2}{\alpha+\kappa}.\end{aligned} \tag{7}$$

It should be mentioned that by using such a Jost function, the extracted np $^3S_1$ phase shift δ in the small k region agrees with the experimental data. However, when k increases (say $T_{\text{lab}} > 100$ MeV), the obtained δ overestimates the data. To roughly compensate this deviation, we employ the experimental data of phase shift instead of δ in Eq. (8), but leave the factor $C(k^2)$ unchanged.

The bare transition matrix includes the contributions from four sub-diagrams plotted in Fig. 1.

The explicit expression, for example, Fig. 1(a), reads

$$\begin{aligned}\mathcal{M}^a(k_1,k_2;p_1) &= \int d^3p_2d^3q[\mathcal{H}\mathcal{S}_f\mathcal{H}]\Psi_{d^*}(q)\\ &\quad\times\delta^3(\vec{p}_1+\vec{k}_1-\vec{q})\delta(\vec{p}_2+\vec{k}_2+\vec{q})\\ &= \int d^3p_2\delta^3(\vec{p}_1+\vec{p}_2+\vec{k}_1+\vec{k}_2)[\mathcal{H}\mathcal{S}_f\mathcal{H}]\\ &\quad\times\Psi_{d^*}(-\vec{p}_2-\vec{k}_2),\end{aligned} \tag{8}$$

where $\mathcal{S}_f$ is the propagator of the intermediate state, and $\mathcal{H}$ is the effective Hamiltonian presented in Eq. (1). Ψ_{d^*} represents the d^* wave function in the momentum space which can be obtained by Fourier transforming the d^* wave functions in the coordinate space in both single $\Delta\Delta$ channel and coupled $\Delta\Delta + CC$ channel approximations.

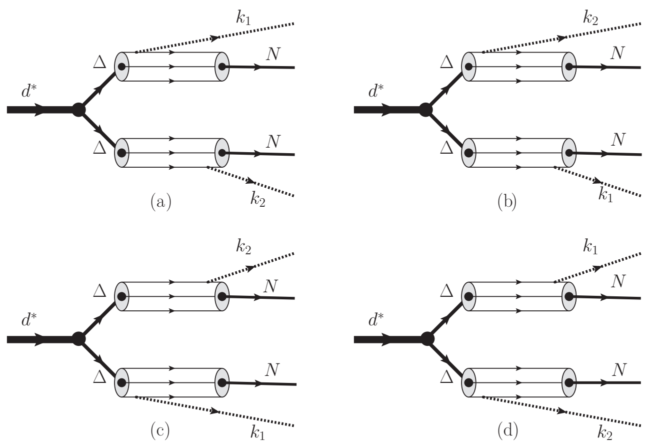

FIG. 1. Four possible emission ways in the decay of the d^* resonance composed of the $\Delta\Delta$ structure only. Two pions with momenta of $\vec{k}_{1,2}$ are emitted from one of the three quarks in 2 Δs, respectively.

In the coupled channel case, we found that there are 31.5% $\Delta\Delta$ component and 68.5% CC component in the d^* wave function shown in Eq. (3) [19]. Because the pion itself is colorless, emission of pion would not change the color structure of the parent particle. On the other hand, the final proton and neutron are, of course, colorless. Therefore, although the pion can be emitted both from the colorless particle and from the colored particle, in the lowest order approximation, the colored parent particle would not contribute, then the contribution from the CC component can be neglected although such a component is the dominant piece in d^*. The major contribution to the decay width comes merely from the $\Delta\Delta$ component.

In the $d^* \to pn\pi^0\pi^0$ process, the obtained partial widths with FSI are about 19.8 MeV and 9.6 MeV in the single channel and coupled channel approximations, respectively; they are tabulated in Table I. Comparing with the corresponding values without FSI, 15.2 MeV and 7.4 MeV for the single channel and coupled channel cases, respectively, one sees that FSI enlarges the widths by a factor of about 30%, which at least qualitatively agrees with the previous calculations [27,28]. From now on, the results are those with FSI, except special notification.

The partial widths of the isoscalar parts of the $d^* \to pp\pi^0\pi^-$ and $d^* \to nn\pi^+\pi^0$ processes can be calculated in terms of our wave function of d^* in the same framework. Because these two processes are mirror states, the widths of these decays should be the same. As shown in Table I, the calculated partial width of 3.5 MeV in the coupled channel approximation is again compatible with our previous estimated value of 3.9 MeV and the data of 4.4 MeV.

The situation for the $d^* \to pn\pi^+\pi^-$ process is somewhat complicated, because both the (pn) pair and $(\pi^+\pi^-)$ pair can be either isoscalar simultaneously or isovector simultaneously, namely $[(pn)_{I_{pn}=0}(\pi^+\pi^-)_{I_{\pi\pi}=0}]_{I=0}$ or $[(pn)_{I_{pn}=1}(\pi^+\pi^-)_{I_{\pi\pi}=1}]_{I=0}$ [4]. According to the isospin relation, the contribution from the former configuration ($I_{pn} = I_{\pi\pi} = 0$) should be twice that from the $d^* \to pn\pi^0\pi^0$ configuration. But because of the isospin violation of pion, our explicit calculation shows that the partial widths of the first isoscalar coupling part, with FSI, are 35.4 MeV and 17.1 MeV in the single channel and coupled channel cases, respectively, which are somewhat smaller than the expected value from the isospin relation, just like that in the $d^* \to d\pi\pi$ case. The ratios of the isoscalar coupling part of the charged pion decay to the chargeless pion decay are 1.79 and 1.78, which is similar to the values of 1.81 and 1.83 in the $d^* \to d\pi\pi$ case, respectively. The later isovector coupling part would also have some contribution. Because both components in this part have isospin 1 ($I_{pn} = I_{\pi^+\pi^-} = 1$), its contribution would be the same as that from the isoscalar part of the $d^* \to pp\pi^0\pi^-$ process. Our calculation gives the partial widths of 7.2 MeV and 3.5 MeV for the $d^* \to pp\pi^0\pi^-$ process in the single channel and coupled channel case, respectively. Adding all these isospin caused effects together, the resultant partial widths of the $d^* \to pn\pi^+\pi^-$ process are about 42.6 MeV and 20.6 MeV, respectively, in the single $\Delta\Delta$

TABLE I. Calculated partial decay widths and corresponding branching ratios of d^* in the two-body, three-body, and four-body decay channels and the total width of d^*. Case I and Case II denote the single channel and coupled channel cases, respectively.

Wave function	This work			Ref. [20][a]		Expt. [2,4,29,30]	
M_{d^*}(MeV)	Case I (100%)$\Delta\Delta$ 2374	Case II (31.5%)$\Delta\Delta$ + (68.5%)CC 2380		(31.5%)$\Delta\Delta$ + (68.5%)CC 2380		2375	
Decay channel	Γ(MeV)	Γ(MeV)	Br(%)	Γ(MeV)	Br(%)	Γ(MeV)	Br(%)
$d^* \to d\pi^0\pi^0$	17.0	9.2	12.8	9.2	13.3	10.2	14(1)
$d^* \to d\pi^+\pi^-$	30.8	16.8	23.4	16.8	24.3	16.7	23(2)
$d^* \to pn\pi^0\pi^0$	19.8	9.6	13.3	7.8	11.3	8.7	12(2)
$d^* \to pn\pi^+\pi^-$	42.6	20.6	28.6	19.2	27.8	21.8	30(4)
$d^* \to pp\pi^0\pi^-$	7.2	3.5	4.9	3.9	5.65	4.4	6(1)
$d^* \to nn\pi^+\pi^0$	7.2	3.5	4.9	3.9	5.65	4.4	6(1)
$d^* \to pn$	8.2	8.7	12.1	8.3	12.0	8.7	12(3)
Total	132.8	71.9	100.0	69.1	100.0	74.9	103

[a]Four-body decay results in this column are obtained by using the ratios of cross section data between relevant decay channels.

and coupled $\Delta\Delta + CC$ channel cases, which are also tabulated in Table I. The calculated partial width in the coupled channel approximation is close to our previous estimation of 19.2 MeV [20] by using observed cross sections, and compatible with the experimental data of 21.8 MeV. If we define the ratio of the partial decay width of the charged double-pion decay to that of the chargeless double-pion decay as

$$R = \frac{\Gamma_{d^* \to pn\pi^+\pi^-}}{\Gamma_{d^* \to pn\pi^0\pi^0}}, \tag{9}$$

the resultant R value is about 2.15 for both the single $\Delta\Delta$ approximation and the coupled channel approximation. Comparing with the value of 2.5 from the isospin relation this ratio is somehow smaller. This is because in the nonfusion double-pion production process, the pion isospin breaking effect caused by the phase space reduction plays a relatively weaker role.

In terms of the branching ratio data, one can obtain the partial width of $d^* \to np$ and consequently the total width of d^*, which are also tabulated in Table I. From this table, one also sees that in the $\Delta\Delta + CC$ coupled-channel approximation, the total width of d^* is about 71.9 MeV, which is close to our previous estimated value of 69.1 MeV and the observed value of 74.9 MeV. Moreover, the calculated branching ratios for these decay processes (shown in Table I) are all close to our previous estimations and in acceptable ranges in comparison with the data.

In short, the partial widths of the three-body and four-body double-pion decays of d^* are all explicitly calculated in the same framework by using our model wave function in the extended chiral SU(3) quark model in a unified way. The total width of d^* is about 71.9 MeV, which is compatible with the value of 69.1 MeV in our previous estimation and the data of 75 MeV.

From this calculation, one again sees that the single $\Delta\Delta$ structure cannot explain the observed data of d^*, but if a CC component is involved, the partial decay widths of the three-body and four-body double pion decays can be reasonably obtained and the mass and width data of d^* can be well understood. All these results support our assertion that one may assign the observed d^* state as a $\Delta\Delta$ bound state with a dominant CC component, namely the d^* state is a six-quark dominated exotic state.

We would like to thank H. Clement, Qiang Zhao, Bing-song Zou, Xu Cao, and Qi-Fang Lü for their useful and constructive discussions. F.H. is grateful for the support of the Youth Innovation Program Association, CAS. This work is partially supported by the National Natural Science Foundation of China under Grants No. 11035006, No. 11165005, No. 11475192, No. 11475181, and No. 11565007, the fund provided to the Sino-German CRC 110 "Symmetries and the Emergence of Structure in QCD" project by the DFG, and the IHEP Innovation Fund under Grant No. Y4545190Y2.

[1] M. Bashkanov, C. Bargholtz, M. Berlowski, D. Bogoslawsky, H. Calen, H. Clement, L. Demiroers, E. Doroshkevich, D. Duniec, C. Ekstrom, K. Fransson, L. Geren, L. Gustafsson, B. Hoistad, G. Ivanov, M. Jacewicz, E. Jiganov, T. Johansson, O. Khakimova, S. Keleta, I. Koch, F. Kren, S. Kullander, A. Kupsc, K. Lindberg, P. Marciniewski, R. Meier, B. Morosov, C. Pauly, H. Pettersson, Y. Petukhov, A. Povtorejko, A. Pricking, R. J. M. Y. Ruber, K. Schonning, W. Scobel, B. Shwartz, T. Skorodko, V. Sopov, J. Stepaniak, P. E. Tegner, P. Thorngren-Engblom, V. Tikhomirov, A. Turowiecki, G. J. Wagner, M. Wolke, J. Zabierowski, I. Zartova, and J. Zlomanczuk, Phys. Rev. Lett. **102**, 052301 (2009).

[2] P. Adlarson *et al.*, Phys. Rev. Lett. **106**, 242302 (2011); Phys. Lett. B **721**, 229 (2013); Phys. Rev. Lett. **112**, 202301 (2014).

[3] A. Abashian, N. E. Booth, and K. W. Crowe, Phys. Rev. Lett. **5**, 258 (1960); N. E. Booth, A. Abashian, and K. M. Crowe, *ibid.* **7**, 35 (1961); F. Plouin *et al.*, Nucl. Phys. A **302**, 413 (1978); J. Banaigs *et al.*, Nucl. Phys. B **105**, 52 (1976).

[4] M. Bashkanov, H. Clement, and T. Skorodko, Eur. Phys. J. A **51**, 87 (2015).

[5] *COSY confirms existence of six-quark states*, CERN COURIER, July 23 (2014).

[6] J. Dyson, Phys. Rev. Lett. **13**, 815 (1964).

[7] A. W. Thomas, J. Phys. G **9**, 1159 (1983).

[8] M. Oka and K. Yazaki, Phys. Lett. B **90**, 41 (1980).

[9] T. Goldman, K. Maltman, G. J. Stephenson, K. E. Schmidt, and F. Wang, Phys. Rev. C **39**, 1889 (1989).

[10] X. Q. Yuan, Z. Y. Zhang, Y. W. Yu, and P. N. Shen, Phys. Rev. C **60**, 045203 (1999).

[11] M. N. Platonova and V. I. Kukulin, Phys. Rev. C **87**, 025202 (2013); Nucl. Phys. A **946**, 117 (2016).

[12] H. X. Huang *et al.*, Phys. Rev. C **79**, 024001 (2009); **89**, 034001 (2014).

[13] A. Gal and H. Garcilazo, Phys. Rev. Lett. **111**, 172301 (2013); Nucl. Phys. A **928**, 73 (2014); A. Gal, Acta Phys. Pol. B **47**, 471 (2016).

[14] P. Adlarson *et al.*, Phys. Rev. C **90**, 035204 (2014).

[15] M. Bashkanov, S. J. Brodsky, and H. Clement, Phys. Lett. B **727**, 438 (2013).

[16] Z. Y. Zhang *et al.*, Nucl. Phys. A **625**, 59 (1997).

[17] L. R. Dai *et al.*, Nucl. Phys. A **727**, 321 (2003).

[18] F. Huang and Z. Y. Zhang, Phys. Rev. C **72**, 024003 (2005).

[19] F. Huang *et al.*, Chin. Phys. C **39**, 071001 (2015), and references therein.

[20] Yubing Dong, Pengnian Shen, Fei Huang, and Zongye Zhang, Phys. Rev. C **91**, 064002 (2015).

[21] K. A. Olive *et al.* (Particle Data Group), Chin. Phys. C **38**, 090001 (2014).

[22] M. Albaladejo and E. Oset, Phys. Rev. C **88**, 014006 (2013); and references therein.

[23] Xu Cao and Xi-Guo Li, Phys. Rev. C **78**, 035207 (2008), and references therein.

[24] Xu Cao, Bing-Song Zou, and Hu-Shan Xu, Phys. Rev. C **81**, 065201 (2010).

[25] F. Hinterberger and A. Sibirtsev, Eur. Phys. J. A **21**, 313 (2004).
[26] A. I. Titov, B. Kämpfer, and B. L. Reznik, Eur. Phys. J. A **7**, 543 (2000); K. M. Watson, Phys. Rev. **88**, 1163 (1952); N. Kaiser, Phys. Rev. C **60**, 057001 (1999); V. Bernard, N. Kaiser, and Ulf-G. Meissner, Eur. Phys. J. A **4**, 259 (1999).
[27] G. Faeldt and C. Wilkin, Phys. Scr. **64**, 427 (2001).
[28] K. Nakayama, J. Speth, and T.-S. H. Lee, Phys. Rev. C **65**, 045210 (2002).
[29] P. Adlarson *et al.*, Phys. Lett. B **743**, 325 (2015).
[30] P. Adlarson *et al.*, Phys. Rev. C **88**, 055208 (2013).