

Effect of deformation on the calculated half-lives of cluster emission using a proximity potential

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The half-lives of deformed cluster emission have been calculated by using the proximity potential Christensen-Winther potential form (1976) and compared to the experimental data and theoretical results of the liquid drop model. Also the calculated results have been compared to the results obtained by a universal form of the proximity potential for spherical cluster emission. The results show reasonable agreement with the experimental data.

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I. INTRODUCTION

Cluster emission is one radioactivity behavior of heavy nuclei. This radioactivity is a cold process which is intermediate between α decay and spontaneous fission. Cluster emission was predicted by Sandulescu *et al.* [1] in 1980 and was first observed experimentally by Rose and Jones [2] in 1984. Studying cluster decays from various nuclei in the trans-lead region marked that the partial half-lives for this region have the values from 10^{11} up to 10^{30} s. This process, cluster emission, has been explained by several theoretical models, such as the α -like approach [3] and fissionlike approach [4–6]. Also for calculating half-lives, one can consider the interacting potential as the sum of Coulomb, nuclear, and centrifugal potentials and use the Wentzel-Kramers-Brillouin (WKB) approximation. Different theoretical formalisms, such as the double-folding formalism [7,8] and the liquid drop model (LDM) [9], can be used as the nuclear part of interacting potential. The other good candidate for describing the nuclear part of the interacting potential is the proximity potential model [10–16]. In this model the shape and geometry of two nuclei and their separation distances play important roles. In this article, we used the Christensen-Winther potential form (CW76) [17] as the nuclear part of the total potential. Different authors [13,15] considered spherical shapes for nuclei, but we considered deformed shapes for cluster and daughter nuclei. We calculated the half-lives of cluster decay and compared them to the experimental data, the results of the liquid drop model, and the results obtained from the proximity potential model for spherical nuclei.

In Sec. II, the interacting potential and the half-life of deformed nuclei are presented, and the results and discussion on the cluster emission are given in Sec. III. The conclusion of this paper is presented in Sec. IV.

II. POTENTIAL ENERGY AND HALF-LIFE

The total nucleus-nucleus potential energy between two nuclei is given by

$$V(R) = V_N(R) + V_C(R) + \frac{\ell(\ell + 1)\hbar^2}{2\mu R^2}, \quad (1)$$

where $V_N(R)$ and $V_C(R)$ are the nuclear and Coulomb potential energies, respectively. The last term is the centrifugal potential energy of the system. The nuclear part of the system can be calculated by different formalisms, such as the double-folding formalism [7,8,18] or the proximity forms. In recent years several versions of the proximity potentials and their modifications have been proposed [10,11,19]. In this paper, we gave our attention to the proximity potentials and used the CW76 [17] for our calculations because we got good results in our previous work by using this form of potential [16]. For the spherical nuclei this proximity potential is given by

$$V_N^{CW}(R) = -50 \frac{R_1 R_2}{R_1 + R_2} \varphi(R - R_1 - R_2) \text{MeV}, \quad (2)$$

where

$$R_i = 1.233 A_i^{1/3} - 0.978 A_i^{-1/3} \text{fm} \quad (i = 1, 2),$$

$$\varphi(S = R - R_1 - R_2) = \exp\left(-\frac{R - R_1 - R_2}{0.63}\right). \quad (3)$$

R_1 and R_2 are the radii of the cluster and daughter nuclei, respectively. For deformed, co-planar, and nonoverlapping nuclei (Fig. 1), the Coulomb potential energy for the system is given by [20]

$$V_C(R) = \frac{Z_1 Z_2 e^2}{R} + 3 Z_1 Z_2 e^2 \sum_{(\lambda, i=1,2)} \frac{1}{2\lambda + 1} \frac{R_i^\lambda(\alpha_i)}{R^{\lambda+1}} Y_\lambda^0(\theta_i) \times \left[\beta_{\lambda i} + \frac{4}{7} \beta_{\lambda i}^2 Y_\lambda^0(\theta_i) \right], \quad (4)$$

with

$$R_i(\alpha_i) = R_{0i} \left[1 + \sum_{(\lambda)} \beta_{\lambda i} Y_\lambda^{(0)}(\alpha_i) \right], \quad (5)$$

where $R_i(\alpha_i)$ ($i = 1, 2$) are the radii of the deformed cluster and daughter nuclei and R_{0i} is equal to R_i in Eq. (3). Also, for deformed nuclei, we used Eq. (5) in Eq. (2) for calculating the nuclear part of the nucleus-nucleus potential. The azimuthal angle between the principal planes of two nuclei φ is equal to zero for co-planar nuclei. The two orientation angles θ_i are shown in Fig. 1.

For the nonzero overlapping region, the power-law interpolation [14] is a good candidate for the case of deformed cluster and daughter nuclei. In this paper, the half-lives of cluster

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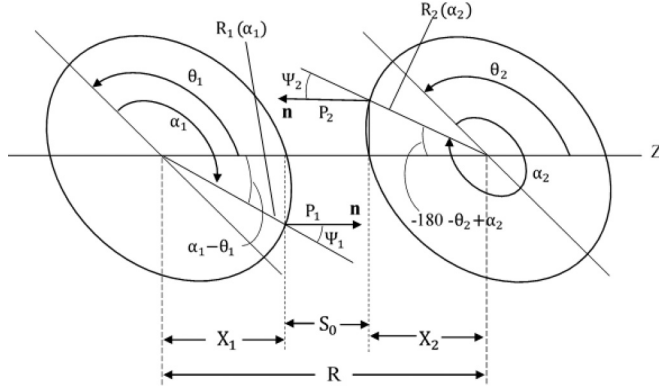


FIG. 1. Schematic of any two axially symmetric deformed oriented nuclei lying on the same plane ($\varphi = 0^\circ$).

decay have been calculated for the head-on orientation, and only the quadrupole deformation has been considered. Using the WKB approximation in one dimension, the half-life $T_{1/2}$

is given by

$$T_{1/2} = \frac{\pi \hbar \ln(2)}{P_0 E_v} [1 + \exp(k)], \quad (6)$$

where

$$k = \frac{2}{\hbar} \int_{R_a}^{R_b} \sqrt{2\mu[V(R) - Q]} dR. \quad (7)$$

In this equation, R_a and R_b are the two turning points of the WKB action integral which is calculated from the condition,

$$E(R_a) = E(R_b) = Q, \quad (8)$$

where Q is the released energy in the cluster emission process. E_v is the zero-point vibration energy which is taken from Ref. [21]. P_0 is a parameter which is obtained by fitting process and has information about the nucleus. To determine the mathematical form of P_0 , we used the least-squares method to determine P_0 . We determined P_0 as $P_0 = 10^{[-0.71A - 12.33\beta(\beta+1) + 1.72]}$ and used this form of P_0 in our calculations. In this form of P_0 , β and A are the

TABLE I. Comparison between the calculated half-lives $\log_{10} T_{1/2}$ of the present paper (Pre.), the experimental data [22] (Expt.), and the results of the LDM [9]. Also, the results of Ref. [13], which considered spherical shapes and proximity potential with a universal form, have been presented. Δ is defined as $\Delta = (\log_{10} T_{1/2})^{\text{Pre.}} - (\log_{10} T_{1/2})^{\text{Expt.}}$. Z_1 and A_1 are the atomic and mass numbers of the cluster. β is the deformation parameter of the cluster.

Parent Nuclei	Z	A	Cluster	Z_1	A_1	ℓ_m	$\beta(\text{cluster})$	Q (Mev)	Expt.	LDM	Ref. [13]	Pre.	Δ
^{226}Th	90	226	^{18}O	8	18	0	0.021	45.73	>16.8	18.95	18.21	17.55	0.75
^{228}Th	90	228	^{20}O	8	20	0	0.003	44.72	20.72	21.61	21.90	21.14	0.42
^{230}U	92	230	^{22}Ne	10	22	0	0.326	61.39	19.57	21.40	20.26	21.05	1.48
^{231}Pa	91	231	^{23}F	9	23	1	-0.202	51.86	26.02	24.26	24.52	24.59	-1.43
^{230}Th	90	230	^{24}Ne	10	24	0	-0.215	57.76	24.61	25.45	24.99	25.21	0.6
^{230}U	92	230	^{24}Ne	10	24	0	-0.215	61.35	>18.2	21.97	22.32	22.57	4.37
^{231}Pa	91	231	^{24}Ne	10	24	1	-0.215	60.41	22.88	21.93	22.29	22.59	-0.29
^{232}Th	90	232	^{24}Ne	10	24	0	-0.215	54.66	>29.20	28.76	28.09	30.07	0.87
^{232}U	92	232	^{24}Ne	10	24	0	-0.215	62.31	21.08	19.99	20.75	20.88	-0.2
^{233}U	92	233	^{24}Ne	10	24	2	-0.215	60.49	24.83	23.36	23.32	23.68	-1.15
^{234}U	92	234	^{24}Ne	10	24	0	-0.215	58.83	25.92	26.54	25.76	25.8	-0.12
^{235}U	92	235	^{24}Ne	10	24	1	-0.215	57.36	27.42	29.40	28.02	27.91	0.49
^{236}U	92	236	^{24}Ne	10	24	0	-0.215	55.95	>25.90	32.18	30.33	30.27	4.37
^{233}U	92	233	^{25}Ne	10	25	2	0	60.73	24.83	23.15	24.03	22.95	-1.88
^{235}U	92	235	^{25}Ne	10	25	3	0	57.71	27.42	29.08	28.69	27.39	-0.03
^{232}Th	90	232	^{26}Ne	10	26	0	0	55.91	>29.20	29.72	29.91	28.77	-0.43
^{234}U	92	234	^{26}Ne	10	26	0	0	59.41	25.92	25.91	26.83	25.62	-0.3
^{236}U	92	236	^{26}Ne	10	26	0	0	56.69	>25.9	31.48	31.20	29.79	3.89
^{232}U	92	232	^{28}Mg	12	28	0	0.323	74.32	>22.26	25.74	25.43	26.57	4.31
^{233}U	92	233	^{28}Mg	12	28	3	0.323	74.23	>27.59	25.78	25.58	25.65	-1.94
^{234}U	92	234	^{28}Mg	12	28	0	0.323	74.11	25.14	25.90	25.48	25.35	0.21
^{235}U	92	235	^{28}Mg	12	28	1	0.323	72.43	>28.09	29.26	28.17	27.73	-0.36
^{236}U	92	236	^{28}Mg	12	28	0	0.323	70.73	27.58	29.34	28.51	29.58	2
^{236}Pu	94	236	^{28}Mg	12	28	0	0.323	79.67	21.67	20.00	21.05	21.08	-0.59
^{238}Pu	94	238	^{28}Mg	12	28	0	0.323	75.91	25.70	26.34	25.69	25.28	-0.42
^{236}U	92	236	^{30}Mg	12	30	0	-0.222	72.27	27.58	29.28	29.43	30.47	2.89
^{237}Np	93	237	^{30}Mg	12	30	2	-0.222	74.79	>27.57	26.56	27.41	28.5	0.93
^{238}Pu	94	238	^{30}Mg	12	30	0	-0.222	76.8	25.70	24.83	25.99	26.93	1.23
^{238}Pu	94	238	^{32}Si	14	32	0	0	91.19	25.27	25.73	24.91	24.84	-0.43
^{240}Pu	94	240	^{34}Si	14	34	0	0	91.03	>25.52	26.08	27.25	26.33	0.81
^{241}Am	95	241	^{34}Si	14	34	3	0	93.92	>24.41	23.32	25.37	24.55	0.14
^{242}Cm	96	242	^{34}Si	14	34	0	0	96.51	23.15	21.11	23.70	22.82	-0.33

TABLE II. Comparison among the calculated results $\log_{10} T_{1/2}$ of the present paper (Pre.), LDM [9], and CPPM [15], which considered the proximity potential with a universal form and Coulomb potential.

Cluster decay	LDM	Reference [15]	Pre.	Cluster decay	LDM	Reference [15]	Pre.
$^{114}\text{Ba} \rightarrow ^{16}\text{O}$	14.51		14.97	$^{223}\text{Th} \rightarrow ^{18}\text{O}$		23.56	21.09
$^{115}\text{Ba} \rightarrow ^{16}\text{O}$	15.86		16.24	$^{224}\text{Th} \rightarrow ^{18}\text{O}$	21.44	22.16	19.72
$^{116}\text{Ba} \rightarrow ^{16}\text{O}$	18.72		19.45	$^{225}\text{Th} \rightarrow ^{18}\text{O}$	19.36	20.05	18.13
$^{117}\text{Ba} \rightarrow ^{16}\text{O}$	21.83		22.34	$^{227}\text{Th} \rightarrow ^{18}\text{O}$	22.00	22.71	20.19
$^{118}\text{Ba} \rightarrow ^{16}\text{O}$	25.87		24.59	$^{228}\text{Th} \rightarrow ^{18}\text{O}$	26.04	26.84	23.25
$^{119}\text{Ce} \rightarrow ^{16}\text{O}$	14.88		15.13	$^{229}\text{Th} \rightarrow ^{18}\text{O}$	29.15	30.05	25.75
$^{120}\text{Ce} \rightarrow ^{16}\text{O}$	17.43		17.20	$^{226}\text{Pa} \rightarrow ^{18}\text{O}$	20.45		19.20
$^{121}\text{Ce} \rightarrow ^{16}\text{O}$	20.08		19.84	$^{227}\text{Pa} \rightarrow ^{18}\text{O}$	19.99		18.55
$^{122}\text{Ce} \rightarrow ^{16}\text{O}$	22.85		22.27	$^{227}\text{U} \rightarrow ^{18}\text{O}$	20.79		19.18
$^{123}\text{Ce} \rightarrow ^{16}\text{O}$	26.18		25.22	$^{228}\text{U} \rightarrow ^{18}\text{O}$	22.15		19.24
$^{124}\text{Ce} \rightarrow ^{16}\text{O}$	29.82		27.55	$^{227}\text{Np} \rightarrow ^{18}\text{O}$	22.04		20.69
$^{124}\text{Pr} \rightarrow ^{16}\text{O}$	25.53		23.25	$^{223}\text{Ra} \rightarrow ^{20}\text{O}$		33.07	32.30
$^{125}\text{Pr} \rightarrow ^{16}\text{O}$	28.48		25.66	$^{224}\text{Ra} \rightarrow ^{20}\text{O}$		30.40	29.64
$^{222}\text{Ac} \rightarrow ^{16}\text{O}$	21.04		18.41	$^{225}\text{Ra} \rightarrow ^{20}\text{O}$	28.53	28.44	27.17
$^{223}\text{Ac} \rightarrow ^{16}\text{O}$	21.00		18.04	$^{226}\text{Ra} \rightarrow ^{20}\text{O}$	27.67	27.55	26.17
$^{224}\text{Ac} \rightarrow ^{16}\text{O}$	24.68		21.45	$^{227}\text{Ra} \rightarrow ^{20}\text{O}$		30.51	28.27
$^{222}\text{Th} \rightarrow ^{16}\text{O}$	18.36		15.81	$^{228}\text{Ra} \rightarrow ^{20}\text{O}$		33.94	31.01
$^{223}\text{Th} \rightarrow ^{16}\text{O}$	16.74		14.56	$^{226}\text{Ac} \rightarrow ^{20}\text{O}$	24.66		24.06
$^{224}\text{Th} \rightarrow ^{16}\text{O}$	16.83		14.43	$^{227}\text{Ac} \rightarrow ^{20}\text{O}$	23.87		23.11
$^{225}\text{Th} \rightarrow ^{16}\text{O}$	20.15		17.33	$^{228}\text{Ac} \rightarrow ^{20}\text{O}$	26.67		25.65
$^{226}\text{Th} \rightarrow ^{16}\text{O}$	23.97		20.31	$^{224}\text{Th} \rightarrow ^{20}\text{O}$		29.87	27.86
$^{227}\text{Th} \rightarrow ^{16}\text{O}$	27.26		23.02	$^{225}\text{Th} \rightarrow ^{20}\text{O}$		27.46	26.18
$^{228}\text{Th} \rightarrow ^{16}\text{O}$	31.50		26.80	$^{226}\text{Th} \rightarrow ^{20}\text{O}$	25.23	25.27	24.15
$^{224}\text{Pa} \rightarrow ^{16}\text{O}$	16.34		14.48	$^{227}\text{Th} \rightarrow ^{20}\text{O}$	22.26	22.31	21.88
$^{225}\text{Pa} \rightarrow ^{16}\text{O}$	16.50		14.33	$^{229}\text{Th} \rightarrow ^{20}\text{O}$	24.54	24.56	23.62
$^{226}\text{Pa} \rightarrow ^{16}\text{O}$	19.71		17.19	$^{230}\text{Th} \rightarrow ^{20}\text{O}$	28.20	28.28	26.34
$^{227}\text{Pa} \rightarrow ^{16}\text{O}$	23.73		20.45	$^{231}\text{Th} \rightarrow ^{20}\text{O}$		31.37	28.72
$^{225}\text{U} \rightarrow ^{16}\text{O}$	15.73		13.74	$^{228}\text{Pa} \rightarrow ^{20}\text{O}$	24.78		24.10
$^{226}\text{U} \rightarrow ^{16}\text{O}$	16.49		14.11	$^{229}\text{Pa} \rightarrow ^{20}\text{O}$	23.89		23.08
$^{227}\text{U} \rightarrow ^{16}\text{O}$	19.76		16.95	$^{226}\text{U} \rightarrow ^{20}\text{O}$		32.13	29.64
$^{225}\text{Np} \rightarrow ^{16}\text{O}$	15.73		14.19	$^{227}\text{U} \rightarrow ^{20}\text{O}$		30.45	28.52
$^{227}\text{Np} \rightarrow ^{16}\text{O}$	16.04		13.64	$^{228}\text{U} \rightarrow ^{20}\text{O}$		29.11	27.21
$^{224}\text{Ac} \rightarrow ^{17}\text{O}$	24.66		22.15	$^{229}\text{U} \rightarrow ^{20}\text{O}$	26.73	26.97	25.33
$^{223}\text{Th} \rightarrow ^{17}\text{O}$	22.18		20.03	$^{230}\text{U} \rightarrow ^{20}\text{O}$	26.69	26.92	25.05
$^{224}\text{Th} \rightarrow ^{17}\text{O}$	23.58		21.11	$^{231}\text{U} \rightarrow ^{20}\text{O}$		29.98	27.73
$^{225}\text{Th} \rightarrow ^{17}\text{O}$	20.30		18.33	$^{232}\text{U} \rightarrow ^{20}\text{O}$		33.01	29.94
$^{226}\text{Th} \rightarrow ^{17}\text{O}$	26.89		23.84	$^{231}\text{Np} \rightarrow ^{20}\text{O}$	28.83	28.83	26.61
$^{227}\text{Th} \rightarrow ^{17}\text{O}$	27.41		24.20	$^{228}\text{Ra} \rightarrow ^{22}\text{O}$	29.11		28.74
$^{225}\text{Pa} \rightarrow ^{17}\text{O}$	23.34		21.11	$^{229}\text{Ac} \rightarrow ^{22}\text{O}$	26.08		26.33
$^{226}\text{Pa} \rightarrow ^{17}\text{O}$	21.15		19.19	$^{227}\text{Th} \rightarrow ^{22}\text{O}$		32.99	32.40
$^{227}\text{Pa} \rightarrow ^{17}\text{O}$	26.48		23.72	$^{228}\text{Th} \rightarrow ^{22}\text{O}$	30.71	30.03	29.98
$^{228}\text{Pa} \rightarrow ^{17}\text{O}$	28.18		25.28	$^{229}\text{Th} \rightarrow ^{22}\text{O}$	26.88	26.25	27.02
$^{227}\text{U} \rightarrow ^{17}\text{O}$	21.08		18.89	$^{230}\text{Th} \rightarrow ^{22}\text{O}$	25.38	24.79	25.62
$^{227}\text{Np} \rightarrow ^{17}\text{O}$	23.26		20.98	$^{231}\text{Th} \rightarrow ^{22}\text{O}$		27.62	28.08
$^{223}\text{Ra} \rightarrow ^{18}\text{O}$	27.80		25.28	$^{232}\text{Th} \rightarrow ^{22}\text{O}$		30.75	30.32
$^{224}\text{Ac} \rightarrow ^{18}\text{O}$	22.75		21.10	$^{233}\text{Th} \rightarrow ^{22}\text{O}$		33.19	32.18
$^{225}\text{Ac} \rightarrow ^{18}\text{O}$	22.26		20.43	$^{231}\text{Pa} \rightarrow ^{22}\text{O}$	29.30		28.83
$^{226}\text{Ac} \rightarrow ^{18}\text{O}$	25.65		23.50	$^{231}\text{U} \rightarrow ^{22}\text{O}$	28.34		33.29
$^{219}\text{Th} \rightarrow ^{18}\text{O}$		31.6	27.23	$^{227}\text{Th} \rightarrow ^{22}\text{Ne}$	28.00		26.64
$^{220}\text{Th} \rightarrow ^{18}\text{O}$		29.47	25.26	$^{228}\text{Th} \rightarrow ^{22}\text{Ne}$	28.68		26.85
$^{221}\text{Th} \rightarrow ^{18}\text{O}$		26.84	23.21	$^{227}\text{Pa} \rightarrow ^{22}\text{Ne}$	24.90		24.16
$^{222}\text{Th} \rightarrow ^{18}\text{O}$		25.47	22.35	$^{228}\text{Pa} \rightarrow ^{22}\text{Ne}$	23.88		23.25
$^{229}\text{Pa} \rightarrow ^{22}\text{Ne}$	24.23		23.63	$^{223}\text{U} \rightarrow ^{28}\text{Mg}$		33.19	32.48
$^{230}\text{Pa} \rightarrow ^{22}\text{Ne}$	27.92		26.54	$^{224}\text{U} \rightarrow ^{28}\text{Mg}$		31.71	31.16
$^{220}\text{U} \rightarrow ^{22}\text{Ne}$		32.26	27.90	$^{225}\text{U} \rightarrow ^{28}\text{Mg}$		30.86	30.87
$^{221}\text{U} \rightarrow ^{22}\text{Ne}$		30.67	26.81	$^{226}\text{U} \rightarrow ^{28}\text{Mg}$		30.05	30.01

TABLE II. (Continued.)

Cluster decay	LDM	Reference [15]	Pre.	Cluster decay	LDM	Reference [15]	Pre.
$^{222}\text{U} \rightarrow ^{22}\text{Ne}$		29.14	25.57	$^{227}\text{U} \rightarrow ^{28}\text{Mg}$		29.38	29.63
$^{223}\text{U} \rightarrow ^{22}\text{Ne}$		27.96	24.66	$^{228}\text{U} \rightarrow ^{28}\text{Mg}$		28.95	29.02
$^{224}\text{U} \rightarrow ^{22}\text{Ne}$		26.8	23.60	$^{229}\text{U} \rightarrow ^{28}\text{Mg}$		28.57	28.60
$^{225}\text{U} \rightarrow ^{22}\text{Ne}$		25.71	22.90	$^{230}\text{U} \rightarrow ^{28}\text{Mg}$		28.28	27.85
$^{226}\text{U} \rightarrow ^{22}\text{Ne}$		25.09	22.74	$^{231}\text{U} \rightarrow ^{28}\text{Mg}$		27.96	27.44
$^{227}\text{U} \rightarrow ^{22}\text{Ne}$		24.33	22.46	$^{234}\text{Np} \rightarrow ^{28}\text{Mg}$	22.41		23.07
$^{228}\text{U} \rightarrow ^{22}\text{Ne}$	22.18	23.83	21.83	$^{235}\text{Np} \rightarrow ^{28}\text{Mg}$	22.57		23.28
$^{229}\text{U} \rightarrow ^{22}\text{Ne}$	20.94	22.52	20.97	$^{236}\text{Np} \rightarrow ^{28}\text{Mg}$	25.91		25.64
$^{231}\text{U} \rightarrow ^{22}\text{Ne}$	24.83	26.56	23.82	$^{228}\text{Pu} \rightarrow ^{28}\text{Mg}$		26.90	24.37
$^{232}\text{U} \rightarrow ^{22}\text{Ne}$		30.56	26.50	$^{229}\text{Pu} \rightarrow ^{28}\text{Mg}$		26.18	23.98
$^{231}\text{Np} \rightarrow ^{22}\text{Ne}$	22.04		21.67	$^{230}\text{Pu} \rightarrow ^{28}\text{Mg}$		25.67	23.39
$^{233}\text{Np} \rightarrow ^{22}\text{Ne}$	29.36		27.43	$^{231}\text{Pu} \rightarrow ^{28}\text{Mg}$		25.17	23.14
$^{232}\text{Pu} \rightarrow ^{22}\text{Ne}$	23.82		21.98	$^{232}\text{Pu} \rightarrow ^{28}\text{Mg}$		24.32	22.91
$^{233}\text{Pu} \rightarrow ^{22}\text{Ne}$	26.04		24.61	$^{233}\text{Pu} \rightarrow ^{28}\text{Mg}$		23.57	22.73
$^{228}\text{Ac} \rightarrow ^{23}\text{F}$	29.88		29.59	$^{234}\text{Pu} \rightarrow ^{28}\text{Mg}$	21.08	22.88	22.00
$^{229}\text{Ac} \rightarrow ^{23}\text{F}$	28.82		28.34	$^{235}\text{Pu} \rightarrow ^{28}\text{Mg}$	20.11	21.24	21.43
$^{230}\text{Th} \rightarrow ^{23}\text{F}$	29.86		29.37	$^{237}\text{Pu} \rightarrow ^{28}\text{Mg}$		25.11	23.46
$^{227}\text{Pa} \rightarrow ^{23}\text{F}$		31.82	30.71	$^{239}\text{Pu} \rightarrow ^{28}\text{Mg}$		31.65	27.34
$^{228}\text{Pa} \rightarrow ^{23}\text{F}$		29.99	29.59	$^{237}\text{Am} \rightarrow ^{28}\text{Mg}$	21.63		22.27
$^{229}\text{Pa} \rightarrow ^{23}\text{F}$	27.64	27.68	27.43	$^{238}\text{Am} \rightarrow ^{28}\text{Mg}$	24.30		24.40
$^{230}\text{Pa} \rightarrow ^{23}\text{F}$	25.52	25.52	25.84	$^{238}\text{Cm} \rightarrow ^{28}\text{Mg}$	22.62		22.57
$^{232}\text{Pa} \rightarrow ^{23}\text{F}$	27.68	27.71	27.52	$^{232}\text{U} \rightarrow ^{30}\text{Mg}$		33.77	35.21
$^{233}\text{Pa} \rightarrow ^{23}\text{F}$		30.7	29.56	$^{233}\text{U} \rightarrow ^{30}\text{Mg}$		33.19	34.56
$^{234}\text{Pa} \rightarrow ^{23}\text{F}$		33.9	32.01	$^{234}\text{U} \rightarrow ^{30}\text{Mg}$		31.77	32.94
$^{223}\text{Th} \rightarrow ^{24}\text{Ne}$		32.74	33.46	$^{235}\text{U} \rightarrow ^{30}\text{Mg}$		30.92	31.16
$^{224}\text{Th} \rightarrow ^{24}\text{Ne}$		31.42	32.10	$^{237}\text{U} \rightarrow ^{30}\text{Mg}$		33.96	32.92
$^{225}\text{Th} \rightarrow ^{24}\text{Ne}$		30.32	31.07	$^{232}\text{Np} \rightarrow ^{30}\text{Mg}$		33.38	33.63
$^{226}\text{Th} \rightarrow ^{24}\text{Ne}$		29.02	29.50	$^{233}\text{Np} \rightarrow ^{30}\text{Mg}$		32.52	32.65
$^{227}\text{Th} \rightarrow ^{24}\text{Ne}$		27.81	28.40	$^{234}\text{Np} \rightarrow ^{30}\text{Mg}$		31.23	32.07
$^{228}\text{Th} \rightarrow ^{24}\text{Ne}$	26.26	26.93	27.20	$^{235}\text{Np} \rightarrow ^{30}\text{Mg}$		29.99	30.82
$^{229}\text{Th} \rightarrow ^{24}\text{Ne}$	25.38	26.00	25.50	$^{236}\text{Np} \rightarrow ^{30}\text{Mg}$		27.53	29.30
$^{231}\text{Th} \rightarrow ^{24}\text{Ne}$	28.93	29.65	27.82	$^{238}\text{Np} \rightarrow ^{30}\text{Mg}$		30.97	31.11
$^{229}\text{Pa} \rightarrow ^{24}\text{Ne}$	23.54	24.22	24.05	$^{239}\text{Np} \rightarrow ^{30}\text{Mg}$		33.4	32.45
$^{230}\text{Pa} \rightarrow ^{24}\text{Ne}$	22.08	22.73	22.80	$^{232}\text{Pu} \rightarrow ^{30}\text{Mg}$		33.86	32.23
$^{223}\text{U} \rightarrow ^{24}\text{Ne}$		32.23	29.61	$^{233}\text{Pu} \rightarrow ^{30}\text{Mg}$		32.66	31.43
$^{224}\text{U} \rightarrow ^{24}\text{Ne}$		30.26	28.03	$^{234}\text{Pu} \rightarrow ^{30}\text{Mg}$		31.31	30.83
$^{225}\text{U} \rightarrow ^{24}\text{Ne}$		28.76	26.96	$^{235}\text{Pu} \rightarrow ^{30}\text{Mg}$		30.22	30.40
$^{226}\text{U} \rightarrow ^{24}\text{Ne}$		27.38	25.66	$^{236}\text{Pu} \rightarrow ^{30}\text{Mg}$		27.32	28.96
$^{227}\text{U} \rightarrow ^{24}\text{Ne}$		26.19	24.83	$^{237}\text{Pu} \rightarrow ^{30}\text{Mg}$		25.63	27.79
$^{228}\text{U} \rightarrow ^{24}\text{Ne}$		25.04	24.28	$^{239}\text{Pu} \rightarrow ^{30}\text{Mg}$		27.92	29.41
$^{229}\text{U} \rightarrow ^{24}\text{Ne}$		23.67	23.55	$^{240}\text{Pu} \rightarrow ^{30}\text{Mg}$		31.83	30.91
$^{231}\text{U} \rightarrow ^{24}\text{Ne}$		21.01	21.35	$^{238}\text{Am} \rightarrow ^{30}\text{Mg}$		28.45	29.89
$^{233}\text{Np} \rightarrow ^{24}\text{Ne}$	21.95		22.53	$^{239}\text{Am} \rightarrow ^{30}\text{Mg}$		27.67	28.93
$^{233}\text{Pu} \rightarrow ^{24}\text{Ne}$	23.30		23.54	$^{240}\text{Cm} \rightarrow ^{30}\text{Mg}$		29.61	30.04
$^{234}\text{Pu} \rightarrow ^{24}\text{Ne}$	23.45		23.38				

deformation parameter and mass number, respectively, of the emitted cluster. It must be emphasized that the conservation laws for the spin and parity imply that a minimum angular momentum ℓ_m be carried away by the system after cluster emission.

III. RESULTS AND DISCUSSIONS

We used the proximity potential CW76 as the nuclear part of the total potential. Deformations of the nuclei were considered in Coulomb and nuclear parts of the interacting

potential. Also, we used the minimum angular momentum, which is carried away in the reaction [13] in our calculations. In Table I, the results of this paper have been compared with the experimental data [22], the results of the LDM [9], and the results of Ref. [13] which considered spherical nuclei. Also, we used experimental data for the Q values [23]. We considered 32 reactions for which the experimental data and the other necessary information, such as the deformation parameter β [24], are available. These 32 reactions are divided in two groups. The first one includes 20 reactions which have

exact experimental data. For this group the absolute values of the differences between the experimental and the theoretical values in this paper are less than 1.5 except for three of them which are more than 1.5 and less than 2.9.

The second group includes 12 reactions for which the minimum values of experimental data of $T_{1/2}$ have been reported. The worst theoretical data for this group belong to the emission of ^{24}Ne from ^{230}U and ^{236}U , which is 4.37 higher than the minimum of the experimental data. The results show that one can use the proximity potential CW76 and consider deformations of the nuclei to calculate the half-lives of $^{18,20}\text{O}$, ^{23}F , $^{24,25,26}\text{Ne}$, $^{28,30}\text{Mg}$, and $^{32,34}\text{Si}$ emission processes. Also, our results are in good agreement with the results of the LDM.

We considered more cases of cluster emission to test our method. The results for 201 reactions are available in Table II for which $\beta \neq 0$. The results of this paper have been compared to the results of the LDM [9] and the result of Ref. [15] which used the Coulomb potential and proximity potential model

(CPPM). In Ref. [15], the half-lives were reported for spherical nuclei. In this part, we ignored the centrifugal part because its effect is very small. Because there are no experimental data for these reactions, one cannot specify which model is better.

IV. CONCLUSION

So many attempts have been made to calculate the half-lives of cluster emission from different nuclei [3–6,9,13–15,25,26]. We also tried to calculate this important parameter. We calculated half-lives of cluster decays using the proximity potential CW76 and considered deformations of the cluster and daughter nuclei. We compared our results with the experimental data, results of the LDM, and the CPPM. The results show that considering deformations of the nuclei in the calculation is a good candidate to improve the theoretical results. One may use the proximity potential with a new universal function instead of CW76 and consider deformation of the nuclei to get better results.

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