Baryon fields with $U_L(3) \times U_R(3)$ chiral symmetry. V. Pion-nucleon and kaon-nucleon Σ terms

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We have previously calculated the pion-nucleon $\Sigma_{\pi N}$ term in the chiral mixing approach with u,d flavors only, and found the lower bound $\Sigma_{\pi N} \geqslant (1+\frac{16}{3}\sin^2\theta)\frac{3}{2}(m_u^0+m_d^0)$, where m_u^0,m_d^0 are the current quark masses, and θ is the mixing angle of the $[(\frac{1}{2},\mathbf{0})\oplus(\mathbf{0},\frac{1}{2})]$ and the $[(\mathbf{1},\frac{1}{2})\oplus(\frac{1}{2},\mathbf{1})]$ chiral multiplets. This mixing angle can be calculated as $\sin^2\theta = \frac{3}{8}(g_A^{(0)}+g_A^{(3)})$, where $g_A^{(0)},g_A^{(3)}$, are the flavor-singlet and the isovector axial couplings. With presently accepted values of current quark masses, this leads to $\Sigma_{\pi N} \geqslant 58.0 \pm 4.5^{+11.4}_{-6.5}$ MeV, which is in agreement with the values extracted from experiments, and substantially higher than most previous two-flavor calculations. The causes of this enhancement are: (1) the large, ($\frac{16}{3} \simeq 5.3$), purely $SU_L(2)\times SU_R(2)$ algebraic factor; (2) the admixture of the $[(\mathbf{1},\frac{1}{2})\oplus(\frac{1}{2},\mathbf{1})]$ chiral multiplet component in the nucleon, whose presence has been known for some time, but that had not been properly taken into account, yet. We have now extended these calculations of $\Sigma_{\pi N}$ to three light flavors, i.e., to $SU_L(3)\times SU_R(3)$ multiplet mixing. Phenomenology of chiral $SU_L(3)\times SU_R(3)$ multiplet mixing demands the presence of three chiral $SU_L(3)\times SU_R(3)$ multiplets, viz. $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$, $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$, and $[(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})]$, in order to successfully reproduce the baryons' flavor-octet and flavor-singlet axial current coupling constants, as well as the baryon anomalous magnetic moments. Here we use these previously obtained results, together with known constraints on the explicit chiral symmetry breaking in baryons to calculate the $\Sigma_{\pi N}$ term, but find no change of $\Sigma_{\pi N}$ from the above successful two-flavor result. The physical significance of these results lies in the fact that they show no need for $q^4\bar{q}$ components, and in particular, no need for an $s\bar{s}$ component in the nucleon, in order to explain the large "observed" $\Sigma_{\pi N}$ value. We also predict the kaon-nucleon σ term Σ_{KN} that is experimentally unknown, but may be calculable in lattice QCD.

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I. INTRODUCTION

For more than 35 years the deviation of the nucleon $\Sigma_{\pi N}$ term extracted from the measured πN scattering partial wave analyses (in the following to be called "measured value", for brevity) from the naive quark model value of 25 MeV was interpreted as an increase of Zweig rule breaking in the nucleon, or equivalently to an increased content of unpolarized $s\bar{s}$ pairs in the nucleon [1–3], defined as $y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{u}u|N\rangle}$. Moreover, the anomalously small measured value of the flavor-singlet axial coupling $g_A^{(0)} = 0.33 \pm 0.06$ [4–6], or the older value 0.28 ± 0.16 [7], as compared with the naive quark model prediction of $g_A^{(0)} = 1$, was long interpreted as evidence for an increased *polarized* $s\bar{s}$ content of the nucleon [7–10]. Yet more recently, both of these conclusions and interpretations were checked directly in low-momentum transfer Q parity-violating elastic electron scattering experiments, however, and were found to be incorrect [11–13].

Whereas this situation is consistent with QCD, it seems in contradiction with earlier expectations, that were based on a combination of quark and chiral effective field theory models [14–16]. The question remains if one can explicitly construct an effective chiral field theory model for nucleons and mesons that is connected to the underlying quark structure of hadrons and reproduces these two "anomalous" results.

Gell-Mann and Lévy's (GML) linear σ model has been the principal example of an effective field theory model of strongly interacting nucleons and pions with spontaneously broken chiral symmetry ever since its inception more than 50 years ago [17,18]. It is well known that this linear σ model does not always reproduce the correct phenomenology, e.g., (a) the value of the isovector axial coupling strength $g_A^{(3)}$ equals unity in this model; (b) the value of the isoscalar pion-nucleon scattering length is too large in this model.

Both of these shortcomings have been removed in an extended linear σ model, proposed by Bjorken and Nauenberg [19] and by Lee [20]: (a) The first one had been fixed by introducing an additional derivative-coupling term that is not renormalizable. (b) Reference [21] showed that consequently the phenomenology is considerably improved in the Bjorken-Nauenberg-Lee (BNL) extended linear σ model, as compared to the original GML model; in particular, the value of the isoscalar pion-nucleon scattering length is reduced to its observed value. This improvement is directly related to the correct value of the isovector axial coupling constant $g_A^{(3)}$ of the nucleon in the BNL model, which in turn is a direct consequence of the new derivative coupling.[1] This shows the

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[1]This BNL extended linear σ model allows one to study the $g_A^{(1)}$ dependence of the πN scattering lengths, $a_{\pi N}$, and of the nucleon σ

phenomenological importance of having the correct value of the isovector axial coupling.

That is not the only axial coupling of the nucleon; however, there is also the isoscalar one $g_A^{(0)}$, whose measured value $g_A^{(0)} = 0.33 \pm 0.08$ or 0.28 ± 0.16 deviates even more from unity, which is the value that the naive nonrelativistic quark model suggests and the GML model postulates. The BNL derivative coupling term does not fix the value of the isoscalar axial coupling strength $g_A^{(0)}$, however. Here one could continue with the BNL stratagem and introduce yet another derivative-coupling term to fix this problem, but clearly that would be *ad hoc* and in no apparent way related to the underlying quark structure.

An alternative approach was attempted with the notion of chiral representation mixing, which is in fact older than the BNL model, but was able to reproduce realistic values of the isovector axial couplings [22–24]. By choosing low-dimensional representations, as corresponding to the ones of the nucleon's three-quark interpolators at present days, that approach turned out to give some constraints on the values of the axial couplings $g_A^{(0,1)}$ without introducing derivative couplings of hadrons [25–27]. In this sense it is rather different from the BNL model, and one should not be surprised if other predictions of the two models are different. What is perhaps not so well known is that there are two-flavor (linear realization) σ model chiral Lagrangians based on the concept of chiral mixing, that reproduce the two-flavor chiral low-energy theorems [20,26–31]. Over the years, these Lagrangians have been extended to three flavors [32–38] and adapted/fitted to the two axial couplings and other nucleon properties, such as the magnetic moments [39,40].

An advantage of the chiral representation mixing is that the possible representations and their mixing may be inferred by the quark structure of the nucleon. For instance, in the Schwinger-Dyson-Faddeev-Bethe-Salpeter approach to QCD [41], different Dirac structures in the Faddeev-Bethe-Salpeter equation are sources of different chiral representation (or components in the Faddeev-Bethe-Salpeter amplitude), thus leading to a mixing of chiral representations in the physical nucleon wave function.

The purpose of the present paper is to coherently and systematically present the calculation of the pion-nucleon σ term $\Sigma_{\pi N}$ and of the isoscalar axial coupling $g_A^{(0)}$ in the chiral-mixing linear σ model. The isovector axial coupling $g_A^{(3)}$ has been studied by many authors as already mentioned above. The isoscalar axial coupling $g_A^{(0)}$ had been calculated in Refs. [30,42], and in Refs. [43,44] we have briefly presented our results for the pion-nucleon σ term $\Sigma_{\pi N}$ in the chiral mixing approach. The result of $\Sigma_{\pi N}$ depends substantially on the isovector axial coupling $g_A^{(3)}$, in contrast to the BNL model one [21], and agrees with the experimental value (almost embarrassingly) well. This phenomenological success has been a source of some open and more hidden criticism. We do not wish to overemphasize this phenomenological agreement here, as it is subject to the time-dependent variation of the free parameters, more specifically, to the current quark masses, which were about 50% larger 15 years ago—in this model, but rather we try and systematically explore the differences among various effective chiral models.

Moreover, we make a systematic exposition of our approach and we record this model's predictions of the kaon-nucleon σ term, which have not been measured as yet, just in case some day they are measured if only on the lattice, and thus open ourselves to potential future criticism. In this way we explicitly show how to construct effective linear chiral model(s) of interacting nucleons and mesons based on the underlying quark structure, that does not need $s\bar{s}$ content in the nucleon to reproduce the two crucial observables, the $\Sigma_{\pi N}$ and the $g_A^{(0)}$.

The crucial assumption here is the systematic implementation of chiral symmetry at all levels, i.e., at both the quark and the hadron levels. In Ref. [21] we have shown how Weinberg's "chiral boost" transformation of the BNL model leads to a nonlinear realization chiral Lagrangian, as corresponding to the leading order of the chiral perturbation theory Lagrangian. The same procedure can be applied to the chiral mixing Lagrangian, with the same result. For this to happen, two different linear Lagrangians lead to the same nonlinear one. That goes to show that the linear-to-nonlinear-realization mapping (Weinberg's chiral boost) is of the many-to-one kind. Thus, it "hides" many details of the underlying dynamics at low values of momentum transfer as compared with $f_\pi = 93$ MeV and $m_\pi = 140$ MeV, and emphasizes the dynamical aspects of chiral symmetry. In that sense, the nonlinear realization can be viewed as being "coarser" than the linear one. These dynamical details become increasingly visible as the momentum transfer is increased, however.

We believe that at least some of the generally valid chiral predictions of all chiral models are most economically obtained in the chiral-mixing model. In particular, we believe that the role of $U_A(1)$ symmetry and its breaking in the baryon sector has been ignored thus far, and our study appears to be the first step in rectifying this lamentable situation.

Throughout this paper we shall use the first Born approximation at the tree level. In order to explore the various possibilities and to facilitate comparison with earlier studies of the Gell-Mann–Levy linear σ model, we introduce three different chiral symmetry breaking [χSB] terms, as in Refs. [45,46].

This paper is the fifth one in a sequence of papers [33–35,47], consequently, we shall repeat here, for the sake of completeness and coherence of presentation, rather than merely cite, several (a bare minimum of) equations and tables that have already appeared in our previous papers.

The paper falls into six sections and three Appendices. In Sec. II we consider the chiral mixing phenomenology. Then in Sec. III, which is devoted to a construction of a

term $\Sigma_{\pi N}$. It is well known that $a_{\pi N}^{(-)}$ depends crucially on the value of $g_A^{(3)}$, whereas the $\Sigma_{\pi N}$ dependence on $g_A^{(3)}$ was not known (and may be model-dependent). We displayed this dependence of $a_{\pi N}^{(-)}$ and showed that a large value of $\Sigma_{\pi N}$ could easily be obtained without recourse to any $s\bar{s}$ component of the nucleon with the values of bare parameters available at the time (which have changed drastically in the meantime, however). We also reproduced the then new, tiny experimental value of the isoscalar πN scattering length $a_0^{(+)}$.

two- and three-flavor chiral Lagrangians that reproduce the chiral mixing phenomenology, we present the χSB terms and the canonical field variables, and show that the Noether charges close the chiral algebra although $g_A \neq 1$. In Sec. IV we examine the pion-nucleon σ term $\Sigma_{\pi N}$—some of these results have been reported at conferences [43,44]. In Sec. V we examine the kaon-nucleon Σ_{KN} term, and in Sec. VI we summarize the results. Technical topics are relegated to the Appendices.

II. PHENOMENOLOGY OF CHIRAL MIXING

The basic premise of the chiral mixing approach is that the chiral $SU_L(3) \times SU_R(3)$ symmetry is spontaneously broken and therefore that the eigenstates do not form irreducible representations of the chiral symmetry group $SU_L(3) \times SU_R(3)$.[2] Rather, the eigenstates are linear superpositions of several (irreducible representations of $SU_L(3) \times SU_R(3)$. In general, such chiral representation mixing theories tend to be most powerful and predictive when only a few chiral multiplets are involved. As the number of admixed multiplets grows, this method becomes increasingly complicated and thus loses its predictive power.

Just which irreducible representations are being admixed, is a question that ultimately ought to be answered by QCD. In the absence of a QCD-based answer, the choice can be (severely) limited by the following mathematical and physical considerations.

A. Chiral representations

Group-theoretical considerations impose limitations on the allowed irreducible representation of $SU_L(3) \otimes SU_R(3)$: any irreducible representation of $SU_L(3) \otimes SU_R(3)$, that is described by two SU(3) irreducible representations, (G_L, G_R), leads to irreducible representations G_F of $SU_F(3)$ as determined by the Clebsch-Gordan series of the tensor product: $G_F \in G_L \otimes G_R$. For example, $\mathbf{10} \oplus \mathbf{8} = \mathbf{6} \otimes \mathbf{3} \in [(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$, $\mathbf{1} \oplus \mathbf{8} = \bar{\mathbf{3}} \otimes \mathbf{3} \in [(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})]$, and $\mathbf{8} = \mathbf{8} \otimes \mathbf{1} \in [(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$.

If one demands that only the experimentally observed irreducible representations G of $SU_F(3)$ appear in these Clebsch-Gordan series, then one is limited to the above three reducible chiral representations: Any other chiral representation, other than the (trivial) chiral-singlet one, $[(\mathbf{1},\mathbf{1})]$, necessarily leads to $SU_F(3)$ exotics.

When we further take into account the so-called "mirror" representations, in which the left- (L) and the right-handed (R) representations are interchanged, $(G_L \leftrightarrow G_R)$, in the chiral multiplet, then the number of allowed chiral multiplets is six. Mathematically, there is no difference between the "naive" (natural?) and "mirror" representations; physically, and historically, the "naive" ones were introduced first, mostly because there were no explicit examples how the "mirror" ones could arise in a three-quark system. That "objection" was finally raised by explicit examples of mirror (three-quark) interpolating fields in Refs. [25–27,47–49]. For octet baryons, this limits the permissible chiral multiplets to $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$, and its "mirror" $[(\mathbf{3},\mathbf{6}) \oplus (\mathbf{6},\mathbf{3})]$, to $[(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})]$, and its "mirror" $[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$, and to $[(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$, and its "mirror" $[(\mathbf{1},\mathbf{8}) \oplus (\mathbf{8},\mathbf{1})]$. Of course, one may have other, "exotic" chiral multiplets that contain manifestly exotic flavor $SU_F(3)$ multiplets, but we exclude them *per fiat*, for lack of observed exotics.

Historically, after the observation, in Refs. [50–54], that several crucial SU(6) algebra results follow from its (smaller) $SU(3) \otimes SU(3)$ subalgebra, the notion of $SU(3) \otimes SU(3)$ representation mixing was proposed as an explanation of the nucleon's (isovector) axial coupling $g_A^{(1)}$. The physical nature of this $SU(3) \otimes SU(3)$ subalgebra was not immediately clear, however, as two options (the conventional chiral charge algebra, and the so-called "collinear" algebra) existed at the time.

Indeed, the "collinear" $SU(3) \otimes SU(3)$ algebra, which was generally assumed in the early work, holds only in a particular (the so-called $p_z \to \infty$) frame of reference, which appears to be in conflict with the general principles of special relativity. Moreover, Adler and Weisberger had also derived their sum rule(s) with the help of the p_∞ frame. It was only after Weinberg's [55] clarification of the Adler-Weisberger sum rule as consisting of two independent statements [viz. (a) the model-independent Goldberger-Miyazawa-Oehme sum rule for the πN scattering lengths; and (b) the chiral symmetry breaking-dependent predictions for the πN scattering lengths] that this matter was settled in favor of chiral symmetry, and thus the way was paved for its later applications in QCD. Thus, only the chiral charge symmetry option leads to Lorentz-invariant quark interaction theories, such as QCD.

B. Chiral mixing

In one of the earliest physical applications of the chiral configuration mixing idea, Harari [22], Bincer [56], Gerstein and Lee [23], and Gatto *et al.* [57] used the mixing of three of the aforementioned six chiral multiplets to fit the nucleon's isovector axial coupling constant $g_A^{(1)}$ value at 1.267, [58] and thus explain its being different from unity, as was seemingly demanded by the Gell-Mann–Lévy model [17]. It turned out, however, that this application was not selective at all: all mixing scenarios could reproduce this value, so long as the $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$ multiplet was involved: (a) Gatto *et al.*—Harari scenario [22,57]

$$|N(8)\rangle = \sin\theta|(\mathbf{6},\mathbf{3})\rangle + \cos\theta(\cos\varphi|(\mathbf{3},\bar{\mathbf{3}})\rangle + \sin\varphi|(\bar{\mathbf{3}},\mathbf{3})\rangle), \quad (1)$$

or (b) Gerstein-Lee scenario [23]

$$|N(8)\rangle = \sin\theta|(\mathbf{6},\mathbf{3})\rangle + \cos\theta(\cos\varphi|(\mathbf{3},\bar{\mathbf{3}})\rangle + \sin\varphi|(\mathbf{8},\mathbf{1})\rangle). \quad (2)$$

For other, more exotic scenarios, see Ref. [59]. Simultaneously, or somewhat later, Refs. [24,40,53,60,61] used the same

[2]Chiral symmetry does not require irreducible representations for parity-conserving interactions.

TABLE I. The Abelian and the non-Abelian axial charges (+ sign indicates "naive", − sign "mirror" transformation properties) and the non-Abelian chiral multiplets of $J^P = \frac{1}{2}$, Lorentz representation $(\frac{1}{2},0)$ nucleon fields. The field denoted by 0 belongs to the $(1,\frac{1}{2}) \oplus (\frac{1}{2},1)$ chiral multiplet and is the basic nucleon field that is mixed with various $(\frac{1}{2},0)$ nucleon fields.

case	field	$g_A^{(0)}$	$g_A^{(1)}$	$SU_L(2) \times SU_R(2)$	F	D	$SU_L(3) \times SU_R(3)$
I	$N_1 - N_2$	-1	$+1$	$(\frac{1}{2},0) \oplus (0,\frac{1}{2})$	0	$+1$	$(3,\bar{3}) \oplus (\bar{3},3)$
II	$N_1 + N_2$	$+3$	$+1$	$(\frac{1}{2},0) \oplus (0,\frac{1}{2})$	$+1$	0	$(8,1) \oplus (1,8)$
III	$N_1' - N_2'$	$+1$	-1	$(0,\frac{1}{2}) \oplus (\frac{1}{2},0)$	0	-1	$(\bar{3},3) \oplus (3,\bar{3})$
IV	$N_1' + N_2'$	-3	-1	$(0,\frac{1}{2}) \oplus (\frac{1}{2},0)$	-1	0	$(1,8) \oplus (8,1)$
0	$\partial_\mu(N_3^\mu + \frac{1}{3}N_4^\mu)$	$+1$	$+\frac{5}{3}$	$(1,\frac{1}{2}) \oplus (\frac{1}{2},1)$	$+\frac{2}{3}$	$+1$	$(6,3) \oplus (3,6)$

approach to saturate the electric dipole operator algebra and calculate the nucleon's anomalous magnetic moments and charge radii. Moreover, other phenomenological applications of the current algebra, e.g., to pion photoproduction can be found in Ref. [53]. All of this was done in the framework of collinear $SU(3) \otimes SU(3)$ algebra, but algebraically these results must be the same as the chiral $SU(3) \otimes SU(3)$ algebra ones. The construction of corresponding chiral multiplets in the $SU(3) \otimes SU(3)$ chiral charge algebra is not as straightforward as in the collinear one, however (see our remarks about interpolators, below).

There is no guarantee that all six of the above chiral (not collinear) multiplets are allowed by the Pauli principle in the ground state of the nucleon, as composed of three Dirac quarks.[3] The (formal) tool for this kind of study was provided around 1980 [62–64], in the form of the so-called nucleon-three-quark interpolating fields.

Studies, in Refs. [25,26,47–49], of local (S wave, therefore ground state candidates), bilocal (P wave and higher), and trilocal (D wave and higher) three-quark interpolators have shown that only $[(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})]$ and $[(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$ are allowed in the local limit and that $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$ appears as a spin 1/2 "complement" to the local Rarita-Schwinger spin-3/2 interpolator. Many other chiral multiplets appear in the nonlocal case, where the Pauli principle is less restrictive.

C. Isoscalar axial coupling

The nucleon has also a flavor singlet axial coupling $g_A^{(0)}$, that has not been measured directly from elastic parity-violating lepton-nucleon scattering, as yet. Rather, it has been extracted indirectly from spin-polarized lepton-nucleon DIS data after 1988 as $g_A^{(0)} = 0.28 \pm 0.16$ [7], or the more recent value of $0.33 \pm 0.03 \pm 0.05$ [16], which is in the nonrelativistic quark model predicted to be unity. Our studies of interpolating fields have shown that each $SU_L(3) \times SU_R(3)$ multiplet carries definite $U_A(1)$ transformation properties and the corresponding $U_A(1)$ charge, see Table I. Then, the next basic question becomes if the same set of chiral mixing angle(s) can simultaneously explain this anomalously low value. The answer, which is in the positive [39], manifestly depends on the $U_A(1)$ chiral transformation properties of the admixed nucleon fields, and leads to the so-called Harari scenario that mixes $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$, with $[(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})]$, and its "mirror" $[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$ field. No admixture of $[(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$, or its "mirror" $[(\mathbf{1},\mathbf{8}) \oplus (\mathbf{8},\mathbf{1})]$ is preferred. This fact confirms the Gatto-Harari scenario, Eq. (1), and eliminates the Gerstein-Lee scenario, Eq. (2), from contention.

Moreover, we note that the above outlined program of fitting the hadron/nucleon observables in order to obtain chiral mixing angles is practically feasible only for the ground state(s): e.g., there is no hope of ever (sufficiently accurately) measuring the isovector axial coupling of the Δ resonance, except, perhaps, on the lattice. The same comments hold for the negative parity, and all of the higher-lying excited states. In this sense, the present scheme is of limited scope, but its potential to explain and illustrate the (fairly complex) QCD physics of baryons is undeniable.

The above no-$[(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$ or $[(\mathbf{1},\mathbf{8}) \oplus (\mathbf{8},\mathbf{1})]$ selection rule is in striking agreement with the results of so-called QCD sum rules and lattice QCD calculations [62,63] that indicate only weak coupling of the physical nucleon ground state to the $[(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$ and/or its "mirror" $[(\mathbf{1},\mathbf{8}) \oplus (\mathbf{8},\mathbf{1})]$ multiplet component. There is no dynamical, or symmetry-based explanation of this fact, as yet.

Specific dynamical models such as the Faddeev-Bethe-Salpeter-Schwinger-Dyson equation approach of Ref. [41], or the Faddeev-Salpeter equation approach of Ref. [65], ought to yield specific predictions for these mixing angles/parameters, and perhaps also to a dynamical explanation of empirical selection rules such as the above one.

Irrespective of specific dynamical model calculations, there ought to exist an effective chiral Lagrangian description of the corresponding hadron degrees of freedom. The task of constructing them was long drawn out: (a) the $(1,\frac{1}{2}) - (0,\frac{1}{2})$ chiral representation mixing Lagrangian with $SU_L(2) \otimes SU_R(2)$ chiral symmetry was first presented by Hara [31]; (b) the first "naive"-"mirror" $(\frac{1}{2},0) - (0,\frac{1}{2})$ chiral-mixing Lagrangian with $SU_L(2) \otimes SU_R(2)$ chiral symmetry was presented by Lee [20], and further extended by a number of researchers [29,66–68], and most recently by Nagata *et al.* [26,27,30,42]; (c) the extension to $SU_L(3) \otimes SU_R(3)$ chiral symmetry has been

[3] That aspect of the problem could be safely neglected in the collinear approach, which allows arbitrary values of the orbital angular momentum L and restricts only its z-projection L_z.

accomplished in Refs. [32,34,35,37,38] and will be briefly reviewed in Sec. III.

D. Consistency of chiral algebra

Before we do that, however, we must show that the axial charges still obey the same $SU_L(3) \otimes SU_R(3)$ chiral algebra even after chiral mixing has occurred. A problem of the axial charge in the spontaneously broken vacuum and in the chiral limit is that it leads to singularities in matrix elements due to the massless pion pole. Once the chiral symmetry is explicitly broken this problem generally disappears, as the Nambu-Goldstone bosons acquire mass. In the present context, the effect of the broken-symmetry vacuum is included by the representation mixing, among other things. It is therefore useful to show that the $SU_L(3) \otimes SU_R(3)$ chiral algebra holds for the baryon axial and vector charge matrix elements with chiral mixing included. That will ensure that the chiral symmetry-breaking Dashen double commutator can be straightforwardly calculated.

A basic feature of the linear chiral realization is that the axial couplings are determined by the chiral representations. In Ref. [47], we found that for the nucleon octet, the three-quark chiral representations of $SU_L(3) \times SU_R(3)$, $(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})$, $(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})$, and $(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})$ provide the nucleon isovector axial coupling $g_A^{(3)} = 1$, 1 and 5/3, respectively. Then in Ref. [33], we found that the mixing of chiral $(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})$, $(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})$, and $(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})$ nucleons leads to the observed axial couplings (the case III-I in Ref. [33]):

$$\begin{aligned} g_A^{(3)} &= g_A^{(3)}(\mathbf{6},\mathbf{3}) \sin^2\theta \\ &\quad + \cos^2\theta \left(g_A^{(3)}(\mathbf{3},\bar{\mathbf{3}})\cos^2\varphi + g_A^{(3)}(\bar{\mathbf{3}},\mathbf{3})\sin^2\varphi\right) = 1.267, \end{aligned} \tag{3}$$

$$\begin{aligned} g_A^{(0)} &= g_A^{(0)}(\mathbf{6},\mathbf{3}) \sin^2\theta \\ &\quad + \cos^2\theta \left(g_A^{(0)}(\mathbf{3},\bar{\mathbf{3}})\cos^2\varphi + g_A^{(0)}(\bar{\mathbf{3}},\mathbf{3}) \sin^2\varphi\right) \\ &= 0.33 \pm 0.08, \end{aligned} \tag{4}$$

where we used

$$\begin{aligned} \langle N|Q_5^a|N\rangle &= \langle N|Q_5^a(\mathbf{6},\mathbf{3})|N\rangle \sin^2\theta \\ &\quad + \cos^2\theta \left(\langle N|Q_5^a(\mathbf{3},\bar{\mathbf{3}})|N\rangle\cos^2\varphi \right. \\ &\quad \left. + \langle N|Q_5^a(\bar{\mathbf{3}},\mathbf{3})|N\rangle\sin^2\varphi\right). \end{aligned} \tag{5}$$

Next we used Table I values of $g_A^{(3)}(\mathbf{6},\mathbf{3}) = \frac{5}{3} = -g_A^{(3)}(\mathbf{3},\mathbf{6})$, $g_A^{(3)}(\mathbf{3},\bar{\mathbf{3}}) = 1 = -g_A^{(3)}(\bar{\mathbf{3}},\mathbf{3})$, and $g_A^{(0)}(\mathbf{3},\bar{\mathbf{3}}) = -1 = -g_A^{(0)}(\bar{\mathbf{3}},\mathbf{3})$, to find

$$g_A^{(3)} = \tfrac{5}{3} \sin^2\theta + \cos^2\theta \cos 2\varphi = 1.267, \tag{6}$$

$$g_A^{(0)} = \sin^2\theta - \cos^2\theta \, \cos 2\varphi = 0.33 \pm 0.08, \tag{7}$$

whose solutions are

$$\theta = 50.7^{\circ} \pm 1.8^{\circ}, \quad \varphi = 66.1^{\circ} \pm 2.9^{\circ}. \tag{8}$$

Of course, this mixing appears to affect the $SU_L(3) \otimes SU_R(3)$ chiral algebra, as well, so we must first check that we did not spoil this algebra. The main "problematic" part of the $SU_L(3) \otimes SU_R(3)$ chiral algebra is the double-axial commutator

$$\left[Q_5^a, Q_5^b\right] = i f^{abc} Q^c. \tag{9}$$

We shall check this commutation rule in the nucleon subspace of the full Hilbert space:

$$\begin{aligned} \langle N|\left[Q_5^a, Q_5^b\right]|N\rangle &= \langle N|\left[Q_5^a(\mathbf{6},\mathbf{3}), Q_5^b(\mathbf{6},\mathbf{3})\right]|N\rangle \sin^2\theta \\ &\quad + \langle N|\left[Q_5^a(\mathbf{3},\bar{\mathbf{3}}), Q_5^b(\mathbf{3},\bar{\mathbf{3}})\right]|N\rangle \cos^2\theta\cos^2\varphi \\ &\quad + \langle N|\left[Q_5^a(\bar{\mathbf{3}},\mathbf{3}), Q_5^b(\bar{\mathbf{3}},\mathbf{3})\right]|N\rangle \cos^2\theta\sin^2\varphi. \end{aligned} \tag{10}$$

Next, we may use the commutators $[Q_5^a, Q_5^b] = i f^{abc} Q^c$ for the $(\mathbf{3},\bar{\mathbf{3}})$ and the $(\mathbf{6},\mathbf{3})$ chiral multiplets worked out in Ref. [33] and listed in Appendix B, which all lead to the same SU(3) vector charges Q^c:

$$\left[Q_5^a(\mathbf{3},\bar{\mathbf{3}}), Q_5^b(\mathbf{3},\bar{\mathbf{3}})\right] = i f^{abc} Q^c(\mathbf{3},\bar{\mathbf{3}}) = i f^{abc} Q^c, \tag{11}$$

$$\left[Q_5^a(\mathbf{6},\mathbf{3}), Q_5^b(\mathbf{6},\mathbf{3})\right] = i f^{abc} Q^c(\mathbf{6},\mathbf{3}) = i f^{abc} Q^c. \tag{12}$$

Thus we find

$$\begin{aligned} &\langle N|\left[Q_5^a, Q_5^b\right]|N\rangle \\ &\quad = i f^{abc}\langle N|Q^c|N\rangle(\sin^2\theta + \cos^2\theta(\cos^2\varphi + \sin^2\varphi)) \\ &\quad = i f^{abc}\langle N|Q^c|N\rangle, \end{aligned} \tag{13}$$

which confirms the chiral charge $SU_L(3) \otimes SU_R(3)$ algebra. This ensures that the chiral symmetry-breaking Dashen double commutator can be safely and reliably calculated in the chiral mixing approach.

III. THE LINEAR σ MODEL FOR CHIRAL MIXING

The next step is to try and reproduce this phenomenological mixing starting from a model interaction, rather than *per fiat*. As the first step in that direction we must look for a dynamical source of chiral mixing. One, perhaps the simplest, such mechanism is the chirally symmetric *nonderivative* one-(σ,π)-meson interaction Lagrangian, which induces baryon masses via its σ-meson coupling. For this reason we need to know the form of the most general such Lagrangian(s); that problem was solved in Ref. [34] for three flavors and in Refs. [30,42] for two flavors.

There is a significant difference between $N_f = 2$ and $N_f = 3$ chirally symmetric linear σ models of chiral mixing, as only in the latter case there are strongly restrictive selection rules.

For example, most $U_A(1)$ symmetry-breaking and $SU_L(3) \times SU_R(3)$ chiral symmetry-conserving interactions are forbidden, see Tables II and III taken from Ref. [34]. In particular only one $SU_L(3) \times SU_R(3)$ symmetric, but $U_A(1)$ symmetry-breaking interaction (the $[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$-$[(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$ and its Hermitian conjugate $[(\mathbf{1},\mathbf{8}) \oplus (\mathbf{8},\mathbf{1})]$[mir]-$[(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}}])$[mir]) is allowed. These results stand in marked contrast to the two-flavor case [30,42], where all of the $SU_L(2) \times SU_R(2)$ symmetric interactions have both a $U_A(1)$

TABLE II. Allowed chiral invariant interaction Lagrangian with one pseudoscalar meson field, denoted by either M or $M^\dagger$ as corresponding to Eqs. (19) and (20). The symbol – indicates that chiral invariant construction is not allowed. All cases are both $\mathrm{SU}_L(3)\times\mathrm{SU}_R(3)$ and $\mathrm{U}_A(1)$ invariant except for the last (third) group where $\mathrm{U}_A(1)$ is broken.

	$(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})$[mir]	$(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})$[mir]	$(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})$	$(\mathbf{1},\mathbf{10})\oplus(\mathbf{10},\mathbf{1})$[mir]
$\overline{(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})}$[mir]	–	$M^\dagger$	$M^\dagger$	–
$\overline{(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})}$[mir]	$M^\dagger$	M	M	–
$\overline{(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})}$	$M^\dagger$	M	M	$M^\dagger$
$\overline{(\mathbf{1},\mathbf{10})\oplus(\mathbf{10},\mathbf{1})}$[mir]	–	–	$M^\dagger$	–
	$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$	$(\mathbf{3},\mathbf{6})\oplus(\mathbf{6},\mathbf{3})$[mir]	$(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})$
$\overline{(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})}$	–	M	M	–
$\overline{(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})}$	M	$M^\dagger$	$M^\dagger$	–
$\overline{(\bar{\mathbf{3}},\bar{\mathbf{6}})\oplus(\bar{\mathbf{6}},\bar{\mathbf{3}})}$[mir]	M	$M^\dagger$	$M^\dagger$	M
$\overline{(\overline{\mathbf{10}},\mathbf{1})\oplus(\mathbf{1},\overline{\mathbf{10}})}$	–	–	M	–
	$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	$(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})$[mir]		
$\overline{(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})}$	–	$M^\dagger$; $U_A(1)$ broken		
$\overline{(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})}$[mir]	M; $U_A(1)$ broken	–		

symmetry-conserving and a $\mathrm{U}_A(1)$ symmetry-breaking version. This is due to the fact that in the $\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)$ limit both the $[(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})]$ and the $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$ multiplet reduce to the same multiplet $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$, albeit with different $\mathrm{U}_A(1)$ symmetry properties.

Although, the $\mathrm{SU}_L(3)\times\mathrm{SU}_R(3)$ symmetry is rather badly explicitly broken, we may expect that in the corresponding $\pi-N$ sector, the $\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)$ symmetry may remain more-or-less conserved. So, although we shall be primarily interested in the pion-nucleon case, i.e., in $N_f=2$, we shall use the $N_f=3$ selection rules for guidance.

A. A brief summary of $N_f = 3$ interactions

In this section, we introduce a shorthand notation:

$$N_{(8m)}\sim[(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})][\mathrm{mir}],\quad N_{(9m)}\sim[(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})][\mathrm{mir}],$$
$$N_{(18)}\sim(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6}),\quad N_{(10m)}\sim[(\mathbf{1},\mathbf{10})\oplus(\mathbf{10},\mathbf{1})][\mathrm{mir}], \tag{14}$$

and similar for their mirror and naive representations. The scalar (σ) and pseudoscalar (π) mesons are introduced and transformed under the chiral transformations as

$$M=\sigma+i\gamma_5\pi\sim(\mathbf{3},\bar{\mathbf{3}}),\quad M^\dagger=\sigma-i\gamma_5\pi\sim(\bar{\mathbf{3}},\mathbf{3}). \tag{15}$$

Now the chiral structure of the Lagrangians for Yukawa-type interactions is

$$\bar{N}MN'+\bar{N}'M^\dagger N\sim\bar{N}_L M N'_R+\bar{N}'_R M^\dagger N_L, \tag{16}$$

where N and N' may belong to different chiral representations. Our task is to form chiral singlet combinations for these interactions. For instance,

$$\bar{N}_{(9m)}MN_{(18)}\sim\overline{(\bar{\mathbf{3}},\mathbf{3})}\otimes(\mathbf{3},\bar{\mathbf{3}})\otimes(\mathbf{3},\mathbf{6})+\overline{(\mathbf{3},\bar{\mathbf{3}})}\otimes(\bar{\mathbf{3}},\mathbf{3})\otimes(\mathbf{6},\mathbf{3}) \tag{17}$$

can make the $\mathrm{SU}_L(3)\times SU_R(3)$ chiral singlet ($\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}\to\mathbf{1}$, $\bar{\mathbf{3}}\otimes\bar{\mathbf{3}}\otimes\mathbf{6}\to\mathbf{1}$). This corresponds to the cell at the second row and third column of Table II. Contrary, a combination like

$$\bar{N}_{(8m)}MN_{(8m)}\sim\overline{(\mathbf{1},\mathbf{8})}\otimes(\mathbf{3},\bar{\mathbf{3}})\otimes(\mathbf{8},\mathbf{1})+\overline{(\mathbf{8},\mathbf{1})}\otimes(\bar{\mathbf{3}},\mathbf{3})\otimes(\mathbf{1},\mathbf{8}) \tag{18}$$

cannot make the chiral invariant interaction as corresponding to the cell at the first row and column of Table II. We can also consider other possible combinations, all of which are listed in Table II.

TABLE III. Allowed chiral invariant mass terms as denoted by 1, while the symbol – indicates that chiral invariant construction is not allowed. All cases are both $\mathrm{SU}_L(3)\times\mathrm{SU}_R(3)$ and $\mathrm{U}_A(1)$ invariant.

$[\mathrm{SU}_A(3),\mathrm{U}_A(1)]$	$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$	$(\mathbf{3},\mathbf{6})\oplus(\mathbf{6},\mathbf{3})$[mir]	$(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})$
$\overline{(\mathbf{1},\mathbf{8})}\oplus\overline{(\mathbf{8},\mathbf{1})}$[mir]	1	–	–	–
$\overline{(\mathbf{3},\bar{\mathbf{3}})}\oplus\overline{(\bar{\mathbf{3}},\mathbf{3})}$[mir]	–	1	–	–
$\overline{(\mathbf{6},\bar{\mathbf{3}})}\oplus\overline{(\bar{\mathbf{3}},\mathbf{6})}$	–	–	1	–
$\overline{(\mathbf{1},\overline{\mathbf{10}})}\oplus\overline{(\overline{\mathbf{10}},\mathbf{1})}$[mir]	–	–	–	1

The results are also expressed explicitly in the form of the Lagrangian which is given by

$$\mathcal{L} = (\overline{N}_{(8m)} \quad \overline{N}_{(9m)} \quad \overline{N}_{(18)} \quad \overline{N}_{(10m)}) \left(M \begin{pmatrix} \mathbf{0}_{8\times8} & \mathbf{0}_{8\times9} & \mathbf{0}_{8\times18} & \mathbf{0}_{8\times10} \\ \mathbf{0}_{9\times8} & g_{(9)}\mathbf{D}^a_{(9)} & g_{(9/18)}\mathbf{T}^a_{(9/18)} & \mathbf{0}_{9\times10} \\ \mathbf{0}_{18\times8} & g^*_{(9/18)}\mathbf{T}^{\dagger a}_{(9/18)} & g_{(18/18)}\mathbf{D}^a_{(18)} & \mathbf{0}_{18\times10} \\ \mathbf{0}_{10\times8} & \mathbf{0}_{10\times9} & \mathbf{0}_{10\times18} & \mathbf{0}_{10\times10} \end{pmatrix} + M^\dagger \begin{pmatrix} \mathbf{0}_{8\times8} & g_{(8/9)}\mathbf{T}^a_{(8/9)} & g_{(8/18)}\mathbf{T}^a_{(8/18)} & \mathbf{0}_{8\times10} \\ g^*_{(8/9)}\mathbf{T}^{\dagger a}_{(8/9)} & \mathbf{0}_{9\times9} & \mathbf{0}_{9\times18} & \mathbf{0}_{9\times10} \\ g^*_{(8/18)}\mathbf{T}^{\dagger a}_{(8/18)} & \mathbf{0}_{18\times9} & \mathbf{0}_{18\times18} & g^*_{(10/18)}\mathbf{T}^{\dagger a}_{(10/18)} \\ \mathbf{0}_{10\times8} & \mathbf{0}_{10\times9} & g_{(10/18)}\mathbf{T}^a_{(10/18)} & \mathbf{0}_{10\times10} \end{pmatrix} \right) \begin{pmatrix} N_{(8m)} \\ N_{(9m)} \\ N_{(18)} \\ N_{(10m)} \end{pmatrix}. \tag{19}$$

Here $\mathbf{0}_{A\times B}$ is the null matrix of dimension $A \times B$, and $\mathbf{D}^a_{(A)}, \mathbf{T}^a_{(A/B)}$ are flavor transition matrices of a dimension as indicated by their subscripts, which are defined in Ref. [33]. Similarly the mirror counterparts are given as

$$\mathcal{L}_{(m)} = (\overline{N}_{(8)} \quad \overline{N}_{(9)} \quad \overline{N}_{(18m)} \quad \overline{N}_{(10)}) \left(M^\dagger \begin{pmatrix} \mathbf{0}_{8\times8} & \mathbf{0}_{8\times9} & \mathbf{0}_{8\times18} & \mathbf{0}_{8\times10} \\ \mathbf{0}_{9\times8} & g'_{(9)}\mathbf{D}^a_{(9)} & g'_{(9/18)}\mathbf{T}^a_{(9/18)} & \mathbf{0}_{9\times10} \\ \mathbf{0}_{18\times8} & g'^*_{(9/18)}\mathbf{T}^{\dagger a}_{(9/18)} & g'_{(18/18)}\mathbf{D}^a_{(18)} & \mathbf{0}_{18\times10} \\ \mathbf{0}_{10\times8} & \mathbf{0}_{10\times9} & \mathbf{0}_{10\times18} & \mathbf{0}_{10\times10} \end{pmatrix} + M \begin{pmatrix} \mathbf{0}_{8\times8} & g'_{(8/9)}\mathbf{T}^a_{(8/9)} & g'_{(8/18)}\mathbf{T}^a_{(8/18)} & \mathbf{0}_{8\times10} \\ g'^*_{(8/9)}\mathbf{T}^{\dagger a}_{(8/9)} & \mathbf{0}_{9\times9} & \mathbf{0}_{9\times18} & \mathbf{0}_{9\times10} \\ g'^*_{(8/18)}\mathbf{T}^{\dagger a}_{(8/18)} & \mathbf{0}_{18\times9} & \mathbf{0}_{18\times18} & g'^*_{(10/18)}\mathbf{T}^{\dagger a}_{(10/18)} \\ \mathbf{0}_{10\times8} & \mathbf{0}_{10\times9} & g'_{(10/18)}\mathbf{T}^a_{(10/18)} & \mathbf{0}_{10\times10} \end{pmatrix} \right) \begin{pmatrix} N_{(8)} \\ N_{(9)} \\ N_{(18m)} \\ N_{(10)} \end{pmatrix}. \tag{20}$$

Besides these, there is another single-term Lagrangian which is also chiral invariant:

$$\mathcal{L}_{(B)} = g_{(B)}\overline{N}_{(8)}M^\dagger\mathbf{T}^a_{(B)}N_{(9m)} + \text{H.c.}, \tag{21}$$

together with its mirror counterpart

$$\mathcal{L}_{(Bm)} = g'_{(B)}\overline{N}_{(8m)}M\mathbf{T}^a_{(B)}N_{(9)} + \text{H.c.} \tag{22}$$

These correspond to the third (bottom) group in Table II.

We note that the Lagrangians (19) and (20) are also invariant under $U_A(1)$ chiral transformation, while Eqs. (21) and (22) are not. This is verified by counting the $U_A(1)$ charge $g_A^{(0)}$ in the interaction Lagrangian. Recall that the meson fields M and $M^\dagger$ carry $g_A^{(0)} = -2$ and $+2$, respectively. Therefore, for the interaction (17) as an example, by using the result of Table I we have the net $U_A(1)$ charge as

$$g_A^{(0)} = +1 - 2 + 1 = 0, \tag{23}$$

where we have used the fact that the $U_A(1)$ charge of the Dirac conjugate is the same as the original one because of the interchange of the left and right components.

These results stand in marked contrast to the two-flavor case [30,42]. Namely, for $SU_L(3) \times SU_R(3)$ chiral invariant Lagrangians with a certain representation structure as given in Table II [or to one term in Eqs. (19)–(22)] are either $U_A(1)$ symmetry-conserving or $U_A(1)$ symmetry-breaking. In contrast, for $SU_L(2) \times SU_R(2)$ chiral invariant Lagrangians with the same representation structure have both a $U_A(1)$ symmetry-conserving and a $U_A(1)$ symmetry-breaking version. This is due to the fact that the two $SU_L(3) \times SU_R(3)$ representations, $(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})$ and $(\mathbf{3},\overline{\mathbf{3}}) \oplus (\overline{\mathbf{3}},\mathbf{3})$ reduce to the same $(\frac{1}{2},0) \oplus (0,\frac{1}{2})$ representation of $SU_L(2) \times SU_R(2)$. Thus, generally speaking, the three-flavor chiral symmetry is more restrictive than the two-flavor one.

Besides the interaction Lagrangians (19)–(22), the so-called "naive"-"mirror" mass terms are also chiral invariant:

$$\mathcal{L}_{(\text{mass})} = m_{(8)}\overline{N}_{(8m)}\gamma_5 N_{(8)} + m_{(9)}\overline{N}_{(9m)}\gamma_5 N_{(9)} + m_{(18)}\overline{N}_{(18m)}\gamma_5 N_{(18)} + m_{(10)}\overline{N}_{(10m)}\gamma_5 N_{(10)}, \tag{24}$$

where $m_{(8)},\ldots,m_{(10)}$ are the mass parameters. The chiral structures of these terms are summarized in Table III.

B. Baryon masses in the chiral limit

Chiral symmetry is spontaneously broken through the "condensation" of the σ field $\sigma \to \sigma_0 = \langle\sigma\rangle_0 = f_\pi$, which leads to the dynamical generation of baryon masses, as can be seen from the linearized chiral invariant interaction Lagrangians (19)–(22).

In this section, we study the masses of the octet baryons. There are altogether six types of octet baryon fields: N_+ ($N_{(8)}$), N_- (contained in $N_{(9)}$) and N_μ (contained in $N_{(18)}$), as well as their mirror fields N'_+ ($N_{(8m)}$), N'_- (contained in $N_{(9m)}$), N'_μ (contained in $N_{(18m)}$). The nucleon mass matrix is already in a simple block-diagonal form when the nucleon fields form the

TABLE IV. The values of the Δ and Λ baryon masses predicted from the isovector axial coupling $g_{A\mathrm{mix}}^{(1)} = g_{A\,\mathrm{exp}}^{(1)} = 1.267$ and $g_{A\mathrm{mix}}^{(0)} = 0.33 \pm 0.08$ due to $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})] - [(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})] - [(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$ mixing.

No.	g_1	g_2	g_3	g_4	g_5	Λ_1^P (MeV)	Λ_2^P (MeV)	Δ^P (MeV)
1	−4.7	8.4	−3.4	2.9	9.8	1370^-	1850^+	2170^-
2	−7.2	4.6	7.9	9.1	−4.2	1940^+	2430^-	1200^-

following mass matrix:

$$M = \frac{1}{\sqrt{6}}\bar{N}\begin{pmatrix} 0 & f_\pi g_{(8/9)} & f_\pi g_{(8/18)} & m_{(8)}\gamma_5 & f_\pi g_B & 0 \\ f_\pi g^*_{(8/9)} & f_\pi g_{(9/9)} & f_\pi g_{(9/18)} & f_\pi g^*_B & m_{(9)}\gamma_5 & 0 \\ f_\pi g^*_{(8/18)} & f_\pi g^*_{(9/18)} & f_\pi g_{(18/18)} & 0 & 0 & m_{(18)}\gamma_5 \\ m_{(8)}\gamma_5 & f_\pi g'_B & 0 & 0 & f_\pi g'_{(8/9)} & g'_{(8/18)} \\ f_\pi g'^*_B & m_{(9)}\gamma_5 & 0 & f_\pi g'^*_{(8/9)} & f_\pi g'_{(9/9)} & f_\pi g'_{(9/18)} \\ 0 & 0 & m_{(18)}\gamma_5 & f_\pi g'^*_{(8/18)} & f_\pi g'^*_{(9/18)} & f_\pi g'_{(18/18)} \end{pmatrix} N, \tag{25}$$

where

$$N = (N'_+, N'_-, N_\mu, N_+, N_-, N'_\mu)^T. \tag{26}$$

Since there are three nucleon fields as well as their mirror fields, there can be a nonzero phase angle. However, for simplicity, we assume all the axial couplings are real.

C. Masses due to $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$–$[(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})]$–$[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$ mixing

As shown in Sec. II, the mixing of chiral $(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})$, $(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})$, and $(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})$ nucleons leads to the observed axial couplings (case III-I in Ref. [33]). Accordingly, we investigate the following three nucleon chiral multiplets:

$$\begin{aligned} (B_2,\Delta) &\in (\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6}), \\ (B_1,\Lambda_1) &\in (\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})[\mathrm{mir}], \\ (B_3,\Lambda_2) &\in (\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3}), \end{aligned} \tag{27}$$

and one meson multiplet

$$(\sigma,\pi) \in (\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3}).$$

Here all baryons have spin 1/2, while the isospin of B_1 and B_2 is 1/2 and that of Δ is 3/2. The Δ field is then represented by an isovector, Dirac-spinor field Δ^i, $(i = 1,2,3)$, which should not be confused with the spin-$\frac{3}{2}$ $\Delta(1232)$ resonance.

In writing down the Lagrangians (19), we have implicitly assumed that the parities of B_1, B_2, Λ, and Δ are the same. In principle, they are arbitrary, except for the ground state nucleon, which must be even. For instance, if B_2 has odd parity, the first term in the interaction Lagrangian (19) must include another γ_5 matrix [67].

Having established the mixing interactions as well as the diagonal terms in Ref. [34], we calculated the masses of the baryon states, as functions of the pion decay constant f_π and the coupling constants $g_1 \sim g_{(9/9)}, g_2 \sim g_{(18/18)}$, and $g_3 \sim g_{(9/18)}$:

$$\begin{aligned} \mathcal{L}_{(9)} &= -g_1 f_\pi(\bar{B}_1 B_1 - 2\bar{\Lambda}\Lambda) + \cdots, \\ \mathcal{L}_{(18)} &= -g_2 f_\pi(\bar{B}_2 B_2 - 2\bar{\Delta}^i\Delta^i) + \cdots, \\ \mathcal{L}_{(9/18)} &= -g_3 f_\pi(\bar{B}_1 B_2) + \cdots, \end{aligned} \tag{28}$$

$$\begin{aligned} \mathcal{L}'_{(9)} &= -g_4 f_\pi(\bar{B}_3 B_3 - 2\bar{\Lambda}_1\Lambda_1) + \cdots, \\ \mathcal{L}_{(9/9)} &= -g_5 f_\pi \bar{B}_1 B_3 - g_5 f_\pi \bar{\Lambda}_1\Lambda_2 + \cdots. \end{aligned} \tag{29}$$

We note that B_1 and B_3 couple with each other through the naive combinations: $m_{(9)}\overline{N}_{(9m)}\gamma_5 N_{(9)}$. Altogether we have

$$\mathcal{L} = -f_\pi(\bar{B}_1,\bar{B}_3,\bar{B}_2)\begin{pmatrix} g_1 & g_5 & g_3 \\ g_5 & g_4 & 0 \\ g_3 & 0 & g_2 \end{pmatrix}\begin{pmatrix} B_1 \\ B_3 \\ B_2 \end{pmatrix} - f_\pi(\bar{\Lambda}_1,\bar{\Lambda}_2)\begin{pmatrix} -2g_1 & g_5 \\ g_5 & -2g_4 \end{pmatrix}\begin{pmatrix} \Lambda_1 \\ \Lambda_2 \end{pmatrix} + 2g_2 f_\pi \bar{\Delta}^i\Delta^i. \tag{30}$$

Let us now diagonalize the mass matrix and express the mixing angle in terms of diagonalized masses. We use the three nucleon candidates $N(940), N(1440)$, and $N^*(1535)$ as well as the two mixing angles $\theta = 50.7^\circ$ and $\varphi = 66.1^\circ$, and finally find that there are two possibilities as shown in Table IV [34]. The odd-parity Δ option appears as the better one. Now, the first flavor-singlet Λ lies at 1370 MeV, substantially closer to 1405 MeV. Flavor-singlet Λ lies at 1850 MeV, very close to the (three star Particle Data Group [69]) $P_{01}(1810)$ resonance. This is our best candidate in the $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$–$[(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})]$–$[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$ mixing scenario.

A comment about the comparatively high value of the Δ mass is in order: In the mid-1960s Hara [31] noticed that the chiral transformation rules for a $(1,\frac{1}{2})$ multiplet impose a strict and seemingly improbable mass relation among its two members: $m_\Delta = 2m_N$. The mixing with the $(\frac{1}{2},0)$ multiplet only makes things worse, i.e., it makes the Δ even heavier. For this reason, the lowest-lying spin-1/2 Δ resonance cannot be a chiral partner of the lowest-lying nucleon $N(940)$, whereas, $\Delta(2150)$ seems to be a viable candidate for the $N(940)$'s chiral partner. Of course, $\Delta(2150)$ may contain components of (i.e., mix with) other high-lying resonances that do not significantly mix with $N(940)$.

D. Chiral symmetry breaking

1. Chiral symmetry breaking: Bare quark masses

In QCD one expects $\mathcal{H}_{\chi\mathrm{SB}}$ to be determined solely by the current quark masses m_u^0,m_d^0,m_s^0 (modulo EM effects), i.e., $\mathcal{H}_{\chi SB} = \mathcal{H}^{\mathrm{q}}_{\chi SB}$:

$$\mathcal{H}^{\mathrm{q}}_{\chi SB} = m_u^0\bar{u}u + m_d^0\bar{d}d + m_s^0\bar{s}s = \sum_{i=u,d,s}\bar{q}_i m^0_{q_i} q_i = \sum_{a=0,3,8} m_a^0(\bar{q}\lambda^a q), \tag{31}$$

where

$$m_0 = \frac{m_d + m_s + m_u}{\sqrt{6}},$$
$$m_3 = \frac{1}{2}(m_u - m_d),$$
$$m_8 = \frac{m_d - 2m_s + m_u}{2\sqrt{3}}.$$

2. Chiral symmetry breaking: Bare baryon masses

We introduce, following Refs. [21,70], an explicit χSB diagonal[4] "bare" nucleon mass and the corresponding χSB Hamiltonian density:

$$\mathcal{H}^{\mathrm{N}}_{\chi SB} = \sum_{i=1}^{3}\bar{N}_i M^0_{N_i} N_i + \bar{\Delta}_{(1,\frac{1}{2})} M^0_{\Delta(1,\frac{1}{2})}\Delta_{(1,\frac{1}{2})}, \tag{32}$$

where i stands for the three chiral multiplets $(1,\frac{1}{2})$,$(\frac{1}{2},0)$, and $(0,\frac{1}{2})$. *A priori*, we do not know the values of the "current" nucleon masses, except for a lower limit—they cannot be smaller than three isospin-averaged current quark masses: $M^0_{N_i} \geqslant 3\bar{m}^0_q = \frac{3}{2}(m^0_u + m^0_d) \simeq 23$ MeV [71], or 14 MeV [72].

To see how this bound comes about, note that the isospin-averaged "bare" nucleon mass term,

$$\mathcal{H}_{\chi\mathrm{SB}}(0) = M^0_N \bar{N}N, \tag{33}$$

where $\bar{M}^0_N = \frac{1}{2}(M^0_p + M^0_n)$, can be readily expressed in terms of the current quark mass term, Eq. (31), with $M^0_N = 3\bar{m}^0_q = \frac{3}{2}(m^0_u + m^0_d)$.

It seems clear that the same "current" (or bare) nucleon mass M^0_N ought to hold for any of the three chiral multiplets $(1,\frac{1}{2})$,$(\frac{1}{2},0)$, and $(0,\frac{1}{2})$, so long as they all correspond to three-quark interpolating fields. Of course, the same chiral multiplets may arise as five-quark interpolators, in which case their bare mass ought to be $\frac{5}{2}(m^0_u + m^0_d)$, i.e., larger than the above value $\frac{3}{2}(m^0_u + m^0_d)$. That explains the inequality in $M^0_{N_i} \geqslant 3\bar{m}^0_q = \frac{3}{2}(m^0_u + m^0_d)$.

For simplicity's sake, we shall assume, as a first approximation, that all three chiral components have the same "current" nucleon mass $M^0_N = M^0_{N(6,3)} = m_N^{(1,\frac{1}{2})} = m_\Delta^{(1,\frac{1}{2})} = M^0_{N(3,\bar{3})} = m_N^{(\frac{1}{2},0)} = M^0_{N(\bar{3},3)} = m_N^{(0,\frac{1}{2})} = \frac{3}{2}(m^0_u + m^0_d)$. In principle, the nucleon bare mass value may differ from one chiral multiplet to another, albeit not by much, e.g., in the three-flavor case it may contain different F and D components, due to different F and D structures of the chiral multiplets, see below. This difference may be important in the three-flavor extension(s) of the model, but not in the two-flavor case.

The model is easily extended to broken SU(3) symmetry case: the explicit χSB "bare" nucleon mass and the corresponding χSB Hamiltonian density are

$$\mathcal{H}^{\mathrm{N}}_{\chi SB} = \sum_{i=1}^{3}\bar{B}_i M^0_{B_i} B_i + \bar{\Delta}_{(6,3)} M^0_{\Delta(6,3)}\Delta_{(6,3)}, \tag{34}$$

where i stands for the three chiral multiplets $(\mathbf{6},\mathbf{3})$, $(\bar{\mathbf{3}},\mathbf{3})$, and $(\mathbf{3},\bar{\mathbf{3}})$, and the nucleon-octet mass matrix $M^0_{B_i}$ in the "physical" basis reads

$$M^0_{B_i} = m_3(d_3 + f_3) - \frac{3}{2\sqrt{3}}d_8(M_\Lambda - M_\Sigma) + 2f_8 m_8 + \sqrt{6}m_0 U, \tag{35}$$

$$M^0_{B_i} = \mathrm{diagonal}\begin{pmatrix} \frac{3(M_\Lambda - M_\Sigma)}{4} + m_d + 2m_u, \\ \frac{3(M_\Lambda - M_\Sigma)}{4} + 2m_d + m_u, \\ \frac{1}{2}(m_d + 2m_s + 3(-M_\Lambda + M_\Sigma + m_u)) - \frac{3M_\Lambda}{2} + \frac{3M_\Sigma}{2} + m_d + m_s + m_u, \\ \frac{1}{2}(-3M_\Lambda + 3M_\Sigma + 3m_d + 2m_s + m_u), \\ \frac{1}{4}(3M_\Lambda - 3M_\Sigma + 2m_d + 8m_s + 2m_u), \\ \frac{1}{4}(3M_\Lambda - 3M_\Sigma + 2m_d + 8m_s + 2m_u), \\ \frac{3(M_\Lambda - M_\Sigma)}{2} + m_d + m_s + m_u \end{pmatrix} + \text{off-diagonal}. \tag{36}$$

[4]Formally, one may include off-diagonal terms here, as well. Physically there are many open questions associated with such terms, however. For example: (1) What dependence of the off-diagonal nucleon components on the current quark mass should one expect? (2) Would it be (linearly) proportional to the average current quark mass, or to the difference of up and down current quark masses? (3) Which chiral multiplets would be admixed and why? There are many unknowns, that we wish to keep at a minimum here, however, for the sake of simplicity.

The off-diagonal term in Eq. (36), given by

$$\text{off-diagonal} = \left(\frac{m_u - m_d}{2\sqrt{3}}\right)\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0 \end{pmatrix}$$

determines the $\Lambda - \Sigma_0$ mixing and mass splitting. As we shall not concern ourselves with hyperons in this paper, this term is of no interest here.

IV. THE PION-NUCLEON $\Sigma_{\pi N}$ TERM

A. $\Sigma_{\pi N}$ at quark level—a brief review

The Σ operator, defined as the double commutator

$$\Sigma = \tfrac{1}{3}\delta^{ab}\big[Q_5^a,\big[Q_5^b,\mathcal{H}_{\chi\mathrm{SB}}(0)\big]\big], \tag{37}$$

was introduced by Dashen as a measure of explicit chiral $\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)$ symmetry breaking [73–76]. It is sensitive to the flavor indices of the axial charges Q_5^a and the form of the $\mathrm{SU}_L(3)\times\mathrm{SU}_R(3)$ chiral symmetry breaking Hamiltonian density $\mathcal{H}_{\chi SB}$: Other choices of summed over indices a,b probe different parts of symmetry breaking Hamiltonian. Its nucleon matrix element is the pion-nucleon $\Sigma_{\pi N}$ term

$$\Sigma_{\pi N} = \tfrac{1}{3}\delta^{ab}\langle N|\big[Q_5^a,\big[Q_5^b,\mathcal{H}_{\chi\mathrm{SB}}(0)\big]\big]|N\rangle, \tag{38}$$

is of importance for the determination of the flavor content, in particular of the $s\bar{s}$ content of the nucleon [1–3,73]. In QCD one expects $\mathcal{H}_{\chi SB}$ to be determined solely by the current quark masses m_u^0,m_d^0,m_s^0 (modulo EM effects) Eq. (31). Then, the axial charges Q_5^a are then also constructed from the quark fields:

$$Q_5^a = \int d\mathbf{x} q^\dagger(x)\gamma_5\frac{1}{2}\lambda^a q(x), \tag{39}$$

which leads, after some basic algebraic manipulations, to

$$\Sigma_{\pi N} = \frac{m_u^0 + m_d^0}{2}\langle N|\bar{u}u + \bar{d}d|N\rangle + m_s^0\langle N|\bar{s}s|N\rangle, \tag{40}$$

and thus the value of the $\pi N \Sigma_N$ term that is given by the sum of the current quark masses in the nucleon. (This is reflected in a nonzero "bare" or "current" nucleon mass on the hadronic level.) Assuming the nucleon contains no, or little strange quark component, i.e., $\langle N|\bar{s}s|N\rangle \sim 0$, one has

$$\Sigma_{\pi N} = \frac{m_u^0 + m_d^0}{2}\langle N|\bar{u}u + \bar{d}d|N\rangle. \tag{41}$$

The matrix element $\langle N|\bar{u}u + \bar{d}d|N\rangle$ counts the number of u and d quarks and/or antiquarks in the nucleon, so that $\Sigma_{\pi N} \simeq \frac{3}{2}(m_u^0 + m_d^0) = 3\hat{m}^0 \simeq 23$ MeV (with the current quark mass estimates that were valid at the time in PDG1998 [71]; these have dropped in the meantime significantly down to roughly $3\hat{m}^0 \simeq 14$ MeV in PDG2012 [72]). At the same time the nucleon mass shift due to the SU(3)-breaking Hamiltonian was evaluated at the baryonic level, assuming the contribution of strange quark to be zero, as

$$\Sigma_{\pi N} = \frac{3\hat{m}^0}{m_s^0 - \hat{m}^0}(M_\Xi - M_\Lambda) \simeq 26\ \mathrm{MeV}, \tag{42}$$

where M_Ξ,M_Λ are the hyperon ground state masses. As these two essentially independent estimates yielded basically one and the same number, any deviation of $\Sigma_{\pi N}$ from the value of 25 MeV seemed to indicate some $\bar{s}s$ content in the nucleon (this agreement between these two methods has disappeared with time, however: with the PDG2012 [72] values one finds $\Sigma_{\pi N} = \frac{3\hat{m}^0}{m_s^0-\hat{m}^0}(M_\Xi - M_\Lambda) \simeq 22$ MeV vs. $\Sigma_{\pi N} \simeq \frac{3}{2}(m_u^0 + m_d^0) \simeq 13$ MeV). But, all estimates of $\Sigma_{\pi N}$ from the πN scattering data yielded substantially larger values, ranging from 55 MeV to 80 MeV [77–80]. Consequently, the importance of the $\Sigma_{\pi N}$ term cannot be exaggerated for the $s\bar{s}$ content of the nucleon. These arguments go back to 1976 [1], and have, by now, found their way into textbooks on particle physics [14,15].

In the meantime there has been a large number of attempts at a theoretical explanation, most of which rely on the enlarged $s\bar{s}$ content of the nucleon. More recently a number of lattice calculations with (almost physical) pions have also reached an enlarged value of $\Sigma_{\pi N}$ [81–84]. But, there have also been many experimental searches for the $s\bar{s}$ contributions to the nucleon observables, none of which produced a significant result (meaning larger than 1% of the $u\bar{u}$ and $d\bar{d}$ contributions; otherwise they are compatible with isospin violating corrections) thus making $s\bar{s}$ effectively negligible [11–13]. Thus the enigma deepens: how is it possible to have such a large $\Sigma_{\pi N}$ term without $s\bar{s}$ content?

B. $\Sigma_{\pi N}$ at the baryon level

The results obtained at the quark level are not always the same as those obtained at the hadronic level, however. The purpose of this study is to lay bare the dependence of the $\Sigma_{\pi N}$ term on the mixing of chiral multiplets, i.e., on the isovector axial coupling $g_A^{(3)}$, and the flavor-singlet axial coupling $g_A^{(0)}$.

The axial current coupling constants of the baryon flavor octet are well known [58]. The zeroth (time-like) components of these axial currents are generators of the $\mathrm{SU}_L(3)\times\mathrm{SU}_R(3)$ chiral symmetry of QCD. The general flavor $\mathrm{SU}_F(3)$ symmetric form of the nucleon axial current contains two free parameters, called the F and D couplings, that are empirically determined as $F = 0.459 \pm 0.008$ and $D = 0.798 \pm 0.008$ [58]. The nucleon also has a flavor singlet axial coupling $g_A^{(0)}$, that has been estimated from spin-polarized lepton-nucleon DIS data as $g_A^{(0)} = 0.28 \pm 0.16$ [7], or more recently as $0.33 \pm 0.03 \pm 0.05$ [9,16], subject to certain assumptions about hyperon decays (the axial F and D values).

In the chiral mixing approach the value $g_A \neq 1$ is achieved naturally by way of mixing different chiral multiplets, without derivative couplings. We shall display the $\Sigma_{\pi N}$ term's dependence on $g_A^{(3)}$ and show that a large value of $\Sigma_{\pi N}$ is easily obtained even with the present day (significantly smaller) current quark masses and with a vanishing $s\bar{s}$ component of the nucleon.

In order to show this, we must first evaluate and discuss the nucleon $\Sigma_{\pi N}$ term as obtained from the Σ double commutators. That will be done in Sec. IV B 1. We shall adopt different chiral symmetry breaking [χSB] terms, in accordance with Refs. [45,46]. Then in Sec. IV B 2 we evaluate the $\Sigma_{\pi N}$ term in the chiral mixing approach.

1. Chiral double commutators

In order to evaluate the double commutators of chiral charges with the Hamiltonian, we shall use the single commutator results from Appendix C:

$$\begin{aligned}\left[Q_5^b,\left[Q_5^a,\bar{N}_{(1,\frac{1}{2})}N_{(1,\frac{1}{2})}\right]\right] &= \left(\frac{5}{3}\right)^2\delta^{ab}\bar{N}_{(1,\frac{1}{2})}N_{(1,\frac{1}{2})} + \left(\frac{5}{3}\right)\frac{1}{\sqrt{3}}\left(\bar{N}_{(1,\frac{1}{2})}\tau^b T^a\Delta_{(1,\frac{1}{2})} + \bar{\Delta}_{(1,\frac{1}{2})}T^{\dagger a}\tau^b N_{(1,\frac{1}{2})}\right)\\ &\quad + \left[Q_5^b,\frac{2}{\sqrt{3}}\left(\bar{N}_{(1,\frac{1}{2})}\gamma_5 T^a\Delta_{(1,\frac{1}{2})} + \bar{\Delta}_{(1,\frac{1}{2})}\gamma_5 T^{\dagger a}N_{(1,\frac{1}{2})}\right)\right]\\ &= \left(\frac{25+16}{9}\right)\delta^{ab}\bar{N}_{(1,\frac{1}{2})}N_{(1,\frac{1}{2})} + \left(\frac{4}{3}\right)\bar{\Delta}_{(1,\frac{1}{2})}\left(\frac{3}{2}\delta^{ab} - \frac{1}{3}\left\{t^a_{(3/2)},t^b_{(3/2)}\right\}\right)\Delta_{(1,\frac{1}{2})} + \cdots, \qquad (43)\end{aligned}$$

where $\cdots$ stands for the off-diagonal terms, such as $\bar{N}_{(1,\frac{1}{2})}(\cdots)\Delta_{(1,\frac{1}{2})}$, and their Hermitian conjugates. Similarly for the Δ-field double commutator

$$\begin{aligned}&\left[Q_5^b,\left[Q_5^a,\bar{\Delta}_{(1,\frac{1}{2})}\Delta_{(1,\frac{1}{2})}\right]\right]\\ &\quad= \left(\tfrac{16}{9}\right)\delta^{ab}\bar{N}_{(1,\frac{1}{2})}N_{(1,\frac{1}{2})} + 2\delta^{ab}\bar{\Delta}_{(1,\frac{1}{2})}\Delta_{(1,\frac{1}{2})}\\ &\qquad - \left(\tfrac{2}{9}\right)\bar{\Delta}_{(1,\frac{1}{2})}\left\{t^a_{(3/2)},t^b_{(3/2)}\right\}\Delta_{(1,\frac{1}{2})} + \cdots, \qquad (44)\end{aligned}$$

where $\cdots$ again stands for the off-diagonal terms. The $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$ chiral multiplets double commutators are much simpler

$$\left[Q_5^b,\left[Q_5^a,\bar{N}_{(\frac{1}{2},0)}N_{(\frac{1}{2},0)}\right]\right] = \delta^{ab}\bar{N}_{(\frac{1}{2},0)}N_{(\frac{1}{2},0)}, \qquad (45)$$

$$\left[Q_5^b,\left[Q_5^a,\bar{N}_{(0,\frac{1}{2})}N_{(0,\frac{1}{2})}\right]\right] = \delta^{ab}\bar{N}_{(0,\frac{1}{2})}N_{(0,\frac{1}{2})}. \qquad (46)$$

Finally, we contract these equations (43)–(46) with $\frac{1}{3}\delta^{ab}$ (where summation over repeated indices is understood) to find

$$\begin{aligned}&\tfrac{1}{3}\delta^{ab}\left[Q_5^b,\left[Q_5^a,\bar{N}_{(1,\frac{1}{2})}N_{(1,\frac{1}{2})}\right]\right]\\ &\quad= \left(\tfrac{41}{9}\right)\bar{N}_{(1,\frac{1}{2})}N_{(1,\frac{1}{2})} + \left(\tfrac{8}{9}\right)\bar{\Delta}_{(1,\frac{1}{2})}\Delta_{(1,\frac{1}{2})} + \cdots, \qquad (47)\end{aligned}$$

and similarly

$$\begin{aligned}&\tfrac{1}{3}\delta^{ab}\left[Q_5^b,\left[Q_5^a,\bar{\Delta}_{(1,\frac{1}{2})}\Delta_{(1,\frac{1}{2})}\right]\right]\\ &\quad= \left(\tfrac{16}{9}\right)\bar{N}_{(1,\frac{1}{2})}N_{(1,\frac{1}{2})} + \left(\tfrac{13}{9}\right)\bar{\Delta}_{(1,\frac{1}{2})}\Delta_{(1,\frac{1}{2})} + \cdots, \qquad (48)\end{aligned}$$

where $\cdots$ again stands for the off-diagonal terms. Here we have used the identity $t^a_{(3/2)}t^a_{(3/2)} = \frac{15}{4}\mathbf{1}_{4\times 4}$. Similarly, from Eqs. (45) and (46) we find

$$\tfrac{1}{3}\delta^{ab}\left[Q_5^b,\left[Q_5^a,\bar{N}_{(\frac{1}{2},0)}N_{(\frac{1}{2},0)}\right]\right] = \bar{N}_{(\frac{1}{2},0)}N_{(\frac{1}{2},0)}, \qquad (49)$$

$$\tfrac{1}{3}\delta^{ab}\left[Q_5^b,\left[Q_5^a,\bar{N}_{(0,\frac{1}{2})}N_{(0,\frac{1}{2})}\right]\right] = \bar{N}_{(0,\frac{1}{2})}N_{(0,\frac{1}{2})}, \qquad (50)$$

which are all the double commutators that we need for the evaluation of $\Sigma_{\pi N}$.

2. Nucleon matrix elements of chiral double commutators

As shown in Sec. II B the physical nucleon field is an admixture of (at least) three chiral multiplet components:

$$|N\rangle = \sin\theta|(6,3)\rangle + \cos\theta(\cos\varphi|(3,\bar{3})\rangle + \sin\varphi|(\bar{3},3)\rangle). \qquad (51)$$

We use the identities

$$N_{(1,\frac{1}{2})}|N(p)\rangle = N_{(1,\frac{1}{2})}(\sin\theta|(6,3)\rangle + \cos\theta(\cos\varphi|(3,\bar{3})\rangle + \sin\varphi|(\bar{3},3)\rangle)) = u(p)_{(1,\frac{1}{2})}\sin\theta, \qquad (52)$$

$$N_{(\frac{1}{2},0)}|N(p)\rangle = N_{(\frac{1}{2},0)}(\sin\theta|(6,3)\rangle + \cos\theta(\cos\varphi|(3,\bar{3})\rangle + \sin\varphi|(\bar{3},3)\rangle)) = u(p)_{(\frac{1}{2},0)}\cos\theta\cos\varphi, \qquad (53)$$

$$N_{(0,\frac{1}{2})}|N(p)\rangle = N_{(0,\frac{1}{2})}(\sin\theta|(6,3)\rangle + \cos\theta(\cos\varphi|(3,\bar{3})\rangle + \sin\varphi|(\bar{3},3)\rangle)) = u(p)_{(0,\frac{1}{2})}\cos\theta\sin\varphi, \qquad (54)$$

and their Dirac conjugates

$$\langle N(p)|\bar{N}_{(1,\frac{1}{2})} = (\sin\theta\langle(6,3)| + \cos\theta(\cos\varphi\langle(3,\bar{3})| + \sin\varphi\langle(\bar{3},3)|))\bar{N}_{(1,\frac{1}{2})} = \bar{u}(p)_{(1,\frac{1}{2})}\sin\theta, \qquad (55)$$

$$\langle N(p)|\bar{N}_{(\frac{1}{2},0)} = (\sin\theta\langle(6,3)| + \cos\theta(\cos\varphi\langle(3,\bar{3})| + \sin\varphi\langle(\bar{3},3)|))\bar{N}_{(\frac{1}{2},0)} = \bar{u}(p)_{(\frac{1}{2},0)}\cos\theta\cos\varphi, \qquad (56)$$

$$\langle N(p)|\bar{N}_{(0,\frac{1}{2})} = (\sin\theta\langle(6,3)| + \cos\theta(\cos\varphi\langle(3,\bar{3})| + \sin\varphi\langle(\bar{3},3)|))\bar{N}_{(0,\frac{1}{2})} = \bar{u}(p)_{(0,\frac{1}{2})}\cos\theta\sin\varphi, \qquad (57)$$

which lead to

$$\langle N(p)|\bar{N}_{(1,\frac{1}{2})}N_{(1,\frac{1}{2})}|N(p)\rangle = \bar{u}(p)_{(1,\frac{1}{2})}u(p)_{(1,\frac{1}{2})}\sin^2\theta = \frac{E_p}{m}\sin^2\theta, \tag{58}$$

$$\langle N(p)|\bar{N}_{(\frac{1}{2},0)}N_{(\frac{1}{2},0)}|N(p)\rangle = \bar{u}(p)_{(\frac{1}{2},0)}u(p)_{(\frac{1}{2},0)}\cos^2\theta\cos^2\varphi = \frac{E_p}{m}\cos^2\theta\cos^2\varphi, \tag{59}$$

$$\langle N(p)|\bar{N}_{(0,\frac{1}{2})}N_{(0,\frac{1}{2})}|N(p)\rangle = \bar{u}(p)_{(0,\frac{1}{2})}u(p)_{(0,\frac{1}{2})}\cos^2\theta\sin^2\varphi = \frac{E_p}{m}\cos^2\theta\sin^2\varphi, \tag{60}$$

which in the $p \to 0$ limit implies

$$\lim_{p\to 0}\langle N(p)|\bar{N}_{(1,\frac{1}{2})}N_{(1,\frac{1}{2})}|N(p)\rangle = \sin^2\theta, \tag{61}$$

$$\lim_{p\to 0}\langle N(p)|\bar{N}_{(\frac{1}{2},0)}N_{(\frac{1}{2},0)}|N(p)\rangle = \cos^2\theta\cos^2\varphi, \tag{62}$$

$$\lim_{p\to 0}\langle N(p)|\bar{N}_{(0,\frac{1}{2})}N_{(0,\frac{1}{2})}|N(p)\rangle = \cos^2\theta\sin^2\varphi. \tag{63}$$

Now take the definition

$$\Sigma^i_{\pi N} = \tfrac{1}{3}\delta^{ab}\langle N_\alpha|\big[Q^a_5,\big[Q^b_5,\mathcal{H}_{\chi\text{SB}}(0)\big]\big]|N_\alpha\rangle, \tag{64}$$

and evaluate it with the chiral symmetry breaking Hamiltonian $\mathcal{H}_{\chi\text{SB}}(0)$ in Eq. (32) and using Eqs. (47)–(50), for different chiral representations denoted by $i = (1,\frac{1}{2}),(\frac{1}{2},0),(0,\frac{1}{2})$, to find

$$\begin{aligned}\Sigma^{(1,\frac{1}{2})}_{\pi N} &= \big(\tfrac{41}{9}m_N^{(1,\frac{1}{2})} + \tfrac{16}{9}m_\Delta^{(1,\frac{1}{2})}\big),\\ \Sigma^{(\frac{1}{2},0)}_{\pi N} &= m_N^{(\frac{1}{2},0)},\\ \Sigma^{(0,\frac{1}{2})}_{\pi N} &= m_N^{(0,\frac{1}{2})}.\end{aligned} \tag{65}$$

Thus we find

$$\begin{aligned}\Sigma_{\pi N} &= \tfrac{1}{3}\delta^{ab}\langle N|\big[Q^a_5,\big[Q^b_5,\mathcal{H}_{\chi\text{SB}}(0)\big]\big]|N\rangle\\ &= \sin^2\theta\,\Sigma^{(1,\frac{1}{2})}_{\pi N} + \cos^2\theta\big(\cos^2\varphi\,\Sigma^{(\frac{1}{2},0)}_{\pi N} + \sin^2\varphi\,\Sigma^{(0,\frac{1}{2})}_{\pi N}\big)\\ &= \sin^2\theta\big(\tfrac{41}{9}m_N^{(1,\frac{1}{2})} + \tfrac{16}{9}m_\Delta^{(1,\frac{1}{2})}\big)\\ &\quad + \cos^2\theta\big(\cos^2\varphi\, m_N^{(\frac{1}{2},0)} + \sin^2\varphi\, m_N^{(0,\frac{1}{2})}\big),\end{aligned} \tag{66}$$

which is our basic result here.

If we make a simplifying assumption now (for a justification see Sec. III D 2), viz. that all three chiral components have the same "current" nucleon mass $M_N^0 = M^0_{(6,3)} = m_N^{(1,\frac{1}{2})} = m_\Delta^{(1,\frac{1}{2})} = M^0_{(3,\bar{3})} = m_N^{(\frac{1}{2},0)} = M^0_{(\bar{3},3)} = m_N^{(0,\frac{1}{2})}$, then finally one finds

$$\Sigma_{\pi N} = \big(\tfrac{57}{9}\sin^2\theta + \cos^2\theta\big)M_N^0 = \big(1 + \tfrac{16}{3}\sin^2\theta\big)M_N^0. \tag{67}$$

Note that the factor $(1 + \frac{16}{3}\sin^2\theta)$ in front of the current nucleon mass is always larger than unity (for real values of the mixing angle θ).

C. Comparison with experiment

As most "measurements" of $\Sigma_{\pi N}$ have yielded values ranging from 55 MeV to 75 MeV,[5] that are substantially larger than the naively expected 25 MeV, it has consequently appeared that the $s\bar{s}$ content of the nucleon must be (very) large.

The nucleon current mass is $M_N^0 = 3\bar{m}_q^0 = \frac{3}{2}(m_u^0 + m_d^0) \simeq$ 14.4 MeV, i.e., $\frac{1}{2}(m_u^0 + m_d^0) \simeq 4.79$ MeV in PDG2012 [72]. We note that here $m_u^0 = 2.3 \times 1.35$ MeV and $m_d^0 = 4.8 \times 1.35$ MeV, where 1.35 is the rescaling factor due to the change of the energy scale from 2 GeV down to 1 GeV [72], yielding $\frac{1}{2}(m_u^0 + m_d^0) \simeq 4.79$ MeV, substantially lower than 7.6 MeV in PDG1998 [71].

The constraint on the mixing angle θ by the experimental values of the axial couplings has been discussed in Sec. II, which gives $\theta = 50.7^\text{o}$ and $\varphi = 66.1^\text{o}$. Inserting these values into Eq. (67), one finds $\Sigma_{\pi N} = 60.3$ MeV.

The Σ operator, Eq. (37), is often identified with the chiral symmetry breaking (χSB) Hamiltonian itself. In Eq. (34) the nucleon Σ term is a measure of the χSB in the nucleon. In such a case it equals the shift of the nucleon mass δM due to the χSB terms in the Hamiltonian. This reasoning underlies the standard interpretation of the nucleon Σ term as being a measure of the strangeness content of the nucleon Ref. [3].

A large value of $\Sigma_{\pi N}$, such as 65 MeV, has often been interpreted as a sign of a substantial $s\bar{s}$ content of the nucleon. We have shown that in the chiral-mixing approximation large values of $\Sigma_{\pi N}$ can be obtained without any strangeness degrees of freedom in the nucleon as a natural consequence of the rather substantial chiral $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$ multiplet component in the nucleon field.

V. THE KAON-NUCLEON Σ_{KN} TERM

A. SU(3) quark level

To calculate the kaon-nucleon Σ_{KN} term, we use the Σ^{ab} operator defined as the double commutator

$$\Sigma^{ab} = \big[Q^a_5,\big[Q^b_5,H_{\chi\text{SB}}\big]\big], \tag{68}$$

[5]Phenomenologically, the σ term is related to the πN scattering amplitude at a certain nonphysical kinematical point, (at the so-called Cheng-Dashen point $t = +2m_\pi^2$). Its extracted value is typically in the range $\Sigma_{CD} = 70$–90 MeV. After the corrections for the finite value of t are taken into account, which roughly amount to -15 MeV, one obtains $\Sigma_{\pi N} = 55$–75 MeV.

of the axial charges Q_5^a and the chiral symmetry breaking Hamiltonian $H_{\chi SB}$.[6] It was introduced by Dashen [74] as a way of separating out the explicit chiral $SU_L(3) \times SU_R(3)$ symmetry breaking part $H_{\chi SB}$ from the total Hamiltonian.

Its (diagonal) nucleon matrix element $\Sigma_{KN} = \frac{1}{4}\sum_{a=4}^{7}\langle N|\Sigma^{aa}|N\rangle$ is due to the (explicit) chiral symmetry breaking current quark masses [74,75]. Then the kaon-nucleon Σ terms are

$$\Sigma^{44} = \begin{pmatrix} 2(m_s+m_u) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2(m_s+m_u) \end{pmatrix},$$

$$\Sigma^{55} = \begin{pmatrix} 2(m_s+m_u) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2(m_s+m_u) \end{pmatrix},$$

$$\Sigma^{66} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2(m_d+m_s) & 0 \\ 0 & 0 & 2(m_d+m_s) \end{pmatrix},$$

$$\Sigma^{77} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2(m_d+m_s) & 0 \\ 0 & 0 & 2(m_d+m_s) \end{pmatrix}.$$

Summing them up and dividing by 4, we find

$$\Sigma_{KN} = \frac{1}{4}\sum_{a=4}^{7}\Sigma^{aa} = \begin{pmatrix} m_s+m_u & 0 & 0 \\ 0 & m_d+m_s & 0 \\ 0 & 0 & m_d+2m_s+m_u \end{pmatrix}.$$

If we assume that $m_u = m_d$, then

$$\Sigma_{KN} = (m_s + m_{u/d}).$$

B. SU(3) hadron level

Double commutator of the axial charges Q_5^a for $a = 4,5,6,7$ and the current/bare nucleon mass Hamiltonian $H_{\chi SB}$ (also) gives the kaon Σ term operator

$$\Sigma_K = \frac{1}{4}\sum_{a=4,5,6,7}[Q_5^a,[Q_5^a,H_{\chi \mathrm{SB}}]].$$

Kaon-nucleon Σ_{KN} term—matrix element $\Sigma_{KN} = \frac{1}{4}\sum_{a=4,5,6,7}\langle N|[Q_5^a,[Q_5^a,\mathcal{H}_{\chi \mathrm{SB}}(0)]]|N\rangle = \langle N|\Sigma_K|N\rangle$—is also a chiral mixture:

$$\Sigma_{KN} = \sin^2\theta\,\Sigma_{KN(6,3)} + \cos^2\theta(\cos^2\varphi\,\Sigma_{KN(3,\bar{3})} + \sin^2\varphi\,\Sigma_{KN(\bar{3},3)}).$$

Thus, we need three double commutators:

1. *The (6,3) and (3,6) chiral multiplets*

$$\frac{1}{4}\sum_{i=4}^{7}[Q_5^i,[Q_5^i,\bar{N}_{(6,3)}M_N N_{(6,3)}]] = \frac{1}{4}\bar{N}_{(6,3)}\begin{pmatrix} \frac{70}{9}m_u + \frac{41}{9}m_d + \frac{5}{3}m_s & 0 \\ 0 & \frac{41}{9}m_u + \frac{70}{9}m_d + \frac{5}{3}m_s \end{pmatrix}N_{(6,3)},$$

$$\frac{1}{4}\sum_{i=4}^{7}[Q_5^i,[Q_5^i,\bar{\Delta}_{(6,3)}M_\Delta \Delta_{(6,3)}]] = \frac{1}{4}\bar{N}_{(6,3)}\begin{pmatrix} \frac{20}{9}m_u + \frac{4}{9}m_d + \frac{4}{3}m_s & 0 \\ 0 & \frac{4}{9}m_u + \frac{20}{9}m_d + \frac{4}{3}m_s \end{pmatrix}N_{(6,3)},$$

thus leading to

$$\Sigma_{KN(6,3)} = \frac{1}{4}(10m_u + 5m_d + 3m_s).$$

2. *The ($\bar{3}$,3) and (3,$\bar{3}$) chiral multiplets*

$$\frac{1}{4}\sum_{i=4}^{7}[Q_5^i,[Q_5^i,\bar{N}_{(\bar{3},3)}M_N N_{(\bar{3},3)}]] = \frac{1}{4}\bar{N}_{(\bar{3},3)}\begin{pmatrix} \frac{26}{3}m_u + \frac{11}{3}m_d + \frac{5}{3}m_s & 0 \\ 0 & \frac{11}{9}m_u + \frac{26}{9}m_d + \frac{5}{3}m_s \end{pmatrix}N_{(\bar{3},3)},$$

$$\frac{1}{4}\sum_{i=4}^{7}[Q_5^i,[Q_5^i,\bar{\Lambda}_{(\bar{3},3)}M_\Lambda \Lambda_{(\bar{3},3)}]] = \frac{1}{4}\bar{N}_{(\bar{3},3)}\begin{pmatrix} \frac{4}{3}m_u + \frac{4}{3}m_d + \frac{4}{3}m_s & 0 \\ 0 & \frac{4}{3}m_u + \frac{4}{3}m_d + \frac{4}{3}m_s \end{pmatrix}N_{(\bar{3},3)},$$

thus leading to

$$\Sigma_{KN(\bar{3},3)} = \frac{1}{4}(10m_u + 5m_d + 3m_s).$$

[6] For normalization and notational conventions see Ref. [21].

3. The (8,1) and (1,8) chiral multiplets

$$\frac{1}{4}\sum_{i=4}^{7}([Q_5^i,[Q_5^i,\bar{N}_{(8,1)}M_N N_{(8,1)}]] = \frac{1}{4}\bar{N}_{(8,1)}\begin{pmatrix}10m_u+5m_d+3m_s & 0\\ 0 & 5m_u+10m_d+3m_s\end{pmatrix}N_{(8,1)},$$

thus leading to

$$\Sigma_{KN(8,1)} = \tfrac{1}{4}(10m_u+5m_d+3m_s).$$

C. Numerical results

One can see that the kaon-nucleon Σ_{KN} terms are identical in these three chiral multiplets, $\Sigma_{KN(6,3)} = \Sigma_{KN(\bar{3},3)} = \Sigma_{KN(8,1)}$, so that the Σ_{KN} term of their admixture also equals the same number:

$$\begin{aligned}\Sigma_{KN} &= \Sigma_{KN(6,3)}(\sin^2\theta+\cos^2\theta(\cos^2\varphi+\sin^2\varphi))\\ &= \Sigma_{KN(6,3)} = \Sigma_{KN(\bar{3},3)} = \Sigma_{KN(8,1)}\\ &= \tfrac{1}{4}(10m_u+5m_d+3m_s).\end{aligned}$$

The 2012 edition of the Particle Data Group, Ref. [72] has $m_u^0 = 2.3\times1.35$ MeV and $m_d^0 = 4.8\times1.35$ MeV, i.e., $\frac{1}{2}(m_u^0+m_d^0) = 4.79$ MeV and $m_s^0 = (93.5\pm2.5)\times1.35 = (126.225\pm3.375)$ MeV, yielding

$$\Sigma_{KN} = 111 \text{ MeV}.$$

Note that these values are substantially lower than before, see, e.g., the PDG1998 values, Ref. [71].

One would like to compare this value with the "experimental" one. In this case, the status of "experimental" Σ_{KN} is even worse than that of "experimental" $\Sigma_{\pi N}$: the kaon-nucleon scattering data are nowhere near of pion-nucleon ones in terms of overall quality, abundance, kinematic range, precision, and accuracy.

Only some very old "experimental" estimates are available: (1) $\Sigma_{KN} \simeq 170$ MeV from 1970, Ref. [85], (2) $\Sigma_{KN} = -370\pm110$ MeV from 1972, Ref. [86], $\Sigma_{KN}\simeq 170$ MeV, from 1973, Ref. [87], (3) $\Sigma_{KN} = 540\pm160$ MeV, from 1973, Ref. [88], (4) $\Sigma_{KN} = 246$ MeV, from 1976, Ref. [1]. The most recent reviews, Refs. [3,89] have calculated Σ_{KN} at zero strangeness content $y_N = 0$ of the nucleon as being 170 MeV, but with the 1987 values of current quark masses. Their formulas translate to the value of 110 MeV with the 2012 values of masses, Ref. [72]. This is (very) close to our predicted value (111 MeV) of the same quantity.

VI. SUMMARY AND CONCLUSIONS

In this paper we have calculated the pion-nucleon $\Sigma_{\pi N}$ term in the chiral mixing approach, first with two light (u,d) flavors, and then we extended it to the case with three light (u,d,s) flavors, i.e., to $SU_L(3)\times SU_R(3)$ multiplet mixing, which we then used to calculate the kaon-nucleon σ term Σ_{KN}. We based our calculations on the chiral mixing formalism and the phenomenology developed previously in Refs. [33,34,43,44,47].

The physical significance of our present work is that it shows that there is no need to introduce $s\bar{s}$ components in addition to the three-quark "core", so as to agree with the observed values of the pion-nucleon Σ term, the baryon axial couplings, and the nucleon magnetic moments: the phenomenologically necessary $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$ chiral component and the $[(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})]$ "mirror" component exist as bilocal three-quark fields, Refs. [39,48]. Thus, we have shown that there is no need for "meson cloud", or (nonexotic) "pentaquark" components in the Fock expansion of the baryon wave function, to explain (at least) the axial currents, magnetic moments, and the pion-nucleon Σ term, contrary to established opinion, Ref. [2]. This goes to show that the algebraic complexity of three Dirac quark fields is such that it can mimic the presence of $q\bar{q}$ pairs, at least in certain observables. For us this was a surprise.

The present formalism and phenomenology can be used to attack other outstanding issues of baryon chiral dynamics: the hyperon radiative decays, for example, have been a long-standing unsolved problem.

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APPENDIX A: ISOSPIN-$\frac{3}{2}$ GENERATORS

From Ref. [27] we take

$$\delta_5^{a_3}\phi^{\mu}_{\frac{3}{2},\frac{3}{2}} = i\gamma_5 a_3\phi^{\mu}_{\frac{3}{2},\frac{3}{2}}, \tag{A1}$$

$$\delta_5^{a_3}\begin{pmatrix}\phi^{\mu}_{\frac{1}{2},\frac{1}{2}}\\ \phi^{\mu}_{\frac{3}{2},\frac{1}{2}}\end{pmatrix} = i\gamma_5 a_3\begin{pmatrix}\frac{5}{3} & \frac{4\sqrt{2}}{3}\\ \frac{4\sqrt{2}}{3} & \frac{1}{3}\end{pmatrix}\begin{pmatrix}\phi^{\mu}_{\frac{1}{2},\frac{1}{2}}\\ \phi^{\mu}_{\frac{3}{2},\frac{1}{2}}\end{pmatrix} \tag{A2}$$

with the familiar ["$SU_{FS}(6)$"] value $\frac{5}{3}$ for its "nucleon" component N^{μ}. In order to read off the value of g_A, it is convenient to express this as

$$\delta_5^{\vec{a}}\Delta^{\mu} = i\,\gamma_5\left(\frac{1}{3}\boldsymbol{t}_{(\frac{3}{2})}\cdot\boldsymbol{a}\,\Delta^{\mu} + \frac{2}{\sqrt{3}}\boldsymbol{a}\cdot\boldsymbol{T}^{\dagger}N^{\mu}\right), \tag{A3}$$

where $\mathbf{t}^i_{(\frac{3}{2})}$ are the isospin-$\frac{3}{2}$ generators of the SU(2) group and $\mathbf{T}^i$ are the so-called isospurion (4×2) matrices, see Appendix B of Ref. [27].

The $t^i_{(\frac{3}{2})}$ are defined as

$$\mathbf{t}^1_{(\frac{3}{2})} = \begin{pmatrix} 0 & \frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix},$$

$$\mathbf{t}^2_{(\frac{3}{2})} = i \begin{pmatrix} 0 & -\frac{\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & 0 & -1 & 0 \\ 0 & 1 & 0 & -\frac{\sqrt{3}}{2} \\ 0 & 0 & \frac{\sqrt{3}}{2} & 0 \end{pmatrix}, \quad \text{(A4)}$$

$$\mathbf{t}^3_{(\frac{3}{2})} = \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix},$$

which leads to the conventional normalization of the SU(2) Casimir operator. The $\mathbf{T}^i$ are defined by

$$\mathbf{T}^1 = \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{6}} & 0 \\ 0 & -\frac{1}{\sqrt{6}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix},$$

$$\mathbf{T}^2 = i \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{6}} & 0 \\ 0 & -\frac{1}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}, \quad \text{(A5)}$$

$$\mathbf{T}^3 = \begin{pmatrix} 0 & \sqrt{\frac{2}{3}} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{2}{3}} & 0 \end{pmatrix}$$

with the properties

$$i\, \boldsymbol{t}_{(\frac{3}{2})} \cdot \boldsymbol{a} = \tfrac{3}{2}\mathbf{T}^{i\dagger}(i\boldsymbol{\tau}\cdot\boldsymbol{a}\delta^{ik})\mathbf{T}^k = -\tfrac{3}{2}\mathbf{T}^{i\dagger}(\epsilon^{ijk}a^j)\mathbf{T}^k,$$
$$\mathbf{T}^i\mathbf{T}^{k\dagger} = P^{ik}_{\frac{3}{2}}. \quad \text{(A6)}$$

APPENDIX B: CLOSURE OF THE CHIRAL $SU_L(3) \times SU_R(3)$ ALGEBRA

The SU(3) vector charges $Q^a = \int d\mathbf{x} J^a_0(t,\mathbf{x})$ defined as

$$-2\boldsymbol{b}\cdot\boldsymbol{J}_\mu = \sum_i \frac{\partial\mathcal{L}}{\partial\partial^\mu B_i}\delta^{\vec{b}} B_i, \quad \text{(B1)}$$

together with the axial charges $Q^a_5 = \int d\mathbf{x} J^a_{05}(t,\mathbf{x})$, defined as

$$-2\boldsymbol{a}\cdot\boldsymbol{J}_{\mu 5} = \sum_i \frac{\partial\mathcal{L}}{\partial\partial^\mu B_i}\delta^{\vec{a}}_5 B_i, \quad \text{(B2)}$$

ought to close the chiral algebra

$$[Q^a, Q^b] = i f^{abc} Q^c, \quad \text{(B3)}$$
$$\left[Q^a_5, Q^b\right] = i f^{abc} Q^c_5, \quad \text{(B4)}$$
$$\left[Q^a_5, Q^b_5\right] = i f^{abc} Q^c, \quad \text{(B5)}$$

where f^{abc} are the SU(3) structure constants. Equations (B3) and (B4) usually hold automatically, as a consequence of the canonical (anti)commutation relations between Dirac baryon fields B_i, whereas Eq. (B5) is not trivial for the chiral multiplets that are different from the $[(8,1)\oplus(1,8)]$, because of the (nominally) fractional axial charges and the presence of the off-diagonal components. When taking a matrix element of Eq. (B5) by baryon states in a certain chiral representation, the axial charge mixes different flavor states within the same chiral representation. This is an algebraic version of the Adler-Weisberger sum rule [90]. In the following we shall check and confirm the validity of Eq. (B5) in the three multiplets of $SU(3)_L \times SU(3)_R$.

1. Closure of the chiral $SU_L(3) \times SU_R(3)$ algebra in the $(8,1) \oplus (1,8)$ multiplet

Due to the absence of fractional coefficients in the $(8,1)\oplus(1,8)$ multiplet's axial charge $Q^a_5 = \int d\mathbf{x} J^a_{05}(t,\mathbf{x})$ defined by the current given in

$$\mathbf{J}^a_{\mu 5} = \overline{N}\gamma_\mu\gamma_5\mathbf{F}^a_{(8)}N, \quad \text{(B6)}$$

the vector charge $Q^a = \int d\mathbf{x} J^a_0(t,\mathbf{x})$ defined by the current given in

$$\mathbf{J}^a_\mu = \overline{N}\gamma_\mu\,\mathbf{F}^a_{(8)}\,N, \quad \text{(B7)}$$

and the axial charge close the chiral algebra defined by Eqs. (B3)–(B5). The same comments holds for the $(10,1)\oplus(1,10)$ chiral multiplet for the same reasons as in the example shown above.

2. Closure of the chiral $SU_L(3) \times SU_R(3)$ algebra in the $(3,\bar{3}) \oplus (\bar{3},3)$ multiplet

The vector charge $Q^a = \int d\mathbf{x} J^a_0(t,\mathbf{x})$ defined by the current given in

$$\mathbf{J}^a_\mu = \overline{N}\gamma_\mu\,\mathbf{F}^a_{(8)}\,N, \quad \text{(B8)}$$

together with the axial charge $Q^a_5 = \int d\mathbf{x} J^a_{05}(t,\mathbf{x})$, defined by the current given in

$$\mathbf{J}^a_{\mu 5} = \overline{N}\gamma_\mu\gamma_5\left(\mathbf{D}^a N + \sqrt{\frac{2}{3}}\mathbf{T}^{a\dagger}_{1/8}\Lambda_1\right) + \overline{\Lambda}_1\gamma_\mu\gamma_5\sqrt{\frac{2}{3}}\mathbf{T}^a_{1/8}N, \quad \text{(B9)}$$

ought to close the chiral algebra defined by Eqs. (B3)–(B5). Equations (B3) and (B4) hold here, whereas Eq. (B5) is the nontrivial one: the diagonal D charge of N $[Q^a_{5D}(N)]$ axial charge,

$$Q^a_{5D}(N) = \int d\mathbf{x}\,(\overline{N}\gamma_0\gamma_5\,\mathbf{D}^a\,N), \quad \text{(B10)}$$

$$Q^a_D(N) = \int d\mathbf{x}\,(\overline{N}\gamma_0\,\mathbf{D}^a\,N), \quad \text{(B11)}$$

lead to

$$\left[Q^a_{5D}(N), Q^b_{5D}(N)\right] = \int d\mathbf{x}(\overline{N}\gamma_0\,(\mathbf{D}^a\mathbf{D}^b - \mathbf{D}^b\mathbf{D}^a)N). \quad \text{(B12)}$$

It turns out that the off-diagonal terms in the axial charge,

$$Q^a_5(N,\Lambda) = \int d\mathbf{x}\left(\sqrt{\frac{2}{3}}(\overline{N}\gamma_0\gamma_5\,\mathbf{T}^{a\dagger}_{1/8}\,\Lambda + \overline{\Lambda}\gamma_0\gamma_5\,\mathbf{T}^a_{1/8}\,N)\right), \quad \text{(B13)}$$

play a crucial role in the closure of the chiral commutator Eq. (B5). The additional terms in the commutator add up to

$$\begin{aligned}&\left[Q_5^a(N,\Delta),Q_5^b(N,\Delta)\right]\\&\quad=\frac{2}{3}\int d\mathbf{x}\overline{N}\gamma_0\left(\mathbf{T}^{a\dagger}_{1/8}\mathbf{T}^b_{1/8}-\mathbf{T}^{b\dagger}_{1/8}\mathbf{T}^a_{1/8}\right)N,\end{aligned}\tag{B14}$$

which provide the "missing" factors due to the following properties of the off-diagonal isospin operators $\mathbf{T}^i_{1/8}$ and $\mathbf{D}^i$ matrices:

$$i\,f^{ijk}\big(\mathbf{F}^k_{(8)}\big)=(\mathbf{D}^i\mathbf{D}^j-\mathbf{D}^j\mathbf{D}^i)+\tfrac{2}{3}\big(\mathbf{T}^{i\dagger}_{1/8}\mathbf{T}^j_{1/8}-\mathbf{T}^{j\dagger}_{1/8}\mathbf{T}^i_{1/8}\big).\tag{B15}$$

Therefore, the chiral algebra Eqs. (B3)–(B5) close.

3. Closure of the chiral $SU_L(3)\times SU_R(3)$ algebra in the $(3,6)\oplus(6,3)$ multiplet

The vector charge $Q^a=\int d\mathbf{x}J_0^a(t,\mathbf{x})$, defined by the current in

$$\mathbf{J}^a_\mu=\big(\overline{N}\gamma_\mu\,\mathbf{F}^a_{(8)}\,N\big)+\big(\overline{\Delta}\gamma_\mu\,\mathbf{F}^a_{(10)}\,\Delta\big),\tag{B16}$$

together with the axial charge $Q_5^a=\int d\mathbf{x}J_{05}^a(t,\mathbf{x})$, defined by the current in

$$\begin{aligned}\mathbf{J}^a_{\mu5}=&\ \overline{N}\gamma_\mu\gamma_5\left(\left(\mathbf{D}^{\rm a}+\frac{2}{3}\mathbf{F}^{\rm a}_{(8)}\right)N+\frac{2}{\sqrt{3}}\mathbf{T}^a\Delta\right)\\&+\overline{\Delta}\gamma_\mu\gamma_5\left(\frac{2}{\sqrt{3}}\mathbf{T}^{a\dagger}N+\frac{1}{3}\mathbf{F}^a_{(10)}\Delta\right),\end{aligned}\tag{B17}$$

ought to close the chiral algebra defined by Eqs. (B3)–(B5). Equations (B3) and (B4) hold here, whereas Eq. (B5) is once again the nontrivial one: the fractions $\frac{2}{3}$ and $\frac{1}{3}$ in the diagonal F charge of N $[Q_5^a(N)]$ and Δ axial charges, respectively, and the diagonal D charge of N $[Q_5^a(N)]$:

$$Q^a_{5F}(N)=\frac{2}{3}\int d\mathbf{x}\,\big(\overline{N}\gamma_0\gamma_5\,\mathbf{F}^a_{(8)}\,N\big),\tag{B18}$$

$$Q^a_{5F}(\Delta)=\frac{1}{3}\int d\mathbf{x}\,\big(\overline{\Delta}\gamma_0\gamma_5\,\mathbf{F}^a_{(10)}\,\Delta\big),\tag{B19}$$

$$Q^a_{5D}(N)=\int d\mathbf{x}\,(\overline{N}\gamma_0\gamma_5\,\mathbf{D}^a\,N),\tag{B20}$$

lead to

$$\begin{aligned}&\left[Q^a_{5D+F}(N),Q^b_{5D+F}(N)\right]\\&=\int d\mathbf{x}\Bigg(\overline{N}\gamma_0\left(\left(\mathbf{D}^a+\frac{2}{3}\mathbf{F}^a_{(8)}\right)\left(\mathbf{D}^b+\frac{2}{3}\mathbf{F}^b_{(8)}\right)\right.\\&\quad\left.-\left(\mathbf{D}^b+\frac{2}{3}\mathbf{F}^b_{(8)}\right)\left(\mathbf{D}^a+\frac{2}{3}\mathbf{F}^a_{(8)}\right)\right)N\Bigg),\end{aligned}\tag{B21}$$

$$\left[Q^a_{5F}(\Delta),Q^b_{5F}(\Delta)\right]=if^{abc}\frac{1}{9}Q^c(\Delta),\tag{B22}$$

lead to "only" part of the N and Δ vector charges, respectively, on the right-hand side of Eqs. (B21) and (B22).

Once again, it turns out that the off-diagonal terms in the axial charge

$$Q_5^a(N,\Delta)=\int d\mathbf{x}\left(\frac{2}{\sqrt{3}}\big(\overline{N}\gamma_0\gamma_5\,\mathbf{T}^a_{10/8}\,\Delta+\overline{\Delta}\gamma_0\gamma_5\,\mathbf{T}^{a\dagger}_{10/8}\,N\big)\right),\tag{B23}$$

play a crucial role in the closure of the chiral algebra, Eq. (B5). The additional terms in the commutator add up to

$$\begin{aligned}&\left[Q_5^a(N,\Delta),Q_5^b(N,\Delta)\right]\\&=\frac{4}{3}\int d\mathbf{x}\big(\overline{N}\gamma_0\left(\mathbf{T}^a_{10/8}\mathbf{T}^{b\dagger}_{10/8}-\mathbf{T}^b_{10/8}\mathbf{T}^{a\dagger}_{10/8}\right)N\\&\quad+\overline{\Delta}\gamma_0\left(\mathbf{T}^{a\dagger}_{10/8}\mathbf{T}^b_{10/8}-\mathbf{T}^{b\dagger}_{10/8}\mathbf{T}^a_{10/8}\right)\Delta\big),\end{aligned}\tag{B24}$$

which provide the "missing" factors due to the following properties of the off-diagonal flavor operators $\mathbf{T}^i$ and $\mathbf{D}^i$ matrices:

$$\begin{aligned}i\,f^{ijk}\big(\mathbf{F}^k_{(8)}\big)=&\ \big(\big(\mathbf{D}^i+\tfrac{2}{3}\mathbf{F}^i_{(8)}\big)\big(\mathbf{D}^j+\tfrac{2}{3}\mathbf{F}^j_{(8)}\big)\\&-\big(\mathbf{D}^j+\tfrac{2}{3}\mathbf{F}^j_{(8)}\big)\big(\mathbf{D}^i+\tfrac{2}{3}\mathbf{F}^i_{(8)}\big)\big)\\&+\tfrac{4}{3}\big(\mathbf{T}^i_{10/8}\mathbf{T}^{j\dagger}_{10/8}-\mathbf{T}^j_{10/8}\mathbf{T}^{i\dagger}_{10/8}\big),\end{aligned}\tag{B25}$$

$$i\tfrac{2}{3}\,f^{ijk}\mathbf{F}^k_{(10)}=\mathbf{T}^{i\dagger}_{10/8}\mathbf{T}^j_{10/8}-\mathbf{T}^{j\dagger}_{10/8}\mathbf{T}^i_{10/8}.\tag{B26}$$

Therefore, the chiral algebra, Eqs. (B3)–(B5), closes in spite, or perhaps because of the apparent fractional axial charges ($\frac{2}{3}$ and $\frac{1}{3}$).

APPENDIX C: EVALUATION OF THE CHIRAL $SU_L(3)\times SU_R(3)$ COMMUTATORS

1. Chiral $SU_L(3)\times SU_R(3)$ commutators

We note that the matrix calculation $[Q_5^b,N]$ is equivalent (up to a multiplicative factor) to the $SU(3)_A$ chiral transformation which we have found in our previous papers: Eqs. (11) and (13) in Ref. [33], lead to

$$\left[Q_5^a,N_{(6,3)}\right]=\gamma_5\left(\left(\mathbf{D}^{\rm a}_{(8)}+\frac{2}{3}\mathbf{F}^{\rm a}_{(8)}\right)N_{(6,3)}+\frac{2}{\sqrt{3}}\mathbf{T}^{\rm a}_{(8/10)}\Delta_{(6,3)}\right),\tag{C1}$$

$$\begin{aligned}&\left[Q_5^a,\Delta_{(6,3)}\right]=\gamma_5\left(\frac{2}{\sqrt{3}}\mathbf{T}^{\dagger{\rm a}}{}_{(8/10)}N_{(6,3)}+\frac{1}{3}\mathbf{F}^a_{(10)}\Delta_{(6,3)}\right),\\&\left[Q_5^a,N_{(3,\bar{3})}\right]=\gamma_5\mathbf{D}^aN_{(3,\bar{3})},\\&\left[Q_5^a,N_{(\bar{3},3)}\right]=-\gamma_5\mathbf{D}^aN_{(\bar{3},3)}.\end{aligned}\tag{C2}$$

These SU(3)-spurion matrices $\mathbf{T}^a$ (sometimes we use $\mathbf{T}^a_{10/8}$) and $\mathbf{F}^a_{(10)}$ have the following properties:

$$\begin{aligned}\mathbf{F}^a_{(10)}&=-i\,f^{abc}\mathbf{T}^{b\dagger}_{10/8}\mathbf{T}^c_{10/8},\\\mathbf{T}^a_{10/8}\mathbf{T}^{a\dagger}_{10/8}&=\tfrac{5}{2}\times\mathbf{1}_{8\times8},\\\mathbf{T}^{a\dagger}_{10/8}\mathbf{T}^a_{10/8}&=2\times\mathbf{1}_{10\times10}.\end{aligned}\tag{C3}$$

The octet generators $(\mathbf{D}^{\rm a}_{(8)}+\frac{2}{3}\mathbf{F}^{\rm a}_{(8)})$, the transition matrices $\mathbf{T}^a_{10/8}$ and the decuplet generators $\mathbf{F}^a_{(10)}$ are listed in Appendices A1, A2, and A3, respectively, of Ref. [33].

2. Chiral $SU_L(2) \times SU_R(2)$ commutators

For the chiral $SU_L(2) \times SU_R(2)$ subgroup of the $SU_L(3) \times SU_R(3)$ group, i.e., for $a = 1,2,3$ values of index a, Eqs. (C1) and (C2) lead to

$$[Q_5^a, N_{(1,\frac{1}{2})}] = \gamma_5\left(\frac{5}{3}\frac{\tau^a}{2}N_{(1,\frac{1}{2})} + \frac{2}{\sqrt{3}}T^a\Delta_{(1,\frac{1}{2})}\right),$$
$$[Q_5^a, \Delta_{(1,\frac{1}{2})}] = \gamma_5\left(\frac{2}{\sqrt{3}}T^{\dagger a}N_{(1,\frac{1}{2})} + \frac{1}{3}t^a_{(3/2)}\Delta_{(1,\frac{1}{2})}\right),$$
$$[Q_5^a, N_{(\frac{1}{2},0)}] = \gamma_5\frac{\tau^a}{2}N_{(\frac{1}{2},0)},$$
$$[Q_5^a, N_{(0,\frac{1}{2})}] = -\gamma_5\frac{\tau^a}{2}N_{(0,\frac{1}{2})}, \quad \text{(C4)}$$

where $a = 1,2,3$, $\mathbf{t}^i_{(\frac{3}{2})}$ are the isospin-$\frac{3}{2}$ generators of the SU(2) group and $\mathbf{T}^i$ are the so-called isospurion (4×2) matrices, see Appendix A.

Consequently,

$$[Q_5^a, \bar{N}_{(1,\frac{1}{2})}N_{(1,\frac{1}{2})}] = \frac{5}{3}\bar{N}_{(1,\frac{1}{2})}\gamma_5\tau^a N_{(1,\frac{1}{2})} + \frac{2}{\sqrt{3}}(\bar{N}_{(1,\frac{1}{2})}\gamma_5 T^a\Delta_{(1,\frac{1}{2})} + \bar{\Delta}_{(1,\frac{1}{2})}\gamma_5 T^{\dagger a}N_{(1,\frac{1}{2})}),$$
$$[Q_5^a, \bar{N}_{(\frac{1}{2},0)}N_{(\frac{1}{2},0)}] = \bar{N}_{(\frac{1}{2},0)}\gamma_5\tau^a N_{(\frac{1}{2},0)},$$
$$[Q_5^a, \bar{N}_{(0,\frac{1}{2})}N_{(0,\frac{1}{2})}] = -\bar{N}_{(0,\frac{1}{2})}\gamma_5\tau^a N_{(0,\frac{1}{2})}, \quad \text{(C5)}$$

and similarly for the Δ-field commutator

$$[Q_5^a, \bar{\Delta}_{(1,\frac{1}{2})}\Delta_{(1,\frac{1}{2})}] = \frac{2}{3}\bar{\Delta}_{(1,\frac{1}{2})}\gamma_5 t^a_{(3/2)}\Delta_{(1,\frac{1}{2})} + \frac{2}{\sqrt{3}}(\bar{N}_{(1,\frac{1}{2})}\gamma_5 T^a\Delta_{(1,\frac{1}{2})} + \bar{\Delta}_{(1,\frac{1}{2})}\gamma_5 T^{\dagger a}N_{(1,\frac{1}{2})}). \quad \text{(C6)}$$

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