Kondo effect in charm and bottom nuclei

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The Kondo effect for isospin-exchange interaction between a $\bar{D}$, B meson and a valence nucleon in charm and bottom atomic nuclei including the discrete energy levels for valence nucleons is discussed. To investigate the binding energy by the Kondo effect, I introduce the mean-field approach for the bound state of the $\bar{D}$, B meson in charm and bottom nuclei. Assuming a simple model, I examine the validity of the mean-field approximation by comparing the results with the exact solutions. The effect of the quantum fluctuation is estimated beyond the mean-field approximation. The competition between the Kondo effect and the other correlations in valence nucleons, the isospin symmetry breaking and the nucleon pairings, are discussed.

I. INTRODUCTION

Recently in hadron and nuclear physics, the flavors of nuclear systems are extended to the multiflavor direction: strangeness flavor for hypernuclei and $\bar{K}$-mesic nuclei. These days, nuclear systems with charm and bottom flavor (charm and bottom nuclei) are also investigated in many theoretical studies. One of the most interesting properties in charm and bottom nuclei is that the masses of charm and bottom hadrons are much heavier than the nucleon mass. For example, the mass of the lightest charm meson, a $\bar{D}$ meson, is 1870 MeV, which is about twice as large as the nucleon mass. The mass of the lightest bottom meson, a B meson, is 5280 MeV, which is even about 5.6 times as large as the nucleon mass. To investigate the behavior of such a heavy particle in a nuclear system is an interesting problem because of impurity physics. This is important not only for understanding the hadron dynamics (hadron interaction, change of hadron in medium) and the nuclear structure, but also for unveiling the fundamental properties of the strong interaction, such as the spontaneous breaking of chiral symmetry and the color confinement in vacuum, in quantum chromodynamics (QCD). In fact, it is discussed that the "heavy-quark spin symmetry" as the fundamental symmetry in QCD is essentially important in the heavy-hadron interaction and the mass spectroscopy of heavy hadrons [1–4].

The existence of impurities can affect the properties of matter systems. As the famous and important impurity physics in condensed-matter systems, the Kondo effect has been investigated for a long time [5–7]. Let us consider that the valence fermion (the quasiparticle forming the Fermi surface in medium and electrons in metal) and the impurity particle (atom with finite spin) have the spin-dependent force ($\vec{s} \cdot \vec{S}$-type interaction with spin operators $\vec{s}$ and $\vec{S}$ for the valence fermion and the impurity particle, respectively) with SU(2) spin symmetry. Then, the effective interaction in low-energy scattering becomes enhanced owing to the noncancellation of loop effects from multiple number of particle-hole pairs around the Fermi surface, and the system becomes a strongly

coupled one regardless of the small coupling between the valence fermion and the impurity particle. As a result, the impurity particle with spin-exchange interaction can change the transport properties of the condensed-matter systems. This is called the Kondo effect. The conditions for emergence of the Kondo effect are (i) heavy impurity, (ii) existence of a Fermi surface (or degenerate state), (iii) loop contribution as quantum effect, and (iv) non-Abelian interaction, such as the spin-dependent force [7]. As far as those conditions are satisfied, the Kondo effect can emerge in any quantum system for any kind of constituent particle and energy scale.

Recently, the Kondo effect is discussed for the isospin-exchange interaction with SU(2) isospin symmetry between a charm and bottom hadron and a nucleon in nuclear matter and for the color-exchange interaction with SU(3) color symmetry between a light (up, down, strange) quark and a heavy (charm, bottom) quark [8,9]. The Kondo effect in strong magnetic fields is also discussed [10]. Those can be studied in experiments in high-energy accelerator facilities. To research the Kondo effect in charm and bottom atomic nuclei, it is important to consider finite-volume effects and discrete energy levels of valence nucleons. Here it is meant that a valence nucleon is the nucleon which is an active degree of freedom in a model space in shell structure in atomic nuclei. In the present work, focusing on the discrete energy levels of valence nucleons, the Kondo effect in charm and bottom nuclei is studied.

As mentioned, the non-Abelian interaction is one of the essential conditions for the Kondo effect. In general, it is known that there are several kinds of non-Abelian interaction in (charm and bottom) nuclei: (i) interaction changing total angular momentum (sum of spin and orbital angular momentum) of a valence nucleon, (ii) interaction changing heavy-quark spin, and (iii) interaction changing isospin of heavy hadron and valence nucleon.

(i) The first interaction induces the Kondo effect in deformed nuclei whose shape is different from the spherical one. This may be a phenomena irrelevant to heavy impurity. Instead, the coupling of nucleon spin to quantum rotation of the deformed nucleus is important. In Ref. [11], Sugawara-Tanabe and Tanabe argued that the Coriolis force in a deformed nucleus plays the role of non-Abelian interaction. In this case, the Coriolis force compels the spins of valence nucleons aligned

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along the spin of the deformed nuclei in the same direction (antipairing force), and hence the Kondo effect reduces the strength of the effective coupling in the low-energy limit.

(ii) The second is the interaction in the heavy-quark effective theory based on QCD, which is given by $1/m_Q$ expansion for the heavy-quark mass m_Q [1,2]. It is known that the heavy-quark spin is the conserved quantity in the heavy-quark limit ($m_Q \to \infty$), regardless to the nonperturbative interaction to light quarks and gluons. In charm and bottom nuclei with a bound Λ_c, Λ_b baryon (isospin 1/2, spin parity $1/2^+$) [12,13], the heavy-quark spin is carried by the Λ_c, Λ_b baryon, when only the leading order in the $1/m_Q$ expansion is considered. In the heavy-quark limit, the heavy-quark spin, namely the spin of Λ_c, Λ_b baryon, cannot flip and hence cannot induce the Kondo effect in charm and bottom nuclei.

(iii) The third gives the Kondo effect by isospin exchange between a $\bar{D}$, B meson (isospin 1/2, spin-parity 0^-) and a valence nucleon. The isospin exchange is still available in the heavy-quark limit because the isospin degrees of freedom remain to exist for the $\bar{D}$, B meson in this limit. It has been discussed by many theoretical studies whether the $\bar{D}$, B meson can be bound in nuclear matter[1]: the quark-meson coupling model [14,15], the QCD sum rules [16–19], the hadron-interaction model [20–31], and the quark-interaction model [32]. Some of them suggest that a $\bar{D}$, B meson is bound by attractive potential in nuclear matter. Possible production processes in experiments are also discussed [33]. It is interesting that the pion-exchange interaction between a $\bar{D}$, B meson and a nucleon can be attractive enough to form some bound and resonant states [34–38].[2] In the present study, I investigate the $\bar{D}$, B meson as the impurity for the Kondo effect in charm and bottom nuclei. Note that a Λ_c, Λ_b has no isospin and hence does not induce Kondo effect by isospin-exchange interaction.

Theoretically, to obtain correctly the ground state of the system with the Kondo effect is a highly nonperturbative problem, because the system becomes a strongly coupled one owing to the enhancement of the coupling strength in the low-energy scattering. Several theoretical approaches have been developed for this problem [6,7]. One of the most effective methods is the numerical renormalization group initiated by Wilson [42]. In the present analysis, the mean-field approach is adopted [43,44]. This has been applied to the quantum dot systems with the Kondo effect [45]. The mean-field approach provides a useful method for theoretical analysis and gives an intuitive understanding of the Kondo effect. The idea of the mean-field approximation, or the Hartree-Fock approximation, has been known to be very useful in nuclear physics [46]. I expect that the mean-field approach for the Kondo effect in

charm and bottom nuclei will be used as a straightforward extension toward the impurity physics in nuclear theory.

The paper is organized as follows. In Sec. II, I introduce the effective interaction for exchanging isospin between a $\bar{D}$, B meson and a valence nucleon. One of the purposes in this paper is to study the validity of the mean-field approximation for the many-body problem in the Kondo effect, when the valence nucleons occupy the discrete energy levels. In Sec. III, I investigate the approximate solution for the Kondo effect in the mean-field approach. Here the auxiliary fermion field for isospin of $\bar{D}$, B meson is introduced, and the mean-field approximation is applied in the extended Fock space. The quantum fluctuation by the random-phase approximation (RPA) is considered, and it is shown that the approximate solution becomes closer to the exact one. In Sec. IV, I discuss the competition between the Kondo effect and the correlations of valence nucleons. The isospin breaking of the valence nucleons and the nucleon pairings are investigated. The final section is devoted to a conclusion.

II. HAMILTONIAN FOR KONDO EFFECT WITH DISCRETE ENERGY LEVELS

A. Model setup

Let us consider a $\bar{D}$, B meson (isospin 1/2) as a heavy impurity particle in atomic nuclei. To treat the isospin-exchange interaction between a $\bar{D}$, B meson and a valence nucleon in a simple form as far as possible, let us consider the Hamiltonian

$$H = H_0 + H_K, \tag{1}$$

where H_0 is the kinetic term for the valence nucleon

$$H_0 = \sum \epsilon_k c_{k\sigma}^\dagger c_{k\sigma}, \tag{2}$$

and H_K is the isospin-exchange (Kondo) interaction term

$$H_K = g \sum [c_{k'\downarrow}^\dagger c_{k\uparrow} T_+ + c_{k'\uparrow}^\dagger c_{k\downarrow} T_- + (c_{k'\uparrow}^\dagger c_{k\uparrow} - c_{k'\downarrow}^\dagger c_{k\downarrow}) T_3], \tag{3}$$

with the coupling constant g. Here $c_{k\sigma}$ ($c_{k\sigma}^\dagger$) is the annihilation (creation) operator for the valence nucleon in the kth single state, where $k = 1, \ldots, N$ indicates the single-state of the valence nucleon,[3] and $\sigma = \uparrow, \downarrow$ is the up, down component of the isospin. I define $T_\pm$ and T_3 as the raising and/or lowering operators and the z component of the SU(2) isospin operator, and

$$T_1 = \tfrac{1}{2}(T_+ + T_-), \tag{4}$$

$$T_2 = \tfrac{1}{2i}(T_+ - T_-). \tag{5}$$

T_1, T_2, and T_3 satisfy the commutation relation of the SU(2) algebra,

$$[T_a, T_b] = i\epsilon_{abc} T_c, \tag{6}$$

with $a, b, c = 1, 2, 3$.

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[1]The dynamics of a $\bar{D}$, B meson ($q\bar{Q}$; $Q = c, b$) is simpler than that of the antiparticle, a D, $\bar{B}$ meson ($\bar{q}Q$) because the former does not have $q\bar{q}$ annihilation in nuclear matter, while the latter has. The difference in their behaviors is attributable to the charge symmetry breaking at finite baryon number density.

[2]The bound and resonant systems composed of a $\bar{D}$, B meson and a nucleon were originally investigated by the bound-state approach in the Skyrme model as pentaquark states [39–41].

[3]For example, N is given by $N = 2j + 1$ for the j shell in the nuclear shell model.

Note that, in the Hamiltonian (1), there is a quantum fluctuation of the impurity isospin, because the direction of the impurity isospin is not fixed. Therefore, the dynamics of the valence nucleon is always affected by the isospin fluctuation of the impurity, and hence it cannot be reduced to the one-body problem. This is one of the interesting properties of the Kondo effect. The purpose in this study is to obtain the energy eigenvalues of the Hamiltonian (1) by considering the isospin fluctuation.

B. Exact energy eigenvalues

1. Variational method for wave function

For simplicity, let us consider the single-particle states with energy $\epsilon_k = \epsilon$ for the valence nucleons. I use the representations $|\uparrow\rangle_{\text{imp}}$ and $|\downarrow\rangle_{\text{imp}}$ for the impurity states with isospins $\uparrow$ and $\downarrow$, respectively. I also use the representation $|\psi_{I,I_3}^{(n)}\rangle$ for the total state, composed of an impurity and valence nucleons, with isospin I, its z component I_3, and the number of valence nucleon n. For example, the $n = 1, 2, 3$ cases are considered in the following.

a. The $n = 1$ case. Let us consider isospin $I = 0, 1$.

For $I = 0$, I assume the wave function

$$\left|\psi_{0,0}^{(1)}\right\rangle = \sum \Gamma_k (c_{k\uparrow}^\dagger |\downarrow\rangle_{\text{imp}} - c_{k\downarrow}^\dagger |\uparrow\rangle_{\text{imp}}), \quad (7)$$

with unknown coefficients $\{\Gamma_k\} = \{\Gamma_1, \ldots, \Gamma_N\}$. By using $H|\psi_{0,0}^{(1)}\rangle = E|\psi_{0,0}^{(1)}\rangle$, I obtain

$$\epsilon\Gamma_k - \frac{3}{2}g\sum_n \Gamma_n = E\Gamma_k, \quad (8)$$

and hence the energy eigenvalues

$$E = \epsilon - \frac{3}{2}Ng \quad (\text{ndf} = 1), \quad \epsilon \ (\text{ndf} = N - 1). \quad (9)$$

The numbers in the parentheses are the number of degeneracy factor (ndf).

For $I = 1$, considering $I_3 = 1$, I assume the wave function

$$\left|\psi_{1,1}^{(1)}\right\rangle = \sum \Gamma_k c_{k\uparrow}^\dagger |\uparrow\rangle_{\text{imp}}, \quad (10)$$

with the unknown coefficients $\{\Gamma_k\}$. By using $H|\psi_{1,1}^{(1)}\rangle = E|\psi_{1,1}^{(1)}\rangle$, I obtain the relation

$$\epsilon\Gamma_k + \frac{1}{2}g\sum_n \Gamma_n = E\Gamma_k, \quad (11)$$

and hence the energy eigenvalues

$$E = \epsilon + \frac{1}{2}Ng \quad (\text{ndf} = 1), \quad \epsilon \ (\text{ndf} = N - 1). \quad (12)$$

The same values are obtained for $I_3 = 0, -1$.

See Table I for a summary.

b. The $n = 2$ case. Let us consider $I = 1/2, 3/2$.

For $I = 1/2$, considering $I_3 = 1/2$, I assume the wave function

$$\left|\psi_{1/2,1/2}^{(2)}\right\rangle = \sum \left\{ \Gamma_{mn}^0 [c_m^\dagger \otimes c_n^\dagger]_{00}|\uparrow\rangle_{\text{imp}} \right.$$
$$+ \Gamma_{mn}^1 \left(\sqrt{\frac{2}{3}} [c_m^\dagger \otimes c_n^\dagger]_{11}|\downarrow\rangle_{\text{imp}} \right.$$
$$\left. \left. - \frac{1}{\sqrt{3}}[c_m^\dagger \otimes c_n^\dagger]_{10}|\uparrow\rangle_{\text{imp}} \right) \right\}, \quad (13)$$

with unknown coefficients Γ_{mn}^0, Γ_{mn}^1 having the properties $\Gamma_{mn}^0 = \Gamma_{nm}^0$, $\Gamma_{mn}^1 = -\Gamma_{nm}^1$. Here I define

$$[c_m^\dagger \otimes c_n^\dagger]_{00} = \frac{1}{\sqrt{2}}(c_{m\uparrow}^\dagger c_{n\downarrow}^\dagger - c_{m\downarrow}^\dagger c_{n\uparrow}^\dagger), \quad (14)$$

and

$$[c_m^\dagger \otimes c_n^\dagger]_{1I_3} = \begin{cases} c_{m\uparrow}^\dagger c_{n\uparrow}^\dagger & (I_3 = 1) \\ \frac{1}{\sqrt{2}}(c_{m\uparrow}^\dagger c_{n\downarrow}^\dagger + c_{m\downarrow}^\dagger c_{n\uparrow}^\dagger) & (I_3 = 0) \\ c_{m\downarrow}^\dagger c_{n\downarrow}^\dagger & (I_3 = -1) \end{cases}, \quad (15)$$

for pairs of valence nucleons with an isosinglet and an isotriplet, respectively, for short notation. By using $H|\psi_{1/2,1/2}^{(2)}\rangle = E|\psi_{1/2,1/2}^{(2)}\rangle$, I obtain the energy eigenvalues

$$E = \epsilon - \frac{3}{2}Ng \ (\text{ndf} = N - 1), \quad \epsilon \ (\text{ndf} = N^2 - 2N + 2),$$
$$\epsilon + \frac{1}{2}Ng \ (\text{ndf} = N - 1). \quad (16)$$

The same values are obtained for $I_3 = -1/2$.

For $I = 3/2$, considering $I_3 = 3/2$, I assume the wave function

$$\left|\psi_{3/2,3/2}^{(2)}\right\rangle = \sum \Gamma_{mn}^1 [c_m^\dagger \otimes c_n^\dagger]_{11}|\uparrow\rangle_{\text{imp}}, \quad (17)$$

with unknown coefficients Γ_{mn}^1 having the property $\Gamma_{mn}^1 = -\Gamma_{nm}^1$. By using $H|\psi_{3/2,3/2}^{(2)}\rangle = E|\psi_{3/2,3/2}^{(2)}\rangle$, I obtain, through the relation

$$2\epsilon\Gamma_{mn}^1 - \frac{1}{2}g\sum_{1 \leqslant l \leqslant N} (\Gamma_{lm}^1 - \Gamma_{ln}^1) = E\Gamma_{mn}^1, \quad (18)$$

with $m < n$, the energy eigenvalues

$$E = \epsilon \left[\text{ndf} = \frac{1}{2}(N - 2)(N - 1)\right],$$
$$\epsilon + \frac{1}{2}Ng \ (\text{ndf} = N - 1). \quad (19)$$

The same values are obtained for $I_3 = 1/2, 0, -1/2, -3/2$.

See Table II for summary.

TABLE II. Energy eigenvalues of the Hamiltonian (1) for $n = 2$. The notations are the same as Table I.

N	No. of valence nucleon $n = 2$	
	$I = 1/2$	$I = 3/2$
1	2ϵ (1)	−
2	$2\epsilon - \frac{3}{2}Ng$ (1), 2ϵ (2), $2\epsilon + \frac{1}{2}Ng$ (1)	$2\epsilon + \frac{1}{2}Ng$ (1)
3	$2\epsilon - \frac{3}{2}Ng$ (2), 2ϵ (5), $2\epsilon + \frac{1}{2}Ng$ (2)	$2\epsilon + \frac{1}{2}Ng$ (2), 2ϵ (1)
4	$2\epsilon - \frac{3}{2}Ng$ (3), 2ϵ (10), $2\epsilon + \frac{1}{2}Ng$ (3)	$2\epsilon + \frac{1}{2}Ng$ (3), 2ϵ (3)

TABLE I. Energy eigenvalues of the Hamiltonian (1) for $n = 1$. The numbers in parentheses are the numbers of degeneracy factor.

N	No. of valence nucleons $n = 1$	
	$I = 0$	$I = 1$
Any N	$\epsilon - \frac{3}{2}Ng$ (1), ϵ $(N - 1)$	$\epsilon + \frac{1}{2}Ng$ (1), ϵ $(N - 1)$

TABLE III. Energy eigenvalues of the Hamiltonian (1) for $n = 3$. The notations are the same as in Table I.

N	No. of valence nucleon $n = 3$		
	$I = 0$	$I = 1$	$I = 2$
1	$-$	$-$	$-$
2	$3\epsilon - \frac{3}{2}Ng$ (1), 3ϵ (1)	3ϵ (1), $3\epsilon + \frac{1}{2}Ng$ (1)	$-$
3	$3\epsilon - \frac{3}{2}Ng$ (3), 3ϵ (4), $3\epsilon + \frac{1}{2}Ng$ (1)	$3\epsilon - \frac{3}{2}Ng$ (1), 3ϵ (3), $3\epsilon + \frac{1}{2}Ng$ (4)	$3\epsilon + \frac{1}{2}Ng$ (1)
4	$3\epsilon - \frac{3}{2}Ng$ (6), 3ϵ (11), $3\epsilon + \frac{1}{2}Ng$ (3)	$3\epsilon - \frac{3}{2}Ng$ (3), 3ϵ (12), $3\epsilon + \frac{1}{2}Ng$ (9)	3ϵ (1), $3\epsilon + \frac{1}{2}Ng$ (3)

c. The $n = 3$ case. Let us consider $I = 0, 1, 2$.
For $I = 0$, I assume the wave function

$$
\begin{aligned}
\big|\psi_{0,0}^{(3)}\big\rangle &= \sum_{0 \leqslant l < m \leqslant N} \Gamma_{lm} c_{l\uparrow}^{\dagger} c_{l\downarrow}^{\dagger} \frac{1}{\sqrt{2}} (c_{m\uparrow}^{\dagger}|\downarrow\rangle_{\mathrm{imp}} - c_{m\downarrow}^{\dagger}|\uparrow\rangle_{\mathrm{imp}}) \\
&+ \sum_{0 \leqslant l < m \leqslant N} \Gamma'_{lm} c_{m\uparrow}^{\dagger} c_{m\downarrow}^{\dagger} \frac{1}{\sqrt{2}} (c_{l\uparrow}^{\dagger}|\downarrow\rangle_{\mathrm{imp}} - c_{l\downarrow}^{\dagger}|\uparrow\rangle_{\mathrm{imp}}) \\
&+ \sum_{0 \leqslant l < m < n \leqslant N} \Gamma_{lmn}^{00} \frac{1}{2} (c_{l\uparrow}^{\dagger} c_{m\downarrow}^{\dagger} - c_{l\downarrow}^{\dagger} c_{m\uparrow}^{\dagger})(c_{n\uparrow}^{\dagger}|\downarrow\rangle_{\mathrm{imp}} - c_{n\downarrow}^{\dagger}|\uparrow\rangle_{\mathrm{imp}}) \\
&+ \sum_{0 \leqslant l < m < n \leqslant N} \Gamma_{lmn}^{11} \frac{1}{\sqrt{3}} \bigg\{ c_{l\uparrow}^{\dagger} c_{m\uparrow}^{\dagger} c_{n\downarrow}^{\dagger}|\downarrow\rangle_{\mathrm{imp}} - \frac{1}{2}(c_{l\uparrow}^{\dagger} c_{m\downarrow}^{\dagger} + c_{l\downarrow}^{\dagger} c_{m\uparrow}^{\dagger})(c_{n\uparrow}^{\dagger}|\downarrow\rangle_{\mathrm{imp}} + c_{n\downarrow}^{\dagger}|\uparrow\rangle_{\mathrm{imp}}) + c_{l\downarrow}^{\dagger} c_{m\downarrow}^{\dagger} c_{n\uparrow}^{\dagger}|\uparrow\rangle_{\mathrm{imp}} \bigg\}, \quad (20)
\end{aligned}
$$

with unknown coefficients Γ_{lm}, Γ'_{lm}, Γ_{lmn}^{00}, and Γ_{lmn}^{11} with $l < m < n$. By using $H|\psi_{0,0}^{(3)}\rangle = E|\psi_{0,0}^{(3)}\rangle$, I obtain the energy eigenvalues as shown in Table III.

For $I = 1$, considering $I_3 = 1$, I assume the wave function

$$
\begin{aligned}
\big|\psi_{1,1}^{(3)}\big\rangle &= \sum_{0 \leqslant l < m \leqslant N} \Gamma_{lm} c_{l\uparrow}^{\dagger} c_{l\downarrow}^{\dagger} c_{m\uparrow}^{\dagger}|\uparrow\rangle_{\mathrm{imp}} + \sum_{0 \leqslant l < m \leqslant N} \Gamma'_{lm} c_{m\uparrow}^{\dagger} c_{m\downarrow}^{\dagger} c_{l\uparrow}^{\dagger}|\uparrow\rangle_{\mathrm{imp}} \\
&+ \sum_{0 \leqslant l < m < n \leqslant N} \Gamma_{lmn}^{01} \frac{1}{\sqrt{2}} (c_{l\uparrow}^{\dagger} c_{m\downarrow}^{\dagger} - c_{l\downarrow}^{\dagger} c_{m\uparrow}^{\dagger}) c_{n\uparrow}^{\dagger}|\uparrow\rangle_{\mathrm{imp}} + \sum_{0 \leqslant l < m < n \leqslant N} \Gamma_{lmn}^{10} c_{l\uparrow}^{\dagger} c_{m\uparrow}^{\dagger} \frac{1}{\sqrt{2}} (c_{n\uparrow}^{\dagger}|\downarrow\rangle_{\mathrm{imp}} - c_{n\downarrow}^{\dagger}|\uparrow\rangle_{\mathrm{imp}}) \\
&+ \sum_{0 \leqslant l < m < n \leqslant N} \Gamma_{lmn}^{11} \bigg\{ \frac{1}{\sqrt{2}} c_{l\uparrow}^{\dagger} c_{m\uparrow}^{\dagger} \frac{1}{\sqrt{2}} (c_{n\uparrow}^{\dagger}|\downarrow\rangle_{\mathrm{imp}} + c_{n\downarrow}^{\dagger}|\uparrow\rangle_{\mathrm{imp}}) - \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} (c_{l\uparrow}^{\dagger} c_{m\downarrow}^{\dagger} + c_{l\downarrow}^{\dagger} c_{m\uparrow}^{\dagger}) c_{n\uparrow}^{\dagger}|\uparrow\rangle_{\mathrm{imp}} \bigg\}, \quad (21)
\end{aligned}
$$

with unknown coefficients Γ_{lm}, Γ'_{lm}, Γ_{lmn}^{10}, and Γ_{lmn}^{11} with $l < m < n$. By using $H|\psi_{1,1}^{(3)}\rangle = E|\psi_{1,1}^{(3)}\rangle$, I obtain the energy eigenvalues as shown in Table III. The same values are obtained for $I_3 = 0, -1$.

For $I = 2$, considering $I_3 = 2$, I assume the wave function

$$
\big|\psi_{2,2}^{(3)}\big\rangle = \sum_{0 \leqslant l < m < n \leqslant N} \Gamma_{lmn}^{11} c_{l\uparrow}^{\dagger} c_{m\uparrow}^{\dagger} c_{n\uparrow}^{\dagger}|\uparrow\rangle_{\mathrm{imp}}, \quad (22)
$$

with unknown coefficients Γ_{lmn}^{11} with $l < m < n$. By using $H|\psi_{2,2}^{(3)}\rangle = E|\psi_{2,2}^{(3)}\rangle$, I obtain the energy eigenvalues as shown in Table III, the same values are obtained for $I_3 = 1, 0, -1, -2$.

2. Method by pseudoisospin SU(2) algebra

For the single-particle states with $\epsilon_k = \epsilon$, the energy eigenvalues can be obtained especially by the simple method.

For this purpose, I define the operator

$$
C_{N\sigma} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} c_{k\sigma}, \quad (23)
$$

as a coherent sum of $c_{k\sigma}$. This satisfies the commutation relation for fermions

$$
\{C_{N\sigma}, C_{N\sigma'}^{\dagger}\} = \delta_{\sigma\sigma'}. \quad (24)
$$

Defining the raising and lowering operators and the z component

$$
T_+^{\mathrm{c}} = C_{N\uparrow}^{\dagger} C_{N\downarrow}, \quad (25)
$$

$$
T_-^{\mathrm{c}} = C_{N\downarrow}^{\dagger} C_{N\uparrow}, \quad (26)
$$

$$
T_3^{\mathrm{c}} = \tfrac{1}{2}(C_{N\uparrow}^{\dagger} C_{N\uparrow} - C_{N\downarrow}^{\dagger} C_{N\downarrow}), \quad (27)
$$

and x, y components

$$T_1^c = \frac{1}{2}(T_+^c + T_-^c), \tag{28}$$

$$T_2^c = \frac{1}{2i}(T_+^c - T_-^c), \tag{29}$$

I introduce the operator

$$\vec{T}^c = (T_1^c, T_2^c, T_3^c). \tag{30}$$

Those operators satisfy the SU(2) algebra; $[T_a^c, T_b^c] = i\epsilon_{abc}T_c^c$, with $a, b, c = 1, 2, 3$. Hence, I call $\vec{T}^c$ the pseudoisospin. I distinguish this from the conventional isospin operator for each valence nucleon, because $\vec{T}^c$ (or $C_{N\sigma}$) gives the coherent state of $k = 1, \ldots, N$ single-particle states. It is emphasized that the pseudoisospin can be defined only when the single-particle states have the same energy ($\epsilon_k = \epsilon$).[4]

By using the identity

$$T_+^c T_- + T_-^c T_+ + 2T_3^c T_3 = 2\vec{T}^c \cdot \vec{T}, \tag{34}$$

I find that the interaction term (3) can be expressed as

$$H_K = 2Ng\vec{T}^c \cdot \vec{T}. \tag{35}$$

According to the compound isospin $|\vec{T}^c + \vec{T}| = 0, 1$ (i.e., $\vec{T}^c \cdot \vec{T} = -3/4, 1/4$, respectively), I obtain

$$H_K = \begin{cases} -\frac{3}{2}Ng & (|\vec{T}^c + \vec{T}| = 0), \\ \frac{1}{2}Ng & (|\vec{T}^c + \vec{T}| = 1). \end{cases} \tag{36}$$

The original Hamiltonian (1) can be given as

$$H = \sum \epsilon C_{k\sigma}^\dagger C_{k\sigma} + 2Ng\vec{T}^c \cdot \vec{T}, \tag{37}$$

by the fermion operator $C_{N\sigma}$ and the $N - 1$ orthogonal fermion operators $C_{k\sigma}$ ($k = 1, \ldots, N - 1$). $C_{k\sigma}$ with $k = 1, \ldots, N - 1$ are linear combinations of $c_{k\sigma}$, with $k = 1, \ldots, N$, all of which are commutative with $C_{N\sigma}$, and satisfy $\{C_{k\sigma}^\dagger, C_{k'\sigma'}\} = \delta_{kk'}\delta_{\sigma\sigma'}$ ($k, k' = 1, \ldots, N - 1$). From Eq. (37), one can indeed confirm the results in Tables I–III.

III. MEAN-FIELD APPROXIMATION IN KONDO EFFECT

In the previous section, the energy eigenvalues were obtained in the simple case with $\epsilon_k = \epsilon$ for the Hamiltonian (1). In general cases, however, I need to perform diagonalization of large matrices with paying a cost to the numerical computation. Moreover, such direct analysis may not be useful for an intuitive understanding of the result. In this section, the mean-field approximation is introduced based on Refs. [43–45].

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[4]Note that the SU(2) algebra holds for more general case,

$$T_+^c = \frac{1}{N}\sum c_{k'\uparrow}^\dagger \mathcal{S}_{k'k} c_{k\downarrow}, \tag{31}$$

$$T_-^c = \frac{1}{N}\sum c_{k'\downarrow}^\dagger \mathcal{S}_{k'k} c_{k\uparrow}, \tag{32}$$

$$T_3^c = \frac{1}{2N}\sum (c_{k'\uparrow}^\dagger \mathcal{S}_{k'k} c_{k\uparrow} - c_{k'\downarrow}^\dagger \mathcal{S}_{k'k} c_{k\downarrow}), \tag{33}$$

with symmetric $\mathcal{S}_{k'k}$ ($\mathcal{S}_{k'k} = \mathcal{S}_{kk'}$). The present case is $\mathcal{S}_{k'k} = \delta_{k'k}$.

It is discussed how the mean-field approximation gives an easy way to obtain the ground state of the Hamiltonian (1) and the validity of the mean-field approach is investigated by comparing the results with the exact ones. The quantum fluctuation in RPA is also considered beyond the mean-field approximation.

Note that the mean-field approach was applied to the cases with a continuous number density of valence fermions in an infinitely large system in condensed-matter physics [43–45]. As emphasized in the Introduction, the purpose in the present discussion is to investigate the Kondo effect in finite systems with discrete energy levels of valence nucleons in charm and bottom nuclei. For this purpose, I apply the mean-feld approach to the finite-size system with discrete energy levels. As analogy, recall that the BCS theory, which successfully describes the superconducting state with continuous number density in an infinitely large system, can be applied to pairings of valence nucleons in finite nuclei [46].

A. Introducing auxiliary fermion fields

To describe the isospin of the impurity, I introduce the auxiliary fermion field f_σ ($\sigma = \uparrow, \downarrow$) [43–45]. This satisfy the fermion commutation relation $\{f_\sigma, f_{\sigma'}^\dagger\} = \delta_{\sigma\sigma'}$ and $\{f_\sigma, f_{\sigma'}\} = 0$. I rewrite the isospin operators T_+, T_-, T_3 of the impurity by using f_σ as

$$T_+ = f_\uparrow^\dagger f_\downarrow, \tag{38}$$

$$T_- = f_\downarrow^\dagger f_\uparrow, \tag{39}$$

$$T_3 = \frac{1}{2}(f_\uparrow^\dagger f_\uparrow - f_\downarrow^\dagger f_\downarrow). \tag{40}$$

Because the number of the impurity should be always equal to one, it is necessary to impose the constraint condition [43–45]

$$\sum f_\sigma^\dagger f_\sigma = 1. \tag{41}$$

The Fock space satisfying this condition is the physical Fock space that should be obtained. The Fock space with the other impurity numbers, $\sum f_\sigma^\dagger f_\sigma = 0, 2$, which is indeed unphysical, needs to be excluded. In the mean-field approximation, however, it will turn out that an extension of the Fock space to the multiple impurity numbers is useful to analyze the ground state of the Hamiltonian (1). In the following, let us consider separately the two cases of $g > 0$ and $g < 0$ in the Hamiltonian (1).

Note that the above decomposition of the operators T_+, T_-, T_3 can be given by boson fields instead of the fermion fields. In the boson case, however, it is necessary to consider superposed fields of the bosons and the valence nucleons, fermions, in the mean-field approximation, which may lead to some difficulty. Moreover, the Fock space for the boson fields has to be extended to infinite number of bosons in contrast to the fermion case, where fermion numbers are limited to two at most. Therefore, let us consider the fermion fields more convenient than the boson fields in the present analysis.

B. Isosinglet condensate ($g > 0$)

Let us consider the $g > 0$ case. First I discusses the mean-field approximation, and then I investigate the fluctuation by using RPA.

1. Mean-field approximation

I rewrite the Hamiltonian (1) as

$$
H = \sum \epsilon_k c_{k\sigma}^\dagger c_{k\sigma}
$$
$$
+ g \left(\sum f_\sigma^\dagger f_{\sigma'} c_{k'\sigma'}^\dagger c_{k\sigma} - \frac{1}{2} \sum c_{k'\sigma}^\dagger c_{k\sigma} \right)
$$
$$
+ \lambda \left(\sum f_\sigma^\dagger f_\sigma - 1 \right), \tag{42}
$$

by using the relations (38)–(40) and the identity

$$
c_{k'\downarrow}^\dagger c_{k\uparrow} T_+ + c_{k'\uparrow}^\dagger c_{k\downarrow} T_- + (c_{k'\uparrow}^\dagger c_{k\uparrow} - c_{k'\downarrow}^\dagger c_{k\downarrow}) T_3
$$
$$
= \sum f_\sigma^\dagger f_{\sigma'} c_{k'\sigma'}^\dagger c_{k\sigma} - \frac{1}{2} \sum c_{k'\sigma}^\dagger c_{k\sigma}, \tag{43}
$$

where the constraint condition (41) is used.[5] In the last term in the right-hand side in Eq. (42), let us consider the constraint condition (41) by introducing the Lagrange multiplier constant λ. Now I apply the mean-field approximation. I introduce the mean-field $\langle f_\sigma^\dagger c_{k\sigma} \rangle$ as an expectation value of $f_\sigma^\dagger c_{k\sigma}$, sandwiched by the ground state, and define the isosinglet "gap" function [43–45]

$$
\Delta = -g \sum \langle f_\sigma^\dagger c_{k\sigma} \rangle. \tag{44}
$$

Using the relation

$$
g \sum f_\sigma^\dagger f_{\sigma'} c_{k'\sigma'}^\dagger c_{k\sigma}
$$
$$
= g \sum f_\sigma^\dagger f_{\sigma'} (-c_{k\sigma} c_{k'\sigma'}^\dagger + \delta_{kk'} \delta_{\sigma\sigma'})
$$
$$
= -g \sum f_\sigma^\dagger c_{k\sigma} c_{k'\sigma'}^\dagger f_{\sigma'} + N g \sum f_\sigma^\dagger f_\sigma
$$
$$
= -g \sum (f_\sigma^\dagger c_{k\sigma} - \langle f_\sigma^\dagger c_{k\sigma} \rangle + \langle f_\sigma^\dagger c_{k\sigma} \rangle)
$$
$$
\times (c_{k'\sigma'}^\dagger f_{\sigma'} - \langle c_{k'\sigma'}^\dagger f_{\sigma'} \rangle + \langle c_{k'\sigma'}^\dagger f_{\sigma'} \rangle) + N g
$$
$$
= -g \sum (f_\sigma^\dagger c_{k\sigma} - \langle f_\sigma^\dagger c_{k\sigma} \rangle)(c_{k'\sigma'}^\dagger f_{\sigma'} - \langle c_{k'\sigma'}^\dagger f_{\sigma'} \rangle)
$$
$$
- g \sum (\langle f_\sigma^\dagger c_{k\sigma} \rangle c_{k'\sigma'}^\dagger f_{\sigma'} + \langle c_{k'\sigma'}^\dagger f_{\sigma'} \rangle f_\sigma^\dagger c_{k\sigma})
$$
$$
+ g \sum \langle f_\sigma^\dagger c_{k\sigma} \rangle \langle c_{k'\sigma'}^\dagger f_{\sigma'} \rangle + N g, \tag{45}
$$

where the constraint condition (41) is used again, I separate the Hamiltonian (1) into the mean-field part H_{MF} and the fluctuation part H_{fluc} as

$$
H = H_{\mathrm{MF}} + H_{\mathrm{fluc}}, \tag{46}
$$

—————————

[5] The second term on the right-hand side in Eq. (43) does not include the flipping of the isospin of the valence nucleon and hence could be neglected for the Kondo effect [45]. However, I keep this term throughout the analysis because the present discussion is devoted to comparison of the result in the mean-field approximation with the exact solution.

with

$$
H_{\mathrm{MF}} = \sum \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum (\Delta^* f_\sigma^\dagger c_{k\sigma} + \Delta c_{k\sigma}^\dagger f_\sigma)
$$
$$
+ \lambda \sum f_\sigma^\dagger f_\sigma + \frac{|\Delta|^2}{g} - \lambda, \tag{47}
$$

and

$$
H_{\mathrm{fluc}} = -g \sum (f_\sigma^\dagger c_{k\sigma} - \langle f_\sigma^\dagger c_{k\sigma} \rangle)(c_{k'\sigma'}^\dagger f_{\sigma'} - \langle c_{k'\sigma'}^\dagger f_{\sigma'} \rangle)
$$
$$
- \frac{1}{2} g \sum c_{k'\sigma}^\dagger c_{k\sigma} + N g. \tag{48}
$$

In the mean-field approximation, let us consider only the mean-field part H_{MF} and neglect the fluctuation part H_{fluc} [43–45]. I diagonalize H_{MF} and introduce the Slater determinant by single-particle states. Then I perform the variation for the expectation value $\langle H_{\mathrm{MF}} \rangle$ with respect to λ and Δ as

$$
\frac{\partial}{\partial \lambda} \langle H_{\mathrm{MF}} \rangle = 0, \tag{49}
$$

$$
\frac{\partial}{\partial \Delta} \langle H_{\mathrm{MF}} \rangle = 0, \tag{50}
$$

and finally obtain λ and Δ. The ground-state energy is given by substituting the λ and Δ into $\langle H_{\mathrm{MF}} \rangle$.

In the following, to demonstrate the mean-field calculation explicitly, let us consider the simple case of $\epsilon_k = \epsilon$ for all $k = 1, \ldots, N$ because the diagonalization of H_{MF} can be analytically performed. Such simplification does not change the essence of the discussion. With the basis $\{c_{k\sigma}, f_\sigma\}$ ($k = 1, \ldots, N$, $\sigma = \uparrow, \downarrow$), I give the mean-field Hamiltonian H_{MF} in terms of the $2(N + 1) \times 2(N + 1)$ matrix $\mathcal{H}_{cf}$,

$$
\mathcal{H}_{cf} = \begin{pmatrix}
\epsilon & 0 & \cdots & \Delta^* & 0 & 0 & \cdots & 0 \\
0 & \epsilon & \cdots & \Delta^* & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\Delta & \Delta & \cdots & \lambda & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & \epsilon & 0 & \cdots & \Delta^* \\
0 & 0 & \cdots & 0 & 0 & \epsilon & \cdots & \Delta^* \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & \Delta & \Delta & \cdots & \lambda
\end{pmatrix}, \tag{51}
$$

as

$$
H_{\mathrm{MF}} = \psi^\dagger \mathcal{H}_{cf} \psi + \frac{|\Delta|^2}{g} - \lambda, \tag{52}
$$

defining

$$
\psi = \begin{pmatrix}
c_{1\uparrow} \\
\vdots \\
c_{N\uparrow} \\
f_\uparrow \\
c_{1\downarrow} \\
\vdots \\
c_{N\downarrow} \\
f_\downarrow
\end{pmatrix}, \tag{53}
$$

for short notation. It is worth noting that $g > 0$ should be maintained because the stability of the ground state is

guaranteed by the positivity of $|\Delta|^2/g$ in H_{MF}. Then I diagonalizes $\mathcal{H}_{cf}$ analytically as

$$\mathcal{H}_{cf}^{\mathrm{diag}} = \mathrm{diag}\left[\epsilon, \ldots, \epsilon, \tfrac{1}{2}(\epsilon + \lambda - D), \tfrac{1}{2}(\epsilon + \lambda + D),\right.$$
$$\epsilon, \ldots, \epsilon, \tfrac{1}{2}(\epsilon + \lambda - D), \tfrac{1}{2}(\epsilon + \lambda + D)\right]$$
$$= \mathrm{diag}(E_1, \ldots, E_{N-1}, E_N, E_{N+1}, E_1, \ldots,$$
$$E_{N-1}, E_N, E_{N+1}),\qquad(54)$$

with

$$D = \sqrt{(\epsilon - \lambda)^2 + 4N|\Delta|^2}.\qquad(55)$$

Introducing the new fields $\{d_{k\sigma}\}$ $(k = 1, \ldots, N)$

$$d_{1\sigma} = \frac{1}{\sqrt{2}}(c_{1\sigma} - c_{2\sigma}),\qquad(56)$$

$$d_{2\sigma} = \frac{1}{\sqrt{2}}(c_{1\sigma} - c_{3\sigma}),\qquad(57)$$

$$\vdots$$

$$d_{N-1\sigma} = \frac{1}{\sqrt{2}}(c_{1\sigma} - c_{N\sigma}),\qquad(58)$$

$$d_{N\sigma} = \frac{1}{\sqrt{2N}}\sqrt{1 - \frac{\epsilon - \lambda}{D}}(c_{1\sigma} + \cdots + c_{N\sigma})$$
$$- \frac{1}{\sqrt{2}}\sqrt{1 + \frac{\epsilon - \lambda}{D}}f_\sigma,\qquad(59)$$

$$d_{N+1\sigma} = \frac{1}{\sqrt{2N}}\sqrt{1 + \frac{\epsilon - \lambda}{D}}(c_{1\sigma} + \cdots + c_{N\sigma})$$
$$+ \frac{1}{\sqrt{2}}\sqrt{1 - \frac{\epsilon - \lambda}{D}}f_\sigma,\qquad(60)$$

I represent the mean-field Hamiltonian H_{MF} by

$$H_{\mathrm{MF}} = \phi^\dagger \mathcal{H}_{cf}^{\mathrm{diag}} \phi + \frac{|\Delta|^2}{g} - \lambda$$
$$= \sum E_k d_{k\sigma}^\dagger d_{k\sigma} + \frac{|\Delta|^2}{g} - \lambda,\qquad(61)$$

with defining

$$\phi = \begin{pmatrix} d_{1\uparrow} \\ \vdots \\ d_{N\uparrow} \\ d_{N+1\uparrow} \\ d_{1\downarrow} \\ \vdots \\ d_{N\downarrow} \\ d_{N+1\downarrow} \end{pmatrix}.\qquad(62)$$

I remark that the isospin components $\uparrow$ and $\downarrow$ for the valence nucleons are separated in the matrix $\mathcal{H}_{cf}$, and the mixing part in the off-diagonal components is absorbed into the fluctuation part H_{fluc}. This separation indeed makes it possible to introduce the mean field for the valence nucleons.

Now let us consider the variation of $\langle H_{\mathrm{MF}}\rangle$ with respect to λ and Δ. As a simple case, the system with one valence

nucleon is considered. The extension to n valence nucleons is straightforward, as discussed later. In the present case, there are two degrees of freedom: an impurity and a valence nucleon. To describe this system by the fields $d_{N\uparrow}$ and $d_{N\downarrow}$ having the minimum energy E_N, let us consider the ground state,

$$|\psi_0\rangle = d_{N\uparrow}^\dagger d_{N\downarrow}^\dagger |0\rangle,\qquad(63)$$

as the most stable state. Performing the variation for

$$E_{\mathrm{MF}}(\lambda, \Delta) = \langle\psi_0|H_{\mathrm{MF}}|\psi_0\rangle = 2E_N + \frac{|\Delta|^2}{g} - \lambda,\qquad(64)$$

with respect to λ and Δ,

$$\frac{\partial}{\partial\lambda}E_{\mathrm{MF}} = 0, \quad \frac{\partial}{\partial\Delta}E_{\mathrm{MF}} = 0,\qquad(65)$$

I obtain the values of λ and Δ

$$\lambda = \epsilon, \quad \Delta = \sqrt{N}g.\qquad(66)$$

The ground-state energy for the mean-field Hamiltonian H_{MF} is

$$E_{\mathrm{MF}}(\epsilon, \sqrt{N}g) = \epsilon - Ng.\qquad(67)$$

Because I need to consider the energy shift $\langle\psi_0|H_{\mathrm{fluc}}|\psi_0\rangle = 0$ by the fluctuation part H_{fluc}, I finally obtain the ground-state energy for the original Hamiltonian (1),

$$E_{\mathrm{MF+shift}} = \langle\psi_0|H_{\mathrm{MF}}|\psi_0\rangle + \langle\psi_0|H_{\mathrm{fluc}}|\psi_0\rangle$$
$$= E_{\mathrm{MF}}(\epsilon, \sqrt{N}g) + 0 = \epsilon - Ng,\qquad(68)$$

in the mean-field approximation. The binding energy $-Ng$ is different by about 33%, in contrast to the exact value $E_{\mathrm{exact}} = \epsilon - 3Ng/2$ in Sec. II B. This difference originates from the limit of the mean-field approximation. I expect that the correction by the fluctuation, which is not included in the mean-field approximation, makes it possible to get the value close to the exact solution. In the next section, the energy correction by RPA is discussed. The result in the case whare the fluctuation is completely included is discussed in the Appendix.

There is a comment on the obtained wave function $|\psi_0\rangle$. Representing $|\psi_0\rangle$ by the original fields $\{c_{k\sigma}, f_\sigma\}$, I find that $|\psi_0\rangle$ is a superposition of multiple number of impurities, i.e., $\sum f_\sigma^\dagger f_\sigma = 0, 1, 2$. However, it should be remembered that only one impurity is allowed to exist, owing to the condition (41). In fact, I confirm that this is satisfied as the average by

$$\langle\psi_0|\sum f_\sigma^\dagger f_\sigma|\psi_0\rangle = 1,\qquad(69)$$

in the present mean-field approximation [43–45]. Note also that the ground state $|\psi_0\rangle$ is a state superposed coherently by many states of valence nucleon $k = 1, \ldots, N$.

There is also a comment about the gap function (44). In the mean-field approximation, I introduced the new fields $\{d_{k\sigma}\}$ and considered the single-particle state for them. In this basis, the gap function gives the strength of the binding energy in the system. However, in the original fields $\{c_{k\sigma}, f_\sigma\}$, the gap function gives the strength of the state mixing between the valence nucleon $(c_{k\sigma})$ and the impurity (f_σ), as seen in the matrix (51) (see also Refs. [43–45]). Although the gap function gives the different physical meaning (the binding

energy or the strength of the state mixing) according to the difference of the basis fields, they give essentially the same result.

2. Fluctuation effect: RPA

The mean-field approximation does not include the fluctuation effect. In this section, I investigate the fluctuation effect based on RPA [46,47] (see also Refs. [48,49] for application to the Hartree-Fock states and the BCS states in atomic nuclei). The Hamiltonian (1) is rewritten in terms of $\{d_{k\sigma}\}$ instead of $\{c_{k\sigma}, f_\sigma\}$ as

$$
H = \left(\epsilon - \frac{3}{4}Ng\right)
$$
$$
\times \{(a_{0\uparrow}^\dagger a_{0\uparrow} + a_{0\downarrow}^\dagger a_{0\downarrow}) + (a_{1\uparrow}^\dagger a_{1\uparrow} + a_{1\downarrow}^\dagger a_{1\downarrow})\}
$$
$$
+ \frac{1}{4}Ng\{(a_{0\uparrow}^\dagger a_{1\uparrow} + a_{0\downarrow}^\dagger a_{1\downarrow}) + (a_{1\uparrow}^\dagger a_{0\uparrow} + a_{1\downarrow}^\dagger a_{0\downarrow})\}
$$
$$
+ \frac{1}{2}Ng(a_{0\uparrow}^\dagger a_{0\downarrow}^\dagger - a_{1\uparrow}^\dagger a_{1\downarrow}^\dagger)(a_{0\uparrow}a_{0\downarrow} - a_{1\uparrow}a_{1\downarrow})
$$
$$
+ (-1)Ng(a_{0\uparrow}^\dagger a_{1\uparrow}^\dagger a_{0\uparrow}a_{1\uparrow} + a_{0\downarrow}^\dagger a_{1\downarrow}^\dagger a_{0\downarrow}a_{1\downarrow})
$$
$$
+ \left(-\frac{1}{2}\right)Ng(a_{0\uparrow}^\dagger a_{1\downarrow}^\dagger + a_{0\downarrow}^\dagger a_{1\uparrow}^\dagger)(a_{0\uparrow}a_{1\downarrow} + a_{0\downarrow}a_{1\uparrow})
$$
$$
+ (-\epsilon + Ng) + \sum_{k=1}^{N-1} E_k d_{k\sigma}^\dagger d_{k\sigma}, \tag{70}
$$

where $a_{0\sigma} = d_{N\sigma}$ and $a_{1\sigma} = d_{N+1\sigma}$ are defined for short notation. Now let us consider the RPA correlation energy by using the ground state $|\psi_0\rangle = a_{0\uparrow}^\dagger a_{0\downarrow}^\dagger |0\rangle$ in the mean-field approximation.

First of all, I calculate energy eigenvalues of the RPA modes. Noting that the fluctuation exists near the ground state $|\psi_0\rangle = a_{0\uparrow}^\dagger a_{0\downarrow}^\dagger |0\rangle$, I solve the RPA equation

$$
\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \Omega_\nu \begin{pmatrix} X \\ Y \end{pmatrix}, \tag{71}
$$

with

$$
A_{\mu\nu\rho\sigma} = \langle\psi_0|[a_{0\nu}^\dagger a_{1\mu}, [H, a_{1\rho}^\dagger a_{0\sigma}]]|\psi_0\rangle,
$$
$$
= \frac{1}{2}Ng\,\delta_{\mu\rho}\delta_{\nu\sigma}
$$
$$
+ Ng(\delta_{\mu\uparrow}\delta_{\nu\downarrow}\delta_{\rho\uparrow}\delta_{\sigma\downarrow} + \delta_{\mu\downarrow}\delta_{\nu\uparrow}\delta_{\rho\downarrow}\delta_{\sigma\uparrow})
$$
$$
+ \frac{1}{2}Ng(\delta_{\mu\uparrow}\delta_{\nu\uparrow} - \delta_{\mu\downarrow}\delta_{\nu\downarrow})(\delta_{\rho\uparrow}\delta_{\sigma\uparrow} - \delta_{\rho\downarrow}\delta_{\sigma\downarrow}), \tag{72}
$$

and

$$
-B_{\mu\nu\rho\sigma} = \langle\psi_0|[a_{0\nu}^\dagger a_{1\mu}, [H, a_{0\sigma}^\dagger a_{1\rho}]]|\psi_0\rangle
$$
$$
= \frac{1}{2}Ng(\delta_{\mu\uparrow}\delta_{\rho\downarrow} - \delta_{\mu\downarrow}\delta_{\rho\uparrow})(\delta_{\nu\uparrow}\delta_{\sigma\downarrow} - \delta_{\nu\downarrow}\delta_{\sigma\uparrow}) \tag{73}
$$

and obtain the RPA energy eigenvalues

$$
\{\Omega_\nu\} = \{\Omega_{\pm 1}, \Omega_{\pm 2}, \Omega_{\pm 3}, \Omega_0\}
$$
$$
= \{\pm\sqrt{2}Ng, \pm\sqrt{2}Ng, \pm\sqrt{2}Ng, 0\}. \tag{74}
$$

The zero-energy mode with $\Omega_0 = 0$ is attributable to the energy degeneracy of the first term in the Hamiltonian (70) for $|\psi_0\rangle = a_{0\uparrow}^\dagger a_{0\downarrow}^\dagger |0\rangle$ and $|\psi_1\rangle = a_{1\uparrow}^\dagger a_{1\downarrow}^\dagger |0\rangle$. This degeneracy is special in

the mean-field approximation and hence should be regarded as the spurious one. Indeed, it is expected that such degeneracy will be resolved when higher-order fluctuations are included in the Appendix.

From the above result, I obtain the RPA correlation energy [46–49]

$$
\Delta E_{\text{RPA}} = \frac{1}{2}\sum_{\nu>0}\Omega_\nu - \frac{1}{2}\text{Tr}A
$$
$$
= \frac{1}{2}(3\sqrt{2} - 5)Ng \simeq -0.378Ng \tag{75}
$$

as the energy difference between the mean-field state and the fluctuating state. Thus, the RPA correlation energy gives the correction to the ground-state energy in the mean-field approximation. Therefore, the ground-state energy in the mean-field approximation and the RPA is

$$
E_{\text{MF+shift+RPA}} = E_{\text{MF+shift}} + \Delta E_{\text{RPA}}
$$
$$
= \epsilon - \frac{1}{2}(7 - 3\sqrt{2})Ng \simeq \epsilon - 1.378Ng. \tag{76}
$$

This is the result for one valence nucleon. For n valence nucleons ($n \leqslant 2N$), one nucleon participates in the binding as described above and the other $n - 1$ valence nucleons do not [see Eq. (54)]. Therefore, the energy becomes

$$
E_{\text{MF+shift+RPA}}(n) \simeq n\epsilon - 1.378Ng. \tag{77}
$$

The binding energy $-1.378Ng$ is about 92% of the exact solution $-3Ng/2$ in Sec. II B. Thus, by including the fluctuation in the RPA, I get the energy close to the exact solution. It is expected that a closer value can be obtained when higher-order fluctuations are taken into account. In fact, I can diagonalize completely the Hamiltonian (70), owing to its simplicity, and obtain the ground-state energy, which is precisely the same as the exact solution presented in the Appendix.

3. Correspondence between exact solution and mean-field+RPA solution

Let us see the correspondence between the mean-field+RPA solution and the exact solution (Tables I–III; $g > 0$). Concerning the ground state, I find that the former reproduces the latter in a good approximation.

Let us consider the $n = 1$ case. For N single-particle states of valence nucleon, there is one single-particle state which is coupled to impurity (coupling orbital) and $N - 1$ single-particle states which are not coupled (noncoupling orbital). In the ground state, one valence nucleon occupies the coupling orbital and forms the isosinglet state as combined to the impurity isospin as the most stable state. Therefore, the number of degeneracy factor is 1. This corresponds to the ground state of $I = 0$, 1 with energy $\epsilon - 3Ng/2$ in Table I.

Let us consider the $n = 2$ case. In this case, one of the two valence nucleons occupies the coupling orbital and forms the isosinglet state combined with the impurity isospin. Another valence nucleon occupies one of the $N - 1$ noncoupling orbitals. Because the first valence nucleon forms the isosinglet state with the impurity, the addition of the second valence nucleon gives isodoublet state. The number of degeneracy factor is $N - 1$. This is the same as the number of degeneracy

factor in the $I = 1/2$ ground state with energy $2\epsilon - 3Ng/2$ in Table II.

Let us consider the $n = 3$ case. In this case, one valence nucleon occupies the coupling orbital and forms the isosinglet state combined with the impurity isospin. The other two valence nucleons occupy one or two of the $N - 1$ noncoupling orbitals and form the isosinglet or isotriplet state. For the isosinglet state, the number of degeneracy factor is $N(N - 1)/2$, because the second two valence nucleons can occupy the same single-particle states ($N - 1$ patterns) or can occupy the different single-particle states [$(N - 1)(N - 2)/2$ patterns]. For the isotriplet state, the number of degeneracy factor is $(N - 1)(N - 2)/2$, because the two valence nucleons should occupy the different single-particle states [$(N - 1)(N - 2)/2$ patterns]. I confirm that those numbers of the degeneracy factor are consistent with those in the ground state of $I = 0, 1$ with energy $3\epsilon - 3Ng/2$ in Table III.

C. Isotriplet condensate ($g < 0$)

As mentioned previously, there is no stable isosinglet condensate for $g < 0$. In this case, I need to consider the isotriplet condensate.

1. Mean-field approximation

I rewrite the Hamiltonian (1) as

$$H = \sum \epsilon_k c_{k\sigma}^\dagger c_{k\sigma}$$
$$+ g \sum f_\sigma^\dagger (\sigma^i)_{\sigma\rho} c_{k\rho} c_{k'\sigma'}^\dagger (\sigma^i)_{\sigma'\rho'} f_{\rho'}$$
$$+ \frac{3}{2} g \sum c_{k'\sigma}^\dagger c_{k\sigma} - 3Ng, \tag{78}$$

by using the identity

$$\sum \{c_{k'\downarrow}^\dagger c_{k\uparrow} T_+ + c_{k'\uparrow}^\dagger c_{k\downarrow} T_- + (c_{k'\uparrow}^\dagger c_{k\uparrow} - c_{k'\downarrow}^\dagger c_{k\downarrow}) T_3\}$$
$$= \sum f_\sigma^\dagger (\sigma^i)_{\sigma\rho} c_{k\rho} c_{k'\sigma'}^\dagger (\sigma^i)_{\sigma'\rho'} f_{\rho'} + \frac{3}{2} \sum c_{k'\sigma}^\dagger c_{k\sigma}$$
$$- 3N, \tag{79}$$

where the constraint condition (41) is used. Defining the isotriplet "gap" function

$$\Delta^i = g \sum \langle f_\alpha^\dagger (\sigma^i)_{\alpha\beta} c_{k\beta} \rangle, \tag{80}$$

I separate the Hamiltonian (1) into the mean-field part H_{MF}' and the fluctuation part H_{fluc}'

$$H = H_{\text{MF}}' + H_{\text{fluc}}', \tag{81}$$

with

$$H_{\text{MF}}' = \sum \epsilon_k c_{k\sigma}^\dagger c_{k\sigma}$$
$$+ \sum [\Delta^i c_{k\sigma}^\dagger (\sigma_{\sigma\rho}^i) f_\rho + \Delta^{i*} f_\sigma^\dagger (\sigma_{\sigma\rho}^i) c_{k\rho}]$$
$$- \frac{1}{g} \sum |\Delta^i|^2, \tag{82}$$

and

$$H_{\text{fluc}}' = g \sum f_\sigma^\dagger (\sigma^i)_{\sigma\rho} c_{k\rho} c_{k'\sigma'}^\dagger (\sigma^i)_{\sigma'\rho'} f_{\rho'}$$
$$- \sum [\Delta^i c_{k\sigma}^\dagger (\sigma_{\sigma\rho}^i) f_\rho + \Delta^{i*} f_\sigma^\dagger (\sigma_{\sigma\rho}^i) c_{k\rho}]$$
$$+ \frac{1}{g} \sum |\Delta^i|^2 + \frac{3}{2} \sum c_{k'\sigma}^\dagger c_{k\sigma} - 3Ng. \tag{83}$$

Note that Δ^i is given by the matrix form

$$\Delta^i (\sigma^i)_{\alpha\beta} = \begin{pmatrix} \Delta^3 & \Delta^1 - i\Delta^2 \\ \Delta^1 + i\Delta^2 & -\Delta^3 \end{pmatrix}_{\alpha\beta}. \tag{84}$$

In the mean-field approximation, let us consider only the mean-field part H_{MF}' and neglect the fluctuation part H_{fluc}', as performed in the isosinglet condensate in Sec. III B. I diagonalizes H_{MF}' and introduce the Slater determinant by single-particle states. Then I perform the variation for the expectation value $\langle H_{\text{MF}}' \rangle$ with respect to λ and Δ^i as

$$\frac{\partial}{\partial \lambda} \langle H_{\text{MF}}' \rangle = 0, \tag{85}$$

$$\frac{\partial}{\partial \Delta^i} \langle H_{\text{MF}}' \rangle = 0, \tag{86}$$

and obtain λ and Δ^i. The ground-state energy is given by substituting the obtained λ and Δ^i into $\langle H_{\text{MF}}' \rangle$.

In the following, to demonstrate the mean-field calculation explicitly, I set $\epsilon_k = \epsilon$ as a simple case, where the diagonalization of H_{MF}' can be performed analytically. Such simplification does not change the essence of the discussion. By using $\{c_{k\sigma}, f_\sigma\}$ ($k = 1, \ldots, N$, $\sigma = \uparrow, \downarrow$), I write the mean-field Hamiltonian H_{MF}', with the $2(N + 1) \times 2(N + 1)$ matrix $\mathcal{H}_{cf}'$,

$$\mathcal{H}_{cf}' = \begin{pmatrix} \epsilon & 0 & \cdots & \Delta^{3*} & 0 & 0 & \cdots & \Delta^{1*} - i\Delta^{2*} \\ 0 & \epsilon & \cdots & \Delta^{3*} & 0 & 0 & \cdots & \Delta^{1*} - i\Delta^{2*} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \Delta^3 & \Delta^3 & \cdots & \lambda & \Delta^{1*} - i\Delta^{2*} & \Delta^{1*} - i\Delta^{2*} & \cdots & 0 \\ 0 & 0 & \cdots & \Delta^1 + i\Delta^2 & \epsilon & 0 & \cdots & -\Delta^{3*} \\ 0 & 0 & \cdots & \Delta^1 + i\Delta^2 & 0 & \epsilon & \cdots & -\Delta^{3*} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \Delta^1 + i\Delta^2 & \Delta^1 + i\Delta^2 & \cdots & 0 & -\Delta^3 & -\Delta^3 & \cdots & \lambda \end{pmatrix}, \tag{87}$$

as

$$H'_{\mathrm{MF}} = \psi^\dagger \mathcal{H}'_{cf} \psi - \frac{1}{g} \sum |\Delta^i|^2 - \lambda, \tag{88}$$

with ψ in Eq. (53). It is worth noting that $g < 0$ should be satisfied for the triplet condensate because the positivity of $-\sum |\Delta^i|^2/g$ supports the stability of the ground state in H'_{MF}. I diagonalize $\mathcal{H}'_{cf}$ analytically as

$$\mathcal{H}_{cf}'^{\mathrm{diag}} = \mathrm{diag}\big[\epsilon, \ldots, \epsilon, \tfrac{1}{2}(\epsilon + \lambda - D'), \tfrac{1}{2}(\epsilon + \lambda + D'),$$
$$\epsilon, \ldots, \epsilon, \tfrac{1}{2}(\epsilon + \lambda - D'), \tfrac{1}{2}(\epsilon + \lambda + D')\big]$$
$$= \mathrm{diag}(E'_1, \ldots, E'_{N-1}, E'_N, E'_{N+1}, E'_1, \ldots, E'_{N-1},$$
$$E'_N, E'_{N+1}), \tag{89}$$

with

$$D' = \sqrt{(\epsilon - \lambda)^2 + 4N \sum |\Delta^i|^2}. \tag{90}$$

Then I obtain the mean-field Hamiltonian H'_{MF}, as discussed in Sec. III B.

Let us consider the system with one valence nucleon. The extension to n valence nucleons is straightforward, as discussed later. Let us consider the isospin $\uparrow$, $\downarrow$ states with energy E'_N as the most stable state. For example, for the case of $\Delta^1 = \Delta^2 = 0$, $\Delta^3 \neq 0$, I define

$$a_{0\uparrow} = \frac{1}{\sqrt{2N}} \sqrt{1 - \frac{\epsilon - \lambda}{D'}} (c_{1\uparrow} + \cdots + c_{N\uparrow})$$
$$- \frac{1}{\sqrt{2}} \sqrt{1 + \frac{\epsilon - \lambda}{D'}} f_\uparrow, \tag{91}$$

$$a_{1\uparrow} = \frac{1}{\sqrt{2N}} \sqrt{1 + \frac{\epsilon - \lambda}{D'}} (c_{1\uparrow} + \cdots + c_{N\uparrow})$$
$$+ \frac{1}{\sqrt{2}} \sqrt{1 - \frac{\epsilon - \lambda}{D'}} f_\uparrow, \tag{92}$$

$$a_{0\downarrow} = \frac{1}{\sqrt{2N}} \sqrt{1 - \frac{\epsilon - \lambda}{D'}} (c_{1\downarrow} + \cdots + c_{N\downarrow})$$
$$+ \frac{1}{\sqrt{2}} \sqrt{1 + \frac{\epsilon - \lambda}{D'}} f_\downarrow, \tag{93}$$

$$a_{1\downarrow} = \frac{1}{\sqrt{2N}} \sqrt{1 + \frac{\epsilon - \lambda}{D'}} (c_{1\downarrow} + \cdots + c_{N\downarrow})$$
$$- \frac{1}{\sqrt{2}} \sqrt{1 - \frac{\epsilon - \lambda}{D'}} f_\downarrow. \tag{94}$$

The ground state is given as $|\psi'_0\rangle = a_{0\uparrow}^\dagger a_{0\downarrow}^\dagger |0\rangle$. In general cases, the mean-field energy $E'_{\mathrm{MF}}(\lambda, \{\Delta^i\}) = \langle H'_{\mathrm{MF}} \rangle$ is represented as

$$E'_{\mathrm{MF}}(\lambda, \{\Delta^i\}) = 2E'_N - \frac{1}{g} \sum |\Delta^i|^2 - \lambda. \tag{95}$$

Performing the variation for $E'_{\mathrm{MF}}(\lambda, \{\Delta^i\})$ with respect to λ and Δ^i,

$$\frac{\partial}{\partial \lambda} E'_{\mathrm{MF}} = 0, \quad \frac{\partial}{\partial \Delta^i} E'_{\mathrm{MF}} = 0, \tag{96}$$

I obtain

$$\lambda = \epsilon, \quad \sqrt{\sum |\Delta^i|^2} = -\sqrt{N} g. \tag{97}$$

Here Δ^i is parametrized by angles θ, φ as

$$\Delta^1 = \Delta_0 \sin\theta \cos\varphi, \tag{98}$$
$$\Delta^2 = \Delta_0 \sin\theta \sin\varphi, \tag{99}$$
$$\Delta^3 = \Delta_0 \cos\theta, \tag{100}$$

with $\Delta_0 = -\sqrt{N} g$. The ground-state energy E'_{MF} in the mean-field approximation is

$$E'_{\mathrm{MF}}(\epsilon, \Delta_0, \theta, \varphi) = \epsilon + Ng. \tag{101}$$

Note that there is degeneracy for changing the angle parameter (θ, φ). Therefore, the isospin symmetry $\mathrm{SU}(2) \simeq \mathrm{SO}(3)$ is broken to the $\mathrm{U}(1)$ symmetry in the ground state with the isotriplet condensate. For example, in the case of $\Delta^1 = \Delta^2 = 0$, $\Delta^3 \neq 0$, the ground state has the $\mathrm{U}(1)$ symmetry in which the element is given by $e^{\alpha T_3} \in \mathrm{U}(1)$ for generator T_3 with α a real-number parameter. Note that there is no symmetry breaking of isospin in isosinglet condensate in Sec. III B.

The ground-state energy is given by including the energy shift $\langle \psi'_0 | H_{\mathrm{fluc}} | \psi'_0 \rangle = -Ng$ as

$$E'_{\mathrm{MF+shift}} = \langle \psi'_0 | H_{\mathrm{MF}} | \psi'_0 \rangle + \langle \psi'_0 | H_{\mathrm{fluc}} | \psi'_0 \rangle$$
$$= (\epsilon + Ng) + (-Ng) = \epsilon, \tag{102}$$

which is quite different from the exact value $\epsilon + Ng/2$ in Sec. II B. Therefore, the mean-field approach does not give the good approximation for the isotriplet condensate with $g < 0$. The correction is given by RPA, as will be shown.

There is a comment. In the mean-field approximation, I obtain no bound state as shown in Eq. (102) for $g < 0$. It may be worthwhile to compare this result with the behavior of the effective coupling strength of the Kondo interaction in the infrared limit [50,51] (see also Refs. [6,7]). It is known that the effective coupling for $g < 0$ becomes zero in the low-energy limit in the renormalization-group method, when the coupling strength is renormalized by including the loop effect dressed by particle-hole pairs near the Fermi surface. This means that the interaction for $g < 0$ vanishes in the low-energy limit and that no bound state is formed. In the case of $g > 0$, however, the effective coupling strength becomes large in the low-energy limit, leading to the formation of the bound state. This is consistent with the existence of the bound state in the mean-field approximation for $g > 0$, as shown in Eq. (68). In the literature, the dependence of the existence and/or nonexistence of the bound state on the sign of the coupling constant in the Kondo interaction was presented in Ref. [52]. It is interesting to see that the present analysis in the mean-field approximation gives the same result with the known results.

2. Fluctuation effect: RPA

Let us consider the fluctuation effect by RPA. In the following, let us consider the case of $\Delta^1 = \Delta^2 = 0$, $\Delta^3 = -\sqrt{N} g$ ($\theta = 0$). The other case can be discussed similarly.

The Hamiltonian (1) is rewritten as

$$
\begin{aligned}
H = {}& \left(\epsilon + \frac{7}{4}Ng\right)(a_{0\uparrow}^\dagger a_{0\uparrow} + a_{0\downarrow}^\dagger a_{0\downarrow}) \\
& + \left(\epsilon + \frac{7}{4}Ng\right)(a_{1\uparrow}^\dagger a_{1\uparrow} + a_{1\downarrow}^\dagger a_{1\downarrow}) \\
& + Ng(a_{0\uparrow}^\dagger a_{1\uparrow}^\dagger a_{0\uparrow} a_{1\uparrow} + a_{0\downarrow}^\dagger a_{1\downarrow}^\dagger a_{0\downarrow} a_{1\downarrow}) \\
& + \frac{1}{2}Ng(a_{0\uparrow}^\dagger - a_{1\uparrow}^\dagger)(a_{0\downarrow}^\dagger + a_{1\downarrow}^\dagger)(a_{0\uparrow} - a_{1\uparrow}) \\
& \times (a_{0\downarrow} + a_{1\downarrow}) \\
& + \frac{1}{2}Ng(a_{0\uparrow}^\dagger + a_{1\uparrow}^\dagger)(a_{0\downarrow}^\dagger - a_{1\downarrow}^\dagger)(a_{0\uparrow} + a_{1\uparrow}) \\
& \times (a_{0\downarrow} - a_{1\downarrow}) \\
& + \left(-\frac{1}{4}\right)Ng(a_{0\uparrow}^\dagger - a_{1\uparrow}^\dagger)(a_{0\downarrow}^\dagger + a_{1\downarrow}^\dagger)(a_{0\uparrow} + a_{1\uparrow}) \\
& \times (a_{0\downarrow} - a_{1\downarrow}) \\
& + \left(-\frac{1}{4}\right)Ng(a_{0\uparrow}^\dagger + a_{1\uparrow}^\dagger)(a_{0\downarrow}^\dagger - a_{1\downarrow}^\dagger)(a_{0\uparrow} - a_{1\uparrow}) \\
& \times (a_{0\downarrow} + a_{1\downarrow}) \\
& + \left(-\frac{1}{4}\right)Ng(a_{0\uparrow}^\dagger a_{1\uparrow} + a_{0\downarrow}^\dagger a_{1\downarrow} + a_{1\uparrow}^\dagger a_{0\uparrow} + a_{1\downarrow}^\dagger a_{0\downarrow}) \\
& + (-\epsilon - 3Ng) + \sum_{k=1}^{N-1} \epsilon d_{k\sigma}^\dagger d_{k\sigma}. \qquad (103)
\end{aligned}
$$

Considering the fluctuation around the ground state $|\psi_0'\rangle$, I solve the RPA equation

$$
\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = \Omega_\nu \begin{pmatrix} X \\ Y \end{pmatrix}, \qquad (104)
$$

with

$$
\begin{aligned}
A_{\mu\nu\rho\sigma} = {}& \langle \psi_0' | [a_{0\nu}^\dagger a_{1\mu}, [H, a_{1\rho}^\dagger a_{0\sigma}]] | \psi_0' \rangle \\
= {}& \tfrac{3}{2}Ng(\delta_{\mu\uparrow}\delta_{\nu\uparrow}\delta_{\rho\downarrow}\delta_{\sigma\downarrow} + \delta_{\mu\downarrow}\delta_{\nu\downarrow}\delta_{\rho\uparrow}\delta_{\sigma\uparrow}) \\
& + (-1)Ng(\delta_{\mu\uparrow}\delta_{\nu\uparrow}\delta_{\rho\uparrow}\delta_{\sigma\uparrow} + \delta_{\mu\downarrow}\delta_{\nu\downarrow}\delta_{\rho\downarrow}\delta_{\sigma\downarrow}) \\
& + (-\tfrac{1}{2})Ng(\delta_{\mu\uparrow}\delta_{\nu\downarrow}\delta_{\rho\uparrow}\delta_{\sigma\downarrow} + \delta_{\mu\downarrow}\delta_{\nu\uparrow}\delta_{\rho\downarrow}\delta_{\sigma\uparrow}), \quad (105)
\end{aligned}
$$

and

$$
\begin{aligned}
-B_{\mu\nu\rho\sigma} = {}& \langle \psi_0' | [a_{0\nu}^\dagger a_{1\mu}, [H, a_{0\sigma}^\dagger a_{1\rho}]] | \psi_0' \rangle \\
= {}& \left(-\tfrac{1}{2}\right)Ng(\delta_{\mu\uparrow}\delta_{\nu\uparrow}\delta_{\rho\downarrow}\delta_{\sigma\downarrow} - \delta_{\mu\uparrow}\delta_{\nu\downarrow}\delta_{\rho\downarrow}\delta_{\sigma\uparrow} \\
& - \delta_{\mu\downarrow}\delta_{\nu\uparrow}\delta_{\rho\uparrow}\delta_{\sigma\downarrow} + \delta_{\mu\downarrow}\delta_{\nu\downarrow}\delta_{\rho\uparrow}\delta_{\sigma\uparrow}). \quad (106)
\end{aligned}
$$

The RPA energy eigenvalues are obtained as

$$
\{\Omega_\nu\} = \{\Omega_{\pm 1}, \Omega_0, \Omega_0, \Omega_0\} = \{\mp\sqrt{6}Ng, 0, 0, 0\}. \quad (107)
$$

Note the ordering of signs in $\Omega_{\pm 1} = \mp Ng$ because of $g < 0$. One of three zero-energy modes ($\Omega_0 = 0$) is attributable to the energy degeneracy of $|\psi_0'\rangle = a_{0\uparrow}^\dagger a_{0\downarrow}^\dagger |0\rangle$ and $|\psi_1'\rangle = a_{1\uparrow}^\dagger a_{1\downarrow}^\dagger |0\rangle$ in the first two terms of the Hamiltonian (103). However, this degeneracy is special in the mean-field approximation and

hence should be a spurious one. The other two are the Nambu-Goldstone modes in the coset space SU(2)/U(1), because the isospin symmetry in the ground state is broken from SU(2) to U(1), where, for example, the U(1) symmetry is given by $e^{\alpha T_3} \in U(1)$ in the case of $\Delta^1 = \Delta^2 = 0$, $\Delta^3 \neq 0$.

From the above result, I obtain the RPA correlation energy [46–49],

$$
\begin{aligned}
\Delta E_{\mathrm{RPA}}' &= \frac{1}{2}\sum_{\nu > 0} \Omega_\nu - \frac{1}{2}\mathrm{Tr}A \\
&= \frac{1}{2}(3 - \sqrt{6})Ng \simeq 0.275Ng. \quad (108)
\end{aligned}
$$

Therefore, the ground-state energy in the mean-field approximation and the RPA is given by

$$
\begin{aligned}
E_{\mathrm{MF+shift+RPA}}' &= E_{\mathrm{MF+shift}}' + \Delta E_{\mathrm{RPA}}' \\
&= \epsilon + \tfrac{1}{2}(3 - \sqrt{6})Ng \simeq \epsilon + 0.275Ng. \quad (109)
\end{aligned}
$$

When there are n valence nucleons ($n \leqslant 2N$), one valence nucleon participates in coupling to the impurity and the other $n - 1$ valence nucleons do not. Therefore, the ground-state energy is modified to

$$
E_{\mathrm{MF+shift+RPA}}(n) \simeq n\epsilon + 0.275Ng. \quad (110)
$$

The obtained binding energy is about 55% to the exact one ($Ng/2$). Thus, the fluctuation by RPA cannot be neglected to obtain the ground-state energy in the mean-field approximation.

For both isosinglet and isotriplet condensates, one notes that the binding energy obtained by the mean-field and the RPA is proportional to g. However, I should remark that this proportional relation is accidental by the simple setting of the present model. In fact, the calculation of the energy eigenvalues by the mean-field and RPA approach is the nonperturbative procedure, and hence should be essentially different from the perturbative one. This can be seen in the energy eigenvalues, $E_N^{(\prime)}$ and $E_{N+1}^{(\prime)}$, in Eqs. (54) and (89), because the dependence of the gap $|\Delta^{(i)}|$ in $E_N^{(\prime)}$ and $E_{N+1}^{(\prime)}$ does not have the perturbative form apparently. The accidental proportional relation would originate from the setting $\epsilon_k = \epsilon$ ($k = 1, \ldots, N$). If different values of ϵ_k are considered, it will be seen that the binding energy in the mean field and the RPA is not proportional to g and becomes a more complicated equation.

IV. DISCUSSION: COMPETITION BETWEEN KONDO EFFECT AND NUCLEON CORRELATIONS

So far, I have discussed the correlation between an impurity and a valence nucleon as the Kondo effect and assumed no correlation between valence nucleons. In realistic nuclei, however, there are several correlations in valence nucleons that are not necessarily negligible. In this section, I briefly consider two types of correlation in valence nucleons—the isospin symmetry breaking and the nucleon pairings—and discuss how the Kondo effect is affected by them.[6]

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[6]See, for example, Ref. [53] as a review for competition between the pairing and the Kondo effect in the condensed-matter systems.

A. Competition between Kondo effect and isospin breaking

Let us consider the isospin symmetry breaking in the valence nucleons. I set the valence nucleon energies $\epsilon_\uparrow$ and $\epsilon_\downarrow$ for $\uparrow$ and $\downarrow$ components of isospin, respectively, and modify the kinetic term of the valence nucleon (2),

$$H_0 \rightarrow H_0 = \sum \epsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma}, \qquad (111)$$

to include the isospin breaking in the single-particle states. In the following, I set $\epsilon_{k\sigma} = \epsilon_\sigma$ for simplicity. The calculation procedure in the mean-field approximation is essentially the same as discussed in Sec. III B. I introduce the isospin breaking in the mean-field Hamiltonian Eq. (61). Instead of the matrix (54), I define the generalized matrix with $2(N+1) \times 2(N+1)$ dimensions

$$\tilde{\mathcal{H}}_{cf} = \begin{pmatrix} \epsilon_\uparrow & 0 & \cdots & \Delta^* & 0 & 0 & \cdots & 0 \\ 0 & \epsilon_\uparrow & \cdots & \Delta^* & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \Delta & \Delta & \cdots & \lambda & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \epsilon_\downarrow & 0 & \cdots & \Delta^* \\ 0 & 0 & \cdots & 0 & 0 & \epsilon_\downarrow & \cdots & \Delta^* \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \Delta & \Delta & \cdots & \lambda \end{pmatrix}, \quad (112)$$

and perform the diagonalization as

$$\tilde{\mathcal{H}}_{cf}^{\text{diag}} = \text{diag}\Big[\epsilon_\uparrow, \ldots, \epsilon_\uparrow, \tfrac{1}{2}(\epsilon_\uparrow + \lambda - D_\uparrow), \tfrac{1}{2}(\epsilon_\uparrow + \lambda + D_\uparrow),$$
$$\epsilon_\downarrow, \ldots, \epsilon_\downarrow, \tfrac{1}{2}(\epsilon_\downarrow + \lambda - D_\downarrow), \tfrac{1}{2}(\epsilon_\downarrow + \lambda + D_\downarrow)\Big]$$
$$= \text{diag}(E_{1\uparrow}, \ldots, E_{N-1\uparrow}, E_{N\uparrow}, E_{N+1\uparrow}, E_{1\downarrow}, \ldots, E_{N-1\downarrow},$$
$$E_{N\downarrow}, E_{N+1\downarrow}), \qquad (113)$$

with

$$D_\sigma = \sqrt{(\epsilon_\sigma - \lambda)^2 + 4N|\Delta|^2}, \qquad (114)$$

for $\sigma = \uparrow, \downarrow$. Instead of the original fields $\{c_{k\sigma}, f_\sigma\}$, I define the new fields

$$\tilde{d}_{1\sigma} = \frac{1}{\sqrt{2}}(c_{1\sigma} - c_{2\sigma}),$$

$$\tilde{d}_{2\sigma} = \frac{1}{\sqrt{2}}(c_{1\sigma} - c_{3\sigma}),$$

$$\vdots$$

$$\tilde{d}_{N-1\sigma} = \frac{1}{\sqrt{2}}(c_{1\sigma} - c_{N\sigma}),$$

$$\tilde{d}_{N\sigma} = \frac{1}{\sqrt{2N}}\sqrt{1 - \frac{\epsilon_\sigma - \lambda}{D_\sigma}}(c_{1\sigma} + \cdots + c_{N\sigma})$$

$$- \frac{1}{\sqrt{2}}\sqrt{1 + \frac{\epsilon_\sigma - \lambda}{D_\sigma}} f_\sigma,$$

$$\tilde{d}_{N+1\sigma} = \frac{1}{\sqrt{2N}}\sqrt{1 - \frac{\epsilon_\sigma - \lambda}{D_\sigma}}(c_{1\sigma} + \cdots + c_{N\sigma})$$

$$+ \frac{1}{\sqrt{2}}\sqrt{1 + \frac{\epsilon_\sigma - \lambda}{D_\sigma}} f_\sigma,$$

and rewrite the mean-field Hamiltonian (61) as

$$\tilde{H}_{\text{MF}} = \phi^\dagger \tilde{\mathcal{H}}_{cf}^{\text{diag}} \phi + \frac{|\Delta|^2}{g} - \lambda$$

$$= \sum E_{k\sigma} \tilde{d}_{k\sigma}^\dagger \tilde{d}_{k\sigma} + \frac{|\Delta|^2}{g} - \lambda, \qquad (115)$$

with ϕ in Eq. (62). Supposing the energy of $\downarrow$ component is larger than that of $\uparrow$ component, I parametrize $\epsilon_\uparrow$ and $\epsilon_\downarrow$ by $\epsilon_\uparrow = \epsilon$ and $\epsilon_\downarrow = \epsilon + \delta\epsilon$ with $\delta\epsilon > 0$.

For the system composed of one valence nucleon and an impurity, let us consider the ground state given by

$$|\tilde{\psi}_0\rangle = \tilde{d}_{N\uparrow}^\dagger \tilde{d}_{N\downarrow}^\dagger |0\rangle. \qquad (116)$$

For the ground-state energy,

$$\tilde{E}_{\text{MF}} = \langle\tilde{\psi}_0|\tilde{H}_{\text{MF}}|\tilde{\psi}_0\rangle = E_{N\uparrow} + E_{N\downarrow} + \frac{|\Delta|^2}{g} - \lambda, \quad (117)$$

I perform the variation with respect to λ and Δ,

$$\frac{\partial}{\partial\lambda}\tilde{E}_{\text{MF}} = 0, \quad \frac{\partial}{\partial\Delta}\tilde{E}_{\text{MF}} = 0, \qquad (118)$$

and obtain λ and Δ as

$$\lambda = \epsilon + \frac{1}{2}\delta\epsilon, \quad \Delta = \sqrt{N}g\sqrt{1 - \frac{(\delta\epsilon)^2}{16N^2g^2}}. \qquad (119)$$

Then the ground-state energy is given as

$$\tilde{E}_{\text{MF}}\left[\epsilon + \frac{1}{2}\delta\epsilon, \sqrt{N}g\sqrt{1 - \frac{(\delta\epsilon)^2}{16N^2g^2}}\right]$$

$$= \epsilon - Ng + \frac{1}{2}\delta\epsilon - \frac{(\delta\epsilon)^2}{16Ng}. \qquad (120)$$

Note that the ground-state energy increases for nonzero $\delta\epsilon$ with $0 < \delta\epsilon \leqslant 4Ng$. At the special value $\delta\epsilon = 4Ng$, I get $\tilde{E}_{\text{MF}} = \epsilon$ and find no bound state. According to the change of the ground-state energy by $\delta\epsilon$, I find that the magnitude of the gap function $|\Delta|$ in Eq. (119) decreases and finally it becomes $|\Delta| = 0$ at $\delta\epsilon = 4Ng$, where the bound state disappears.

For $\delta\epsilon > 4Ng$, however, the ground state is the isosinglet or isotriplet state given by $\tilde{d}_{k\sigma}^\dagger$ ($k = 1, \ldots, N-1$) and $\tilde{d}_{N\sigma'}^\dagger$ with the number of degeneracy factor $N - 1$. The ground-state energy is $\tilde{E}_{\text{MF}} = \epsilon$, with $\lambda = \epsilon - Ng$ and $\Delta = 0$. The solution for $\delta\epsilon \leqslant 4Ng$ is connected continuously to the solution for $\delta\epsilon > 4Ng$.

B. Competition between Kondo effect and pairing of valence nucleons

Let us consider the pairing interaction in valence nucleons and discuss the competition between the Kondo effect and the

pairing effect. Let us consider two types of the pairing interaction: the isovector-type ($I = 1$) pairing and the isoscalar-type ($I = 0$) pairing.

Let us consider the isovector-type pairing interaction

$$
\begin{aligned}
H_{\text{pair}}^1 = -G_1 \sum_{i,j:\text{odd}} & \left\{ c_{i\uparrow}^\dagger c_{i+1\uparrow}^\dagger c_{j+1\uparrow} c_{j\uparrow} \right. \\
& + \frac{1}{2}(c_{i\uparrow}^\dagger c_{i+1\downarrow}^\dagger + c_{i\downarrow}^\dagger c_{i+1\uparrow}^\dagger)(c_{j+1\uparrow} c_{j\downarrow} + c_{j+1\downarrow} c_{j\uparrow}) \\
& \left. + c_{i\downarrow}^\dagger c_{i+1\downarrow}^\dagger c_{j+1\downarrow} c_{j\downarrow} \right\},
\end{aligned}
\tag{121}
$$

with the coupling constant $G_1 > 0$, where the pairing is given between the ith and $i + 1$th single-particle states, with i, j being odd numbers, as the simple pairing model. Because of the commutation relation

$$
\left[H_{\text{pair}}^1, H_{\text{K}}\right] \neq 0,
\tag{122}
$$

for the Kondo interaction H_{K} [Eq. (3)], I find that the bound state by the Kondo interaction is affected by the isovector-type pairing.

The situation is different for the isoscalar-type pairing. Let us consider the isoscalar-type pairing interaction

$$
\begin{aligned}
H_{\text{pair}}^0 = \frac{G_0}{2} \sum_{i,j:\text{odd}} & (c_{i\uparrow}^\dagger c_{i+1\downarrow}^\dagger - c_{i\downarrow}^\dagger c_{i+1\uparrow}^\dagger) \\
& \times (c_{j+1\uparrow} c_{j\downarrow} - c_{j+1\downarrow} c_{j\uparrow}),
\end{aligned}
\tag{123}
$$

with the coupling constant $G_0/2 > 0$, where the pairing is supplied again for the ith and $i + 1$th single-particle states, with i, j being odd numbers. In this case, the commutation relation with the Kondo interaction H_{K} [Eq. (3)] is given by

$$
\left[H_{\text{pair}}^0, H_{\text{K}}\right] = 0.
\tag{124}
$$

Therefore, I find that the bound state by the Kondo interaction does not suffer from the isoscalar-type pairing.

Let us consider the eigenstates of the Hamiltonian $H_0 + H_{\text{K}} + H_{\text{pair}}^{0,1}$. I represent $|\psi_{\text{K}}\rangle$ for the eigenstate of the Hamiltonian $H_0 + H_{\text{K}}$ [Eq. (1)] and $|\psi_{\text{pair}}^1\rangle$ for the eigenstate of $H_0 + H_{\text{pair}}^1$. I consider the case of $\epsilon_k = \epsilon$ in Eq. (2). Because H_0, H_{pair}^1, and H_{K} are noncommutative for each other [cf. Eq. (122)], the eigenstate for $H_0 + H_{\text{K}} + H_{\text{pair}}^1$ including both the Kondo effect and the isovector-type pairing is given by the sum of the tensor product of several states, $\sum_{i,j} \gamma_{ij} |\psi_{\text{K}i}\rangle \otimes |\psi_{\text{pair}j}^1\rangle$ with $|\psi_{\text{K}i}\rangle$ and $|\psi_{\text{pair}j}^1\rangle$ being the ith and jth states of $|\psi_{\text{K}}\rangle$ and $|\psi_{\text{pair}}^1\rangle$, respectively, and γ_{ij} being appropriate coefficients. Therefore, $|\psi_{\text{K}}\rangle$ and $|\psi_{\text{pair}}^1\rangle$ are entangled by the Hamiltonian $H_0 + H_{\text{K}} + H_{\text{pair}}^1$. However, in the case of the isoscalar-type pairing, because H_0, H_{pair}^1, and H_{K} are commutative [cf. Eq. (124)], the eigenstate of the Hamiltonian $H_0 + H_{\text{K}} + H_{\text{pair}}^0$ is given by a simple tensor product: $|\psi_{\text{K}}\rangle \otimes |\psi_{\text{pair}}^0\rangle$, where $|\psi_{\text{pair}}^0\rangle$ is the eigenstate of

$H_0 + H_{\text{pair}}^1$. Therefore, $|\psi_{\text{K}}\rangle$ and $|\psi_{\text{pair}}^0\rangle$ are disentangled by the Hamiltonian $H_0 + H_{\text{K}} + H_{\text{pair}}^0$.

There is a comment for the case that the interaction between an impurity and a valence nucleon has no isospin exchange, in contrast to the Kondo interaction (3). For the interaction with no isospin-exchange $H_{\text{NK}} = g' \sum c_{i\sigma}^\dagger c_{j\sigma}$ with the coupling constant g', I find that the commutations with the pairing interactions are given by $[H_{\text{pair}}^1, H_{\text{NK}}] \neq 0$ and $[H_{\text{pair}}^0, H_{\text{NK}}] \neq 0$. Therefore, the eigenstate of $H_0 + H_{\text{NK}}$, $|\psi_{\text{NK}}\rangle$, is affected both by the isoscalar-type pairing and by the isovector-type pairing, and hence $|\psi_{\text{NK}}\rangle$ becomes entangled with $|\psi_{\text{pair}}^{0,1}\rangle$ by the Hamiltonian $H_0 + H_{\text{NK}} + H_{\text{pair}}^{0,1}$.

The above properties of the entanglement are obtained as the exact solutions of the Hamiltonians. Because they should be maintained also for the approximate solutions in the mean-field approach, it is recommended to check the validity of the mean-field approximation. Further detailed analysis will be left for future studies.

V. CONCLUSION

The Kondo effect in charm and bottom nuclei with a $\bar{D}$, B meson bound as a heavy impurity is considered and the binding energy for the isospin-exchange interaction between an impurity and a valence nucleon is discussed. I consider the discrete energy levels of valence nucleons in the charm and bottom nuclei. By introducing the auxiliary fermion field for the isospin of the $\bar{D}$, B meson and extending the Fock space to include the multiple numbers of the impurity, I perform the mean-field approximation and the RPA calculation. Based on the simple model which is analytically solvable, it is presented that the approximate energy is comparable with the exact one, and it is found that the mean-field approach is valid as the ground state with the Kondo effect, when the fluctuation effect is included by the RPA. The approach by the mean-field approximation and the RPA is applicable to general cases, for example, where the interaction between an impurity and a valence nucleon is given by more realistic form and the structure of discrete energy levels for valence nucleons are more complex.

In the present discussion, I consider a $\bar{D}$, B meson ($I = 1/2$) as a heavy impurity in charm and bottom nuclei. I may also consider the case of a Σ_c, Σ_b baryon with isospin $I = 1$ [12,13]. In this case, the isospin-exchange interaction between a Σ_c, Σ_b baryon and a valence nucleon will induce the bound state between them. However, owing to the isospin $I = 1$ of the Σ_c, Σ_b baryon, this bound state has still a finite isospin, $I = 1/2$, and hence it can attract another nucleon to form the isosinglet bound state. It may form a three-body bound state composed of a Σ_c, Σ_b baryon and two valence nucleons. Because the two valence nucleons have isospin 1 as a subsystem, I expect that the properties of this three-body bound state can be affected by the isovector-type pairing for valence nucleons, rather than by the isoscalar-type pairing. In any case, to study the Kondo effect in charm and bottom nuclei in various situations will be important for both experimental and theoretical research.

APPENDIX A: INCLUDING COMPLETE FLUCTUATION EFFECT FOR ISOSINGLET CONDENSATE ($g > 0$)

I include the fluctuation effect completely beyond the RPA for the Hamiltonian (1) for $g > 0$ [see also Eq. (70)]. In the general form of the Hamiltonian, such a procedure is not necessarily always possible. However, the present simple model makes it possible to obtain the exact solution by complete diagonalization.

I suppose the ground-state wave function as

$$|\psi\rangle = c_{00} a_{0\uparrow}^\dagger a_{0\downarrow}^\dagger |0\rangle + c_{01}(a_{0\uparrow}^\dagger a_{1\downarrow}^\dagger - a_{0\downarrow}^\dagger a_{1\uparrow}^\dagger)|0\rangle$$
$$+ c_{11} a_{1\uparrow}^\dagger a_{1\downarrow}^\dagger |0\rangle, \tag{A1}$$

with three isosinglet bases $a_{0\uparrow}^\dagger a_{0\downarrow}^\dagger |0\rangle$, $(a_{0\uparrow}^\dagger a_{1\downarrow}^\dagger - a_{0\downarrow}^\dagger a_{1\uparrow}^\dagger)|0\rangle$, $a_{1\uparrow}^\dagger a_{1\downarrow}^\dagger |0\rangle$. I obtain the energy eigenvalue E and the coefficients c_{00}, c_{01}, c_{11} by solving the eigenvalue equation $H|\psi\rangle = E|\psi\rangle$:

$$\begin{pmatrix} \epsilon - Ng & \frac{1}{2}Ng & \frac{1}{2}Ng \\ \frac{1}{4}Ng & \epsilon - \frac{1}{2}Ng & \frac{1}{4}Ng \\ \frac{1}{2}Ng & \frac{1}{2}Ng & \epsilon - Ng \end{pmatrix} \begin{pmatrix} c_{00} \\ c_{01} \\ c_{11} \end{pmatrix} = E \begin{pmatrix} c_{00} \\ c_{01} \\ c_{11} \end{pmatrix}. \tag{A2}$$

The solutions are

$$E^{(0)} = \epsilon - \tfrac{3}{2}Ng, \tag{A3}$$

$$E^{(1)} = \epsilon - Ng, \tag{A4}$$

$$E^{(2)} = \epsilon, \tag{A5}$$

with the coefficients

$$\begin{pmatrix} c_{00}^{(0)} \\ c_{01}^{(0)} \\ c_{11}^{(0)} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \tag{A6}$$

$$\begin{pmatrix} c_{00}^{(1)} \\ c_{01}^{(1)} \\ c_{11}^{(1)} \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \tag{A7}$$

$$\begin{pmatrix} c_{00}^{(2)} \\ c_{01}^{(2)} \\ c_{11}^{(2)} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \tag{A8}$$

and the corresponding wave functions

$$|\psi^{(0)}\rangle = \frac{1}{\sqrt{2N}}[(c_{1\uparrow}^\dagger + \cdots + c_{N\uparrow}^\dagger)f_\downarrow^\dagger$$
$$- (c_{1\downarrow}^\dagger + \cdots + c_{N\downarrow}^\dagger)f_\uparrow^\dagger]|0\rangle, \tag{A9}$$

$$|\psi^{(1)}\rangle = f_\uparrow^\dagger f_\downarrow^\dagger |0\rangle, \tag{A10}$$

$$|\psi^{(2)}\rangle = \frac{1}{N}(c_{1\uparrow}^\dagger + \cdots + c_{N\uparrow}^\dagger)(c_{1\downarrow}^\dagger + \cdots + c_{N\downarrow}^\dagger)|0\rangle, \tag{A11}$$

for respective eigenvalues. Notes that, for n valence nucleons ($n \leqslant 2N$), one of n valence nucleons participates in the coupling to the impurity and the other $n - 1$ valence nucleons do not, as discussed in the text. Therefore, for n valence nucleons, the energies of the Hamiltonian (1) are

$$E^{(0)}(n) = n\epsilon - \tfrac{3}{2}Ng, \tag{A12}$$

$$E^{(1)}(n) = n\epsilon - Ng, \tag{A13}$$

$$E^{(2)}(n) = n\epsilon. \tag{A14}$$

Remember here that the Fock space is extended to multiple numbers of the impurity in the mean-field approach. In the three states $|\psi^{(0)}\rangle$, $|\psi^{(1)}\rangle$, and $|\psi^{(2)}\rangle$, only the ground state $|\psi^{(0)}\rangle$ gives the exact solution as discussed in Sec. II B. The other two excited states $|\psi^{(1)}\rangle$ and $|\psi^{(2)}\rangle$ are the spurious states, because the number of the auxiliary fermions by f_σ are 0 and 2, respectively, and hence they should be discarded as the real solution of the Hamiltonian (1).

The analogous discussion will be applied to the case of $g < 0$ with the isotriplet condensate.

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